Measurement of strong-phase difference between D^0 and $\overline{D^0} \rightarrow K_S^0 K^+ K^-$ decay at BESIII

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March 30, 2021

Outline

Flavour Physics

CKM Matrix

- Measurement of angle γ
- BPGGSZ method- model-dependent and model-independent methods
- Strong-phase coefficients

BEPCII and BESIII

Dalitz plots

- Dalitz plot binning
- Bin yields, corrections, backgrounds etc.

Extraction of strong-phase parameters

- Fitter
- Systematic uncertainties

Impact of strong-phase measurements on γ

- A sneek-peek into LHCb result

Summary

Flavour Physics

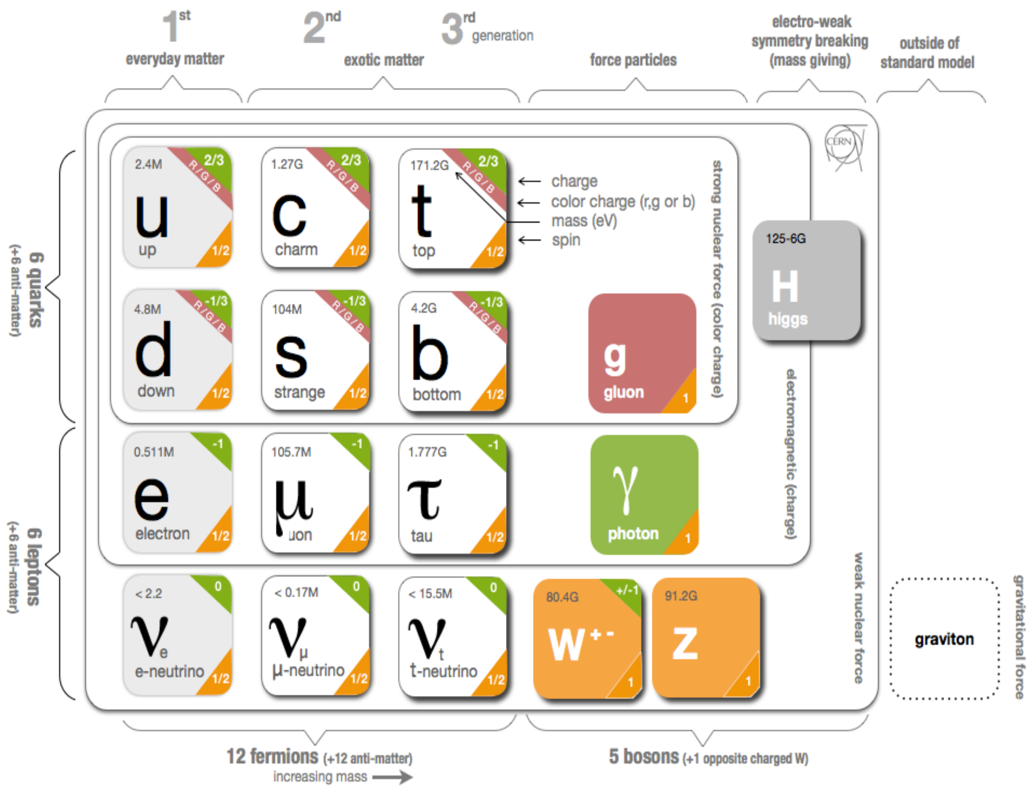
Note: Discussion will be only on quark flavours

Standard Model is an QFT that describes the fundamental particles and its interaction (Weak , em and strong)

All test made on SM has been successful. Not full story!

Flavour sector is less well known (more open questions)

Flavour relates the existence of family of quarks and how they couple to each other



Why flavour physics is interesting?

- Why 3 generations of quarks? Why only 3?
- Extreme hierarchy of masses (2.4 to 1.75$\times$ 10^5) MeV/c^2
- CP violation explained in SM; but not enough to explain matter –antimatter asymmetry
- ….

Charm Factories: BESIII, CLEO-c

B- Factories: BELLE, BELLEII, BaBar, LHCb

These mysteries makes flavour physics of SM of great interest

Cabbibo-Kobayashi-Maskawa (CKM) matrix

2008

Mixing between weak eigenstates and flavor eigenstates in three generations.

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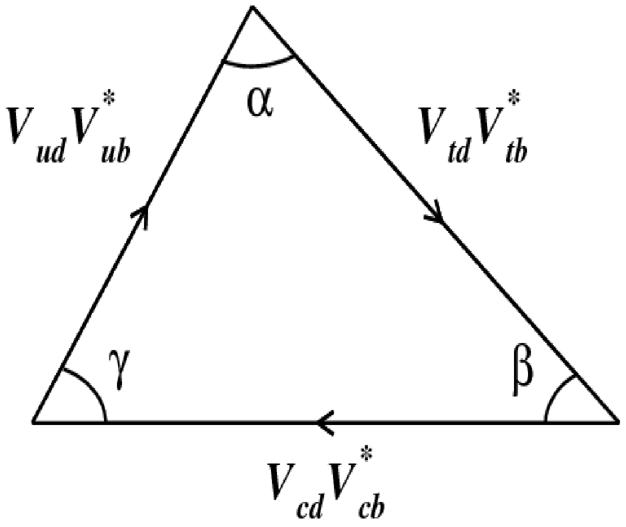
(b-d unitary triangle)



$$\boldsymbol{\alpha/\phi_2 = (84.9^{+5.1}_{-4.5})^{\circ}}$$

$$\boldsymbol{\beta/\phi_1 = (22.2 \pm 0.7)^{\circ}}$$

$$\boldsymbol{\gamma/\phi_3 = (71.1^{+4.6}_{-5.3})^{\circ}}$$

http://ckmfitter.in2p3.fr

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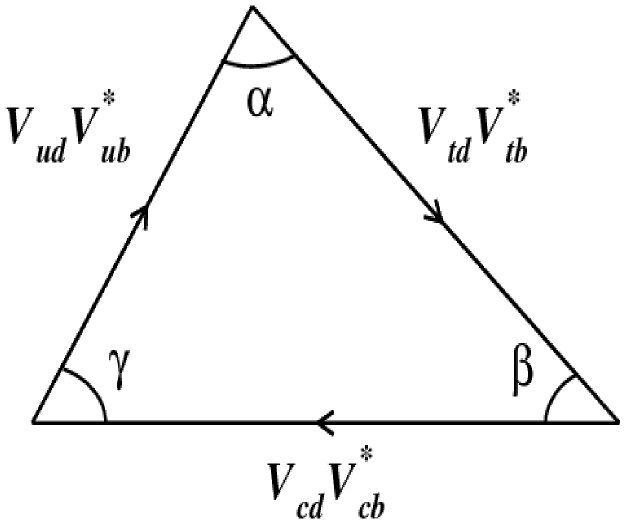

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One aim of flavor physics experiments is to measure CKM parameters precisely

http://ckmfitter.in2p3.fr

CKM angle γ

Direct measurements

Indirect measurements

CKM angle γ

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- Measure it using tree level decays
- Theoretical uncertainty $\mathcal{O}(10^{-7})$

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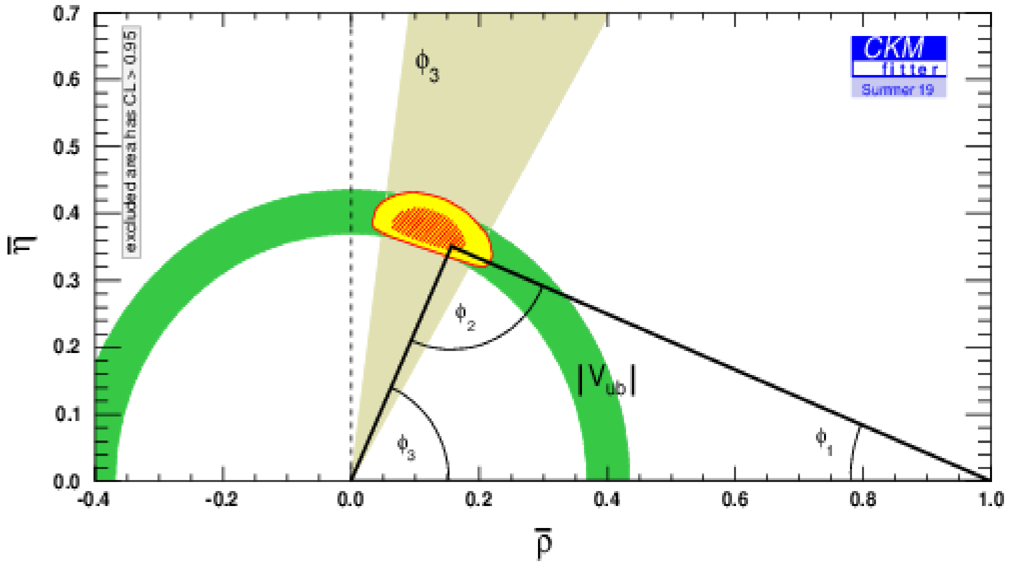

Large experimental uncertainties, potential for further improvement in coming years

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  JHEP 01, 051 (2014)

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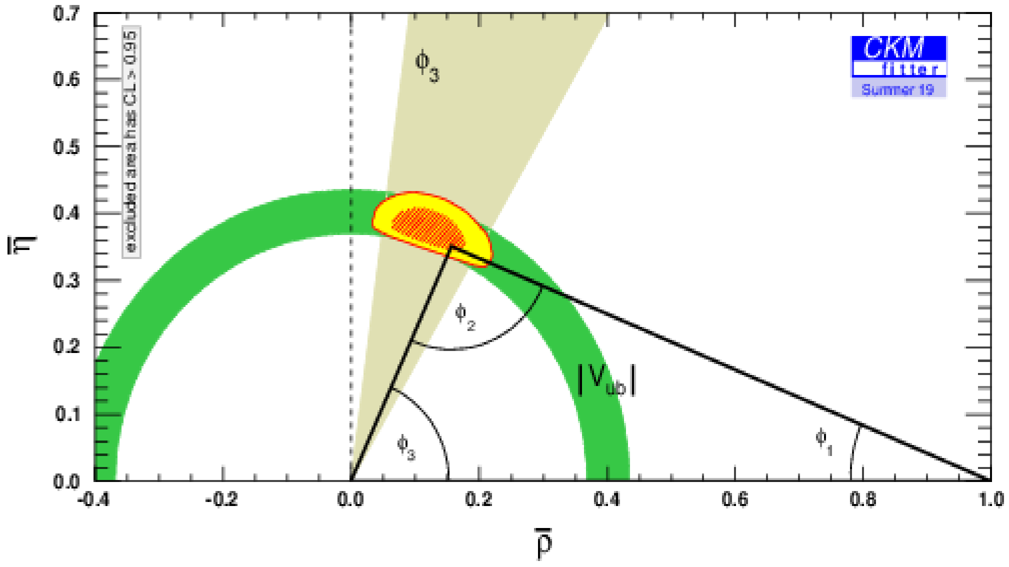

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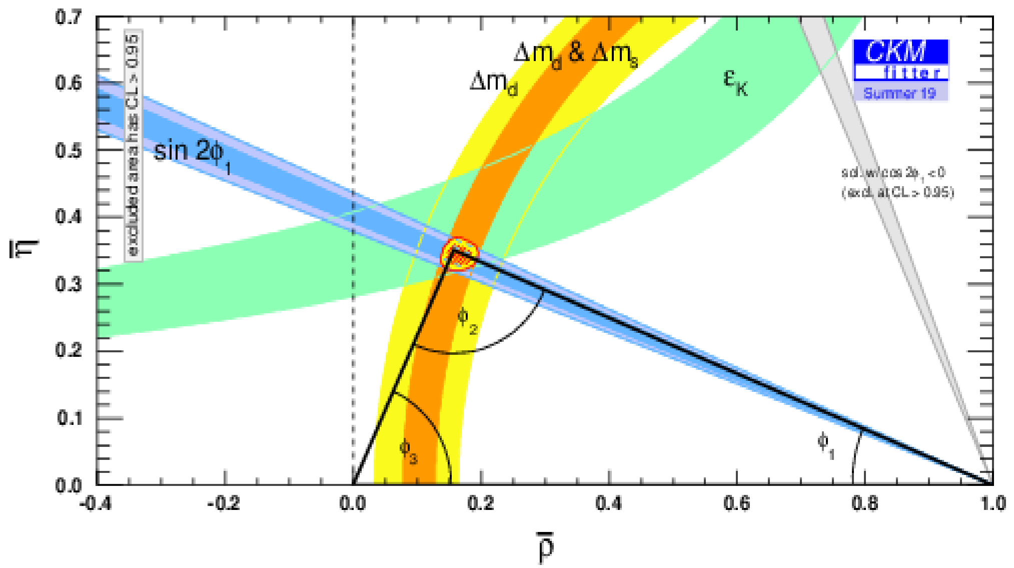
- NP effects can play – potential for different central value

$\boldsymbol{\gamma = (65.3^{+1.0}_{-2.5})^{\circ}}$



Uncertainties from LQCD

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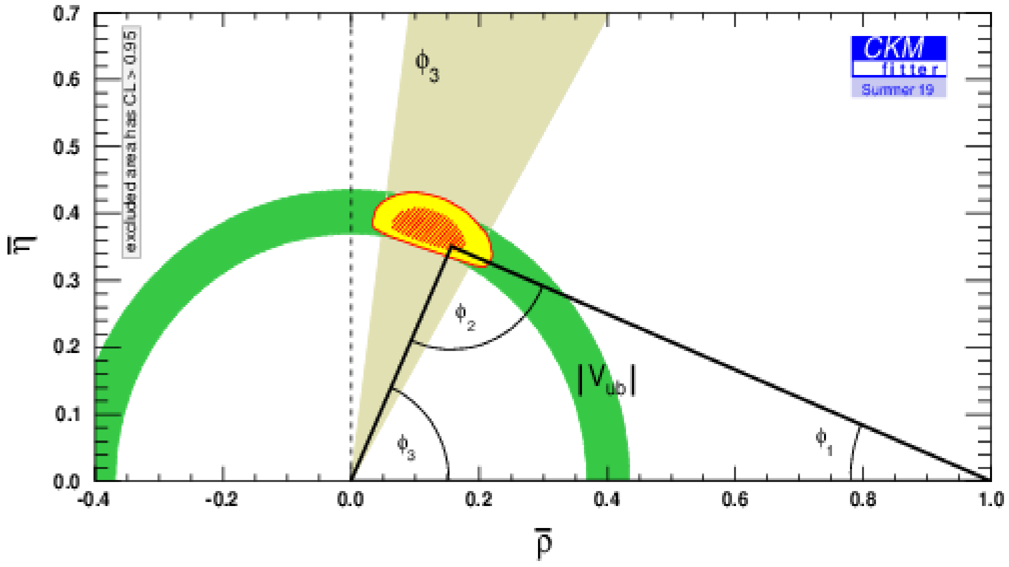

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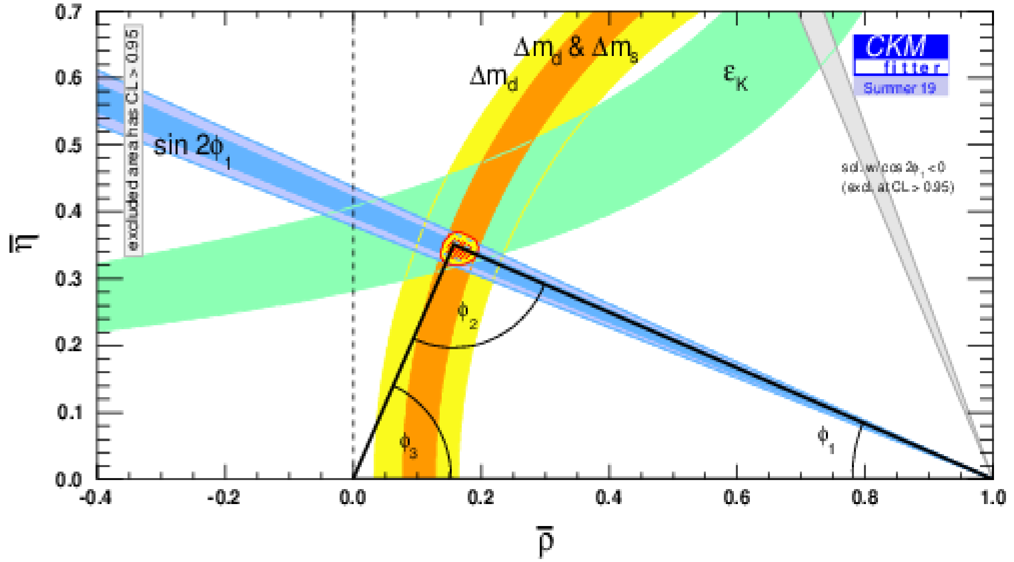
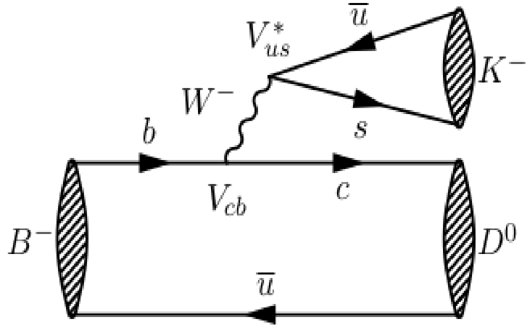
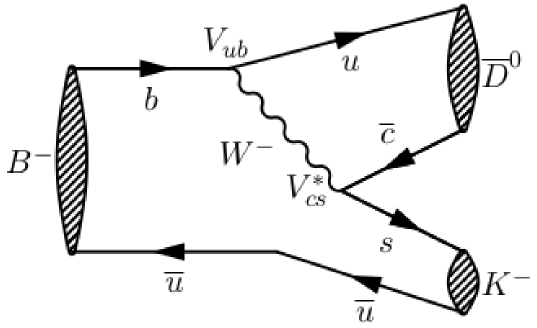


Uncertainties from LQCD

Precise measurement required for meaningful comparison

Measurement of γ/ϕ_3





$\boldsymbol{B^\pm} \rightarrow \boldsymbol{DK^\pm}$ where D = $\boldsymbol{D^0}$ or $\overline{\boldsymbol{D^0}}$ (PLB **265**, 172 (1991))

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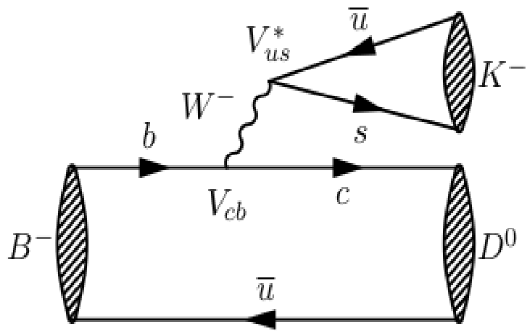

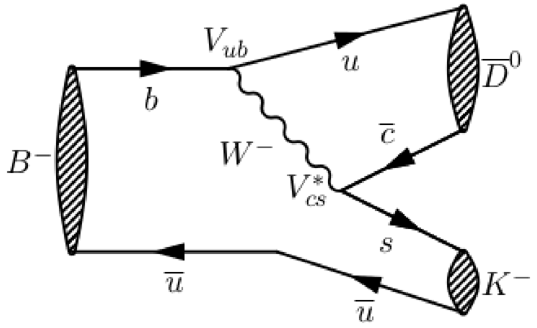

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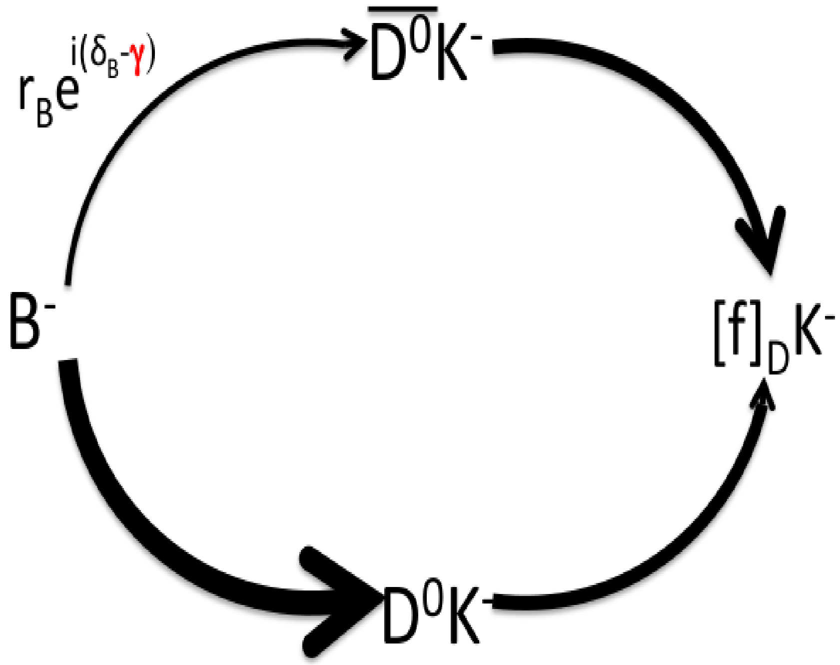
$b \rightarrow c\bar{u}s + b \rightarrow u\bar{c}s$



