DARK MATTER – lectures at University of Ljubljana

↳ why dark matter

↳ searches for dark matter

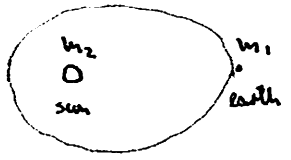
↳ models of dark matter

1) Why dark matter?

How do we measure masses of celestial bodies?

↳ from their gravitational effects

- consider mass of the sun



period $$T = 2\pi a^{3/2} \sqrt{\frac{1}{G_N (m_1 + m_2)}}$$

$$G_N = 6{,}674\,08(31) \cdot 10^{-11} \frac{m^3}{kg\, s^2}$$ Newton constant

a – major axis of the ellipse

⇒ from kinematics and geometry can obtain masses

↳ this underlies the present evidence for dark matter existence

three main probes:

- from individual spiral galaxies
- clusters of galaxies
- CMB and large scale structures at cosmological scales

↳ all point to presence of DM (i.e. not coupling to light)

give

$$\Omega_{DM} = 0.258 \pm 0.011$$

where $\Omega := \rho / \rho_{cr}$ with critical density $\rho_{cr} = \frac{3H^2}{8\pi G_N}$

↳ compare with baryonic DM

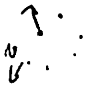
$$\Omega_b = (0.02226 \pm 0.00023) \frac{1}{h^2} = 0.0484(10)$$

⇒ DM constitutes 25% of total matter-energy of the Universe, visible matter 4.8%

↳ astrophysical and cosmological data are reproduced assuming DM is:

- cold: DM behaves as non-relativistic fluid (already at the time structure formation begins)

  if gas of particles



$v \ll c$ (or $|\vec{p}| \ll m_{DM}$)

- non-interacting (or collision-less): interactions between DM particles can be neglected, or DM with other particles. In contrast ordinary matter has significant EM interactions

  ⇒ this also means DM is dissipation-less: DM cannot emit EM radiation ~~[illegible]~~ to dissipate its eng. and cool down

  ↳ the main reason DM and visible (baryonic) matter behave differently

- stable: DM was present since early phases of the Universe and has not disappeared until now

  ↳ present limits ~~[illegible]~~ on DM decay time

  $\tau_{DM} \gtrsim 10^{26}\, s$ compare with $t_{uni} \simeq 13.8\, Gyr = 4.35 \cdot 10^{17}\, s$

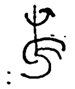
- adiabatic: means that, on cosmological scales, DM has the same primordial density inhomogeneity as other particles

  - DM is denser where ordinary matter and photons are denser

1.1 Evidence for DM from Galaxies

↳ rotation curves of spiral galaxies

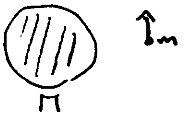
- spiral galaxies rotate around their vertical axis



measure Doppler shift of atomic lines

⇒ circular velocity of stars or hydrogen clouds

↳ circular velocity of test particle

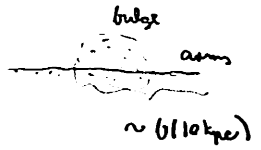


$$\frac{m\, v_c^2(r)}{r} = \frac{G\, m\, M(r)}{r^2}$$

$$\Rightarrow v_c(r) = \sqrt{\frac{G\, M(r)}{r}}$$

$M(r)$ - mass enclosed in radius r

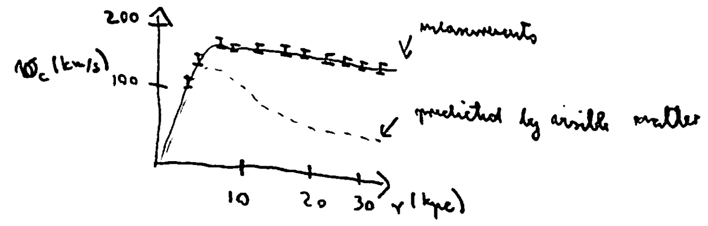
- spiral galaxies : most of the visible mass in a dense central bulge and arms of the disk



