

# Calorimetry

Peter Križan

- Basic principles
- Interaction of charged particles and photons
- Electromagnetic cascades
- Nuclear interactions
- Hadronic cascades
- Homogeneous calorimeters
- Sampling calorimeters

## Calorimetry:

Energy measurement by total absorption, combined with spatial reconstruction.

Calorimetry is a “destructive” method

Detector response  $\propto E$

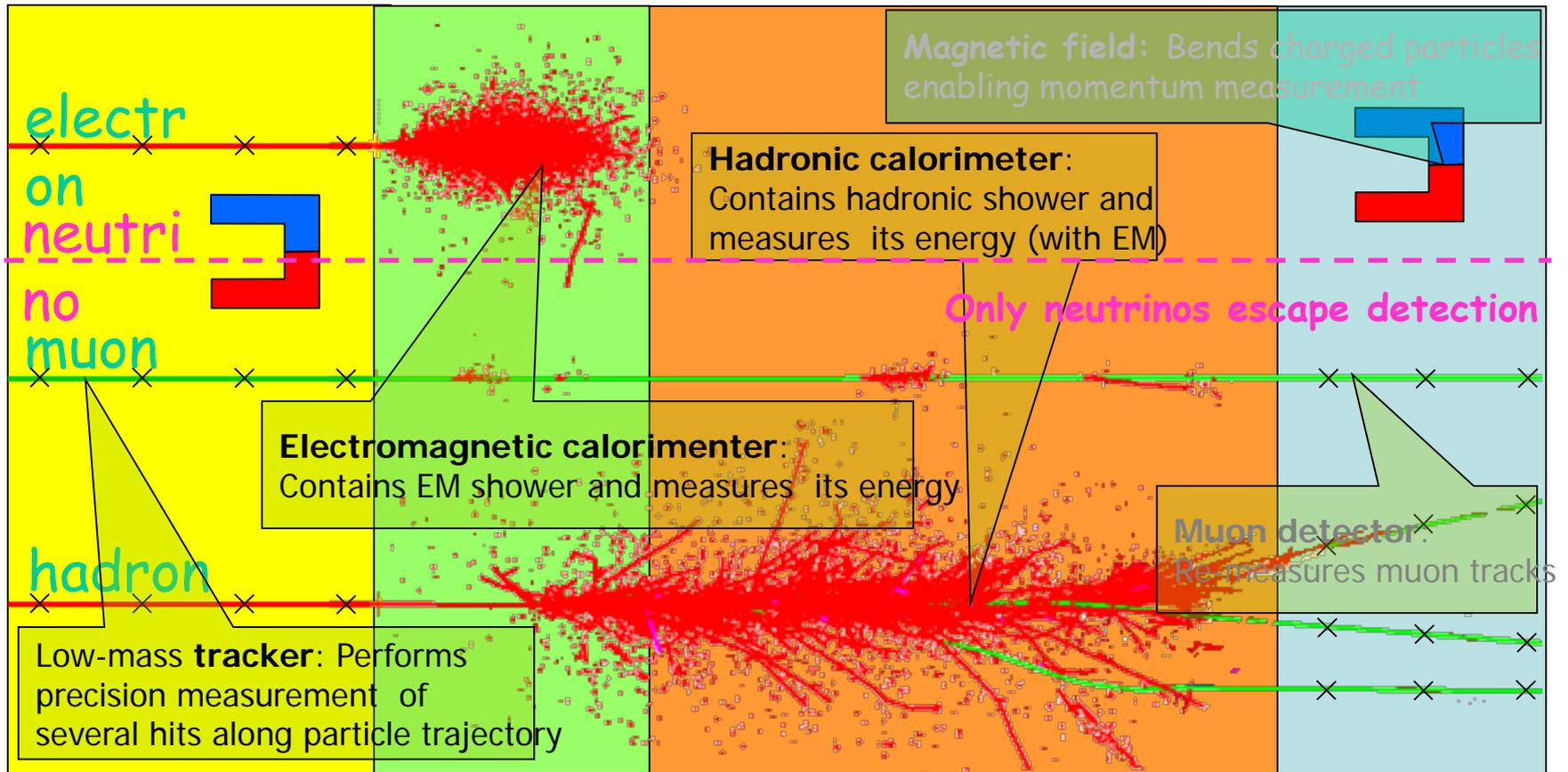
Calorimetry works both for

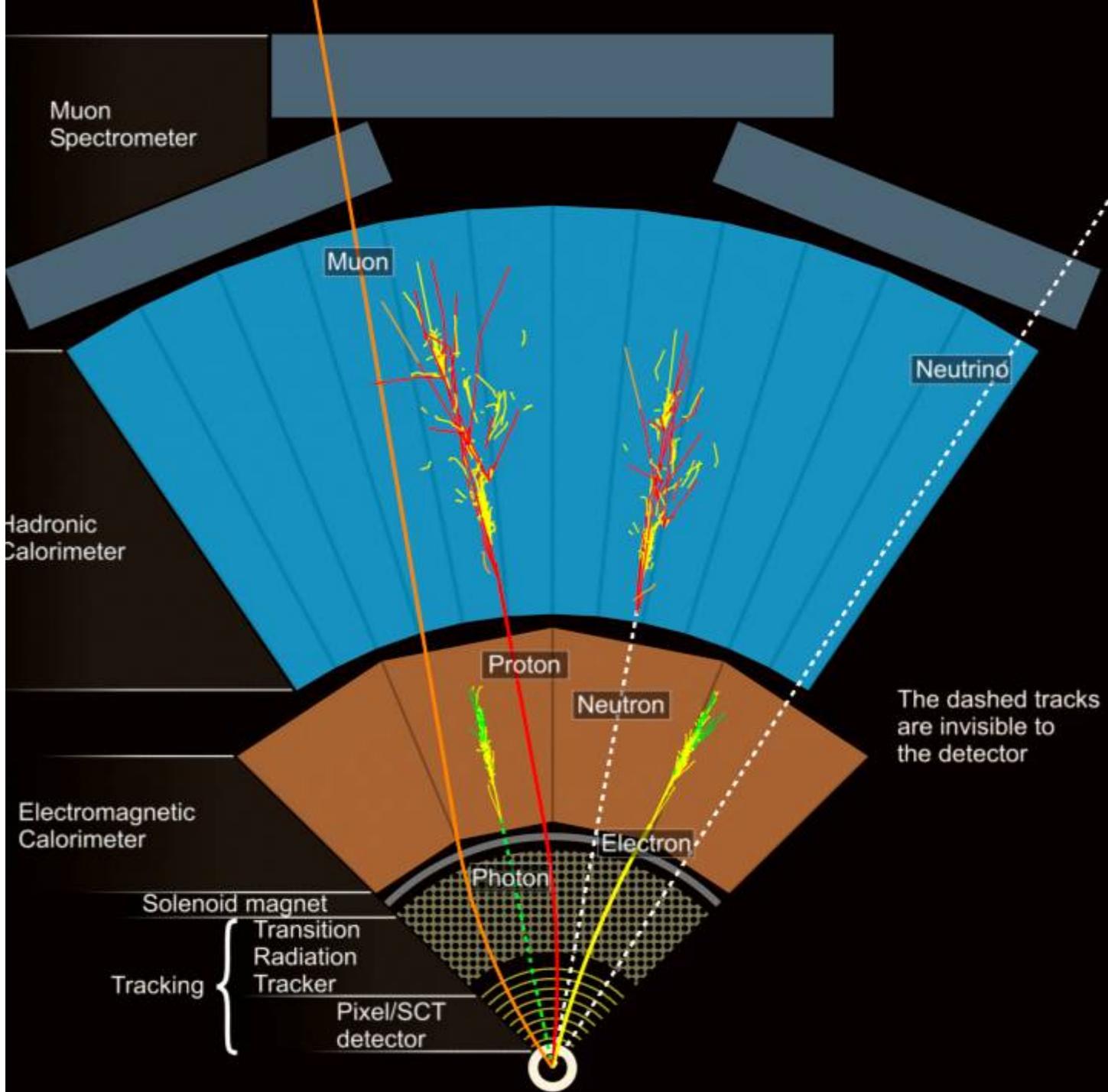
- charged ( $e^\pm$  and hadrons) and
- neutral particles ( $n, \gamma$ )

Basic mechanism: formation of electromagnetic or hadronic showers.

Finally, the energy is converted into ionization or excitation of the matter.

