Using Kalman Filter to Track Particles

Saša Fratina

advisor:

Samo Korpar

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Overview

- Motivation
- Basic principles of Kalman Filter
  $\rightarrow$ example
- Application to particle tracking

No big deal

R. E. Kalman

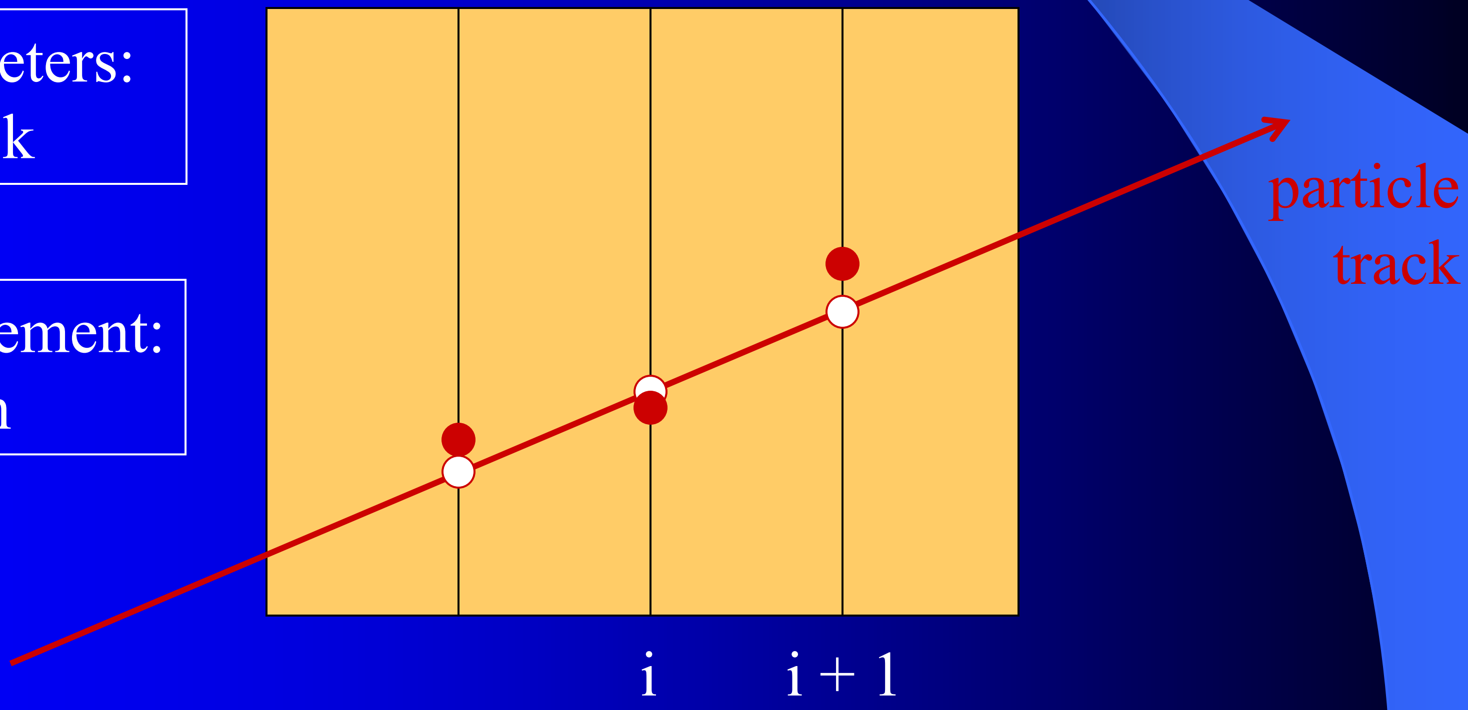
- Born 1930 in Hungary
- Studied at MIT / Columbia
- Developed filter in 1960/61