Common final states-possibility of interference – access to phase term

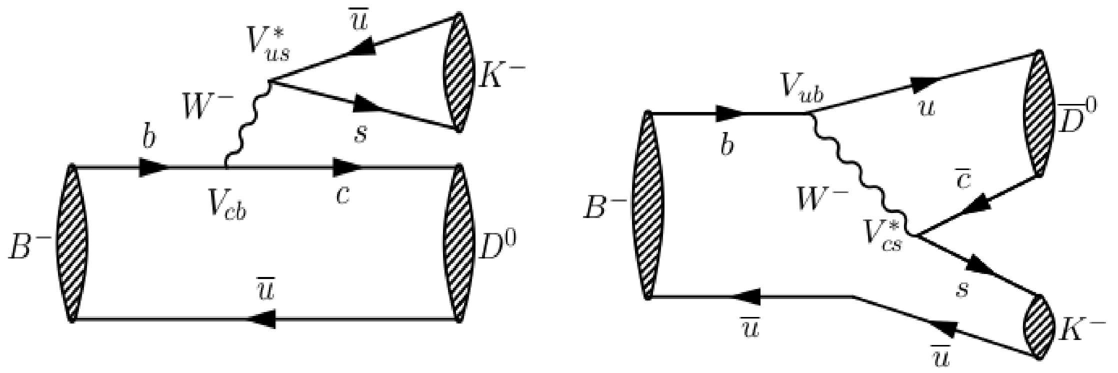
$$\Gamma \propto |f(B^- \rightarrow DK^-)|^2 = A_B^2 + A_B^2 r_B^2 + 2A_B^2 r_B^2 \cos(\delta_B - \gamma)$$

$$r_B = \left|\frac{f\left(B^- \rightarrow \overline{D^0}K^-\right)}{f(B^- \rightarrow D^0K^-)}\right|$$

Sensitivity to γ comes from interference

Other related modes with D^* or K^* in final states can also be used

Measurement of γ/ϕ_3



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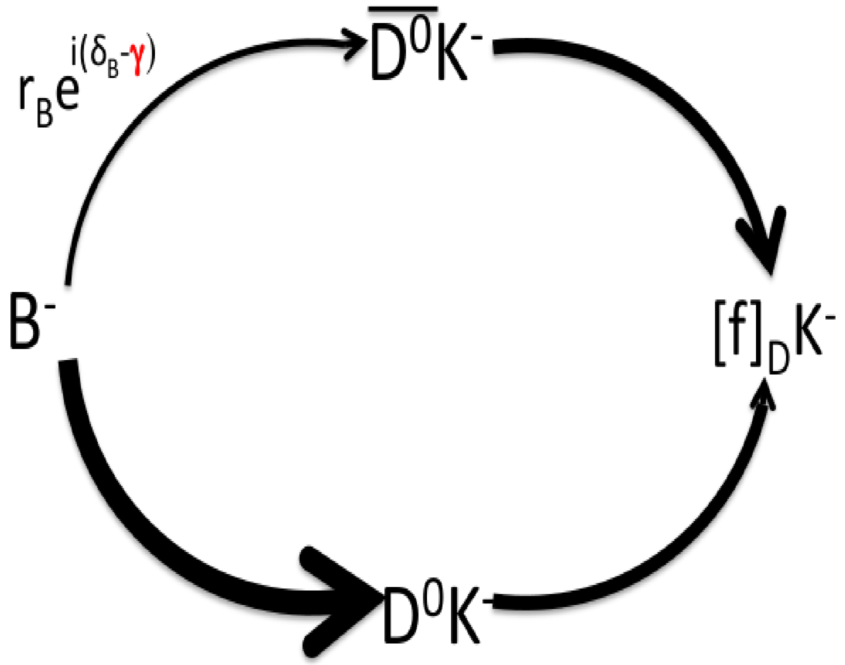

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$\boldsymbol{B^{\pm} \to D\pi^{\pm}}$ modes has larger BF but small value of r_B less sensitive to γ

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Methods to measure γ from $B^{\pm} \to DK^{\pm}$

- **Gronau, London and Wyler method (GLW):** $\boldsymbol{D}$ decay to CP eigenstate, K^-K^+, $K_S^0\pi^0$ … PLB 265, 172 (1991)
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BPGGSZ method

Dalitz plot analysis of multibody final states.

Dalitz plot: A scatter plot of decay in terms of Lorentz invariant quantities Phil. Mag. 44, 1068 (1953)

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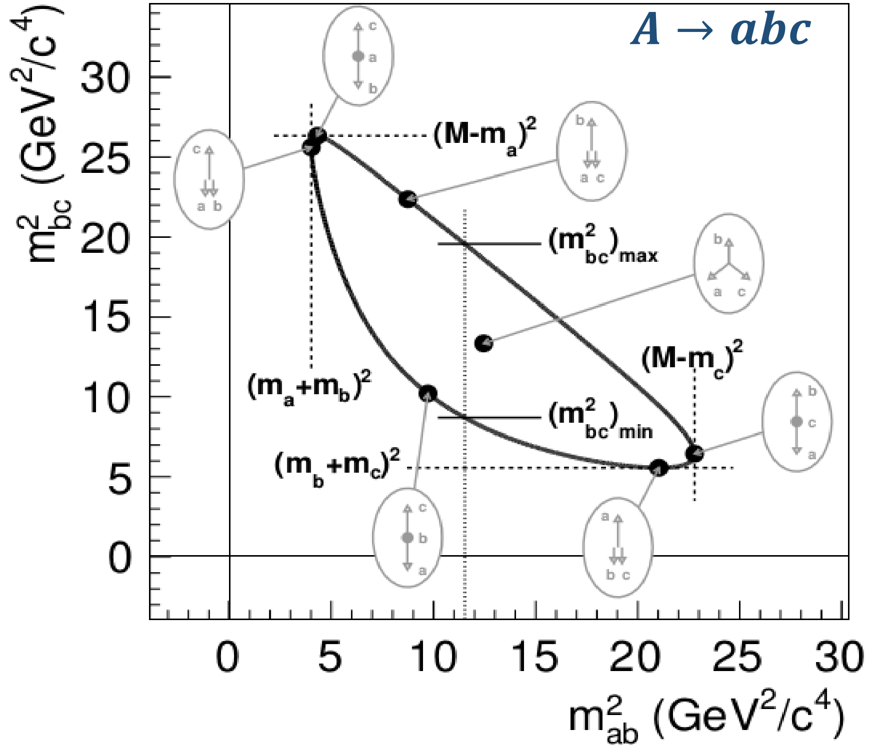

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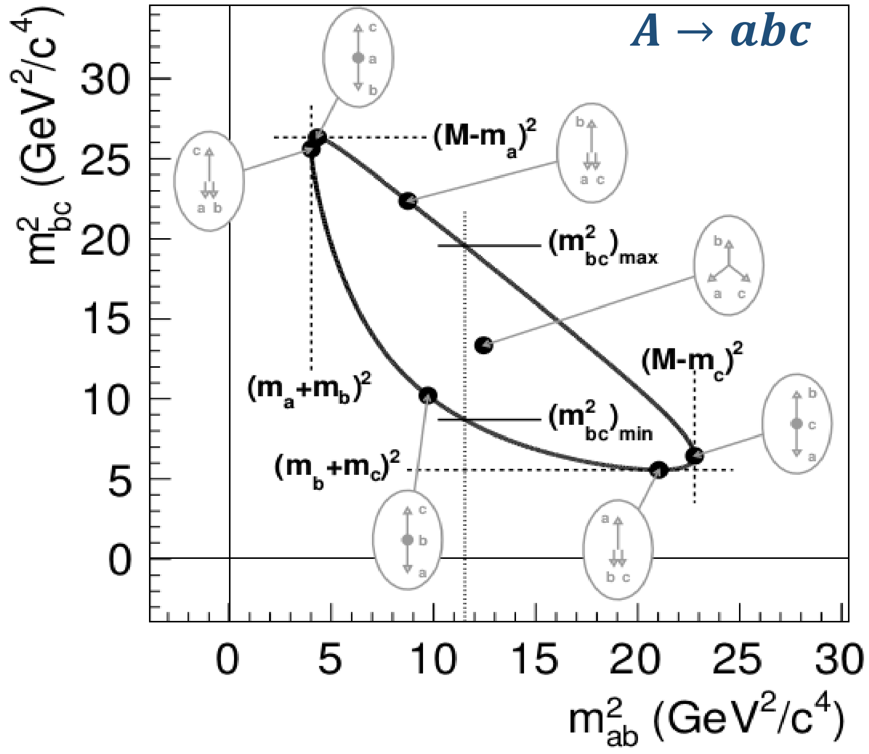

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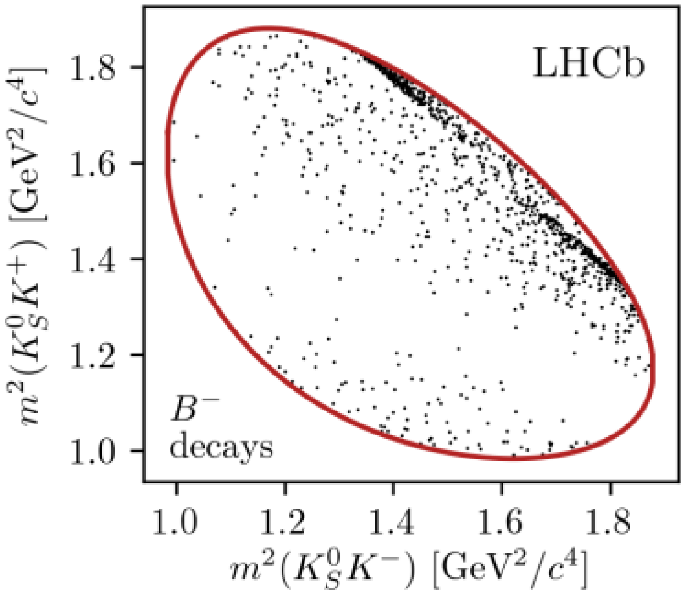

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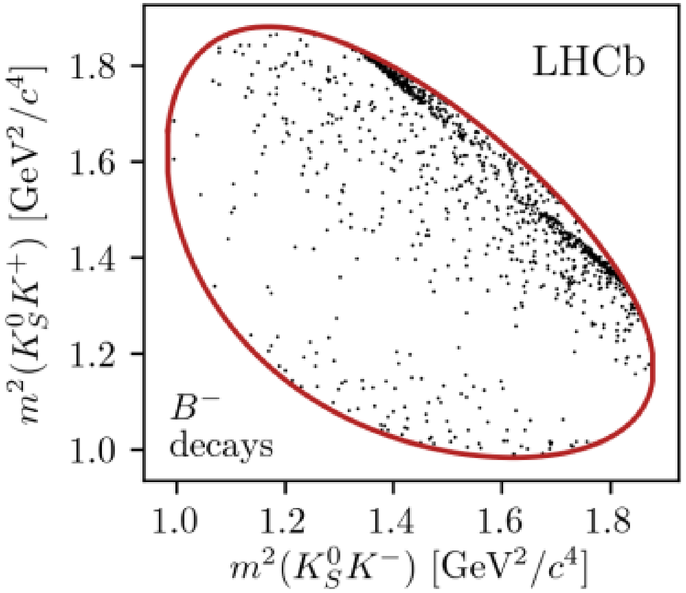

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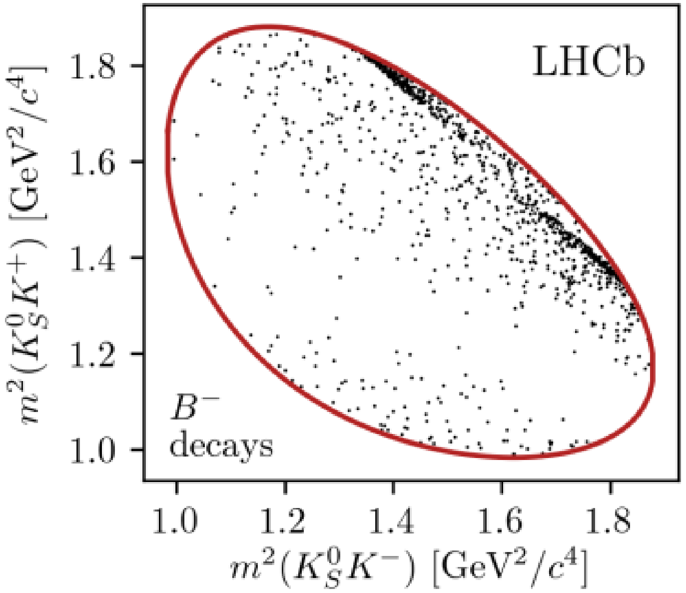

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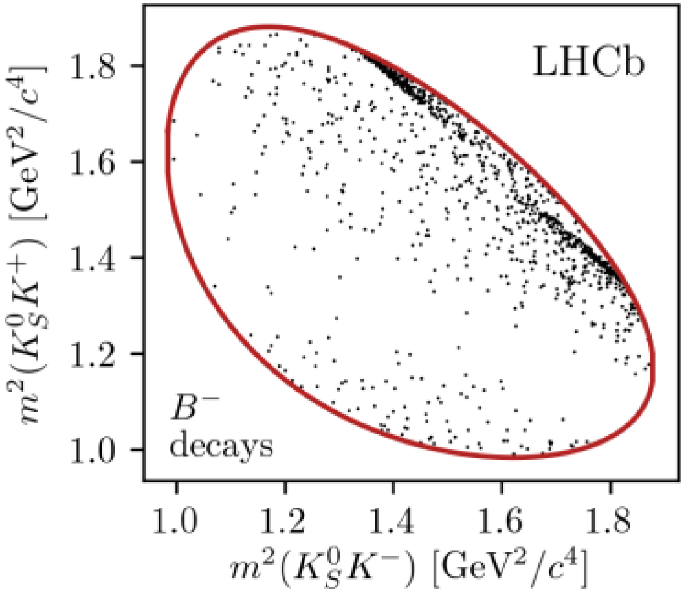

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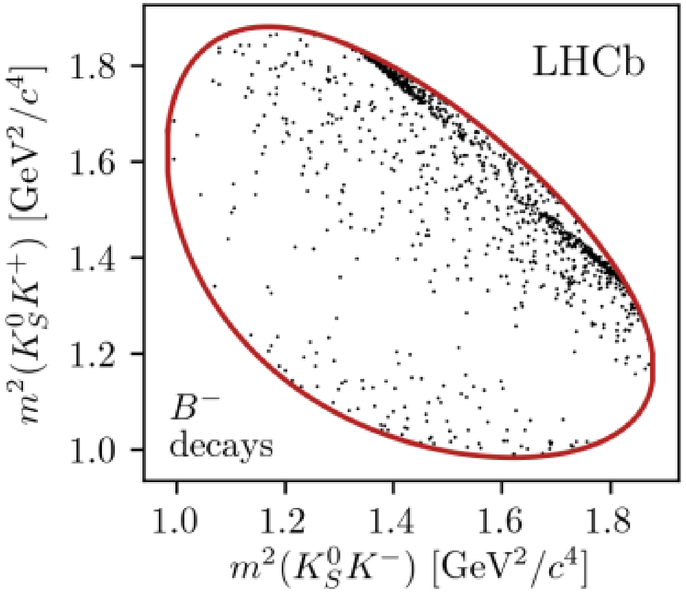

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Knowledge of D decay dynamics crucial

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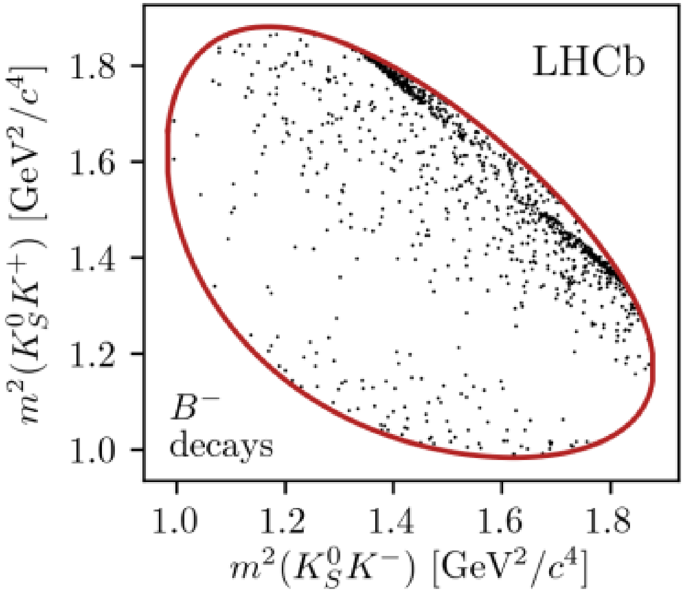

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(Strong-phase difference)

Knowledge of D decay dynamics crucial

Multibody decays proceeds through various intermediate resonant state hence large Strong-phase variations expected over Dalitz plot.