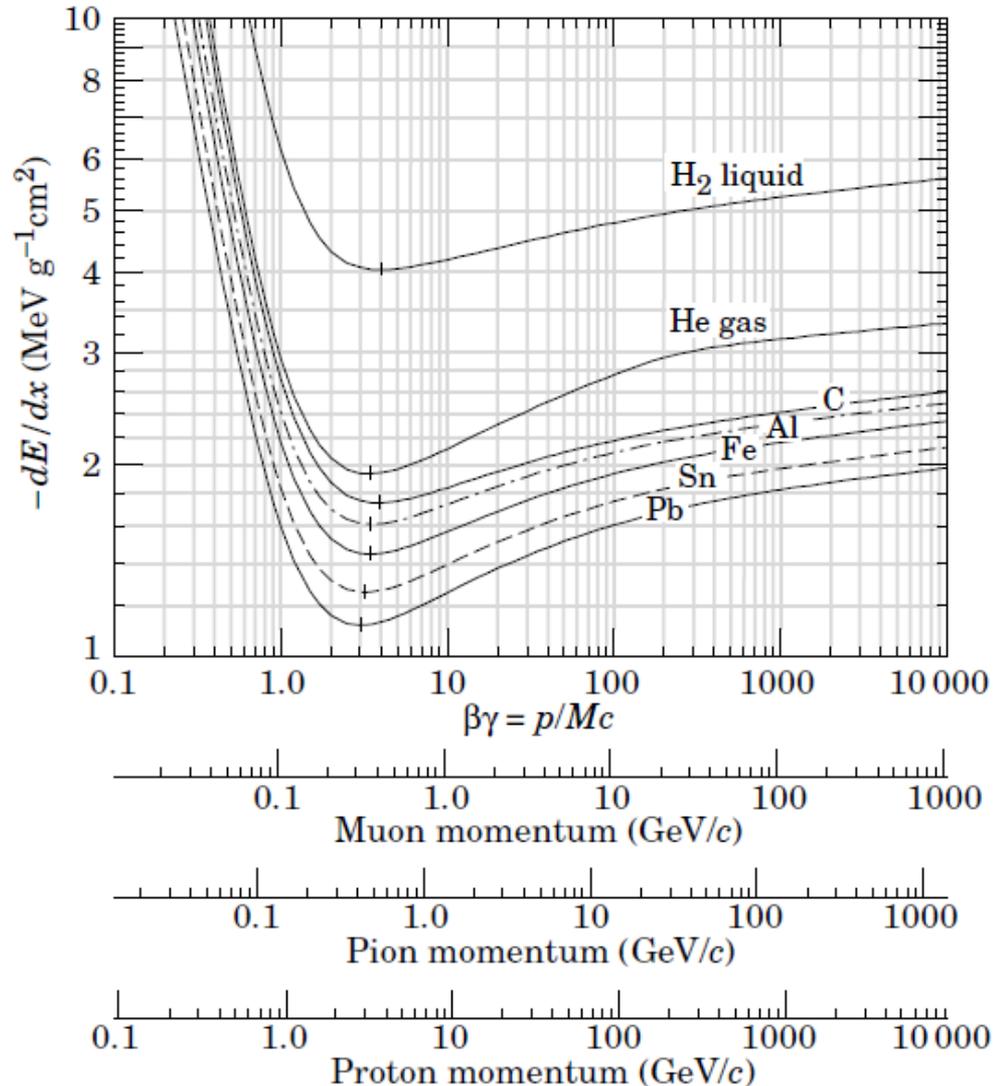
# Generic LHC Detector for all Particles





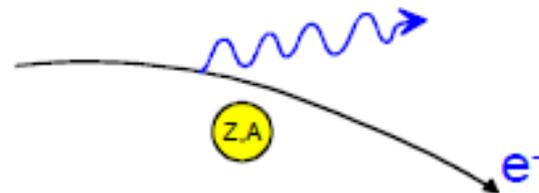
# Mean energy loss by ionisation

Bethe-Bloch formula  
For different materials



**Figure 27.3:** Mean energy loss rate in liquid (bubble chamber) hydrogen, gaseous helium, carbon, aluminum, iron, tin, and lead. Radiative effects, relevant for muons and pions, are not included. These become significant for muons in iron for  $\beta\gamma \gtrsim 1000$ , and at lower momenta for muons in higher- $Z$  absorbers. See Fig. 27.21.

# Energy loss by Bremsstrahlung



Radiation of real photons in the  
Coulomb field of the nuclei of the absorber

$$-\frac{dE}{dx} = 4\alpha N_A \frac{Z^2}{A} z^2 \left( \frac{1}{4\pi\epsilon_0} \frac{e^2}{mc^2} \right)^2 E \ln \frac{183}{Z^{1/3}} \propto \frac{E}{m^2}$$

Effect plays a role only for  $e^\pm$  and ultra-relativistic  $\mu$   
( $>1000$  GeV)

For electrons:

$$-\frac{dE}{dx} = 4\alpha N_A \frac{Z^2}{A} r_e^2 E \ln \frac{183}{Z^{1/3}}$$

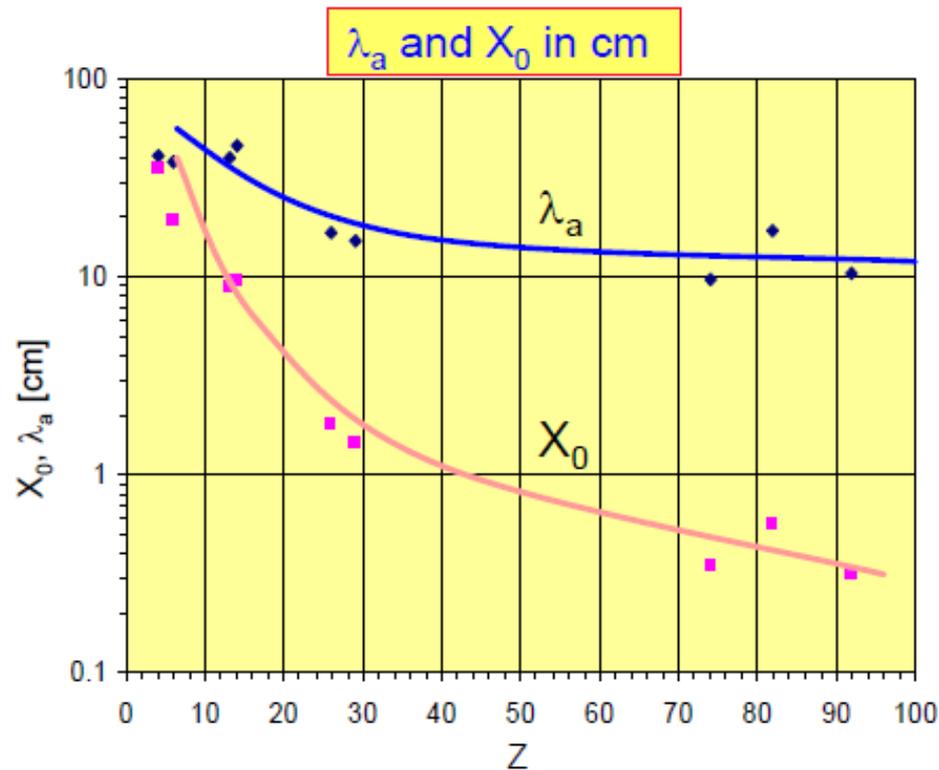
$$\boxed{-\frac{dE}{dx} = \frac{E}{X_0}}$$

$$X_0 = \frac{A}{4\alpha N_A Z^2 r_e^2 \ln \frac{183}{Z^{1/3}}}$$

radiation length [g/cm<sup>2</sup>]

Material	Z	A	$\rho$ [g/cm <sup>3</sup> ]	$X_0$ [g/cm <sup>2</sup> ]	$\lambda_a$ [g/cm <sup>2</sup> ]
Hydrogen (gas)	1	1.01	0.0899 (g/l)	63	50.8
Helium (gas)	2	4.00	0.1786 (g/l)	94	65.1
Beryllium	4	9.01	1.848	65.19	75.2
Carbon	6	12.01	2.265	43	86.3
Nitrogen (gas)	7	14.01	1.25 (g/l)	38	87.8
Oxygen (gas)	8	16.00	1.428 (g/l)	34	91.0
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Iron	26	55.85	7.87	13.9	131.9
Copper	29	63.55	8.96	12.9	134.9
Tungsten	74	183.85	19.3	6.8	185.0
Lead	82	207.19	11.35	6.4	194.0
Uranium	92	238.03	18.95	6.0	199.0