Illustration example

Measuring parameters of a particle track in 2D

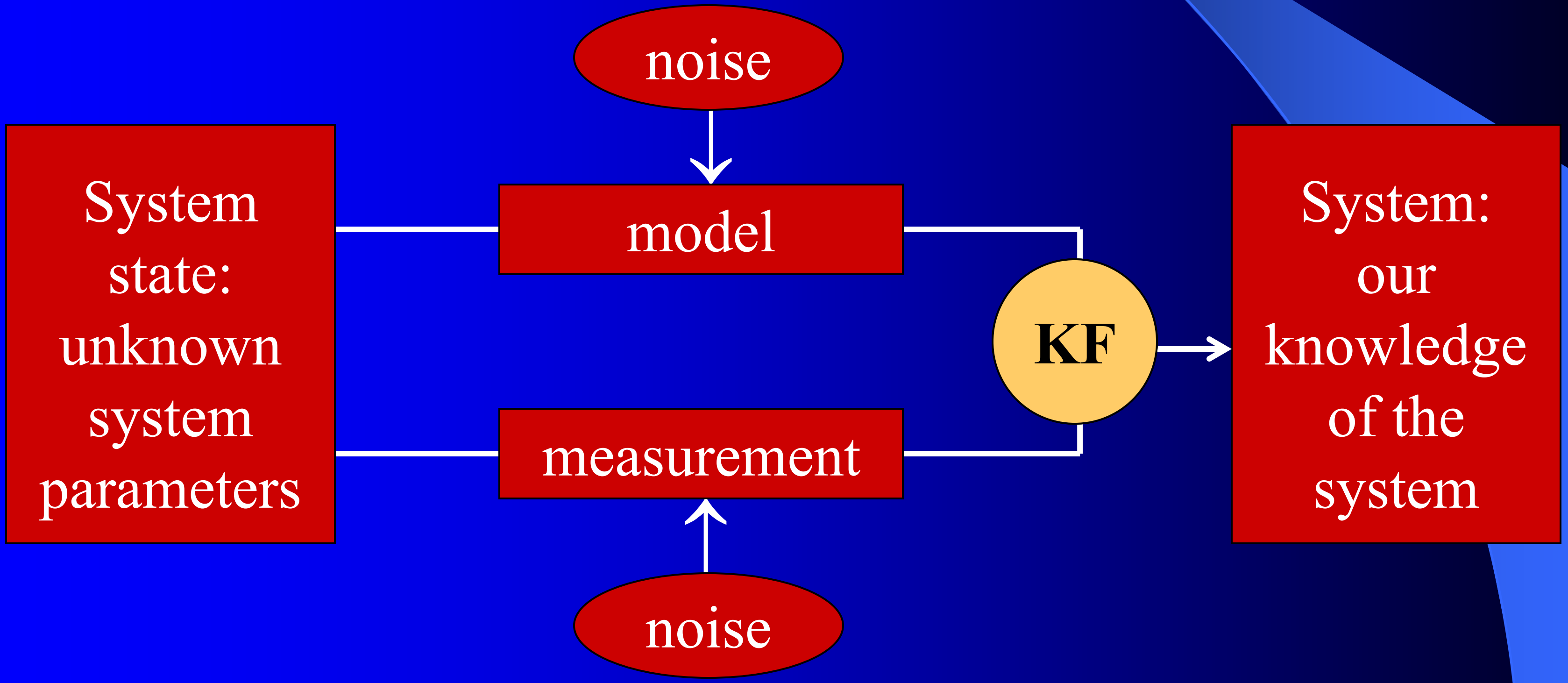
parameters:
y, k

measurement:
m



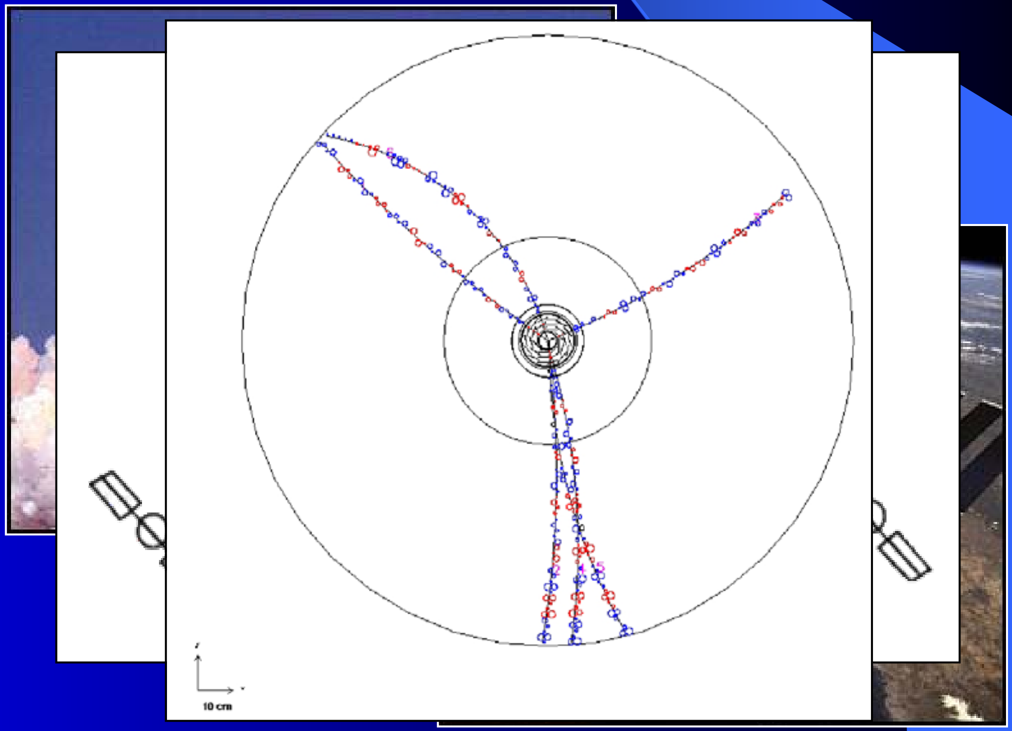
Kalman filter – KF

When and where?



When and where?

- Tracking and navigation
  - Tracking missiles, aircrafts and spacecrafts
  - GPS technology
  - Visual reality
- Tracking in HEP experiments