$\boldsymbol{\mathcal{BF}(D^0 \to K_S^0 K^+ K^-) = (4.45 \pm 0.19) \times 10^{-3}}$

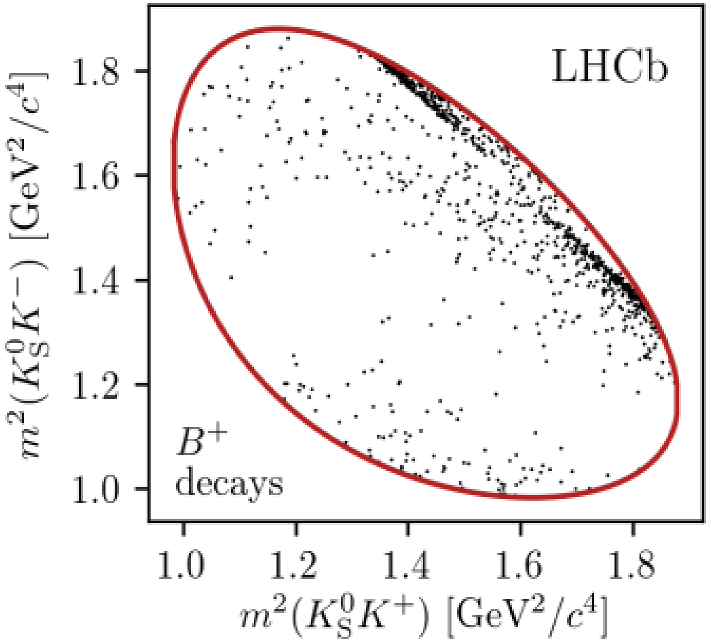
(PDG) PTEP **2020**, 083C01 (2020)

BPGGSZ method

BPGGSZ method

Model-dependent measurements

- $f_D(m_+^2, m_-^2)$ from an amplitude model for $D \to K_S^0 K^+ K^-$



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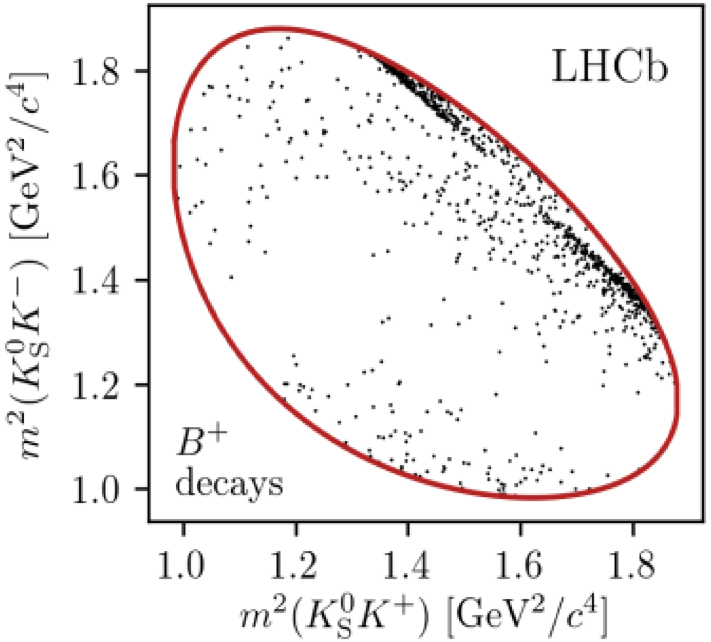
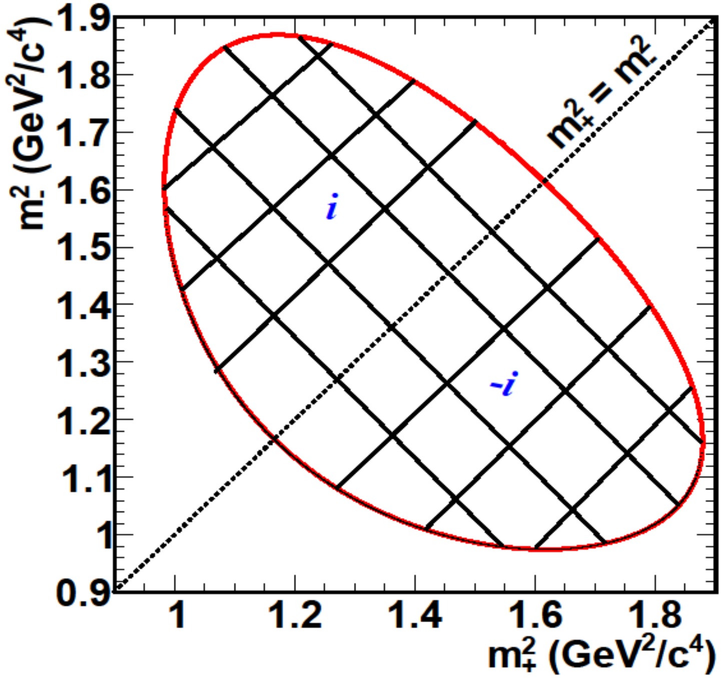


Model-independent measurements

- Binned Dalitz plot
- Requires amplitude weighted average values of $\Delta\boldsymbol{\delta_D}(\boldsymbol{m_+^2}, \boldsymbol{m_-^2})$ in each bins

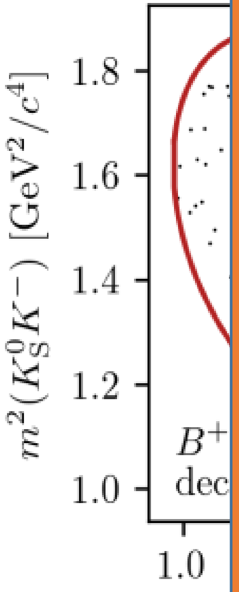


BPGGSZ method

Model-dependent measurements

Model-independent measurements

❑ $f_D(m_+^2$



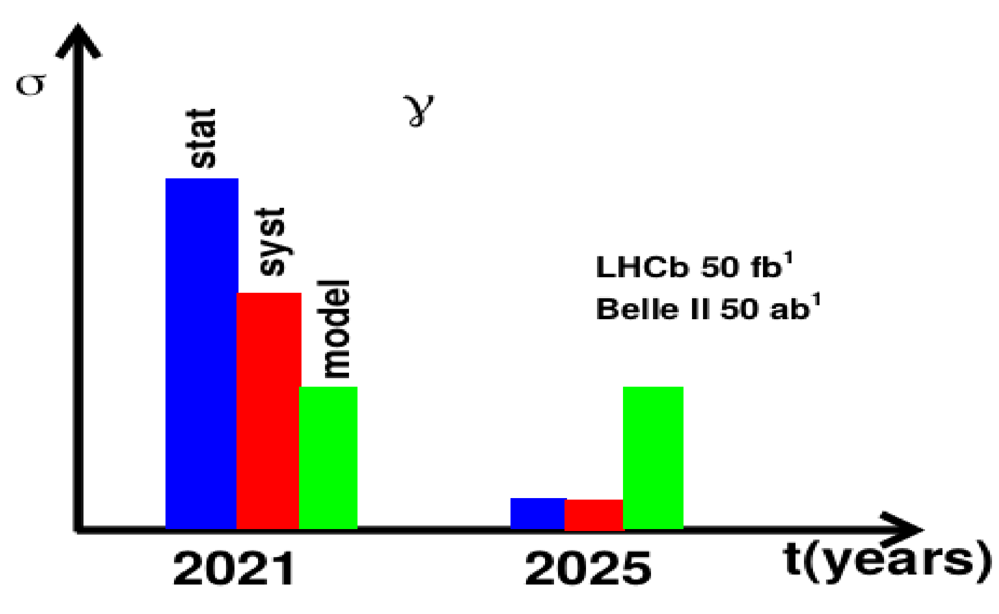
Which one?

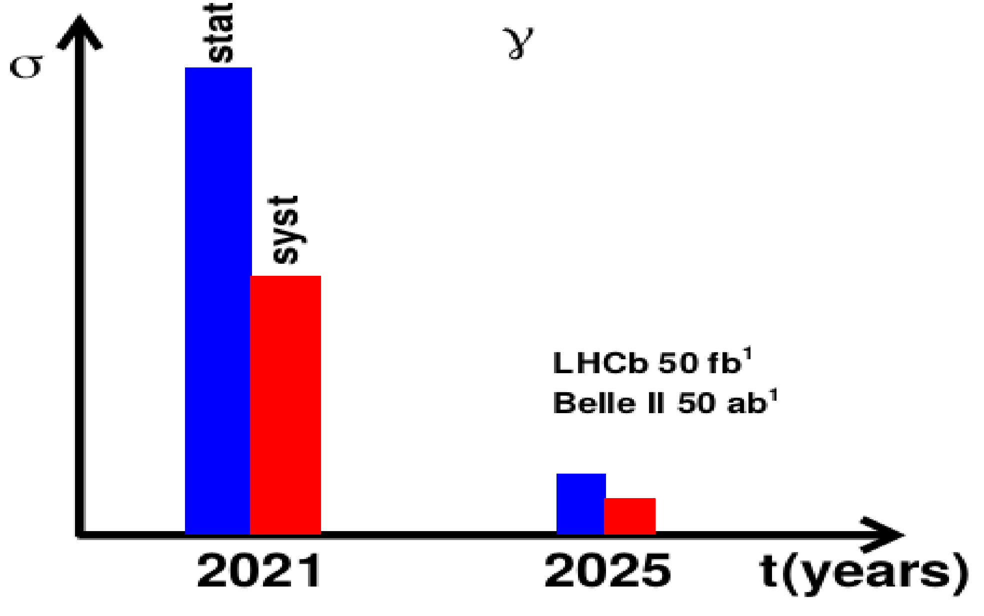
BPGGSZ method

Model-dependent measurements

Model-independent measurements

Which one?





Better statistical sensitivity

Model uncertainty of 3 – 9 degrees

Less statistical sensitivity

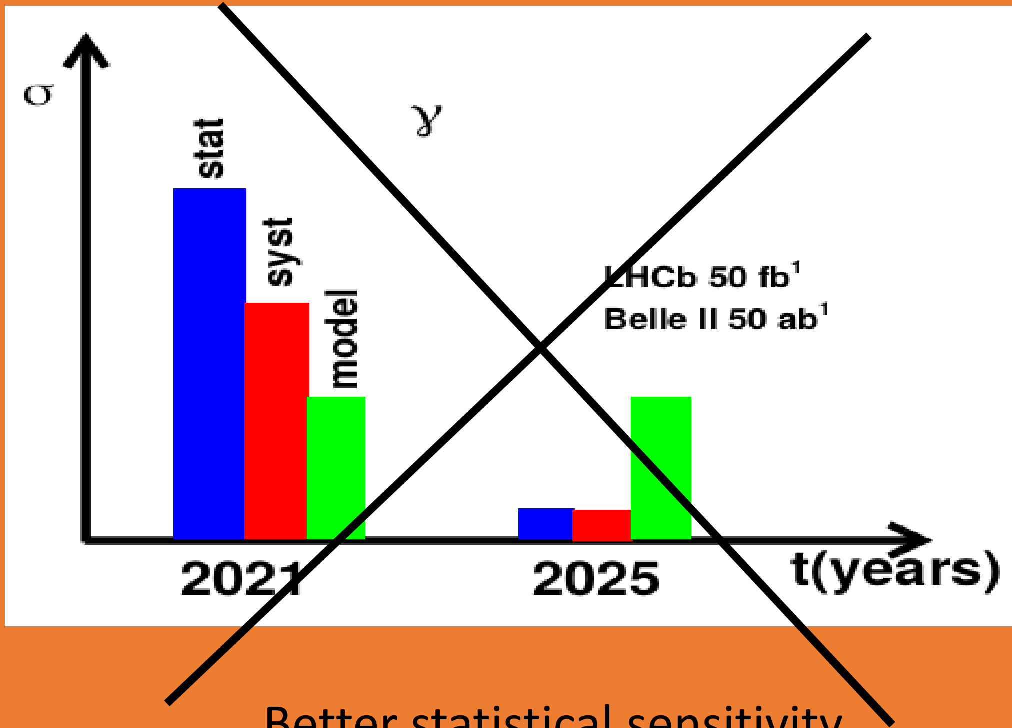
No model-dependent uncertainty

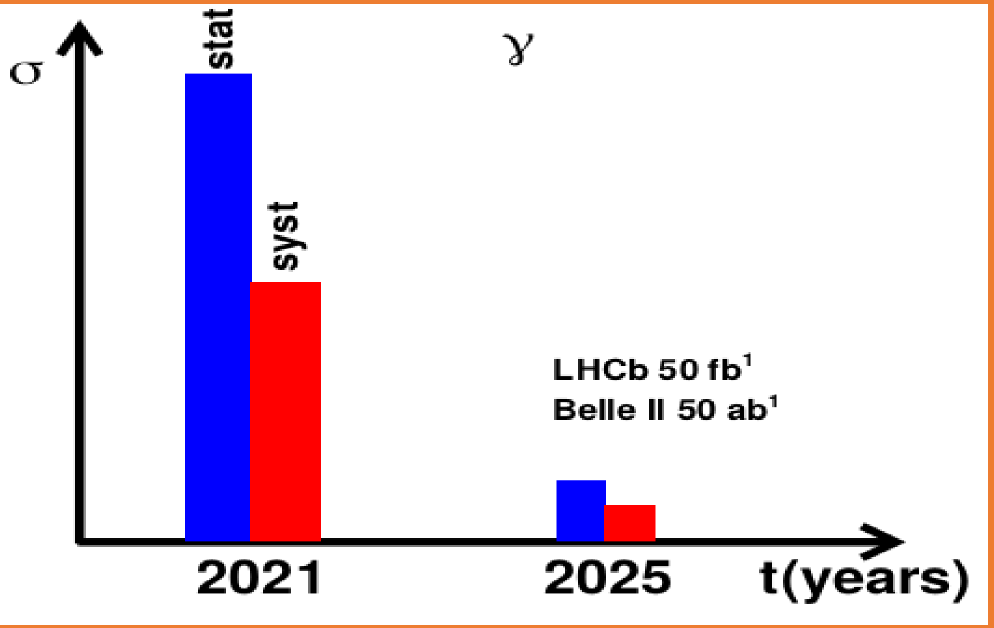
BPGGSZ method

Model-dependent measurements

Model-independent measurements

Which one?





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Model-independent measurement of γ

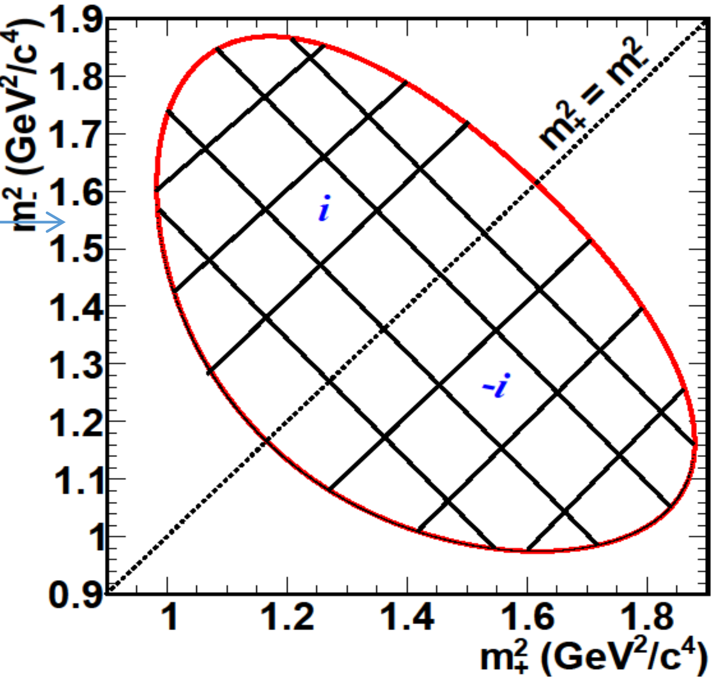
Steps:

Model-independent measurement of γ

Steps:

- Divide Dalitz plot into $2\mathcal{N}$ bins (indexed i), symmetrically around $m_+^2 = m_-^2$ line (**Note**: no assumptions on bin shapes)

PRD **68**, 054018 (2003)



Square binning scheme

Model-independent measurement of γ

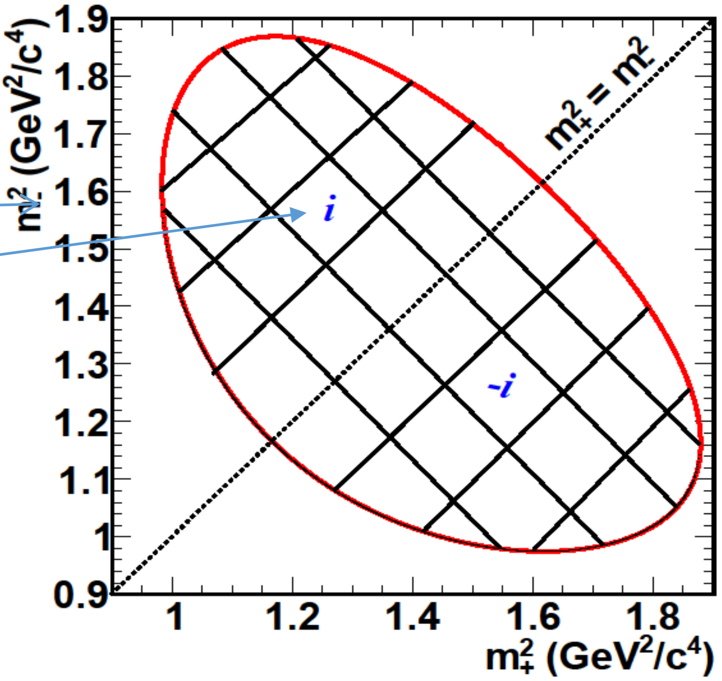
Steps:

- Divide Dalitz plot into $2\mathcal{N}$ bins (indexed i), symmetrically around $m_+^2 = m_-^2$ line (**Note**: no assumptions on bin shapes) PRD **68**, 054018 (2003)
- Yield of $B^{\pm} \to D(K_S^0 K^+ K^-)K^{\pm}$ decay in i^{th} bin

$$\boxed{N_i^{\mp} \propto \left(K_{\pm i} + r_B^2 K_{\mp i} + 2\sqrt{K_i K_{-i}}\,(x_{B\mp} c_i + y_{B\mp} s_i)\right)}$$

$x_{B\pm} = r_B \cos(\delta_B \pm \gamma)$ $\quad y_{B\pm} = r_B \sin(\delta_B \pm \gamma)$ $\quad r_B^2 = x_{B\pm}^2 + y_{B\pm}^2$

K_i: **Flavor-tagged** $D^0 \to K_S^0 K^+ K^-$ **events**



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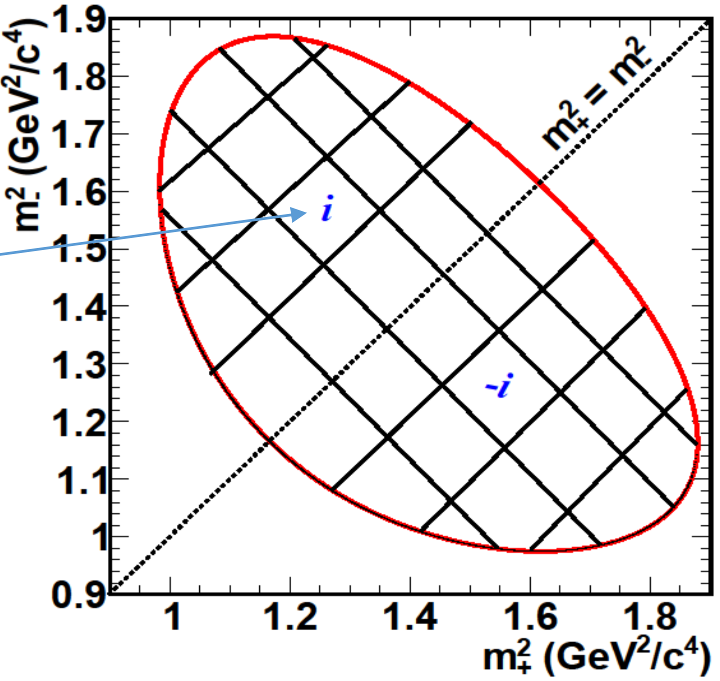

Square binning scheme

$$c_i = \frac{\int_i |f_D(m_+^2, m_-^2)||f_D(m_-^2, m_+^2)| \times \cos\left(\Delta\delta_D(m_+^2, m_-^2)\right) dm_+^2, dm_-^2}{\sqrt{F_i F_{-i}}}$$

$$s_i = \frac{\int_i |f_D(m_+^2, m_-^2)||f_D(m_-^2, m_+^2)| \times \sin\left(\Delta\delta_D(m_+^2, m_-^2)\right) dm_+^2, dm_-^2}{\sqrt{F_i F_{-i}}}$$

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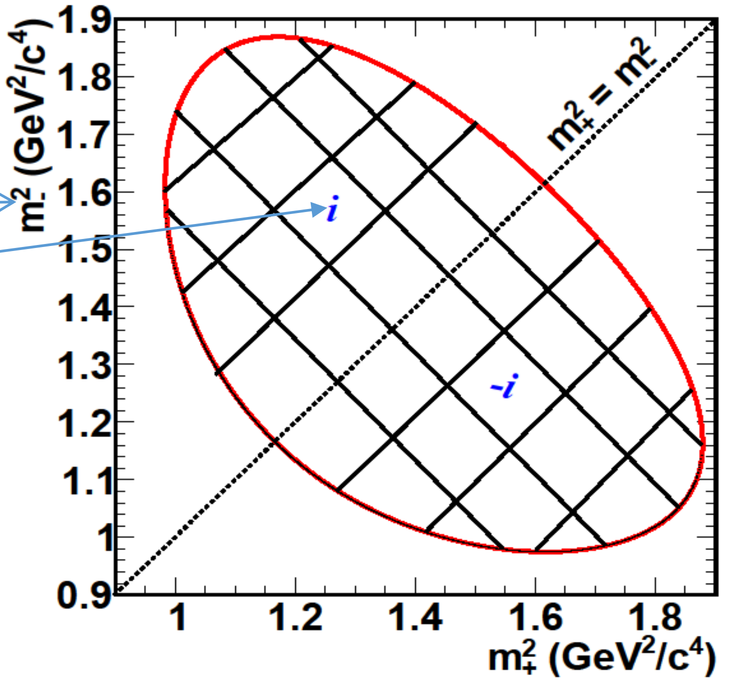

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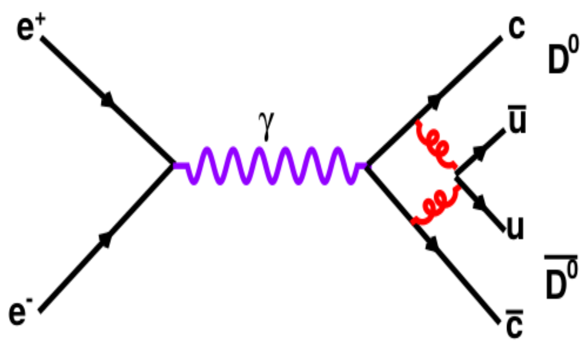
Needs to be measured first.