For  $Z > 6$ :  $\lambda_a > X_0$



# Electrons: fractional energy loss, $1/E \, dE/dx$

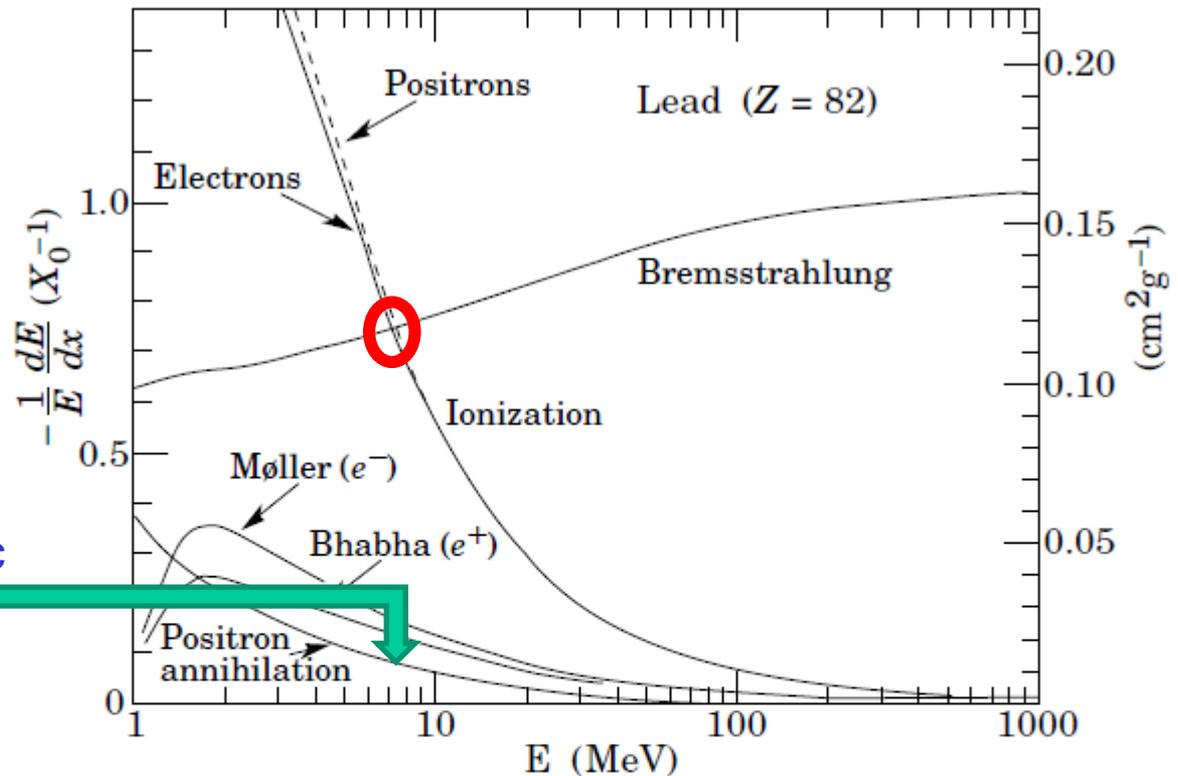
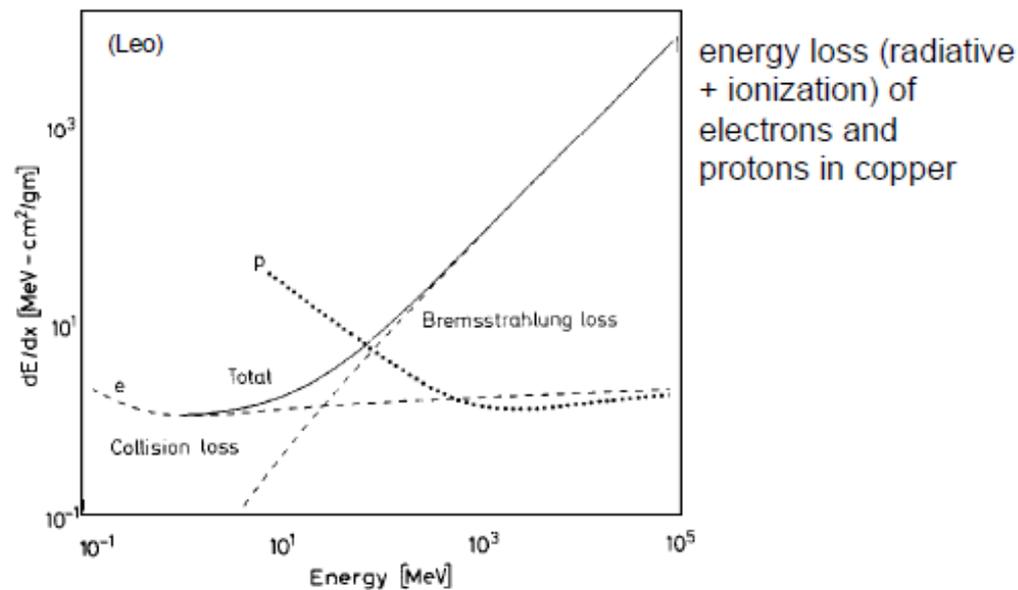


Figure 27.10: Fractional energy loss per radiation length in lead as a function of electron or positron energy. Electron (positron) scattering is considered as ionization when the energy loss per collision is below 0.255 MeV, and as Møller (Bhabha) scattering when it is above. Adapted from Fig. 3.2 from Messel and Crawford, *Electron-Photon Shower Distribution Function Tables for Lead, Copper, and Air Absorbers*, Pergamon Press, 1970. Messel and Crawford use  $X_0(\text{Pb}) = 5.82 \text{ g/cm}^2$ , but we have modified the figures to reflect the value given in the Table of Atomic and Nuclear Properties of Materials ( $X_0(\text{Pb}) = 6.37 \text{ g/cm}^2$ ).



### Critical energy $E_c$

$$\left. \frac{dE}{dx}(E_c) \right|_{Brems} = \left. \frac{dE}{dx}(E_c) \right|_{ion}$$

For electrons one finds approximately:

$$E_c^{solid+liq} = \frac{610MeV}{Z+1.24} \quad E_c^{gas} = \frac{710MeV}{Z+1.24} \quad \text{density effect of } dE/dx(\text{ionisation}) !$$

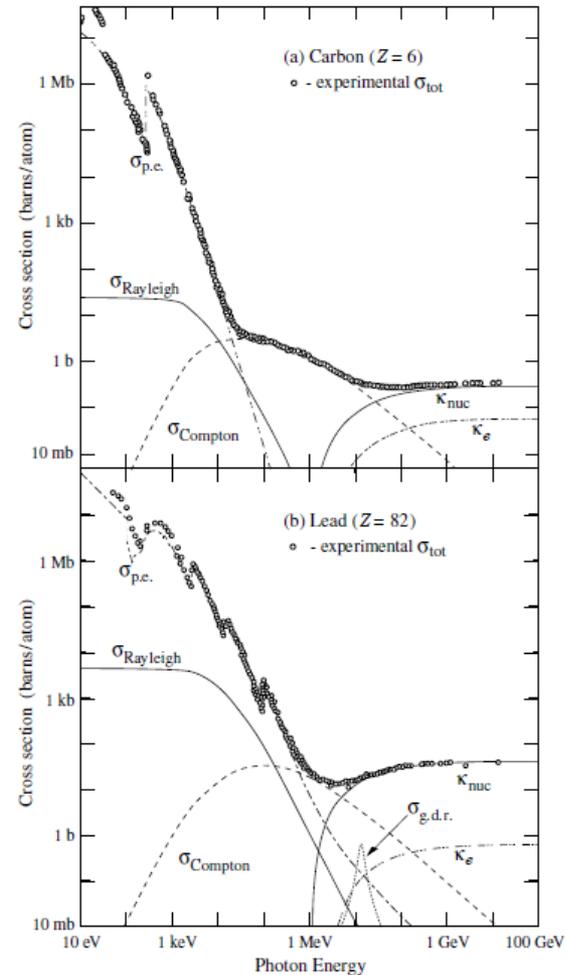
$$E_c(e^-) \text{ in Fe}(Z=26) = 22.4 \text{ MeV}$$

For muons

$$E_c \approx E_c^{elec} \left( \frac{m_\mu}{m_e} \right)^2$$

$$E_c(\mu) \text{ in Fe}(Z=26) \approx 1 \text{ TeV}$$

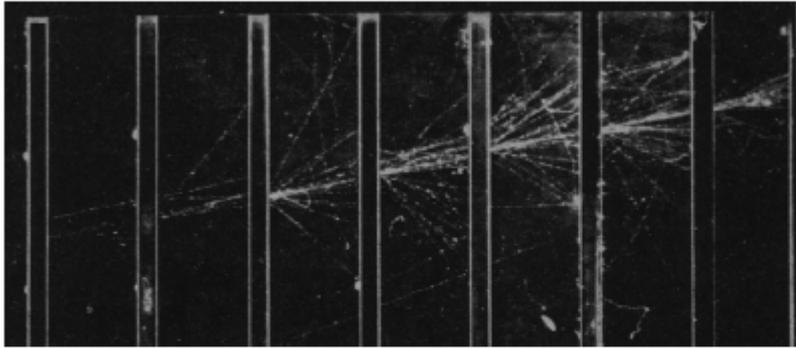
# Interaction of photons with matter



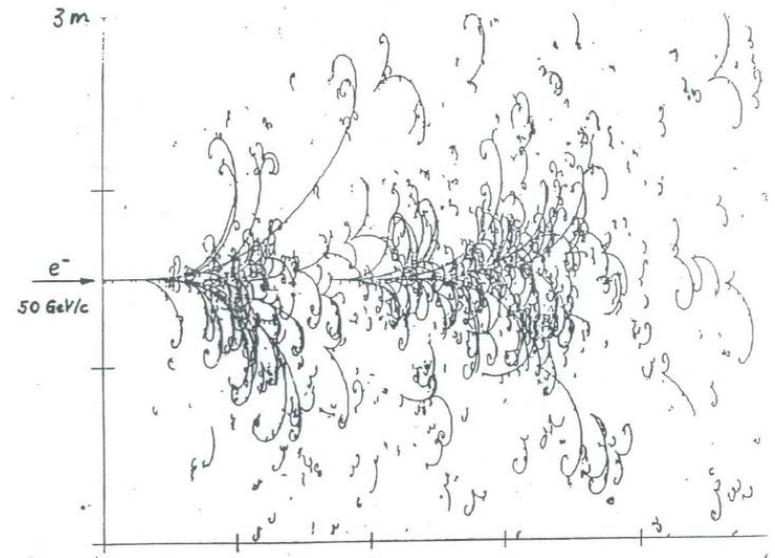
**Figure 27.14:** Photon total cross sections as a function of energy in carbon and lead, showing the contributions of different processes:

- $\sigma_{p.e.}$  = Atomic photoelectric effect (electron ejection, photon absorption)
- $\sigma_{\text{Rayleigh}}$  = Rayleigh (coherent) scattering—atom neither ionized nor excited
- $\sigma_{\text{Compton}}$  = Incoherent scattering (Compton scattering off an electron)
- $\kappa_{\text{nuc}}$  = Pair production, nuclear field
- $\kappa_e$  = Pair production, electron field
- $\sigma_{g.d.r.}$  = Photonuclear interactions, most notably the Giant Dipole Resonance [46]. In these interactions, the target nucleus is broken up.