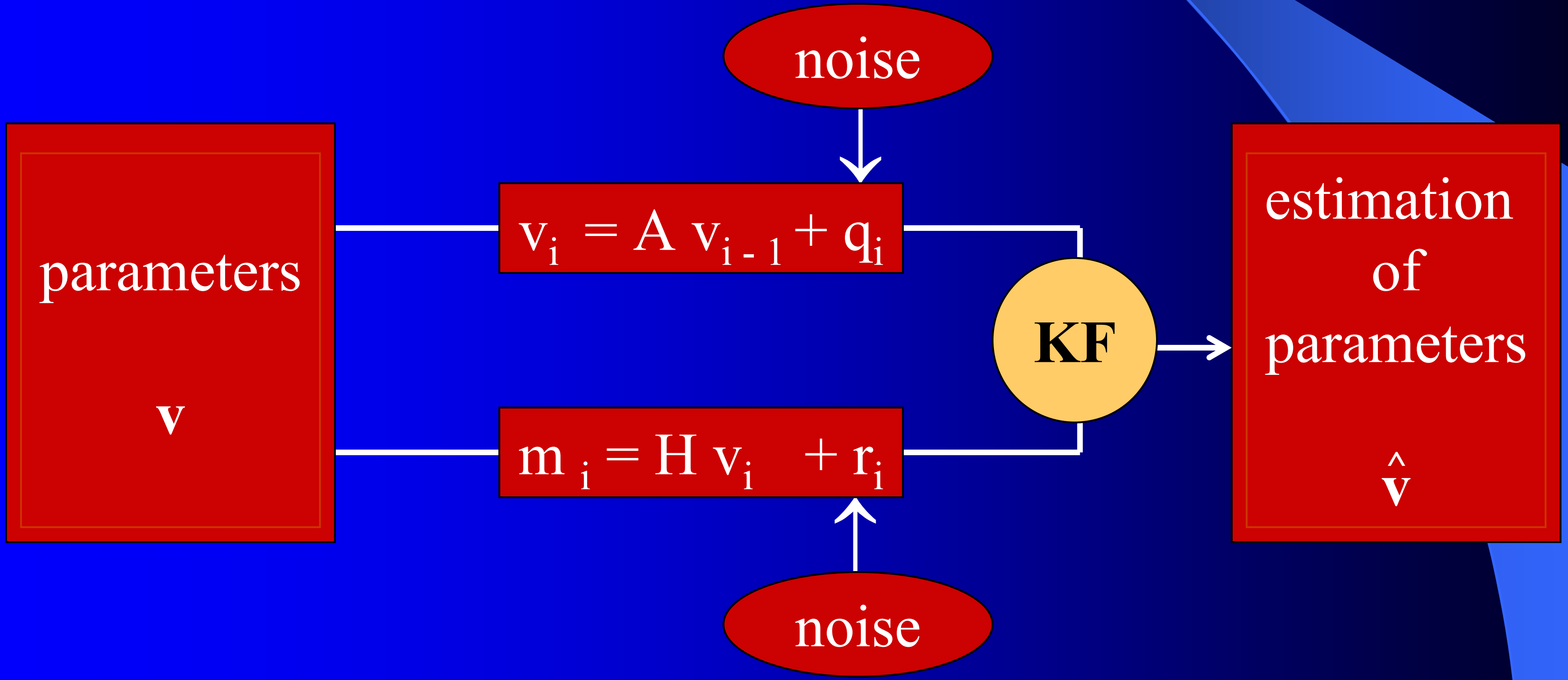
KF assumptions

- Linear system
  - System parameters are linear function of parameters at some previous time
  - Measurements are linear function of parameters
- White Gaussian noise
  - White: uncorrelated in time
  - Gaussian: noise amplitude

⇒ KF is the optimal filter

KF description

using vectors and matrices



KF description: example

- System parameters: **v** → $v_i = \begin{pmatrix} y \\ k \end{pmatrix}_i$
- System model:

  linear motion y = k x