Determination of strong-phase difference from $D^0\overline{D^0}$ sample

c_i and s_i can be measured using $e^+e^- \rightarrow \psi(3770) \rightarrow D^0\overline{D^0}$

Data from charm factories collected at $\sqrt{s}$ = 3.773 GeV



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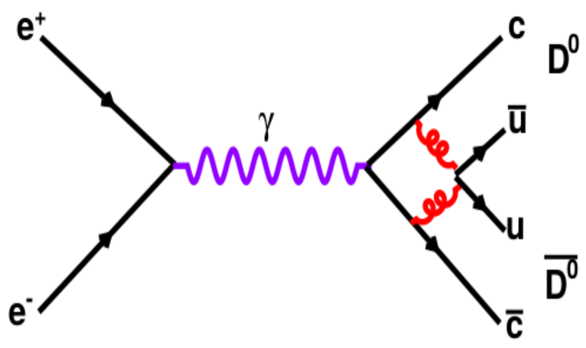

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Both D has opposite CP to each other: reconstructing one D in CP eigenstates gives the CP of other D

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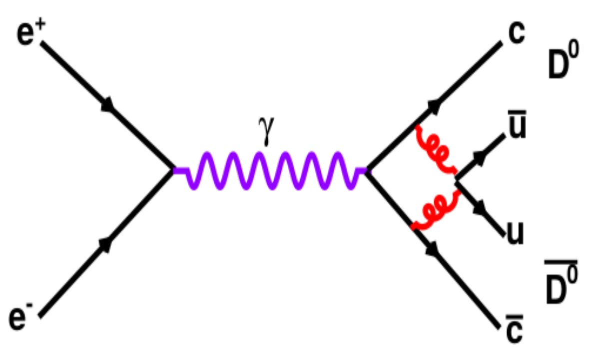

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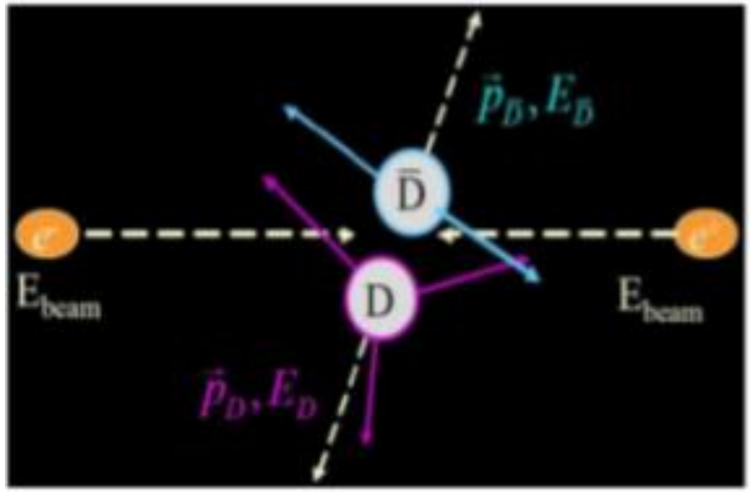

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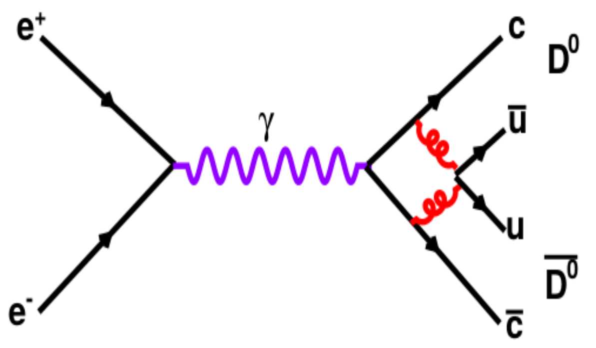

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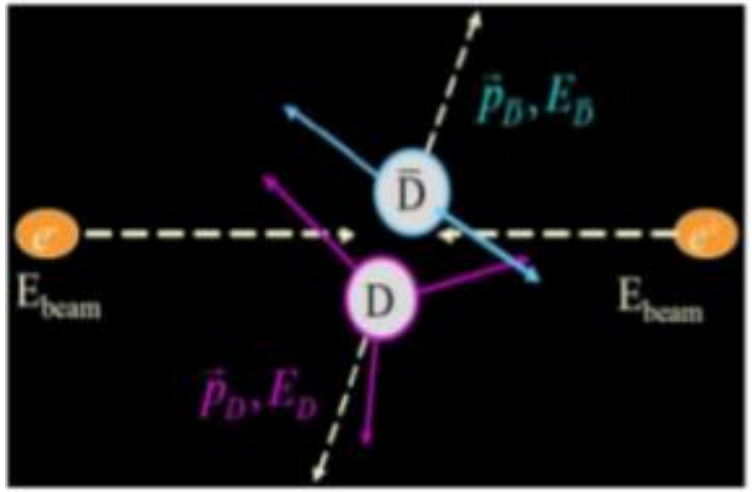
Both *D* has opposite *CP* to each other: reconstructing one *D* in *CP* eigenstates gives the *CP* of other *D*

Tagging

Singletag (ST): Only one ***D*** meson in reconstructed in an event

For eg: $D^0 \rightarrow K_S^0K^+K^-$ vs $\overline{D^0} \rightarrow$ anything

or $D^0 \rightarrow$ anything vs $\overline{D^0} \rightarrow K_S^0K^+K^-$



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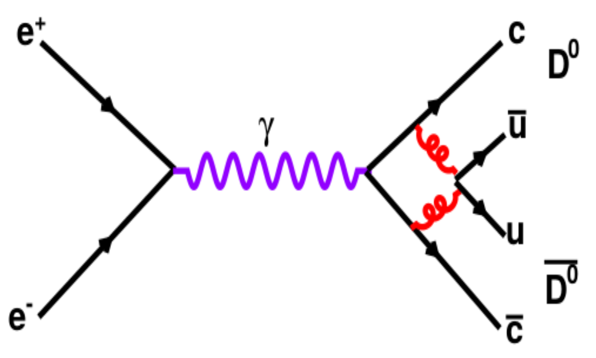

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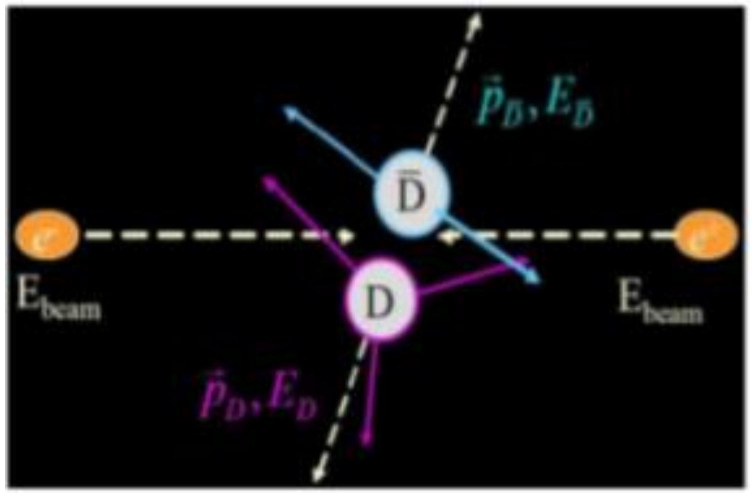

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Double tag (DT): Both ***D*** meson reconstructed in an event

For eg: $D^0 \to K_S^0K^+K^-$ vs $\overline{D^0} \to K^+K^-$

or $D^0 \to K^+K^-$ vs $\overline{D^0} \to K_S^0K^+K^-$

Flavor identification not possible

c_i can determined from *CP*-tagged $K_S^0 K^+ K^-$ events (DT $K_S^0 K^+ K^-$ vs $CP\pm$ tag modes)

F_+ = 1 (0) for pure CP+ (CP-) states

$$\boxed{\langle \boldsymbol{M_i^{\pm}} \rangle = \frac{\boldsymbol{S_\pm}}{\boldsymbol{S_f}} \left(\boldsymbol{K_i - 2c_i(2F_+ - 1)\sqrt{K_i K_{-i}} + K_{-i}}\right) \times \boldsymbol{\epsilon_{\mathrm{DT},i}}}$$

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c_i and s_i can be determined from $K_S^0K^+K^-$ vs $K_S^0h^+h^-$ $(h = K, \pi)$

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For $K_S^0K^+K^-$ vs $K_S^0\pi^+\pi^-$ c_j, s_j corresponds to strong phase parameters of $D^0 \to K_S^0\pi^+\pi^-$

PRL 124, 241802 (2020)
PRD 124, 241802 (2020)

Done !

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PRL 124, 241802 (2020)
PRD 124, 241802 (2020)

Done !

For $D^0 \to K_L^0K^+K^-$ we can measure $c_i'$and s_i'

$c_i'$and s_i' not required for γ measurements

c_i' can determined from *CP*-tagged $K_L^0K^+K^-$ events

$$\langle M_i'^{\pm}\rangle = \frac{S_\pm}{S_f}\left(K_i' + 2c_i'(2F_+ - 1)\sqrt{K_i'K_{-i}'} + K_{-i}'\right) \times \epsilon_{\mathrm{DT},i}$$

Required for improving the precision of c_i and s_i

$c_i'$and s_i' can be determined from $K_L^0K^+K^-$ vs $K_S^0h^+h^-$ $(h = K, \pi)$

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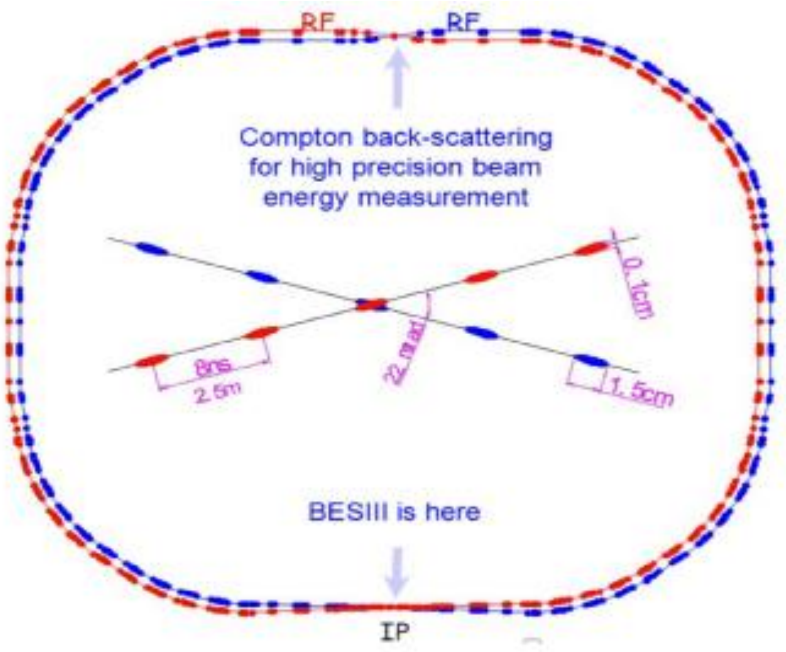
Beijing Spectrometer Experiment (BESIII)



@ IHEP, Beijing

BEPC II

- Two ring e^+e^- symmetric collider;
  circumference: 240 m
- Design $\mathcal{L}$ = 1 x 10^{33} $\mathrm{cm}^{-2}\mathrm{s}^{-1}$
- $\sqrt{s}$ = 2 – 4.6 GeV

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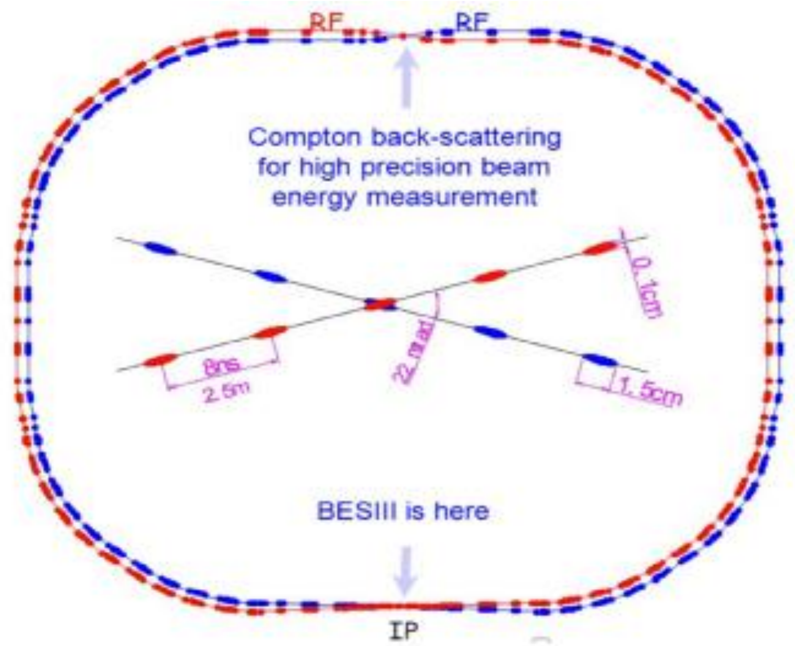

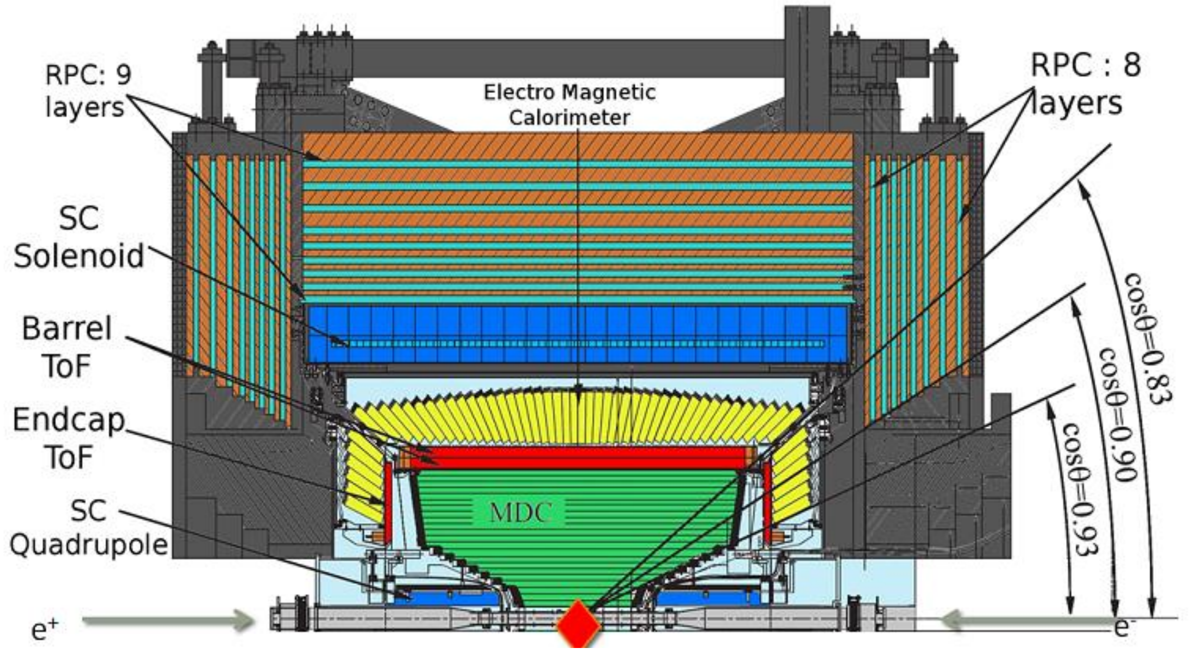

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BESIII

- Hermiticity: 93% of 4π
- MDC: $\frac{\sigma_p}{p}$ = 0.5% @ 1 GeV
- TOF: σ = 80 ps (100 ps) in barrel and end-cap regions
- ECL: $\frac{\sigma_E}{E}$ = 2.5% @ 1 GeV
- SC solenoid: 1T

"Second generation charm factory after CLEO-c" **(Since 2009)**

arXiv:hep-ex/0809.1869

Analysis strategy

Tag modes

CP and flavor modes will be reconstructed both as ST and DT

- **Signal:** $K_S^0K^+K^-$, $K_L^0K^+K^-$
- ***CP* even:** K^+K^-, $\pi^+\pi^-$, $K_S^0\pi^0\pi^0, \pi^+\pi^-\pi^0$, $K_L^0\pi^0$, $K_L^0\omega$, $K_L^0\eta$, $K_L^0\eta'$
- ***CP* odd:** $K_S^0\pi^0$, $K_S^0\omega$, $K_S^0\eta$, $K_S^0\eta'$
- **Flavor:** $K^-\pi^+$, $K^-\pi^+\pi^0$, $K^-e^+\upsilon_e$
- **Mixed *CP*:** $K_S^0\pi^+\pi^-$, $K_L^0\pi^+\pi^-$

Branching fraction (1.43 ± 0.06) %

(PDG) PTEP **2020**, 083C01 (2020)

$F_+ = 0.973 \pm 0.017$

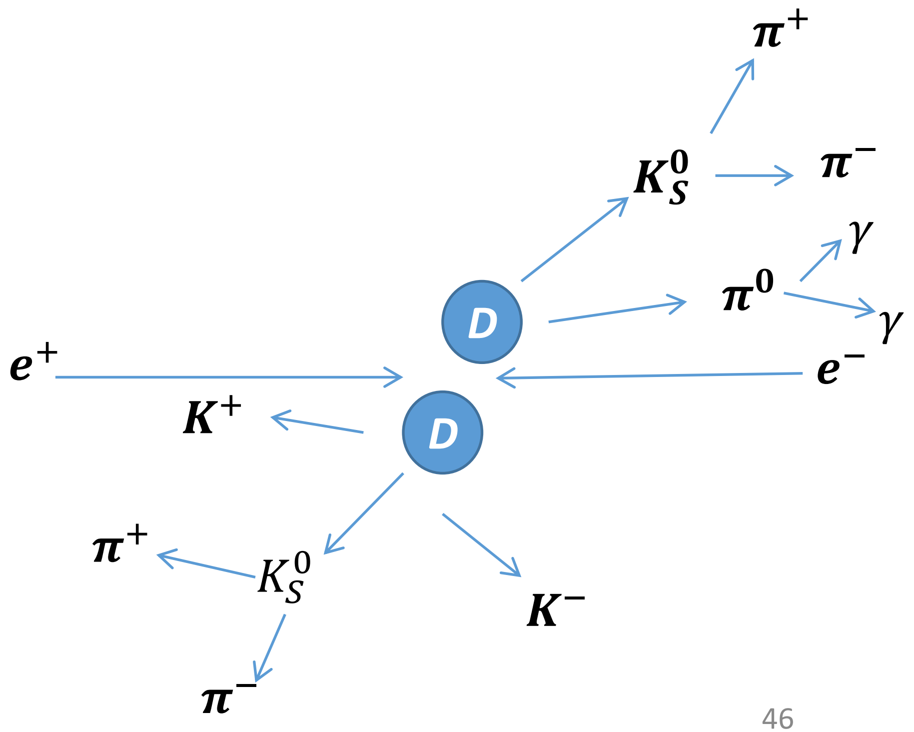
PLB **747**, 9 (2015)

Reconstruction of final state particles

$K_S^0 \to \pi^+\pi^-, \omega \to \pi^+\pi^-\pi^0, \pi^0 \to \gamma\gamma, \eta \to \gamma\gamma,\ \eta' \to \pi^+\pi^-\eta$

Particles are combined to reconstruct final states

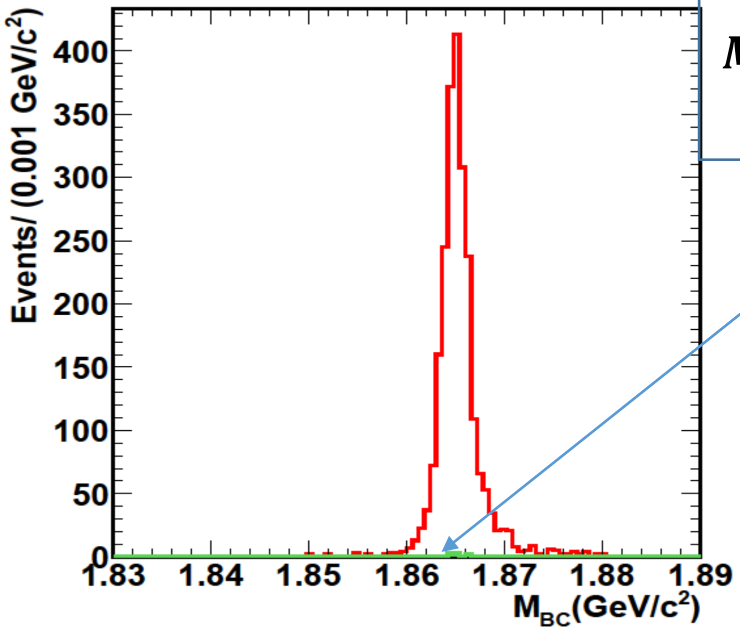
- **Fully reconstructed tags:** Full kinematic reconstruction possible**,**
- **Partially reconstructed tags:** Particles having missing particle in final states; no full reconstruction, inference using missing energy and momentum