# Electromagnetic Cascades (showers)

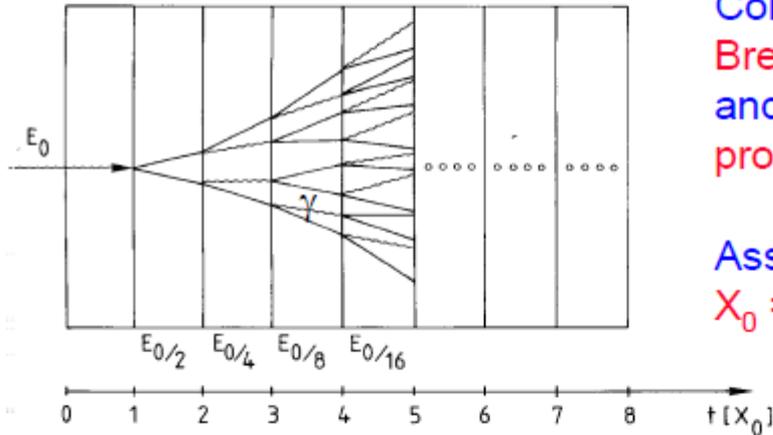


Electron shower in a cloud chamber with lead absorbers



B E B C , Ne / H<sub>2</sub> (70/30%) , B = 3T  
ELECTROMAGNETIC SHOWER DEVEL.

## Simple qualitative model



Consider only  
Bremsstrahlung  
and pair  
production.

Assume:  
 $X_0 = \lambda_{\text{pair}}$

$$N(t) = 2^t \quad E(t) / \text{particle} = E_0 \cdot 2^{-t}$$

Process continues until  $E(t) < E_c$

$$t_{\text{max}} = \frac{\ln E_0 / E_c}{\ln 2} \quad N^{\text{total}} = \sum_{t=0}^{t_{\text{max}}} 2^t = 2^{(t_{\text{max}}+1)} - 1 \approx 2 \cdot 2^{t_{\text{max}}} = 2 \frac{E_0}{E_c}$$

After  $t = t_{\text{max}}$  the dominating processes are ionization, Compton effect and photo effect → absorption.

→ Calorimeter size depends only logarithmically on  $E_0$

Longitudinal shower development:

$$\frac{dE}{dt} \propto t^\alpha e^{-t}$$

Detailed model: "Rossi approximaton B"

Shower maximum at  $t_{\max} = \ln \frac{E_0}{E_c} \frac{1}{\ln 2}$

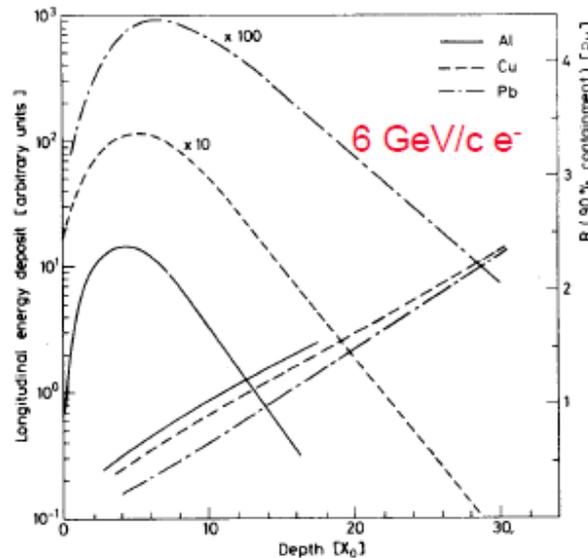
95% containment  $t_{95\%} \approx t_{\max} + 0.08Z + 9.6$

Size of a calorimeter grows only logarithmically with  $E_0$

Transverse shower development: 95% of the shower cone is located in a cylinder with radius  $2 R_M$

Determined mainly by multiple scattering of shower particles

$$R_M = \frac{21 \text{ MeV}}{E_c} X_0 \quad [g/cm^2] \quad \text{Molière radius}$$



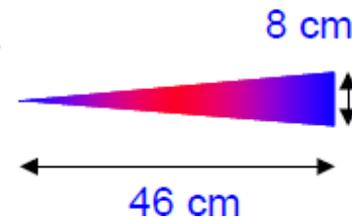
Longitudinal and transverse development scale with  $X_0, R_M$

(C. Fabjan, T. Ludlam, CERN-EP/82-37)

Example:  $E_0 = 100 \text{ GeV}$  in lead glass

$E_c = 11.8 \text{ MeV} \rightarrow t_{\max} \approx 13, t_{95\%} \approx 23$

$X_0 \approx 2 \text{ cm}, R_M = 1.8 \cdot X_0 \approx 3.6 \text{ cm}$



◆ Energy resolution of a calorimeter (intrinsic limit)

$$N^{total} \propto \frac{E_0}{E_c} \quad \text{total number of track segments}$$

$$\frac{\sigma(E)}{E} \propto \frac{\sigma(N)}{N} \propto \frac{1}{\sqrt{N}} \propto \frac{1}{\sqrt{E_0}} \quad \text{holds also for hadron calorimeters}$$

Also spatial and angular resolution scale like  $1/\sqrt{E}$

Relative energy resolution of a calorimeter improves with  $E_0$

More general:

$$\frac{\sigma(E)}{E} = \frac{a}{\sqrt{E}} \oplus b \oplus \frac{c}{E}$$

Stochastic term

Constant term

Noise term

Inhomogenities  
Bad cell inter-calibration  
Non-linearities

Electronic noise  
radioactivity  
pile up

Quality factor !

# Calorimeter types

## ◆ Homogeneous calorimeters:

- ⇒ Detector = absorber
- ⇒ good energy resolution
- ⇒ limited spatial resolution (particularly in longitudinal direction)
- ⇒ only used for electromagnetic calorimetry

## ◆ Sampling calorimeters:

- ⇒ Detectors and absorber separated → only part of the energy is sampled.
- ⇒ limited energy resolution
- ⇒ good spatial resolution
- ⇒ used both for electromagnetic and hadron calorimetry

# Homogeneous calorimeters

Two main types: Scintillator crystals or “glass” blocks (Cherenkov radiation).

→ photons. Readout via photomultiplier, -diode/triode

## ◆ Scintillators (crystals)

Scintillator	Density [g/cm <sup>3</sup> ]	X <sub>0</sub> [cm]	Light Yield $\gamma$ /MeV (rel. yield)	$\tau_1$ [ns]	$\lambda_1$ [nm]	Rad. Dam. [Gy]	Comments
NaI (TI)	3.67	2.59	4×10 <sup>4</sup>	230	415	≥10	hygroscopic, fragile
CsI (TI)	4.51	1.86	5×10 <sup>4</sup> (0.49)	1005	565	≥10	Slightly hygroscopic
CSI pure	4.51	1.86	4×10 <sup>4</sup> (0.04)	10 36	310 310	10 <sup>3</sup>	Slightly hygroscopic
BaF <sub>2</sub>	4.87	2.03	10 <sup>4</sup> (0.13)	0.6 620	220 310	10 <sup>3</sup>	
BGO	7.13	1.13	8×10 <sup>3</sup>	300	480	10	
PbWO <sub>4</sub>	8.28	0.89	≈100	10 10	≈440 ≈530	10 <sup>4</sup>	light yield = f(T)

Relative light yield: rel. to NaI(Tl) readout with PM (bialkali PC)

## ◆ Cherenkov radiators

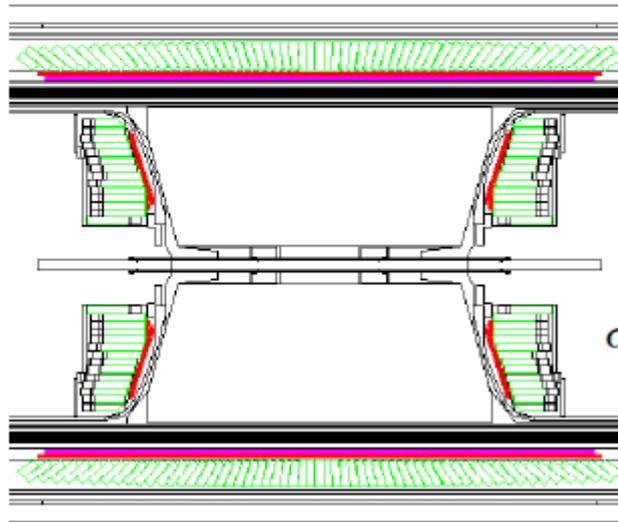
Material	Density [g/cm <sup>3</sup> ]	X <sub>0</sub> [cm]	n	Light yield [p.e./GeV] (rel. p.e.)	$\lambda_{cut}$ [nm]	Rad. Dam. [Gy]	Comments
SF-5 Lead glass	4.08	2.54	1.67	600 (1.5×10 <sup>-4</sup> )	350	10 <sup>1</sup>	
SF-6 Lead glass	5.20	1.69	1.81	900 (2.3×10 <sup>-4</sup> )	350	10 <sup>2</sup>	
PbF <sub>2</sub>	7.66	0.95	1.82	2000 (5×10 <sup>-4</sup> )		10 <sup>3</sup>	Not available in quantity

Relative light yield: rel. to NaI(Tl) readout with PM (bialkali PC)

# Examples

## OPAL Barrel + end-cap: lead glass + pre-sampler

(OPAL collab. NIM A 305 (1991) 275)



≈10500 blocks (10 x 10 x 37 cm<sup>3</sup>, 24.6 X<sub>0</sub>),  
PM (barrel) or PT  
(end-cap) readout.