  $\mathbf{v_i = A\ v_{i-1}}$ → $\begin{pmatrix} y \\ k \end{pmatrix}_i = \begin{pmatrix} 1 & \Delta x \\ 0 & 1 \end{pmatrix} \begin{pmatrix} y \\ k \end{pmatrix}_{i-1}$
- Measurement model:

  $\mathbf{m_i = H\ v_i}$ → $m_i = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} y \\ k \end{pmatrix}_i$

Noise

Noise: e Gaussian $\Rightarrow E(e^2) = \sigma^2$

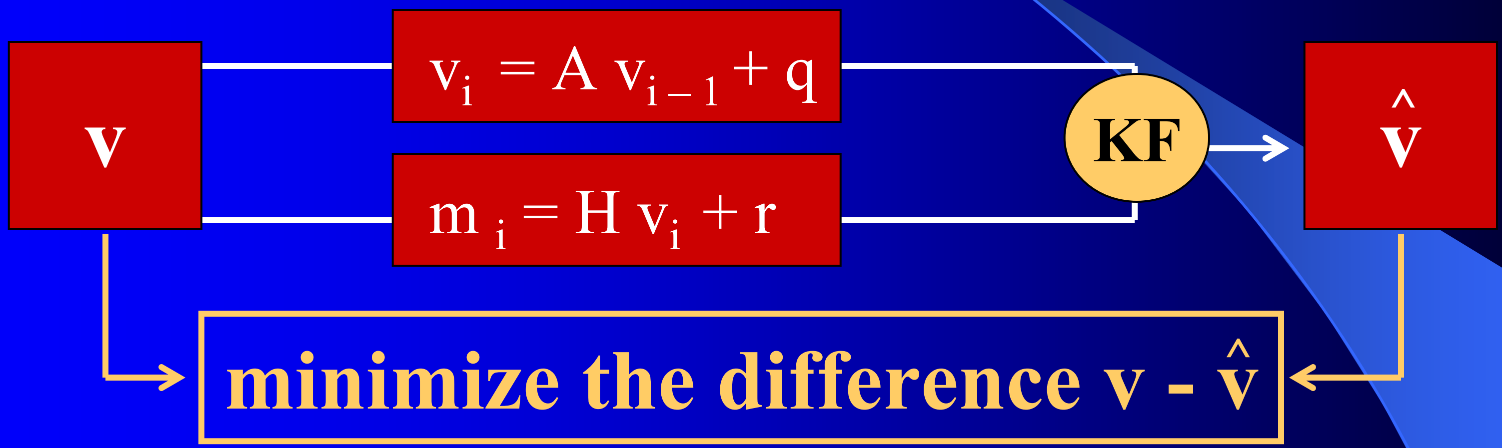
Noise covariance matrix

$$V = E(ee^T) = \begin{pmatrix} E(e_1e_1) & E(e_1e_2) & \cdots \\ E(e_2e_1) & E(e_2e_2) & \\ \vdots & & \ddots \end{pmatrix}$$