D^0 reconstruction

Beam constrained mass:

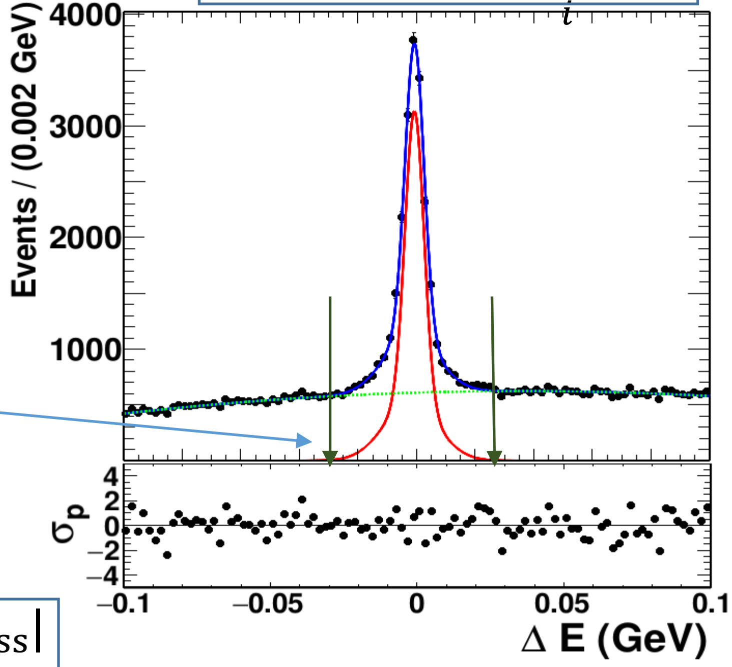
$$M_{BC} = \sqrt{E_{\text{beam}} - \sum_i |\boldsymbol{p}_i|^2}$$



Low background level/high purity samples

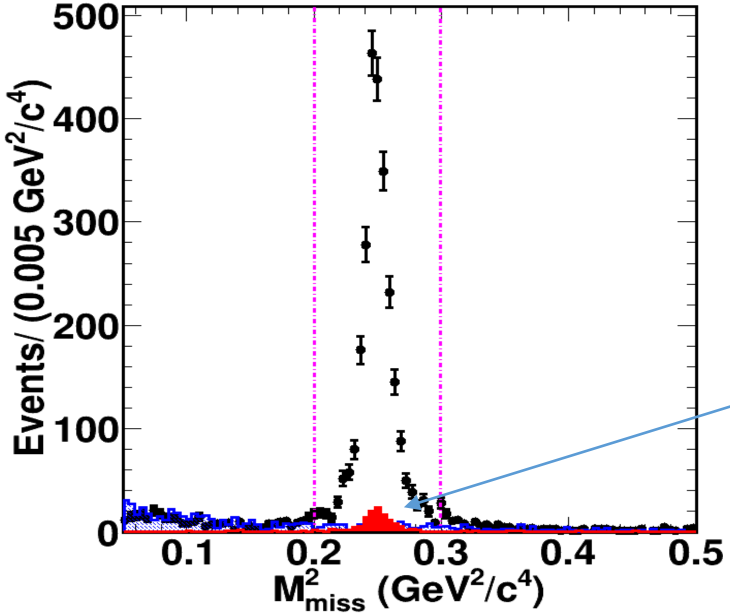
Beam energy difference:

$$\Delta \boldsymbol{E} = E_{\text{beam}} - \sum_i E_i$$

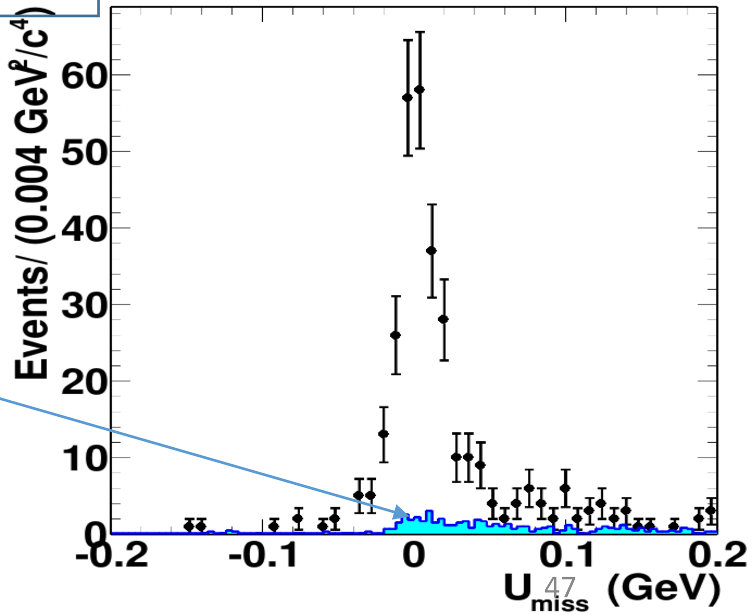


Cuts on ΔE to reduce combinatorial backgrounds

$$M^2_{\text{miss}} = E^2_{\text{miss}} - |\boldsymbol{p}_{\text{miss}}|^2$$



$$U_{\text{miss}} = E_{\text{miss}} - |p_{\text{miss}}|$$



Larger backgrounds due to partial reconstruction

Larger yields !

Data yields

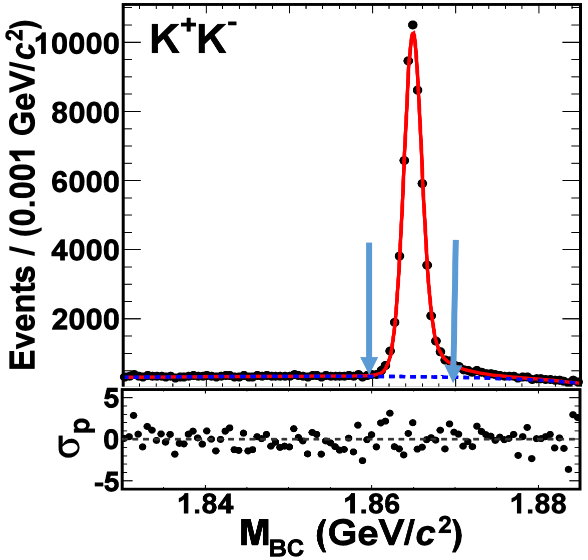
ST Yield

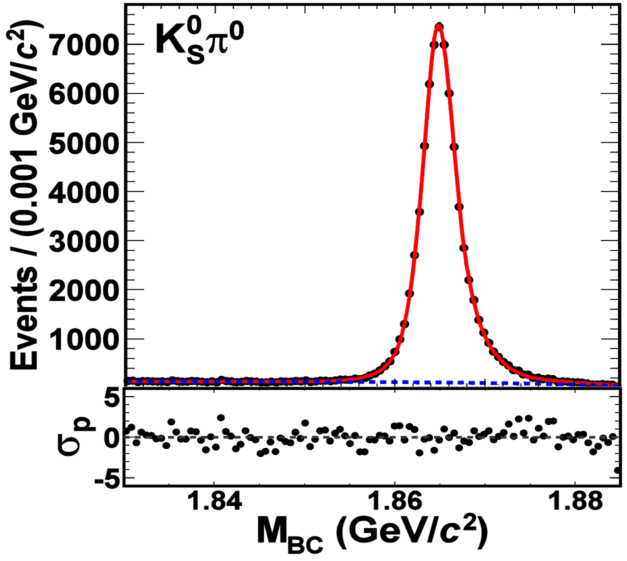
Total PDF = Signal MC shape ⊗ Gaussian + Argus

PLB **241**, 278 (1990)

N_{ST} by integrating signal PDF

Mode	ST	
	N_{ST}	$\epsilon_{\mathrm{ST}}(\%)$
Flavor-tags		
$K^-\pi^+$	524307 ± 742	63.31 ± 0.06
$K^-\pi^+\pi^0$	995683 ± 1117	31.70 ± 0.03
$K^-e^+\nu_e$	752387 ± 12795	
CP-even tags		
K^+K^-	53481 ± 247	61.02 ± 0.11
$\pi^+\pi^-$	19339 ± 163	64.52 ± 0.11
$K^0_{\mathrm{S}}\pi^0\pi^0$	19882 ± 233	14.86 ± 0.08
$\pi^+\pi^-\pi^0$	99981 ± 618	37.65 ± 0.11
$K^0_{\mathrm{L}}\pi^0$	209445 ± 14796	
$K^0_{\mathrm{L}}\eta(\gamma\gamma)$	40009 ± 2543	
$K^0_{\mathrm{L}}\omega$	207376 ± 11498	
$K^0_{\mathrm{L}}\eta'(\pi^+\pi^-\eta)$	33683 ± 1909	
CP-odd tags		
$K^0_{\mathrm{S}}\pi^0$	65072 ± 281	36.92 ± 0.11
$K^0_{\mathrm{S}}\eta(\gamma\gamma)$	9524 ± 134	32.94 ± 0.11
$K^0_{\mathrm{S}}\omega$	19262 ± 157	12.14 ± 0.07
$K^0_{\mathrm{S}}\eta'(\pi^+\pi^-\eta)$	3301 ± 62	12.46 ± 0.07





For partially reconstructed tags

$$\boxed{N_{ST} = 2 \times N_{D^0\overline{D^0}} \times \mathcal{BF}}$$

Note: $\mathcal{BF}(K_L^0X) = \mathcal{BF}(K_S^0X)$ for $(X = \eta, \eta', \omega)$

(PDG) PTEP **2020**, 083C01 (2020)

Difference expected to be around 10%

PLB **349**, 363 (1995)

DT yield

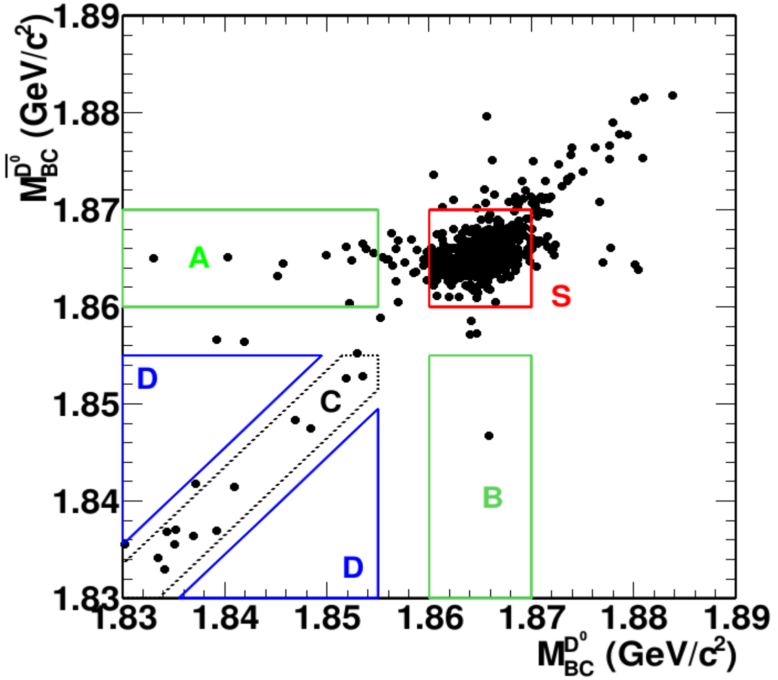
Sideband subtraction method on 2D M_{BC} plane

$$N_{\mathrm{DT}} = N_{\mathrm{S}} - \underbrace{N_{\mathrm{P}}}_{\text{Peaking background}} - \underbrace{\left(\frac{a_{\mathrm{S}}}{a_{\mathrm{D}}} N_{\mathrm{D}} + \sum_{i=\mathrm{A,B,C}} \frac{a_{\mathrm{S}}}{a_i}\left(N_i - \frac{a_{\mathrm{S}}}{a_i} a_{\mathrm{D}}\right)\right)}_{\text{Combinatorial backgrounds}}$$

N_i: Counts in i^{th} region

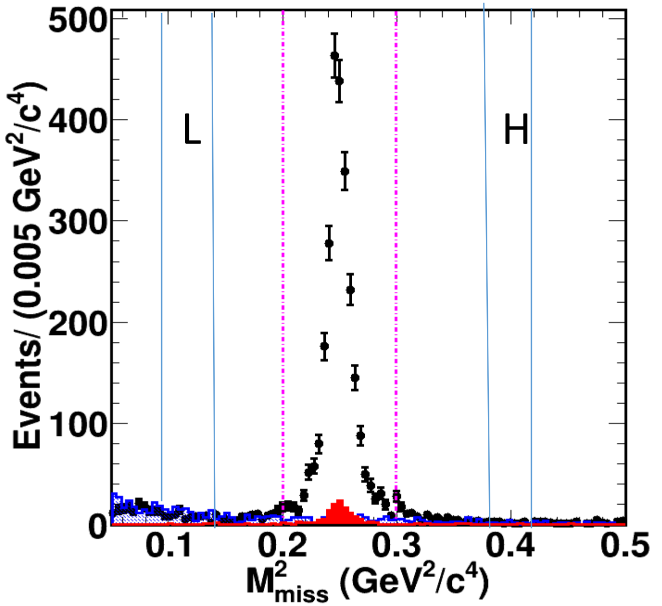
a_i: Area of i^{th}

PRD **78**, 012001 (2008)



For partially reconstructed tag sideband subtraction done on M^2_{miss} or U_{miss} distribution

$$Y_S = \frac{(N_S - N_S^P) - \delta(N_L - N_L^P) - \gamma(N_H - N_H^P)}{1 - \delta\alpha - \gamma\beta}$$

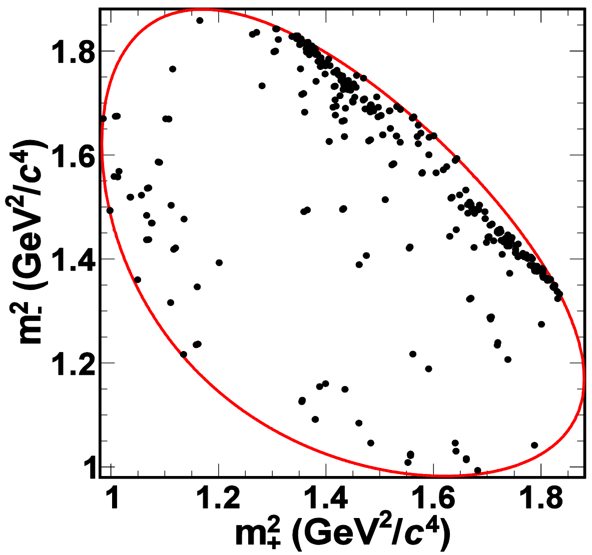


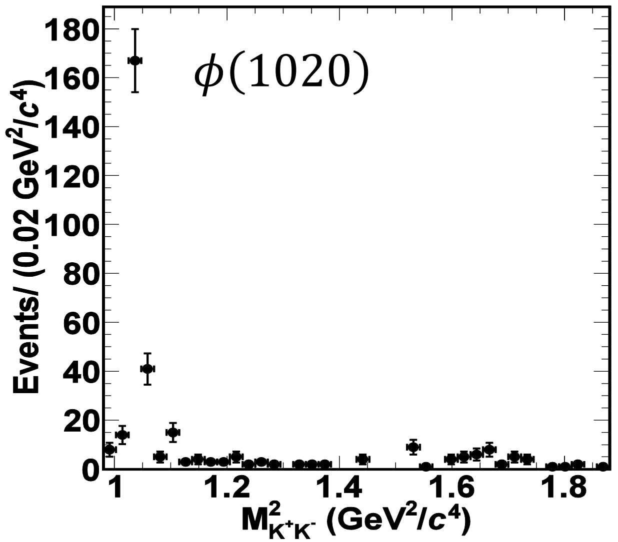
Full DT yields are not required bin-by-bin yields are only required

Sideband-subtraction done on each bin of the Dalitz plot

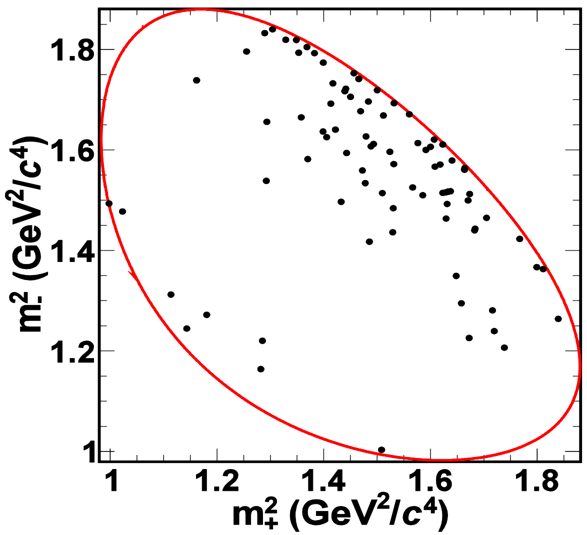
Dalitz Plot

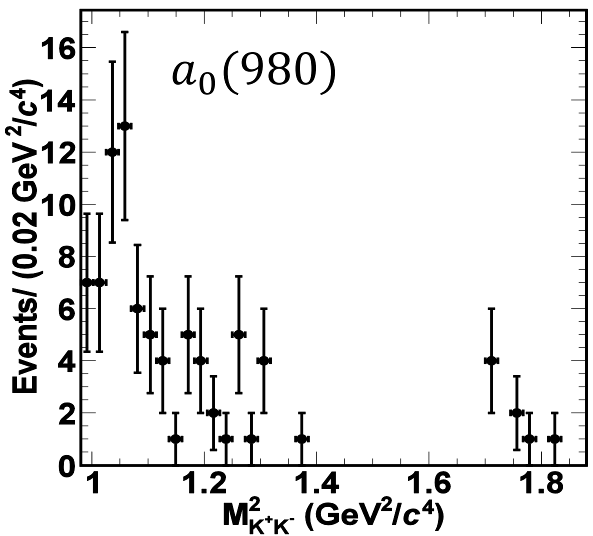
$K_S^0 K^+ K^-$ vs. *CP*-even tags





$K_S^0 K^+ K^-$ vs. *CP*-odd tags





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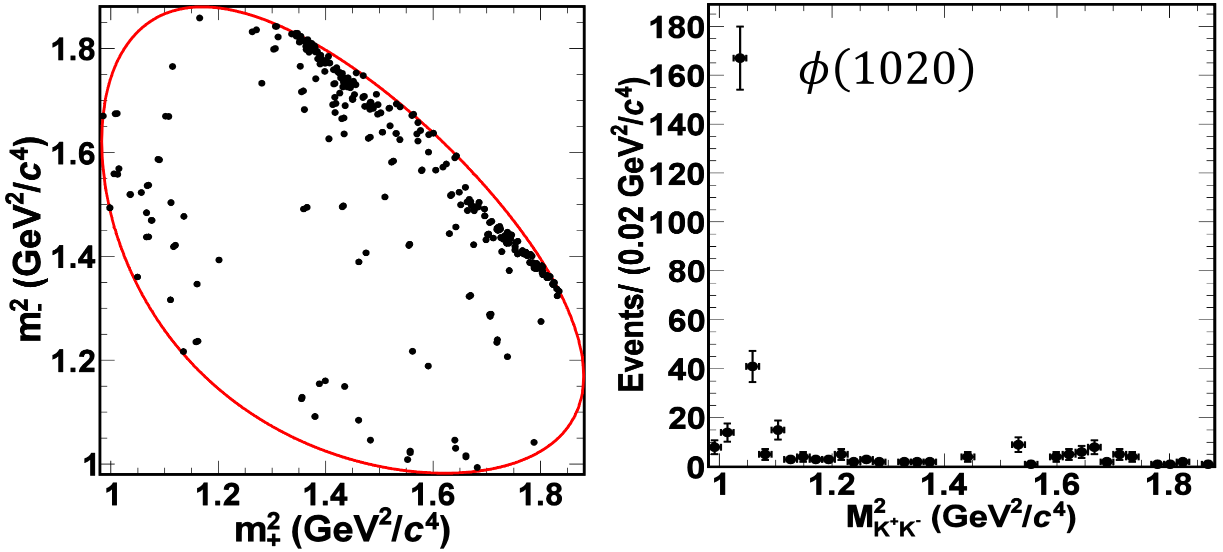
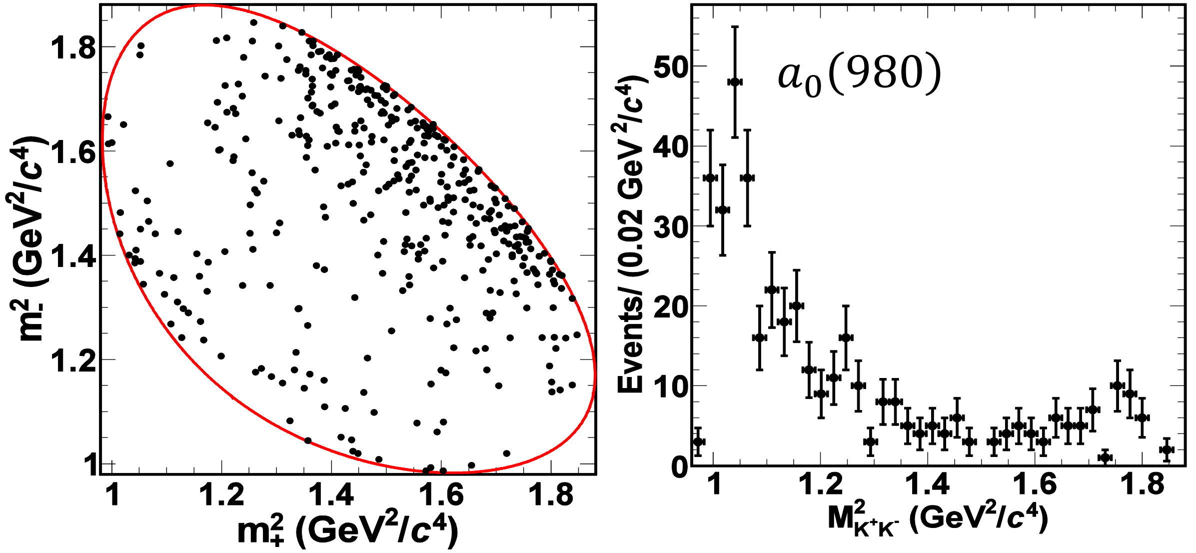