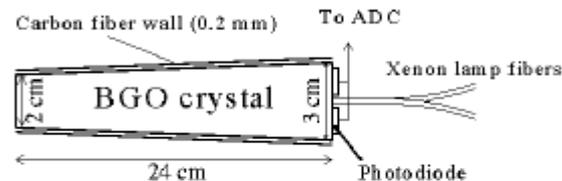
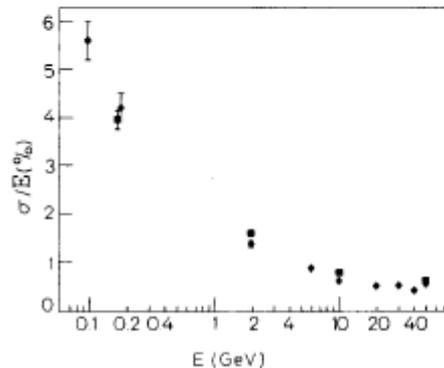
$$\sigma(E)/E = 0.06/\sqrt{E} \oplus 0.002$$

Spatial resolution  
(intrinsic) ≈ 11 mm  
at 6 GeV

## BGO E.M. Calorimeter in L3

(L3 collab. NIM A 289 (1991) 53)

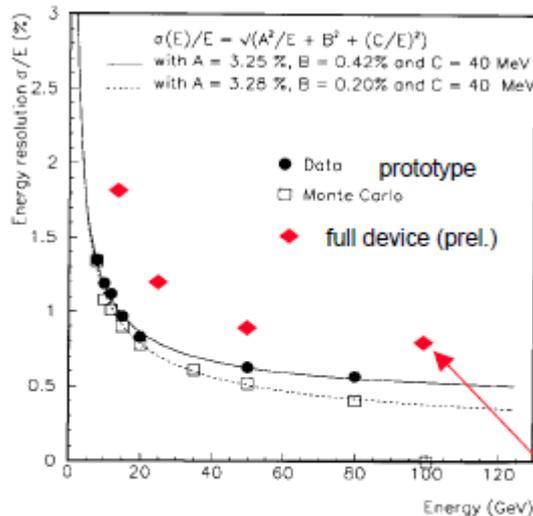
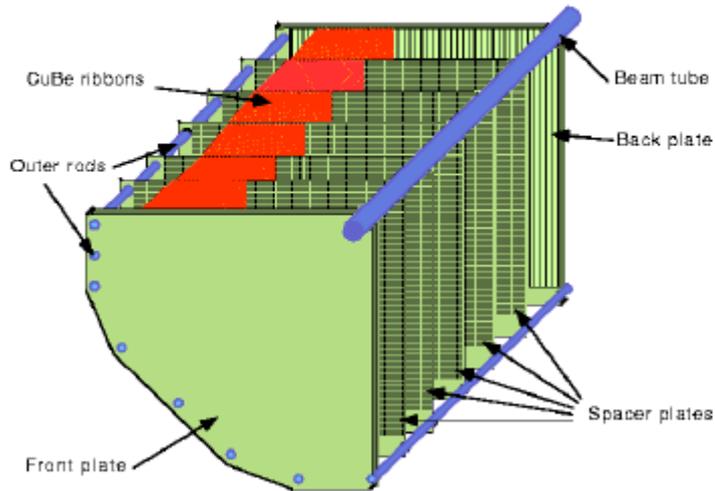
11000 crystals, 21.4 X<sub>0</sub>,  
temperature monitoring +  
control system  
light output -1.55% / °C



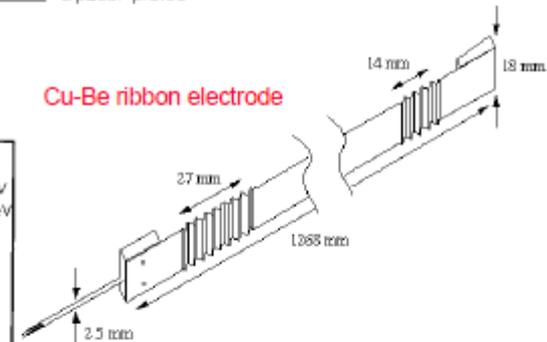
σ<sub>E</sub>/E < 1% for E > 1 GeV  
spatial resolution < 2 mm  
(E > 2 GeV)

Partly test beam results !

NA48: LKr Ionisation chamber (T = 120 K)  
 no metal absorbers → quasi homogenous !



Cu-Be ribbon electrode

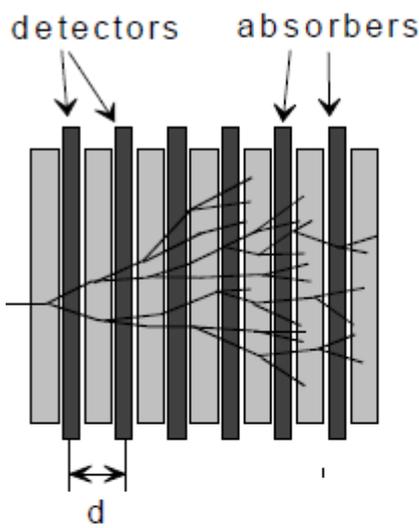


$\sigma_{x,y} \leq 1 \text{ mm}$   
 $\sigma_t \approx 230 \text{ ps}$

97 run: reduced performance  
 due to problems with blocking  
 capacitors → lower driftfield:  
 1.5 kV/cm rather than 5 kV/cm

# Sampling calorimeters

Absorber + detector separated → additional sampling fluctuations

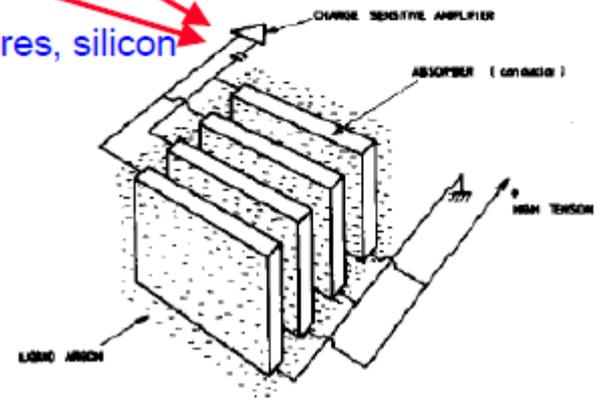
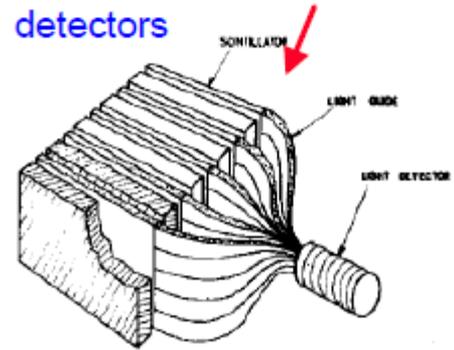
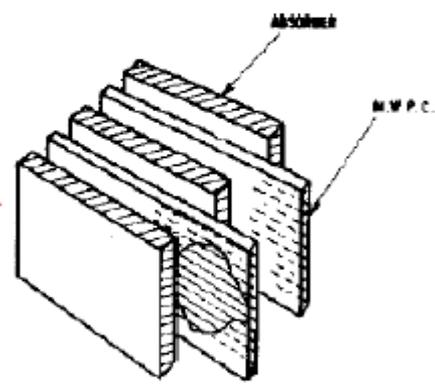


$$N = \frac{T_{\text{det}}}{d} \quad \text{Detectable track segments}$$

$$= F(\xi) \frac{E}{E_c} X_0 \frac{1}{d}$$

$$\frac{\sigma(E)}{E} \propto \frac{\sqrt{N}}{N} \propto \sqrt{\frac{1}{E}} \cdot \sqrt{\frac{d}{X_0}}$$

- MWPC, streamer tubes
- warm liquids  
 TMP = tetramethylpentane,  
 TMS = tetramethylsilane
- cryogenic noble gases:  
 mainly LAr (LXe, LKr)
- scintillators, scintillation fibres, silicon detectors



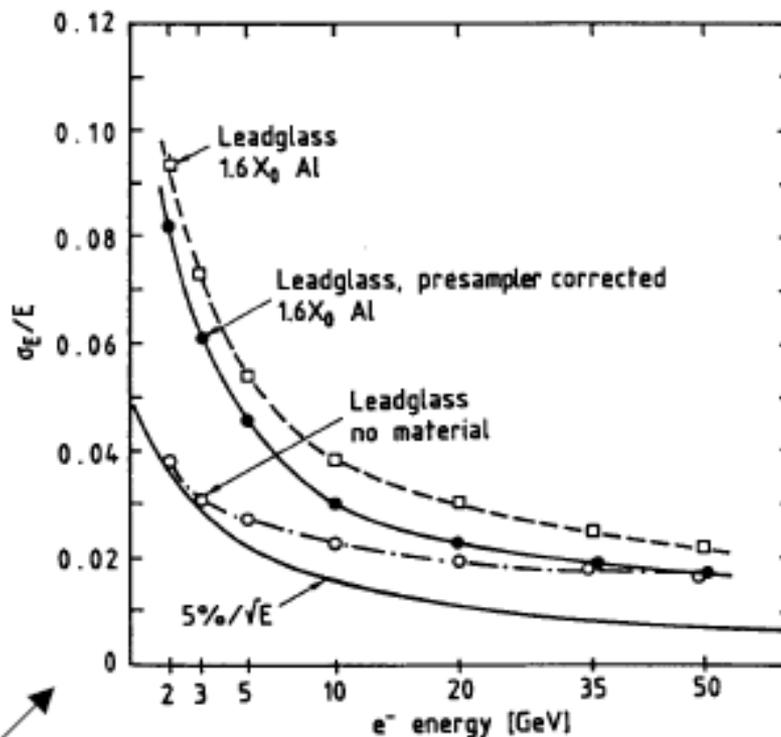


## Material in front of calorimeter

Showers start in 'dead' material in front of calorimeter  
(other detectors, solenoid, support structure)

Install a highly segmented pre-shower detector in front of calorimeter

- recover lost energy
- improved background rejection due to good spatial resolution
- improve angular resolution

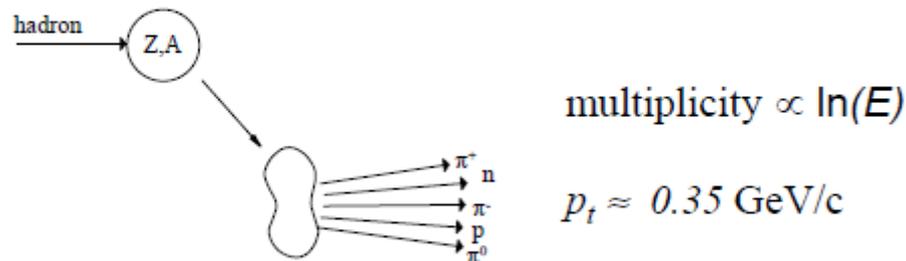


(C. Beard et al., NIM A 286 (1990) 117)

**OPAL end cap calorimeter + pre-shower**

# Nuclear Interactions

The interaction of energetic hadrons (charged or neutral) is determined by **inelastic nuclear processes**.