- System noise: $\mathbf{v_i = A\, v_{i-1} + q_i \Rightarrow Q = E(qq^T)}$
- Measurement noise: $\mathbf{m_i = H\, v_i + r_i}$

  $\Rightarrow \mathbf{R = E(rr^T)}$

KF algorithm



- Prediction: $\hat{v}_i^- = A\, \hat{v}_{i-1}$
- Correction: $\hat{v}_i = \hat{v}_i^- + K\,(m_i - H\, \hat{v}_i^-)$

Kalman gain matrix

Kalman gain matrix

- It is easy to show

$$K = V^{-}H^{T}(H V^{-}H^{T} + R)^{-1},$$

$$\text{where } V_i^{-} = A V_{i-1} A^{T} + Q$$

- Minimize the expected error

$$e = v - \hat{v} \;;\; V = E(ee^{T}) = \begin{pmatrix} E(e_1e_1) & E(e_1e_2) & \cdots \\ E(e_2e_1) & E(e_2e_2) & \\ \vdots & & \ddots \end{pmatrix} \Rightarrow \frac{\partial V_{ij}}{\partial K_{ab}} = 0$$

- Limits:
  - system noise << measurement noise $\Rightarrow \hat{v}_i = \hat{v}_i^{-}$
  - system noise >> measurement noise $\Rightarrow \hat{v}_i = H^{-1} m_i$

Error on parameters

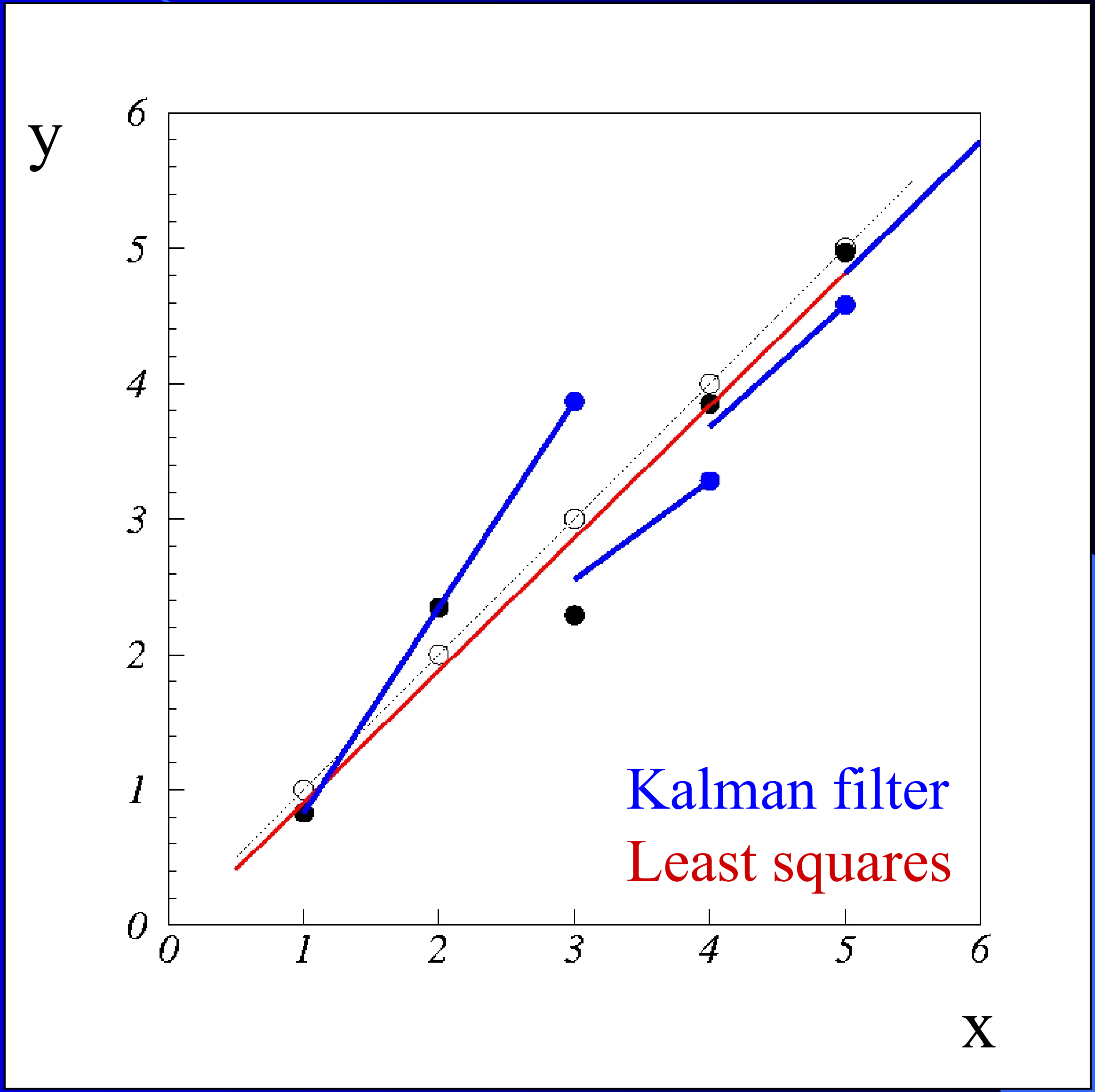
- Predictor: $V_i^- = A V_{i-1} A^T + Q$
  - Q: system noise
- Corrector: $V_i = (I - KH) V_i^-$
  - error reduced