$K_L^0 K^+ K^-$ vs. *CP*-even tags



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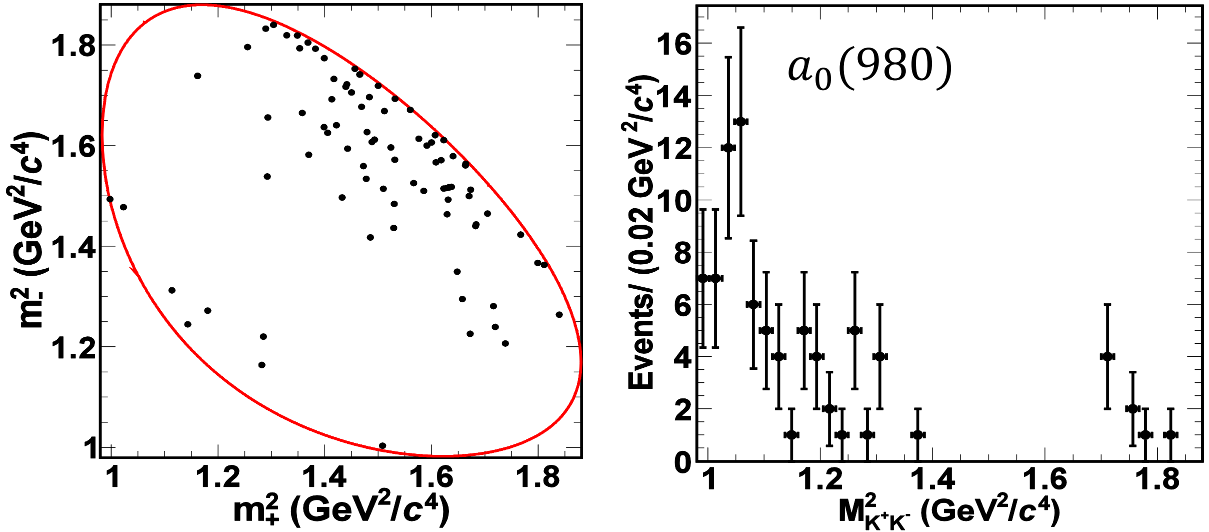

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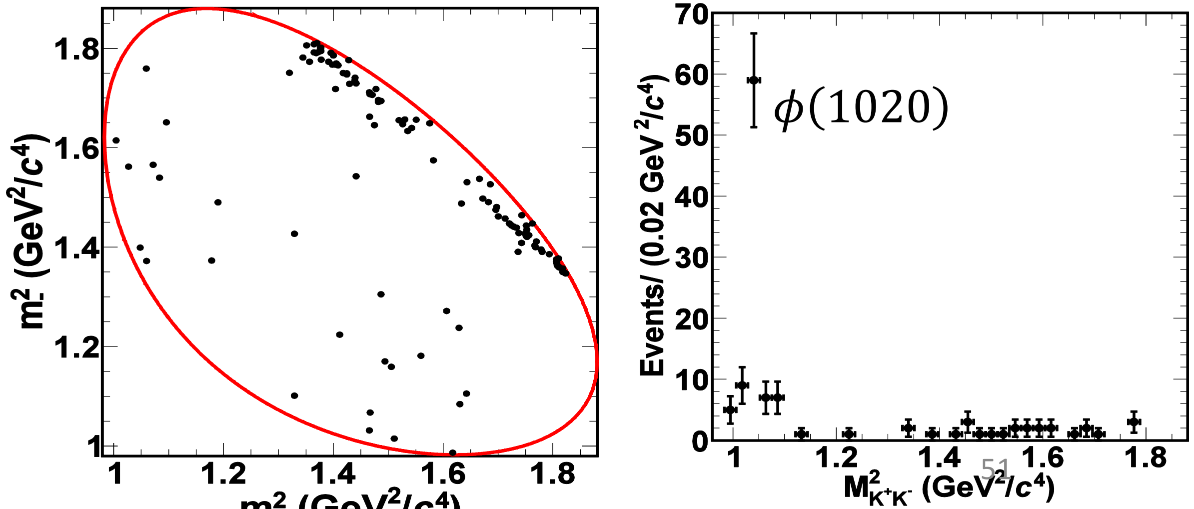
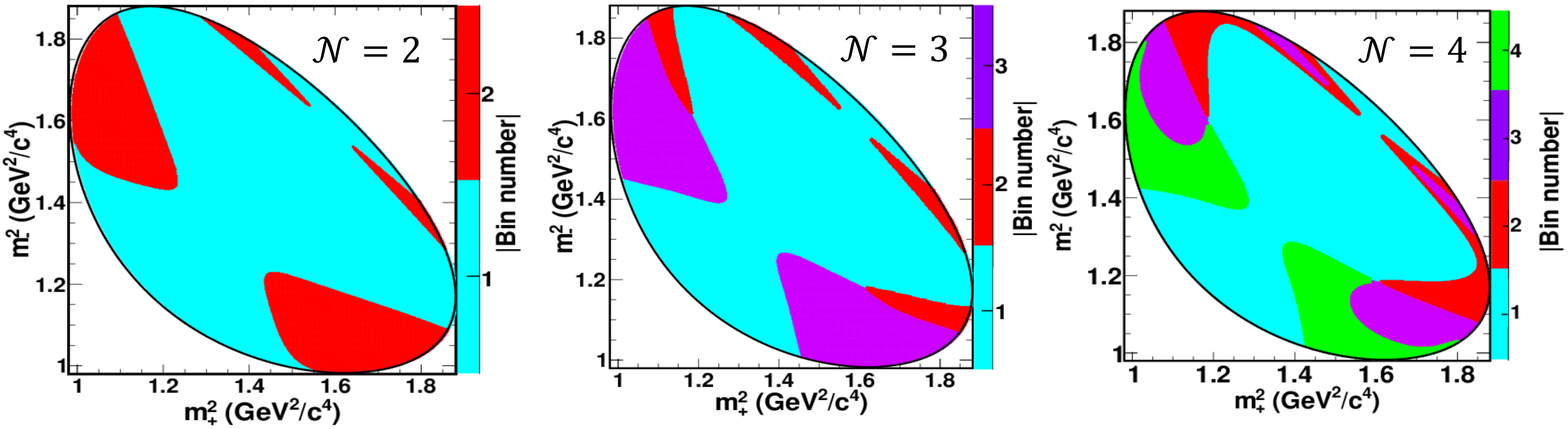


Dalitz plot binning

Equal-$\Delta\delta_D$ binning scheme



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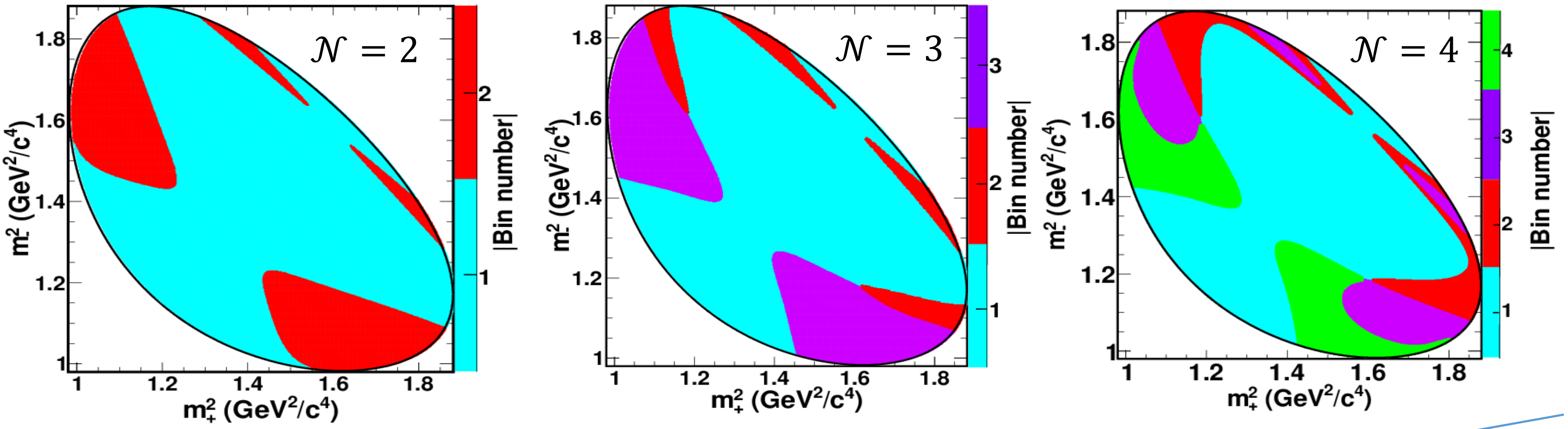

$a^0, a^{\pm}$ Flatte function

Others RBW function

BaBar Isobar model amplitude

(PRL **78**, 034023 (2008))

$$f_D(m_+^2, m_-^2) = \sum_r a_r e^{i\phi_r} f_r(m_+^2, m_-^2) + a_{NR} e^{i\phi_{NR}}$$

$K_S^0\phi(1020)$,
$K_S^0 f_0(980)$,
$K^{\pm}a^{\mp}(980)$,
$K^0 a^0(980)$
$K_S^0 f_0(1370)$,
$K^{\pm}a_0^{\mp}(1450)$,
non resonant

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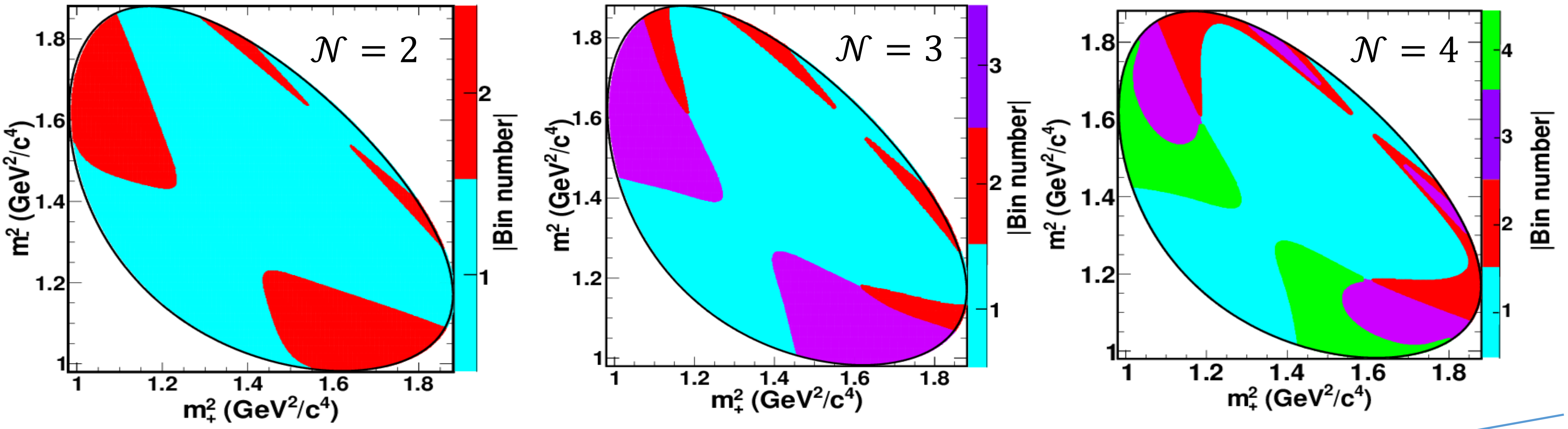

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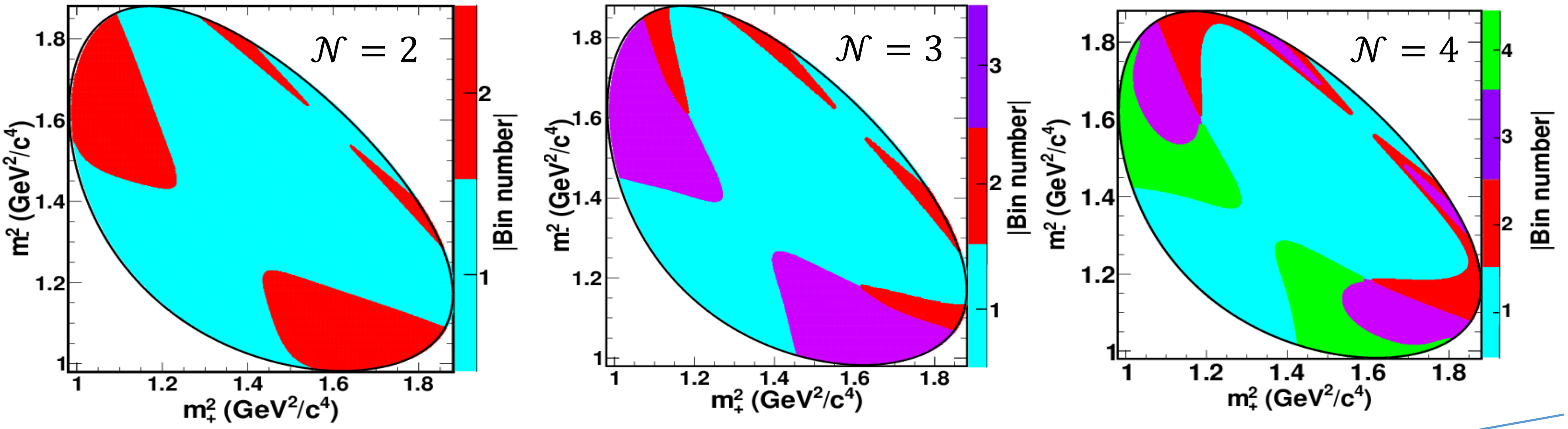

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Equal-$\Delta\delta_D$ binning scheme

Dalitz plot divided into bins satisfying condition

$$\boxed{2\pi(i - {}^3/_2)/\mathcal{N} \leq \Delta\delta_D(m_+^2, m_-^2) < 2\pi(i - {}^1/_2)/\mathcal{N}}$$ for $i = 1, 2, 3 \ldots \mathcal{N}$

$\forall\, \mathcal{N} \geq 2$

Gain in statistical sensitivity by 30 % for Equal-$\Delta\delta_D$ compared to rectangular bins !

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Compare the Q^2 values **EPJC 47, 347 (2006)**

$$\mathcal{Q}^2 = \frac{\sum_i \left[\left(\frac{1}{\sqrt{\Gamma_i}} \frac{d\Gamma_i}{dx} \right)^2 + \left(\frac{1}{\sqrt{\Gamma_i}} \frac{d\Gamma_i}{dy} \right)^2 \right]}{\int \left[\left(\frac{1}{\sqrt{|A_{B^-}|^2}} \frac{d|A_{B^-}|^2}{dx} \right)^2 + \left(\frac{1}{\sqrt{|A_{B^-}|^2}} \frac{d|A_{B^-}|^2}{dy} \right)^2 \right] dm_+^2 dm_-^2}$$

$\mathcal{N} = 2\ (0.94^{+0.16}_{-0.06})$ $\mathcal{N} = 3\ (0.87^{+0.14}_{-0.06})$ $\mathcal{N} = 4\ (0.94^{+0.21}_{-0.06})$ for the binning used in this analysis

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Effects of Model based binning on γ

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Effects of Model based binning on γ

- No model-dependent systematics
- Wrong model effects sensitivity but no bias on result

Bin yields

Tag	Bin 1	Bin 2
$K_S^0K^+K^-$ vs. K^+K^-	6	36
$K_S^0K^+K^-$ vs. $\pi^+\pi^-$	3	7
$K_S^0K^+K^-$ vs. $K_S^0\pi^0\pi^0$	2	5
$K_S^0K^+K^-$ vs. $\pi^+\pi^-\pi^0$	14	37
$K_S^0K^+K^-$ vs. $K_L^0\pi^0$	25	67
$K_S^0K^+K^-$ vs. $K_L^0\omega$	11	32
$K_S^0K^+K^-$ vs. $K_L^0\eta(\gamma\gamma)$	5	14
$K_S^0K^+K^-$ vs. $K_L^0\eta'(\pi^+\pi^-\eta)$	3	4
$K_S^0K^+K^-$ vs. $K_S^0\pi^0$	30	7
$K_S^0K^+K^-$ vs. $K_S^0\omega$	15	0
$K_S^0K^+K^-$ vs. $K_S^0\eta(\gamma\gamma)$	7	2
$K_S^0K^+K^-$ vs. $K_S^0\eta'(\pi^+\pi^-\eta)$	2	0
$K_L^0K^+K^-$ vs. K^+K^-	95	17
$K_L^0K^+K^-$ vs. $\pi^+\pi^-$	27	4
$K_L^0K^+K^-$ vs. $K_S^0\pi^0\pi^0$	36	9
$K_L^0K^+K^-$ vs. $\pi^+\pi^-\pi^0$	197	57
$K_L^0K^+K^-$ vs. $K_S^0\pi^0$	23	66
$K_L^0K^+K^-$ vs. $K_S^0\omega$	10	17
$K_L^0K^+K^-$ vs. $K_S^0\eta(\gamma\gamma)$	3	7
$K_L^0K^+K^-$ vs. $K_S^0\eta'(\pi^+\pi^-\eta)$	0	5

Bin yields

Tag	Bin 1	Bin 2
$K^0_S K^+K^-$ vs. K^+K^-	6	36
$K^0_S K^+K^-$ vs. $\pi^+\pi^-$	3	7
$K^0_S K^+K^-$ vs. $K^0_S\pi^0\pi^0$	2	5
$K^0_S K^+K^-$ vs. $\pi^+\pi^-\pi^0$	14	37
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Migration correction

Finite detector resolution leads to migration of events

Migration matrix **U** constructed using signal MC sample

$$\mathbf{U}_{ij} = \frac{m_{ji}}{\sum_{k=-\mathcal{N},k\neq 0}^{\mathcal{N}} m_{jk}}$$

m_{ji}: Events generated in bin j and reconstructed in bin i

Vector of corrected yield **N**, related to uncorrected yield $\mathbf{N}^{\mathrm{rec}}$ by