**Excitation and finally breakup up nucleus**  $\rightarrow$  nucleus fragments + production of secondary particles.

For high energies ( $>1 \text{ GeV}$ ) the cross-sections depend only little on the energy and on the type of the incident particle ( $p, \pi, K, \dots$ ).

$$\sigma_{inel} \approx \sigma_0 A^{0.7} \quad \sigma_0 \approx 35 \text{ mb}$$

In analogy to  $X_0$  a **hadronic absorption length** can be defined

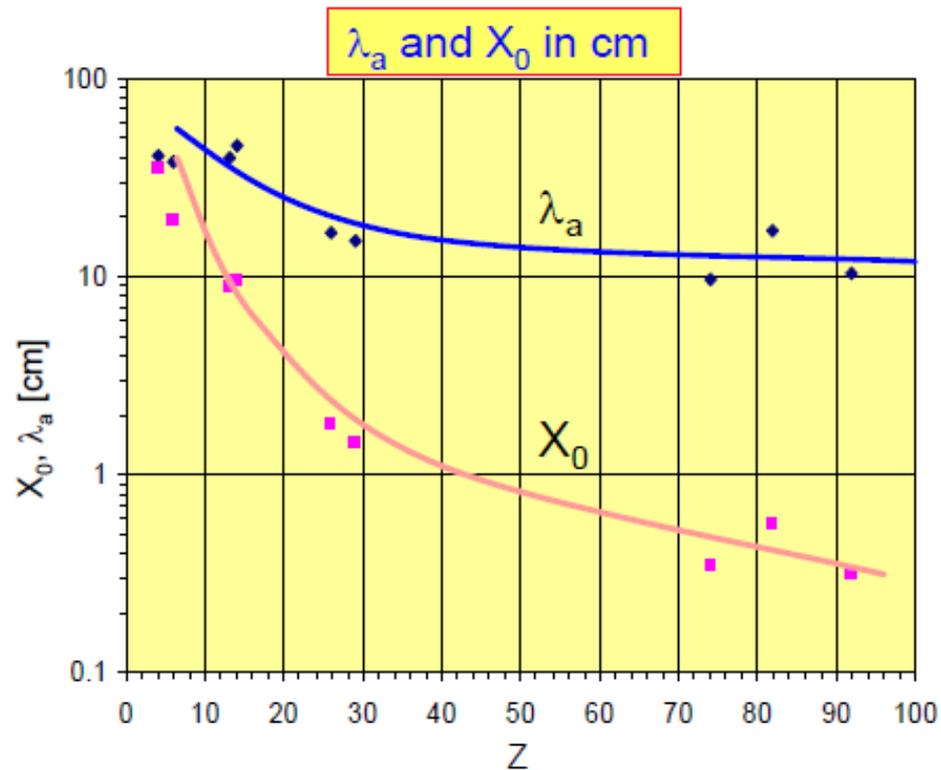
$$\lambda_a = \frac{A}{N_A \sigma_{inel}} \propto A^{\frac{1}{4}} \quad \text{because } \sigma_{inel} \approx \sigma_0 A^{0.7}$$

similarly a **hadronic interaction length**

$$\lambda_I = \frac{A}{N_A \sigma_{total}} \propto A^{\frac{1}{3}} \quad \lambda_I < \lambda_a$$

Material	Z	A	$\rho$ [g/cm <sup>3</sup> ]	$X_0$ [g/cm <sup>2</sup> ]	$\lambda_a$ [g/cm <sup>2</sup> ]
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Lead	82	207.19	11.35	6.4	194.0
Uranium	92	238.03	18.95	6.0	199.0

For  $Z > 6$ :  $\lambda_a > X_0$



# Interaction of neutrons

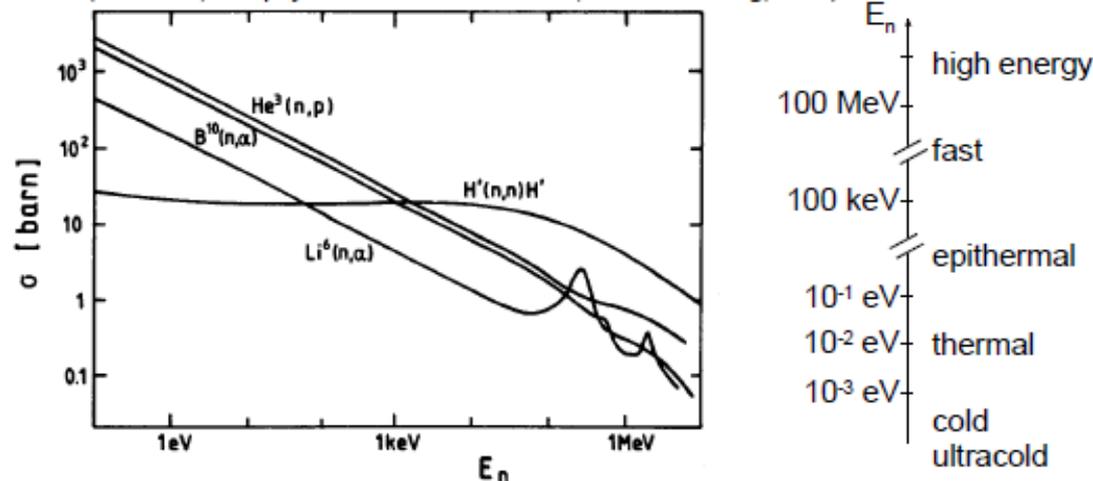
Neutrons have no charge, i.e. their interaction is based only on **strong (and weak) nuclear force**.

To detect neutrons, we have to create charged particles.

## Possible neutron conversion and elastic reactions

- $n + {}^6\text{Li} \rightarrow \alpha + {}^3\text{H}$
  - $n + {}^{10}\text{B} \rightarrow \alpha + {}^7\text{Li}$
  - $n + {}^3\text{He} \rightarrow p + {}^3\text{H}$
  - $n + p \rightarrow n + p$
- }  $E_n < 20 \text{ MeV}$
- $E_n < 1 \text{ GeV}$

(H. Neuert, Kernphysikalische Messverfahren, G. Braun Verlag, 1966)

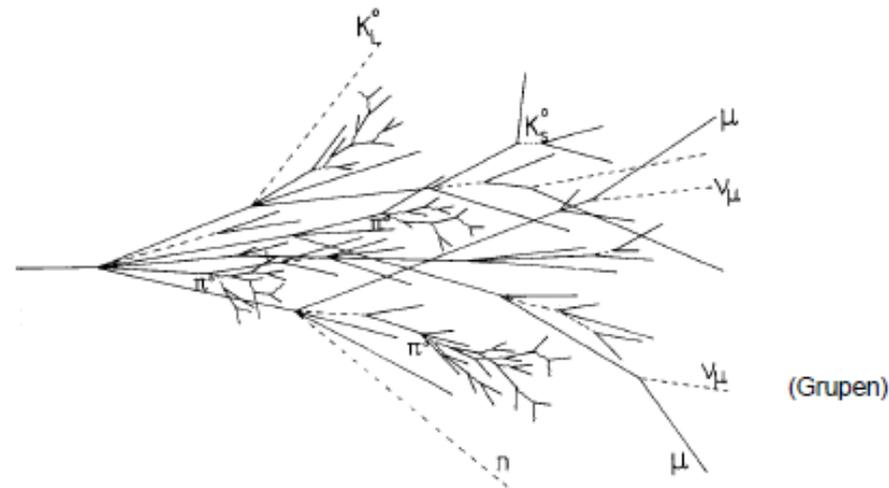


In addition there are

- neutron induced fission  $E_n \approx E_{\text{th}} \approx \frac{1}{40} \text{ eV}$
- hadronic cascades (see below)  $E_n > 1 \text{ GeV}$

# Hadronic cascades

Various processes involved. Much more complex than electromagnetic cascades.