⇒ for $r \gg 10\text{kpc}$ expect $M(r) = \text{const}$

$\Rightarrow v_c(r) \propto \frac{1}{r^{1/2}}$

↳ observations (NGC 6503 Begeman et al 1991, original work Rubin, Ford 1970)

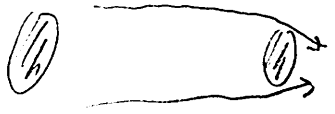


↳ for all spiral galaxies for $r \gg 10\text{kpc}$ velocity curves are flat ! (up to 50kpc and beyond)

- can explain $v_c(r) \simeq \text{const.}$ if there is a dark halo with density

$\rho \propto \frac{1}{r^2}$ to even large radii

↳ galaxy-galaxy lensing



from bending of light ⇒ distortion of background galaxy

⇒ can measure mass of foreground galaxy

↳ from here $M_{\text{Dark}} > M_{\text{visible}}$ observationally

1.2 Evidence for DM from clusters of galaxies

↳ historically the first evidence for DM :

- 1933 Fritz Zwicky observed velocity dispersion in the Coma cluster of galaxies

⇒ found out need extra matter to keep them together

↳ clusters of galaxies : largest gravitationally bound objects in the Universe

$\sim 10^2 - 10^3$ galaxies

~ few Mpc in size

⇒ probe "average" Universe ⇒ give $\Omega_{DM} \approx 0.2$

From velocity dispersion

virial theorem : $\langle K \rangle = -\frac{1}{2} \langle V \rangle$

avr. kin. eg. avr. pot. eng.

toy model : $N \gg 1$ galaxies of mass m at equal distances r

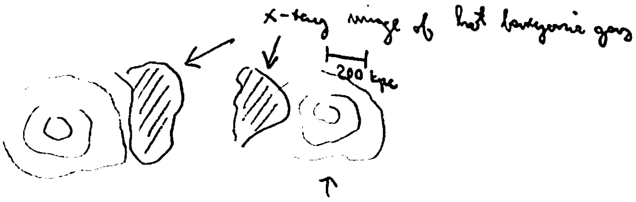
$$N \frac{m v^2}{2} = \frac{1}{2} \frac{N^2}{2} \frac{G m^2}{r} \Rightarrow mN = \frac{2 r v^2}{G} \qquad \Rightarrow \langle v^2 \rangle \text{ gives total mass } mN$$

historically relevant

weak gravitational lensing

↳ from weak lensing can obtain distrib of mass

↳ most striking example bullet cluster (Clowe, Bradac et al 2006)



total mass distribution reproduced from lensing

- two galaxy clusters colliding
- visible and dark matter are spatially separated
  - ↳ visible matter interacts through EM ⇒ collisional shock wave
  - ↳ DM simply passed through each other

↳ more than 70 systems known now

- imply a bound on DM self interaction

$$\frac{\sigma}{m_{DM}} \lesssim 1 \frac{cm^2}{g} = 1.8 \frac{mb}{GeV} \qquad \text{compare } pp \to pp \text{ in QCD} \sim \frac{\sigma}{m_p} \sim \frac{50 mb}{GeV}$$

1.3 Evidence for DM from cosmological scales

- the most convincing and precise evidence for DM comes from the largest scales possible: the entire Universe

↳ without DM world not look the same as it does

↳ DM acts as a catalyzer to form galaxies

- ordinary matter cannot do this since it couples to radiation

large scale structure formation

↳ the Universe is very inhomogeneous today:

30 maps of galaxies show structures like lumps, filaments, walls, voids

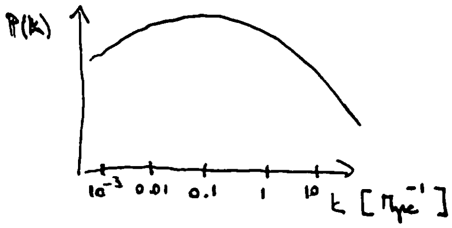
↳ quantitatively, these measurements can be condensed to matter power spectrum $P(k)$

$$\langle \delta_k \delta_{k'} \rangle = (2\pi)^3 P(k) \delta^3(\vec{k}-\vec{k}')$$