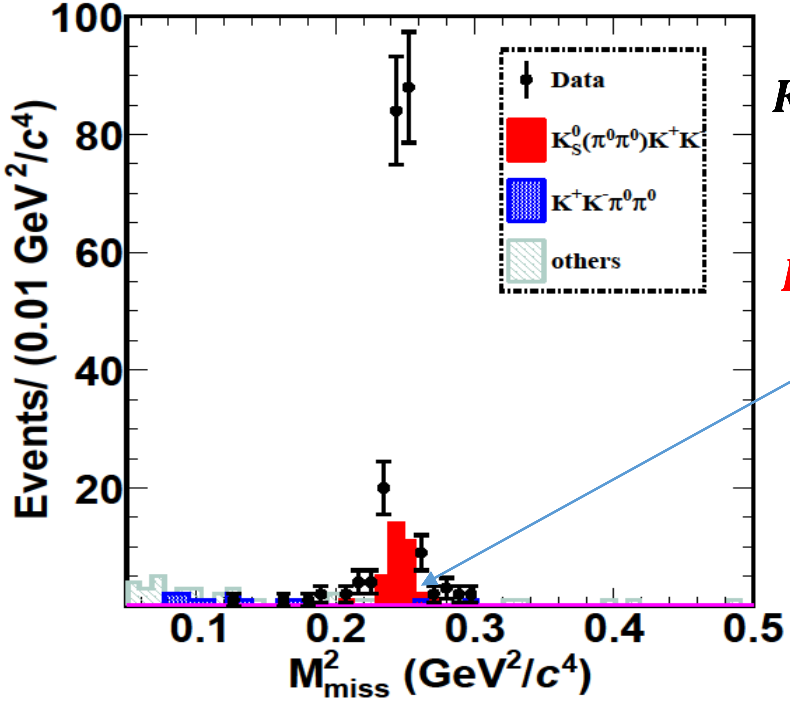
$$\mathbf{N} = \mathbf{U}^{-1}\mathbf{N}^{\mathrm{rec}}$$

i	$U_{i,1}$	$U_{i,2}$	$U_{i,3}$	$U_{i,-1}$	$U_{i,-2}$	$U_{i,-3}$
1	0.968	0.020	0.001	0.011	0.000	0.000
2	0.036	0.967	0.001	0.000	0.001	0.003
3	0.007	0.001	0.992	0.000	0.000	0.000
−1	0.010	0.000	0.000	0.972	0.018	0.000
−2	0.000	0.000	0.000	0.032	0.967	0.001
−3	0.000	0.000	0.000	0.006	0.006	0.988

Background Analysis

All $K_L^0 X$ modes contains backgrounds from $K_S^0 X$ modes

$K_S^0 \rightarrow \pi^0\pi^0$ with both π^0 not reconstructed



$K_L^0 K^+K^- vs\ CP$ even

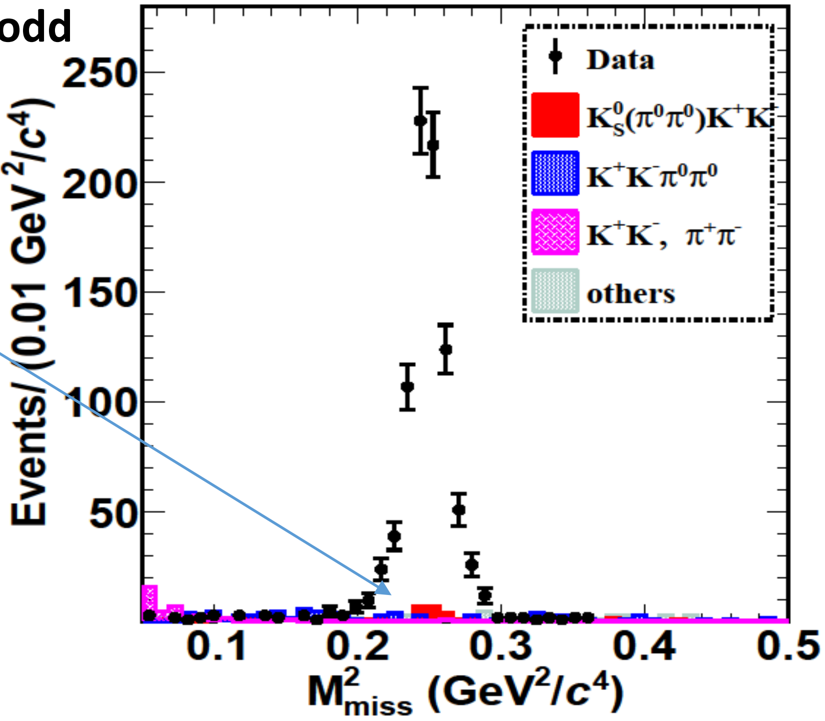
$K_S^0 K^+K^- vs\ CP$ even

$K_L^0 K^+K^- vs\ CP$ odd

$K_S^0 K^+K^- vs\ CP$ odd

$\sim 2-4\ \%$

Flat backgrounds: $K^+K^-\pi^0\pi^0 \sim 2\%$



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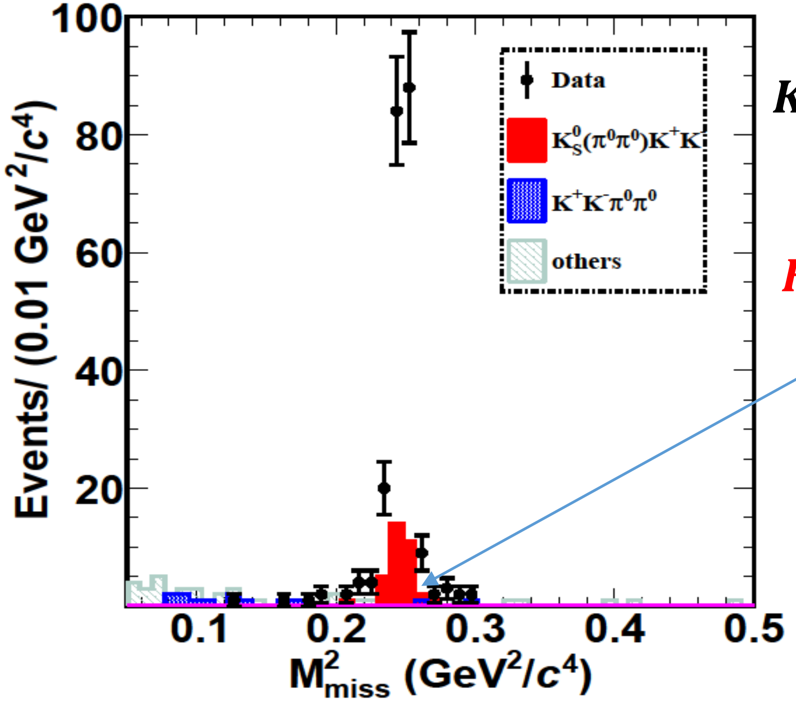
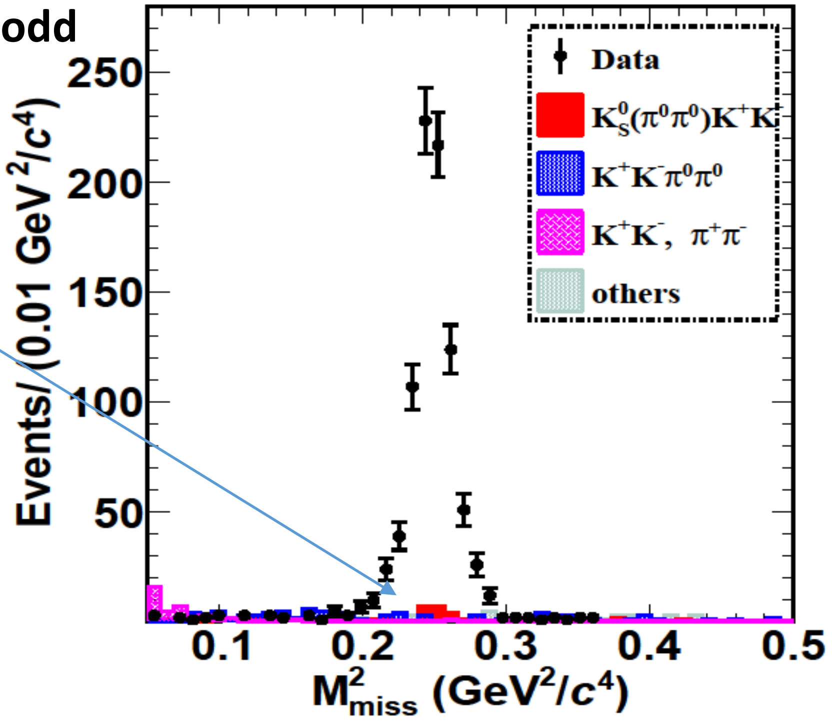


$K_L^0K^+K^- vs\ CP$ even

$K_L^0K^+K^- vs\ CP$ odd



$K_S^0K^+K^- vs\ CP$ even

$K_S^0K^+K^- vs\ CP$ odd

$\sim 2-4\ \%$

Flat backgrounds: $K^+K^-\pi^0\pi^0 \sim 2\%$

Dalitz plot distribution of backgrounds for $K_L^0K^+K^- vs\ CP \pm$ → Dalitz plot distribution of $K_S^0K^+K^- vs\ CP \mp$

Expected background yields can be calculated using c_i and s_i from CLEO-c value → $\langle N_i^{Bkg} \rangle = N_{CLEO}^{Bkg} \times \epsilon_i^{ret}$

PRD **82**, 112006 (2010)

ϵ_i^{ret}: retention efficiency

$$\epsilon_i^{ret} = \frac{\text{Number of Background events selected}}{\text{Number of Background events generated}}$$

Extraction of c_i and s_i

The uncorrected yields in related bins are combined according to the symmetry relations

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$c_i^{(\prime)}, s_i^{(\prime)}$ obtained by minimizing the negative log likelihood expression

$$
\begin{aligned}
-2\ln\mathcal{L} \quad = \quad & -2\sum_i \ln P\left(N_i^{\pm}, \langle N_i^{\pm}\rangle\right)_{K_S^0K^+K^-,\ CP} \\
& -2\sum_i \ln P\left(N_i'^{\pm}, \langle N_i'^{\pm}\rangle\right)_{K_L^0K^+K^-,\ CP} \\
& -2\sum_{i,j} \ln P\left(N_{ij}, \langle N_{ij}\rangle\right)_{K_S^0K^+K^-,\ K_S^0K^+K^-} \\
& -2\sum_{i,j} \ln P\left(N'_{ij}, \langle N'_{ij}\rangle\right)_{K_S^0K^+K^-,\ K_L^0K^+K^-} \\
& -2\sum_{i,j} \ln P\left(N_{ij}, \langle N_{ij}\rangle\right)_{K_S^0K^+K^-, K_S^0\pi^+\pi^-} \\
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& -2\sum_{i,j} \ln P\left(N'_{ij}, \langle N'_{ij}\rangle\right)_{K_L^0K^+K^-,\ K_S^0\pi^+\pi^-} \\
& +\chi^2\ .
\end{aligned}
$$

$\langle N\rangle$ is expected migration corrected yield

$$N = M + B$$
$$\langle N\rangle = \langle M\rangle + \langle B\rangle$$

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\end{aligned}
$$

$\langle N\rangle$ is expected migration corrected yield

$$N = M + B$$
$$\langle N\rangle = \langle M\rangle + \langle B\rangle$$

$$\chi^2 = \sum_i \left(\frac{c_i' - c_i - \Delta c_i}{\sigma_{\Delta c_i}}\right)^2 + \sum_i \left(\frac{s_i' - s_i - \Delta s_i}{\sigma_{\Delta s_i}}\right)^2$$

$$\Delta c_i = c'_{i,\mathrm{BaBar}} - c_{i,\mathrm{BaBar}}$$

Systematic uncertainties

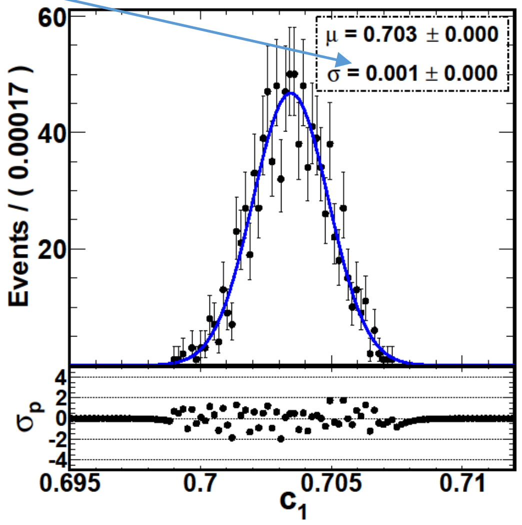
General strategy: smearing the input quantity by a Gaussian with in the measured uncertainty to produce a new value of input quantity.

Repeat fit 1000 times and build a distribution of $c_i^{(\prime)}$ and $s_i^{(\prime)}$ values; width of distribution gives systematic uncertainty

Accounting for correlation: vector of correlated variable

$$\mathbf{X} = \boldsymbol{\mu} + \boldsymbol{AZ}$$

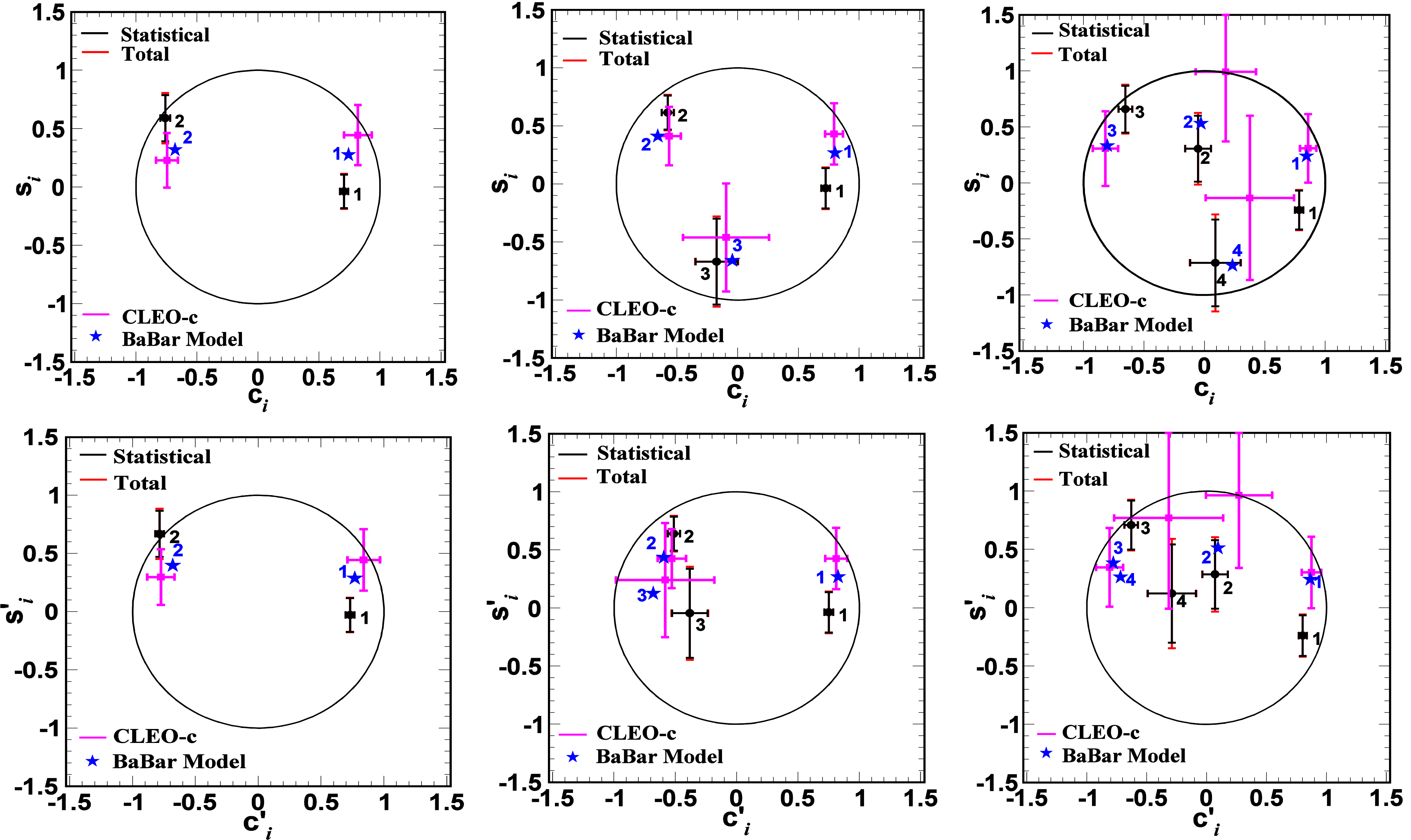
A is Cholesky decomposition of covariance matrix, **Z** is vector of unit Gaussian



Systematic	c_1	c_2	s_1	s_2	c'_1	c'_2	s'_1	s'_2
ST yield	0.002	0.003	0.000	0.000	0.002	0.001	0.000	0.000
$K_i^{(\prime)}$ statistics	0.000	0.003	0.004	0.005	0.001	0.002	0.004	0.004
$K^0\pi^+\pi^-(c_i^{(\prime)}, s_i^{(\prime)})$	0.001	0.002	0.037	0.075	0.001	0.002	0.037	0.076
$K^0\pi^+\pi^-(K_i^{(\prime)})$	0.000	0.001	0.007	0.033	0.000	0.000	0.006	0.033
$N_{D\bar{D}}$	0.000	0.000	0.001	0.001	0.000	0.000	0.001	0.001
MC statistics	0.001	0.002	0.000	0.000	0.001	0.003	0.000	0.000
Background	0.002	0.003	0.011	0.022	0.002	0.003	0.011	0.022
DCS correction	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.001
Stat	0.034	0.040	0.144	0.198	0.035	0.034	0.144	0.198
Syst total	0.003	0.007	0.039	0.085	0.003	0.006	0.039	0.086
Total	0.034	0.041	0.149	0.215	0.035	0.035	0.149	0.216

Systematic uncertainty for $\mathcal{N} = 2$ bins

Fit results



(c_i, s_i) for $\mathcal{N} = 2, 3, 4$

(c'_i, s'_i) for $\mathcal{N} = 2, 3, 4$

Impact on measurement of γ

Simulate $B^\pm \to D(K_S^0 K^+ K^-)K^\pm$ decay in each bins

$$N_i^{\mp} = \frac{a_B}{a_D}\left(\boldsymbol{K_{\pm i}} + \boldsymbol{r_B^2 K_{\mp i}} + \boldsymbol{2\sqrt{K_i K_{-i}}}\,(\boldsymbol{x_{B\mp} c_i} + \boldsymbol{y_{B\mp} s_i})\right)$$

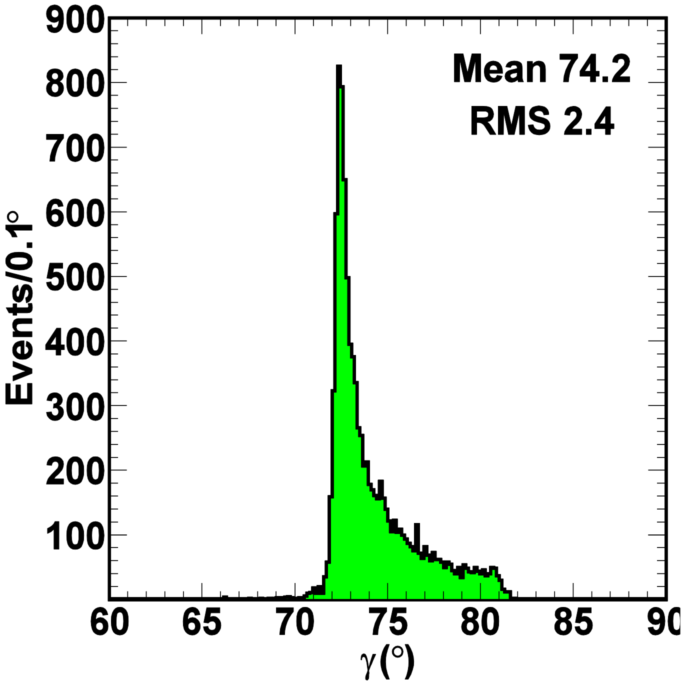
Smear values of c_i and s_i within their uncertainties accounting for correlation