Example

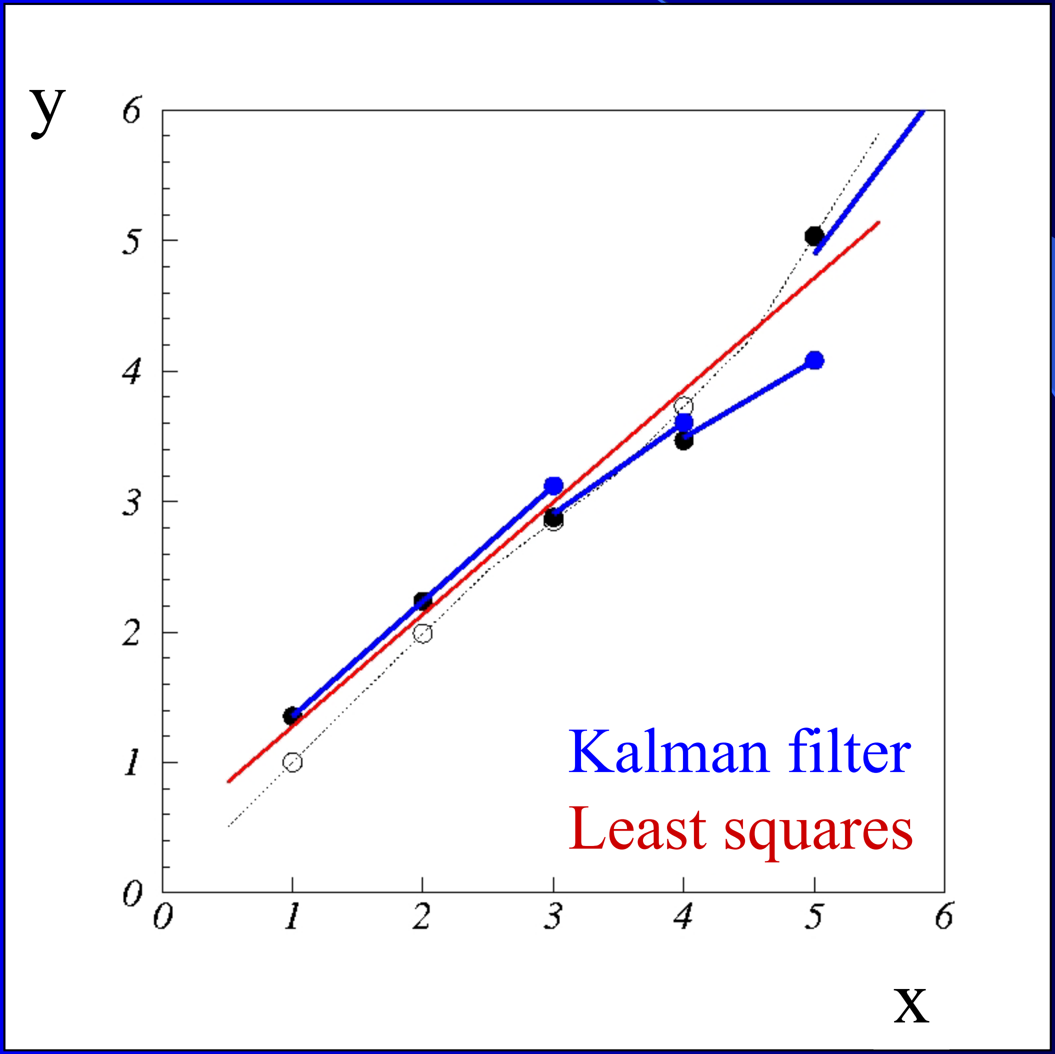
- Simulation

  y = k x

- Implemented KF
  - prediction
  - correction
- Compare with LS method

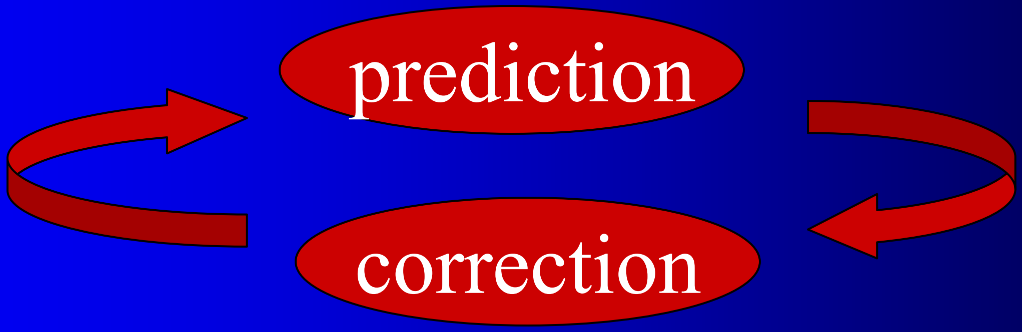


System noise



KF overview

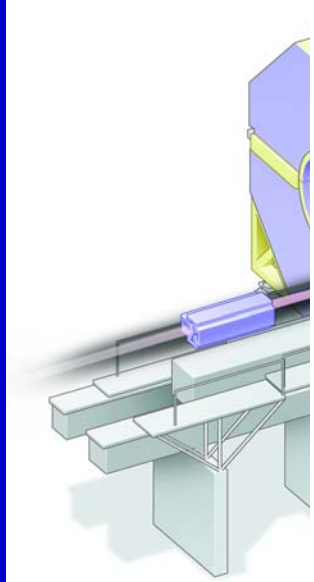