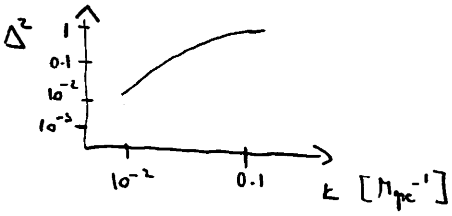
δ_k - Fourier transform of the density contrast $\delta(\vec{r}) = \frac{\rho_1(\vec{r})}{\rho_0(\vec{r})}$

$\langle \ \rangle$ - average over $\vec{k}$ space

$P(k)$ has a form



or in terms of dimensionless quantity $\Delta^2(k) = \frac{k^3 P(k)}{2\pi^2}$



$\Delta^2 \sim 1$ large overdensities

(100% large pert. over average

$\Rightarrow$ at large k (small scales) Universe exhibits large inhomogeneities

↳ in CMB $\delta \sim 10^{-5}$: how does this grow to large inhomogeneities today

↳ will do a more quantitative analysis

- consider Universe filled with generic matter fluid
- we approximate GR with Newtonian limit (ok for length scales << horizon)

↳ non-relativistic fluid described by: density $\rho(\vec{r},t)$

- velocity field $\vec{v}(\vec{r},t)$
- equation of state $p(\rho)$

$\uparrow$ pressure

gravitational interactions: Newtonian potential $\Phi(\vec{r},t)$

↳ they are governed by diff. equations

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0$$ continuity

(*) $$\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{\vec{\nabla}p}{\rho} - \vec{\nabla}\underline{\Phi}$$ Newton laws $\vec{a} = \frac{\vec{F}}{m}$

$$\nabla^2 \underline{\Phi} = 4\pi G_N \rho$$ Poisson eq.

↳ for quasi-homogeneous Universe it is useful to expand in perturbation theory around homogeneous background (0-th order), perturbations are 1-st order

$$\left.\begin{aligned} \rho &= \rho_0(t) + \rho_1(\vec{x},t) \\ p &= p_0(t) + p_1(\vec{x},t) \\ \vec{v} &= \vec{v}_0(t) + v_1(\vec{x},t) \\ \underline{\Phi} &= \underline{\Phi}_0(t) + \underline{\Phi}_1(\vec{x},t) \end{aligned}\right\} \Leftrightarrow \quad \nabla^2 \phi_0 = 4\pi G_N \rho_0 \quad \text{etc}$$

↳ first consider static Universe (no expansion, so $\rho_0(t) = \rho_0$, $p_0(t) = p_0$, ...)

$$(*) \Rightarrow \begin{bmatrix} (1) & \dfrac{\partial \rho_1}{\partial t} + \rho_0 \vec{\nabla}\cdot\vec{v}_1 = 0 \\ (2) & \dfrac{\partial \vec{v}_1}{\partial t} + \dfrac{v_s^2}{\rho_0}\vec{\nabla}\rho_1 + \vec{\nabla}\phi_1 = 0 \\ (3) & \nabla^2\phi_1 = 4\pi G_N \rho_1 \end{bmatrix}$$ (static Universe)

↳ here $v_s^2 = \frac{\partial p}{\partial \rho} = \frac{\partial p_1}{\partial \rho_1}$ sound speed in the fluid

↳ taking $\frac{\partial}{\partial t}(1)$ + insert (2) & (3) gives evolution eq. for ρ_1

$$\boxed{\frac{\partial^2 \rho_1}{\partial t^2} - v_s^2 \nabla^2 \rho_1 = 4\pi G_N \rho_0 \rho_1}$$ Jeans equation