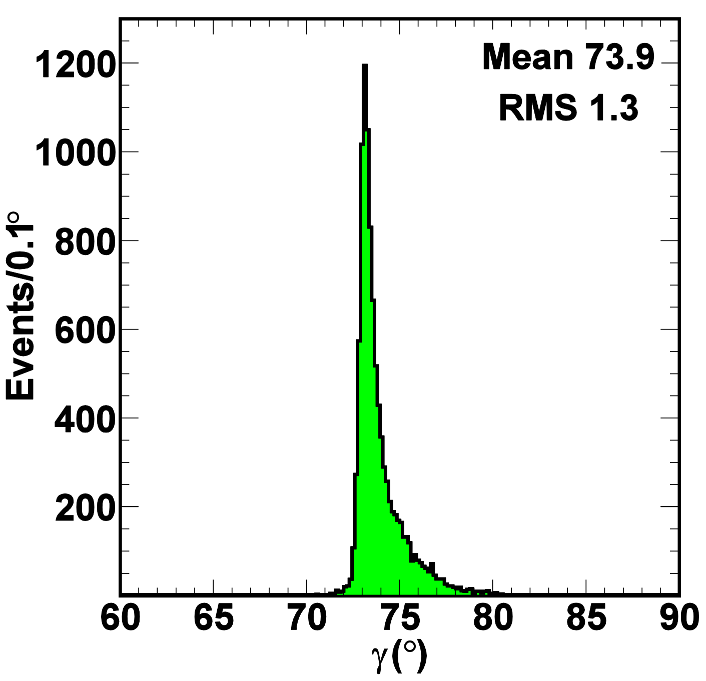
a_B set to a larger value

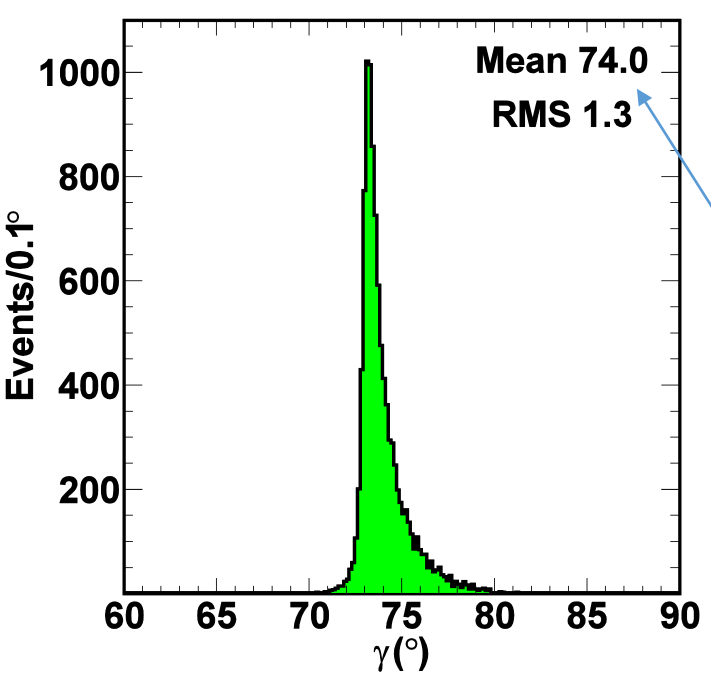
10000 ToyMC sample generated

Values of r_B, δ_B and γ set to latest world averages

http://ckmfitter.in2p3.fr







For $\mathcal{N} = 2, 3, 4$

Bias in fit values due to the values outside the region $c_i^2 + s_i^2 = 1$

Summary

- ❑ Explained the model-independent BPGGSZ method for measuring CKM angle γ
- ❑ Role of charm factories in the model independent measurement of γ explained
- ❑ c_i and s_i measured with a better precision to date
- ❑ Measurements are statistically dominated no irreducible systematic uncertainty
- ❑ Uncertainty on γ due to c_i and s_i are 2.4°, $1.3^\circ, 1.3^\circ$ for $\mathcal{N} = 2, 3, 4$ Equal-$\Delta\delta_D$ bins

Phys. Rev. D 102, 052008 (2020) [arXiv: hep-ex/2007.07959]

Measurements of γ using $B^\pm \to D(K_S^0 h^+ h^-)K^\pm$ at LHCb (2020)

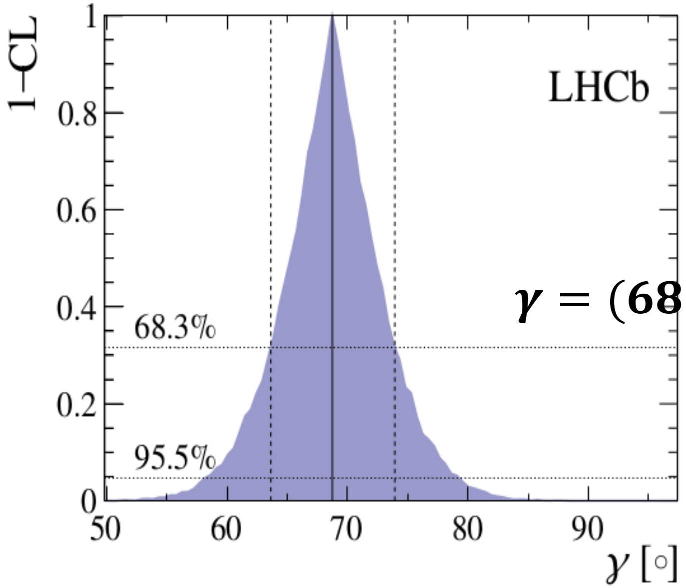
JHEP 02 , 169 (2021)

Uncertainty due to c_i and $s_i \sim 1^\circ$ $\rightarrow$ 50% improvement with BESIII value

9 fb^{-1} $P\bar{P}$ data @ 7,8 , 13 TeV

1900 $K_S^0 K^+ K^-$events

$\boldsymbol{\mathcal{N} = 2}$



$\boldsymbol{\gamma = (68.7^{+5.2}_{-5.1})^\circ}$

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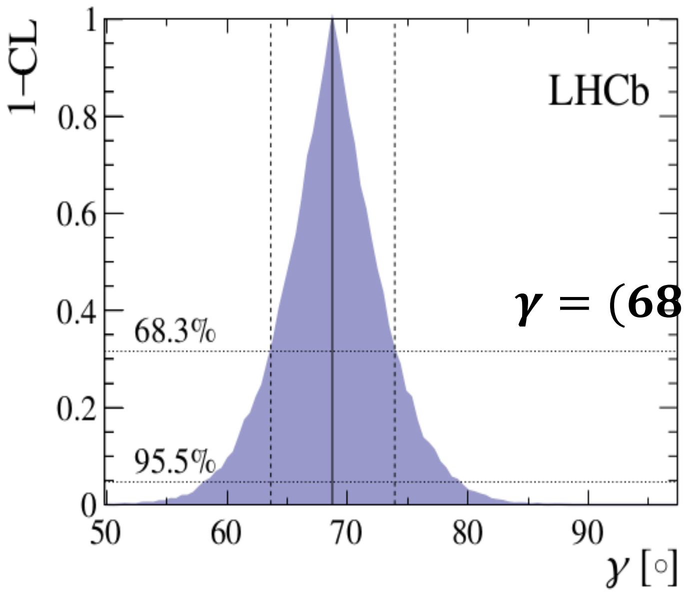
Similar analysis using Belle+BelleII data under progress!



$\gamma = (68.7^{+5.2}_{-5.1})°$

Other areas:

The value of c_i and s_i required for model-independent determination of charm mixing parameters and for searching CP violation in $D \rightarrow K_S^0 K^+ K^-$ decays

Phys.Rev.Lett 122, 231802 (2019) [arXiv:hep-ex/1903.03074]

New $D \rightarrow K_S^0 K^+ K^-$ model at BESIII

arXiv:hep-ex/2006.02800

Other activities at BESIII

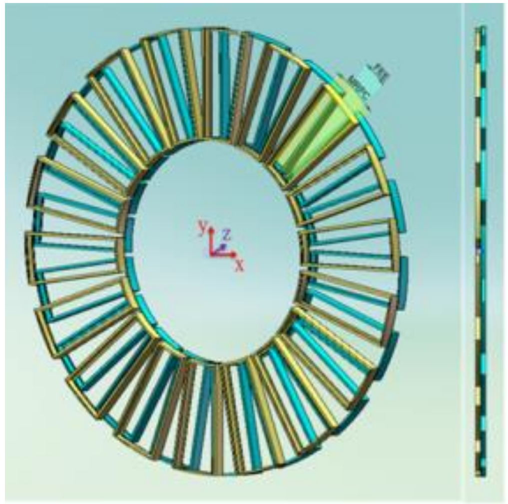
- Involved in the similar measurements of c_i and s_i for $D \rightarrow K_S^0 \pi^+ \pi^-$ with groups at Oxford University
- Quantum-correlated studies of $D \rightarrow K_S^0 \pi^+ \pi^- \pi^0$ decays (undergraduate project of Mr. Pratyush Anand , Master student at ETH Zurich)

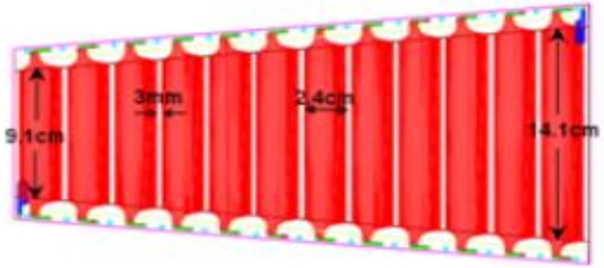
TOF Calibration work at BESIII

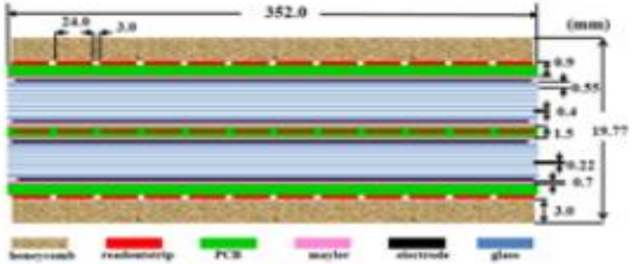
Plastic based scintillator was replaced by MRPC in 2015

ETOF has 72 modules, each modules has 8 MRPC strips

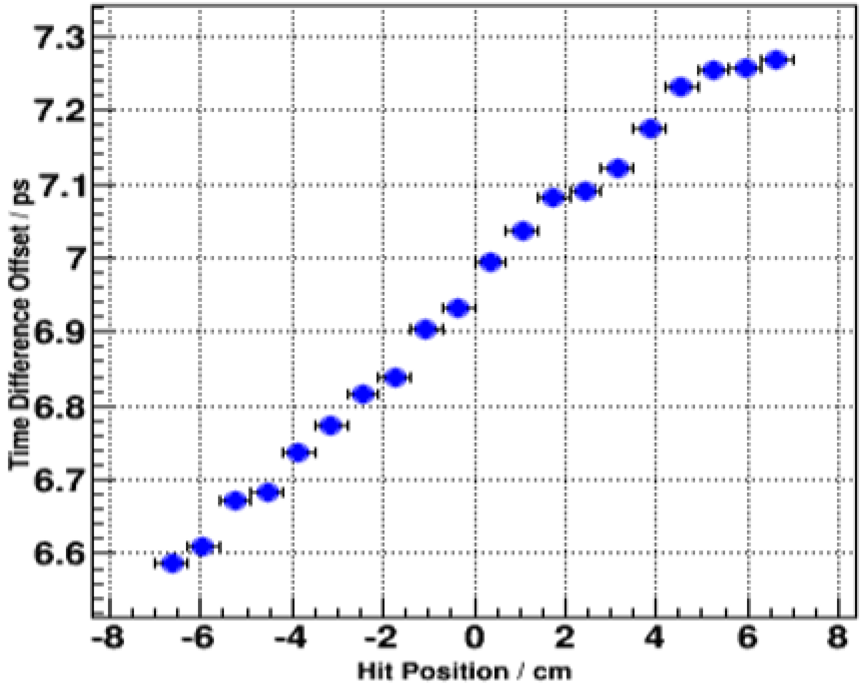
Need to calibrate each strip







$$t_{raw} = TOF + \underbrace{t_{prop}}_{\text{Position of hit}} + \underbrace{t_{cable} + t_{elec} + t_{timewalk}}_{\text{constant}}$$



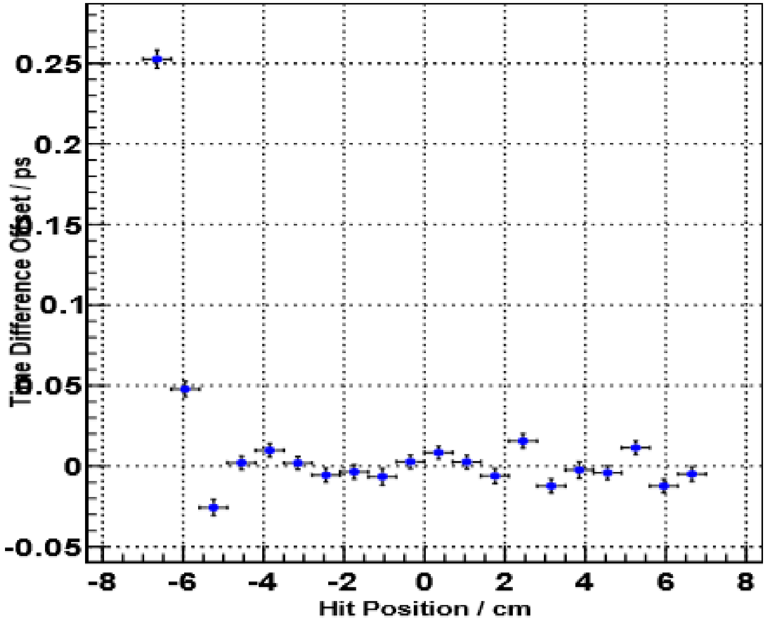
$t_{pred} - t_{raw}$

3[rd] strip 5[th] module

$$f(x) = c_1 + \frac{c_2}{x^3} + c_3 x + c_4 x^2 + c_5 x^3 + c_6 x^4$$

Not appropriate needs to be improved

Nucl. Instrum and Method in Phys. A 953, 163053 (2020)



Hvala

Fit results: First uncertainty is statistical and second uncertainty is systematic

$\mathcal{N}$	i	c_i	s_i	c_i'	s_i'
2	1	$0.704 \pm 0.034 \pm 0.003$	$-0.038 \pm 0.144 \pm 0.039$	$0.730 \pm 0.035 \pm 0.003$	$-0.028 \pm 0.144 \pm 0.039$
	2	$-0.760 \pm 0.040 \pm 0.007$	$0.590 \pm 0.198 \pm 0.085$	$-0.785 \pm 0.034 \pm 0.006$	$0.669 \pm 0.198 \pm 0.086$
3	1	$0.724 \pm 0.035 \pm 0.003$	$-0.037 \pm 0.174 \pm 0.049$	$0.751 \pm 0.036 \pm 0.003$	$-0.037 \pm 0.174 \pm 0.049$
	2	$-0.576 \pm 0.050 \pm 0.009$	$0.616 \pm 0.146 \pm 0.047$	$-0.512 \pm 0.050 \pm 0.009$	$0.640 \pm 0.146 \pm 0.047$
	3	$-0.174 \pm 0.173 \pm 0.040$	$-0.669 \pm 0.370 \pm 0.119$	$-0.382 \pm 0.145 \pm 0.040$	$0.045 \pm 0.384 \pm 0.116$
4	1	$0.783 \pm 0.034 \pm 0.003$	$-0.242 \pm 0.173 \pm 0.051$	$0.802 \pm 0.034 \pm 0.003$	$-0.239 \pm 0.174 \pm 0.051$
	2	$-0.053 \pm 0.106 \pm 0.017$	$0.306 \pm 0.294 \pm 0.125$	$0.070 \pm 0.106 \pm 0.017$	$0.286 \pm 0.294 \pm 0.124$
	3	$-0.654 \pm 0.057 \pm 0.011$	$0.659 \pm 0.210 \pm 0.059$	$-0.630 \pm 0.056 \pm 0.011$	$0.709 \pm 0.210 \pm 0.059$
	4	$0.090 \pm 0.208 \pm 0.041$	$-0.713 \pm 0.387 \pm 0.195$	$-0.290 \pm 0.201 \pm 0.036$	$0.122 \pm 0.422 \pm 0.206$

Average of CLEO and BESIII results

CLEO and BESIII results are compatible: Weighted average will be equally good

Just add a term to fit $\chi^2_{\mathrm{avg}} = (\mathbf{P} - \mathbf{P}^{\mathrm{CLEO}})^T V^{-1} (\mathbf{P} - \mathbf{P}^{\mathrm{CLEO}})$

Average of CLEO and BESIII results, uncertainties statistical and systematic added in quadratures

$\mathcal{N}$	i	c_i	s_i	c_i'	s_i'
2	1	0.713 ± 0.032	0.107 ± 0.132	0.737 ± 0.032	0.116 ± 0.132
	2	-0.758 ± 0.037	0.394 ± 0.173	-0.782 ± 0.033	0.473 ± 0.174
3	1	0.738 ± 0.030	0.112 ± 0.102	0.765 ± 0.030	0.111 ± 0.102
	2	-0.573 ± 0.044	0.550 ± 0.113	-0.503 ± 0.044	0.574 ± 0.113
	3	-0.129 ± 0.155	-0.619 ± 0.317	-0.412 ± 0.138	0.089 ± 0.327
4	1	0.796 ± 0.030	-0.082 ± 0.173	0.817 ± 0.030	-0.080 ± 0.173
	2	-0.018 ± 0.099	0.393 ± 0.262	0.105 ± 0.098	0.375 ± 0.261
	3	-0.691 ± 0.048	0.551 ± 0.200	-0.657 ± 0.048	0.601 ± 0.200
	4	0.183 ± 0.182	-0.646 ± 0.415	-0.321 ± 0.185	0.218 ± 0.438

Event pre-selections

Detector described using cylindrical coordinates with IP as the origin

Charged Particle
- Polar angle: $|\cos(\theta)| < 0.93$
- Radial distance: $|V_{xy}| < 1$ cm
- Longitudinal distance: $|V_Z| < 10$ cm

Neutral showers
- $0 <$ time < 700 ns
- Energy: > 0.025 GeV (> 0.050 GeV) for barrel (end-cap)
- Distance from MDC track exit $> 10\sigma$

Particle identification
- $\pi^{\pm}$: $\mathcal{L}_\pi > \mathcal{L}_K$
- $K^{\pm}$: $\mathcal{L}_K > \mathcal{L}_\pi$
- $e^{\pm}$: $\frac{\mathcal{L}_e}{\mathcal{L}_\pi+\mathcal{L}_K+\mathcal{L}_e} > 0.8$

$\pi^0 \to \gamma\gamma$ ($\eta \to \gamma\gamma$) reconstruction
- Kinematic fit: $\chi^2 < 20$
- $0.110 < M(\gamma\gamma) < 0.165$ GeV ($0.480 < M(\gamma\gamma) < 0.580$ GeV)
- No. of γ in end-cap > 1

K_S^0 reconstruction
- Kinematic fit: $\chi^2 < 100$
- $0.487 < M(\gamma\gamma) < 0.511$ GeV
- Flight significance: $L/\sigma_L > 2$
- No track quality requirement

$\omega \to \pi^+\pi^-\pi^0$ and $\eta' \to \pi^+\pi^-\eta$
- $0.760 < M(\pi^+\pi^-\pi^0) < 0.805$ (GeV)
- $0.938 < M(\pi^+\pi^-\eta) < 0.978$ GeV

Particles are combined to reconstruct final states
- ❑ **Fully reconstructed tags:** Full kinematic reconstruction possible,
- ❑ **Partially reconstructed tags:** Particles having missing particle in final states; no full reconstruction, inference using missing energy and momentum

Cabbibo-Kobayashi-Maskawa (CKM) matrix

Mixing between weak eigenstates and flavor eigenstates in three generations.

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$V_{CKM}V_{CKM}^{\dagger}$ = I = $V_{CKM}^{\dagger}V_{CKM}$

Can be parameterized using 3 angles and 1 phase (rephasing quark fields)

Various parameterizations (**Chau and Keung parameterization**)

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}s_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$C_{ij}(S_{ij}) = \cos\theta_{ij}(\sin\theta_{ij})$

δ is the irreducible phase

Wolfenstein parameterization

$$V_{\mathrm{CKM}} \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$s_{12} \equiv \lambda$

$s_{23} \equiv A\lambda^2$

$s_{13}e^{-i\delta} \equiv A\lambda^3(\rho - i\eta)$

$\lambda = 0.22$