- Matrix description of system state, model and measurement
- Progressive method

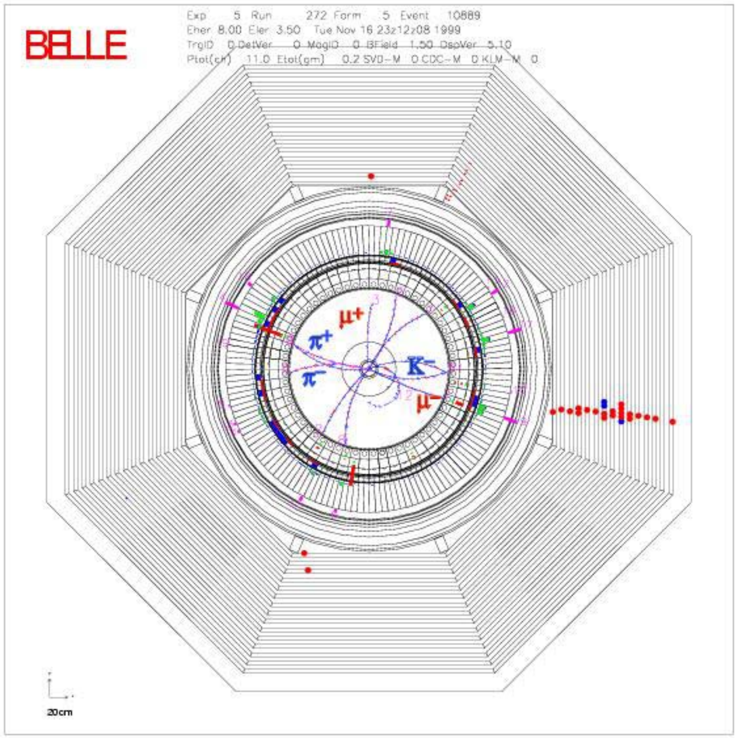


- Proper dealing with noise

Application to particle tracking

- Detector:
  - Silicon vertex detector
  - Central drift chamber
- Description of track: 5 parameters