Hadronic

+

electromagnetic  
component



charged pions, protons, kaons ....  
Breaking up of nuclei  
(binding energy),  
neutrons, neutrinos, soft  $\gamma$ 's  
muons ....  $\rightarrow$  invisible energy



neutral pions  $\rightarrow 2\gamma \rightarrow$   
electromagnetic cascade  
 $n(\pi^0) \approx \ln E(\text{GeV}) - 4.6$   
example 100 GeV:  $n(\pi^0) \approx 18$

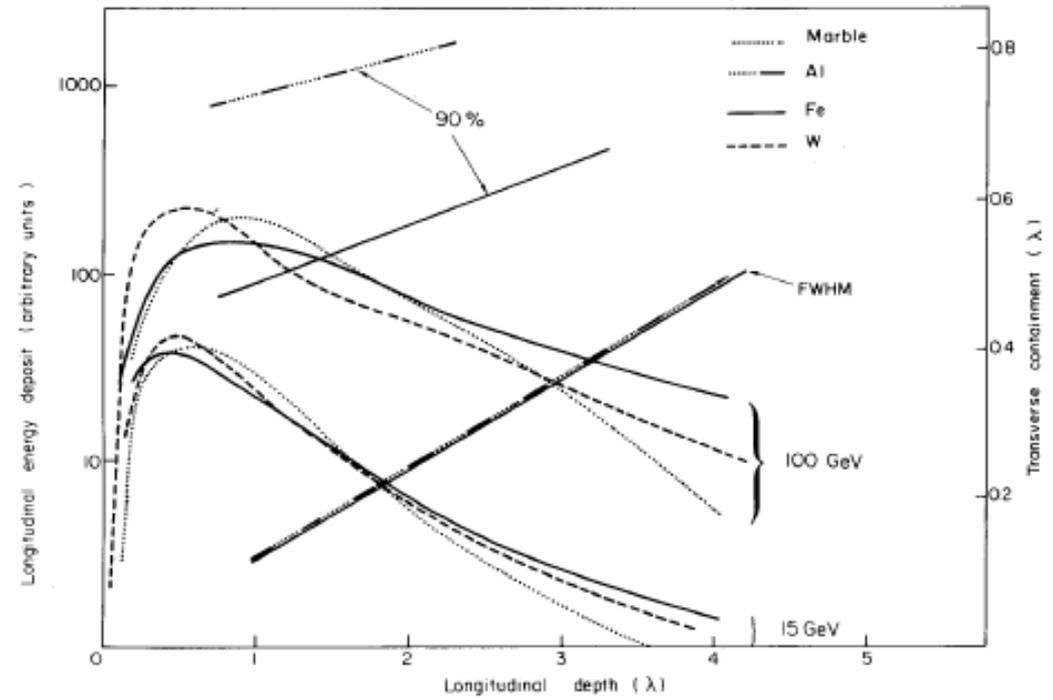
Large energy fluctuations  $\rightarrow$  limited energy resolution

# Longitudinal shower development

$$t_{\max}(\lambda_I) \approx 0.2 \ln E[\text{GeV}] + 0.7$$

$$t_{95\%} \approx a \ln E + b$$

For Iron:  $a = 9.4$ ,  $b = 39$   
 $E = 100 \text{ GeV}$   
 $\rightarrow t_{95\%} \approx 80 \text{ cm}$



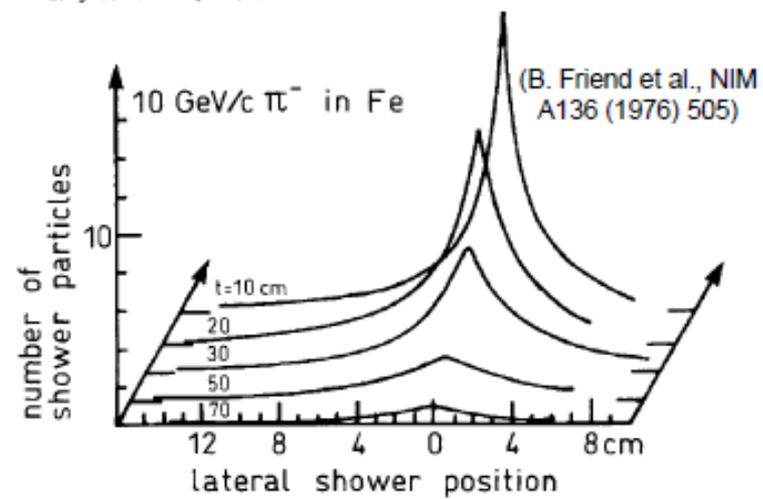
(C. Fabjan, T. Ludlam, CERN-EP/82-37)



Hadronic showers are much longer and broader than electromagnetic ones!

Laterally shower consists of core + halo. 95% containment in a cylinder of radius  $\lambda_I$ .

Iron:  $\lambda_I = 16.7 \text{ cm}$



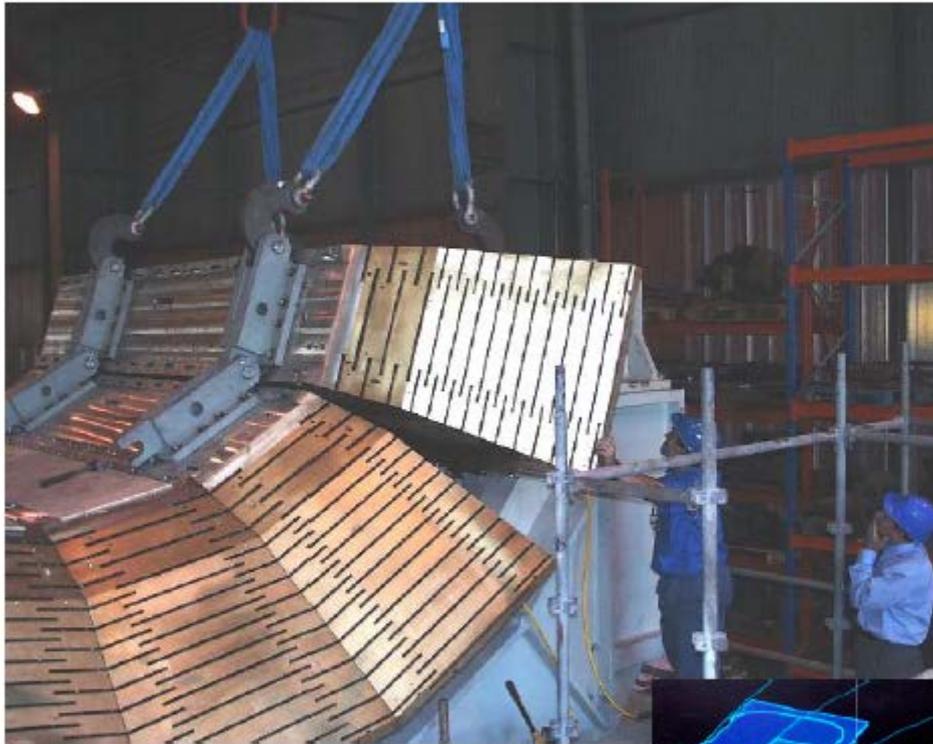
(B. Friend et al., NIM A136 (1976) 505)

◆ CMS Hadron calorimeter

Cu absorber + scintillators



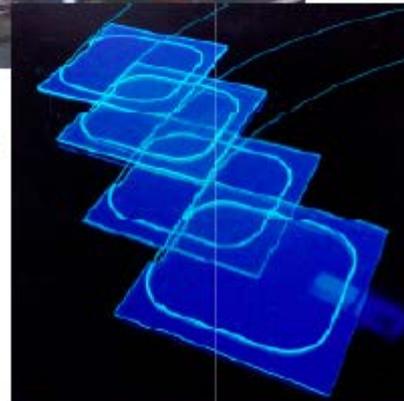
2 x 18 wedges (barrel)  
+ 2 x 18 wedges (endcap)  $\approx$  1500 T absorber



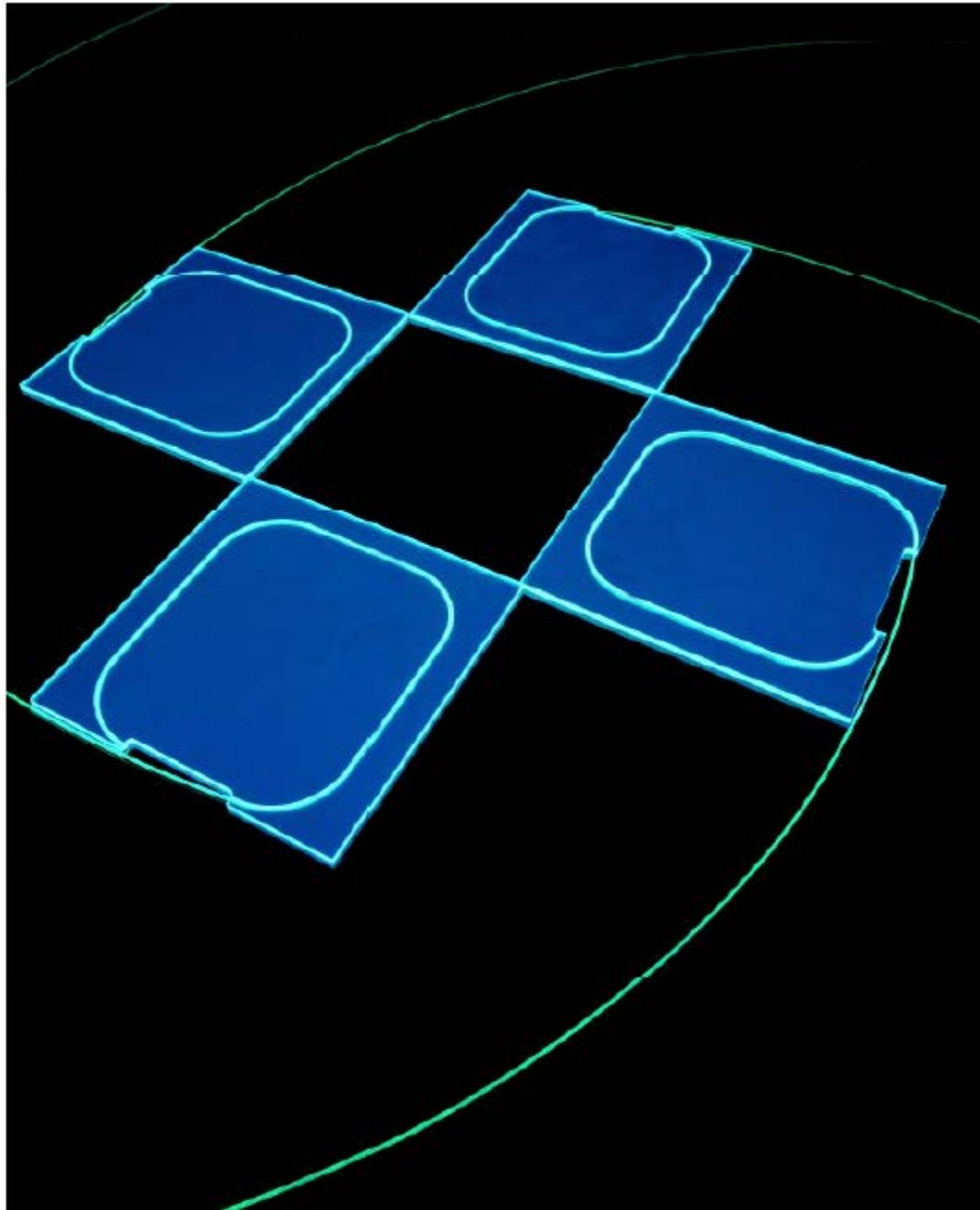
Scintillators fill slots and are read out via fibres by HPDs

