Vertexing and Kinematic Fitting, Part IV: Building Virtual Particles

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Overview of plan

- **4th of several lectures on kinematic fitting**
- **Focus in this lecture on building virtual particles**
- **Plan of lectures**
  - Lecture 1: Basic theory
  - Lecture 2: Introduction to the KWFIT fitting package
  - Lecture 3: Vertex fitting
  - Lecture 4: Building virtual particles
- **References**
  - **KWFIT**
    http://www.phys.ufl.edu/~avery/kwfit/ or
    http://w4.lns.cornell.edu/~avery/kwfit/
  - Several write-ups on fitting theory and constraints
    http://www.phys.ufl.edu/~avery/fitting.html

What is a Virtual Particle?

Basic idea

We want to take a set of particles and build a new particle using a vertex constraint. The 4-momentum of the new "virtual particle" is the sum of the 4-momenta of the tracks (at the vertex location) and the position is the fitted vertex. The 7×7 covariance matrix is also desired.

What's so great about that?

Once the virtual particle and its covariance matrix is built, it behaves like any other track. You can throw away the tracks that made it up and use it in later fits.

For example, you can apply a mass constraint to the new track to further improve the track parameters.

Software for this exists in KWFIT

kvir_add_nofit	No vertex fit. Just add 4-momenta
kvir_vertex_unknown	Vertex location unknown
kvir_vertex_known	Prior vertex and error already exists
kvir_vertex_fixed	Vertex location fixed

Example

Fit the decay sequence shown below (measured particles shown in boldface) by combining particles starting at the bottom and building up the chain:

$$\overline{B}^0 \to D^{*+}\rho^-$$

$$D^{*+} \to D^0\pi^+$$

$$\rho^- \to \pi^-\pi^0$$

$$D^0 \to \mathrm{K}^-\pi^+$$

There are 5 measured particles in this example. Build 4 virtual particles in order:

1. $D^0 \to K^-\pi^+$ (apply mass constraint)
2. $\rho^- \to \pi^-\pi^0$
3. $D^{*+} \to D^0\pi^+$ (apply mass constraint)
4. $\overline{B}^0 \to D^{*+}\pi^-$ (optional)

Theory

The 7 parameters are the 4-momentum and the 3-position. Let **x** be the fitted vertex. The virtual particle track parameters are

$$
\begin{aligned}
p_{Vx} &= \sum_i \left[p_{ix} + a_{zi}\left(y_i - x_y\right) - a_{yi}(z_i - x_z) \right] \\
p_{Vy} &= \sum_i \left[p_{iy} + a_{xi}(z_i - x_z) - a_{zi}(x_i - x_x) \right] \\
p_{Vz} &= \sum_i \left[p_{iz} + a_{yi}(x_i - x_x) - a_{xi}\left(y_i - x_y\right) \right] \\
E_V &= \sum_i E_i \\
\mathbf{x}_V &= \mathbf{x}
\end{aligned}
$$

where, e.g., $a_{xi} = -0.299792458 B_x Q_i$, Q_i is the charge on the particle and B_x is the x component of the B field.

This can be written in matrix form:

$$\mathbf{p}_V = \mathbf{A}\boldsymbol{\alpha} + \mathbf{B}\mathbf{x} = \sum_i \mathbf{A}_i \alpha_i + \mathbf{B}\mathbf{x}$$
$$\mathbf{x}_V = \mathbf{x}$$

where

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} \mathbf{A}_1 & \mathbf{A}_2 & \cdots & \mathbf{A}_n \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 0 & -\sum a_{zi} & \sum a_{yi} \\ \sum a_{zi} & 0 & -\sum a_{xi} \\ -\sum a_{yi} & \sum a_{xi} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

and

$$\mathbf{A}_i = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & a_{zi} & -a_{yi} \\ 0 & 1 & 0 & 0 & -a_{zi} & 0 & a_{xi} \\ 0 & 0 & 1 & 0 & a_{yi} & -a_{xi} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

This part is easy since the track parameters are determined from the vertex constraint. The harder part is getting the covariance matrix right. It can be written generally as

$$\mathbf{V}_{\alpha_V} = \begin{pmatrix} \mathbf{V}_{\mathbf{p}_V} & \mathrm{cov}(\mathbf{p}_V, \mathbf{x}_V) \\ \mathrm{cov}(\mathbf{x}_V, \mathbf{p}_V) & \mathbf{V}_{\mathbf{x}_V} \end{pmatrix}$$

$$\begin{aligned} \mathbf{V}_{\mathbf{p}V} &= \mathbf{A}\mathbf{V}_\alpha \mathbf{A}^T + \mathbf{A}\,\mathrm{cov}(\alpha, \mathbf{x})\mathbf{B}^T + \mathbf{B}\,\mathrm{cov}(\mathbf{x}, \alpha)\mathbf{A}^T + \mathbf{B}\mathbf{V}_\mathbf{x}\mathbf{B}^T \\ \mathrm{cov}(\mathbf{p}_V, \mathbf{x}_V) &= \mathbf{A}\,\mathrm{cov}(\alpha, \mathbf{x}) + \mathbf{B}\mathbf{V}_\mathbf{x} \\ \mathbf{V}_{\mathbf{x}_V} &= \mathbf{V}_\mathbf{x} \end{aligned}$$

From CBX 98–38, we find the covariance matrix for the virtual particle

$$\mathbf{V}_{\alpha_V} = \begin{pmatrix} \sum_i \left(\mathbf{A}_i \mathbf{V}_{0i} \mathbf{A}_i^T - \mathbf{S}_{1i} \mathbf{V}_{Di} \mathbf{S}_{1i}^T \right) + \mathbf{T}\mathbf{V}_\mathbf{x}\mathbf{T}^T & -\mathbf{T}\mathbf{V}_\mathbf{x} \\ -\mathbf{V}_\mathbf{x}\mathbf{T}^T & \mathbf{V}_\mathbf{x} \end{pmatrix}$$

where

$$\begin{aligned} \mathbf{S}_{1i} &= \mathbf{A}_i \mathbf{V}_{0i} \mathbf{D}_i^T \\ \mathbf{S}_{3i} &= \mathbf{A}_i \mathbf{V}_{0i} \mathbf{D}_i^T \mathbf{V}_{Di} \mathbf{E}_i \\ \mathbf{T} &= -\mathbf{B} + \sum_i \mathbf{S}_{3i}\,. \end{aligned}$$

Further issues

Should we incorporate mass constraint directly in the particle building?

NO!

1. The code to do so is much more complicated.
2. The 2 step procedure gives 2 χ^2 values. This provides more discrimination against background.
3. 2 step process is mathematically identical to 1 step process

Flexibility issues

I have implemented a more useful version of the procedure above in KWFIT. Three kinds of tracks can be used:

1. Those that are used to determine the vertex

   Normal tracks

2. Those that are not used to determine the vertex, but are required to pass through it

   Soft pions from D^{*+}

3. Those that only contribute 4-momentum (no vertex info)

   Photons, pi0s

This procedure is mathematically rigorous, as proven in CBX 98–38