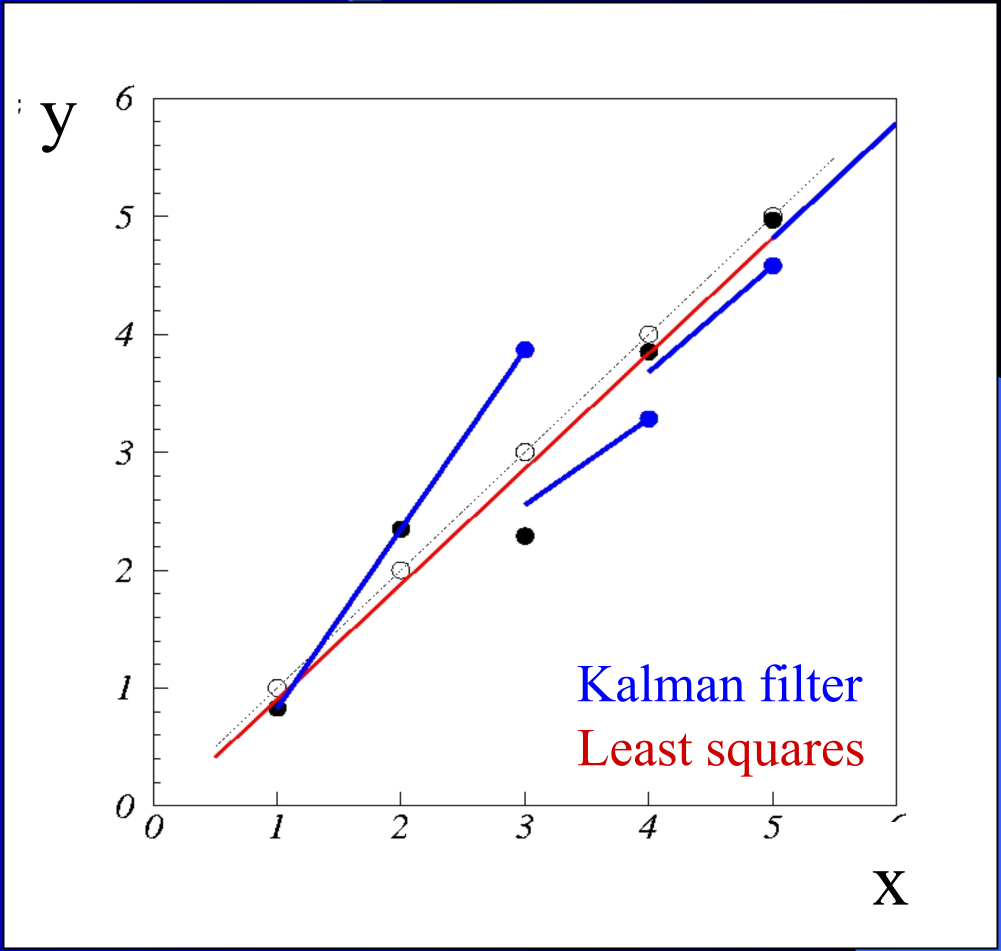


Advantages of using KF
in particle tracking

- Progressive method
  - No large matrices has to be inverted
- Proper dealing with system noise
- Track finding and track fitting
- Detection of outliers
- Merging track from different segments

Modifications of KF

- (!) Non - linear system → extended Kalman filter
- Full precision only after the last step
  - Prediction
  - Correction
  - Smoothing



To sum up. / Conclusion

- We have demonstrated the principles

  predictor – corrector method

  combining model and measurement
- Very useful in tracking
- For given assumptions, KF is **the** optimal filter
- Extensions for non-linear systems
- Extensive application

Tracking in BELLE detector

Track finding

Track fitting

Track managing

Notation overview

- v: vector of parameters
  - $\hat{v}$: our estimation
  - v^{-}: predicted value
- m: vector of measurements
- A: matrix describing linear system → $v_i = A\, v_{i-1}$
- H: matrix describing measurements → $m_i = H\, v_i$
- V: error (on parameter) covariance matrix
- Q: system noise covariance matrix
- R: measurement noise covariance matrix
- K: Kalman gain matrix