Test beam  
resolution for  
single hadrons

$$\frac{\sigma_E}{E} = \frac{65\%}{\sqrt{E}} \oplus 5\%$$



## 4 scintillating tiles of the CMS Hadron calorimeter



# Atmosphere as a calorimeter

Need:

- detect high energy cosmic rays
- Measure their energy
- Determine the identity (gamma or hadron, which hadron)

Idea: use atmosphere as a detector + calorimeter

Virtues:

- A lot of material
- Transparent

Use Cherenkov light emitted by charged particles to determine the energy of the incoming cosmic ray.

Gamma-ray

Particle shower

Detection of high-energy gamma rays

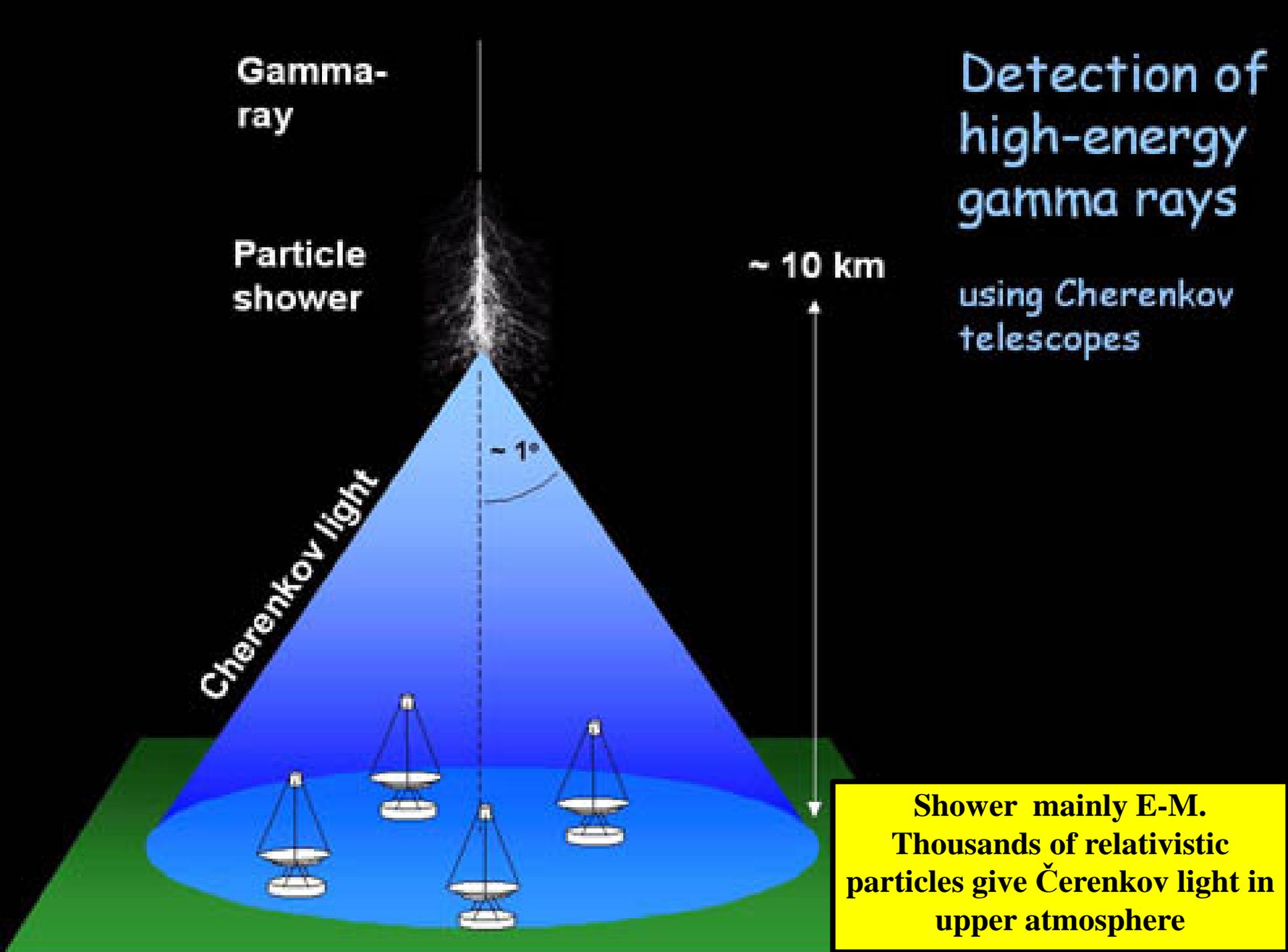
using Cherenkov telescopes

~ 10 km

Cherenkov light

~ 1°

Shower mainly E-M.  
Thousands of relativistic particles give Čerenkov light in upper atmosphere

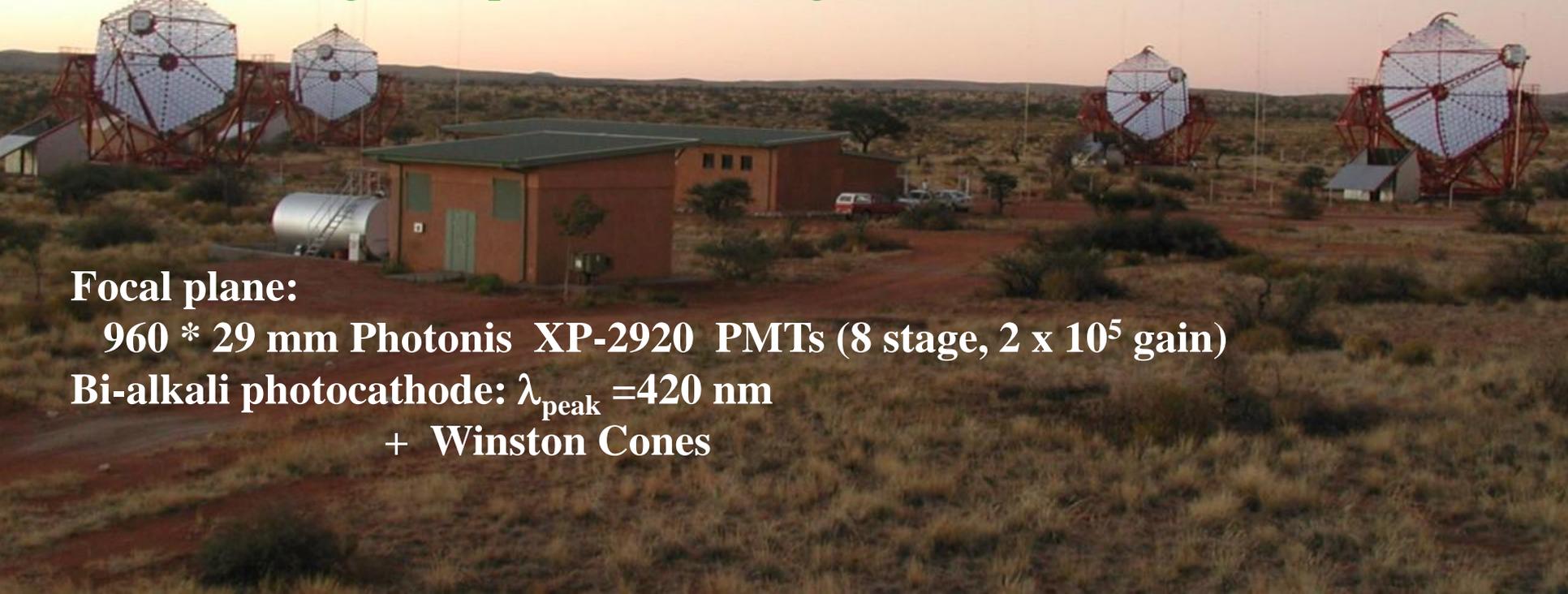


# HESS 1 UHE Gamma Ray Telescope Stereoscopic Quartet

**Khomas Highland, Namibia, (23°16'S, 16°30'E, elev. 1800m)**

**Four  $\emptyset = 12$  m Telescopes (since 12/2003)  $E_{th} \sim 100$  GeV**

**108 m<sup>2</sup> /mirror [382 x  $\emptyset=60$ cm individually steerable (2-motor) facets]  
aluminized glass + quartz overcoating  $R > 80\%$  ( $300 < \lambda < 600$  nm)**



**Focal plane:**

**960 \* 29 mm Photonis XP-2920 PMTs (8 stage,  $2 \times 10^5$  gain)**

**Bi-alkali photocathode:  $\lambda_{peak} = 420$  nm  
+ Winston Cones**