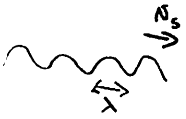
- in the limit of no gravity ($G_N \to 0$) we have

$$\partial_t^2 \rho_1 - v_s^2 \nabla^2 \rho_1 = 0$$

$\Rightarrow$ v_s is the speed of sound

↳ solutions are density waves (sound waves) $\rho_1 = e^{i(\vec{k}\cdot\vec{r} - \omega t)}$

that travel at speed v_s where $\frac{\omega}{k} = v_s$



$k = \frac{2\pi}{\lambda}$

Jeans eq. : $$\partial_t^2 \rho_1 \underbrace{- v_s^2 \nabla^2 \rho_1}_{\text{pressure term}} = \underbrace{4\pi G_N \rho_0 \rho_1}_{\text{collapse term}}$$

– can rewrite as

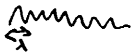
$$\frac{1}{v_s^2} \partial_t^2 \rho_1 - \nabla^2 \rho_1 = \frac{1}{\lambda_J^2} \rho_1 \qquad \lambda_J = \frac{v_s}{\sqrt{4\pi G_N \rho_0}} \quad \text{Jeans length}$$

↳ Two regimes : i) $\lambda > \lambda_J$

can ignore pressure term $\Rightarrow \partial_t^2 \rho_1 \simeq \frac{v_s^2}{\lambda_J^2} \rho_1$

perturbations will exponentially grow $\rho_1 \propto e^{\sqrt{4\pi G_N \rho_0}\, t}$

ii) $\lambda < \lambda_J$

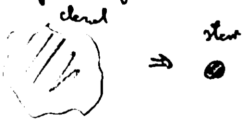


can ignore collapse term $\Rightarrow \partial_t^2 \rho_1 = \nabla^2 \rho_1$

perturbation on small scales are suppressed by pressure

↳ This is the Jeans instability

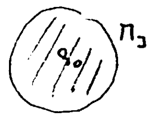
↳ applied to normal matter explains how gas clouds collapse to compact objects such as stars



↳ Jeans scale (λ_J): size of a gas cloud that is too big

- its hydrostatic pressure prevents collapse on time scales $T_{\text{pressure}} \sim \frac{\lambda_J}{v_s}$
- instability if this is too slow to prevent gravitational collapse on, that occurs on time scales $T_{\text{gravity}} \sim (G\rho_0)^{-1/2}$

↳ Jeans mass $$M_J = \frac{4\pi}{3} \rho_0 \lambda_J^3$$



mass of matter enclosed in a sphere of radius λ_J

↳ perturbations with mass $M > M_J$ are "Jeans unstable" and collapse

↳ next case expanding universe

0-th order quantities describe a smooth background with Hubble expansion

$\rho_0(t) = \rho_0(t_0) \cdot \frac{1}{a^3(t)}$ $\qquad a(t)$ scale factor of the Universe

$\vec{v}_0 = H \vec{r}$ $\qquad$ Hubble expansion, Hubble law for the velocity field

$\vec{\nabla} \Phi_0 = \frac{4\pi G_N}{3} \rho_0 \vec{r}$

Taking (*) on p.6 then gives

$$(**)\quad \left[\begin{array}{l} \dfrac{\partial \rho_1}{\partial t} + 3H\rho_1 + H(\vec{r}\cdot\vec{\nabla})\rho_1 + \rho_0 \vec{\nabla}\cdot\vec{v}_1 = 0 \\ \dfrac{\partial \vec{v}_1}{\partial t} + H\vec{v}_1 + H(\vec{r}\cdot\vec{\nabla})\vec{v}_1 + \dfrac{v_s^2}{\rho_0}\vec{\nabla}\rho_1 + \vec{\nabla}\phi_1 = 0 \\ \nabla^2\phi_1 = 4\pi G_N \rho_1 \end{array}\right] \qquad \text{(expanding Universe)}$$

↳ the extra terms due to expansion of the Universe all have H prefactor

↳ next define relative density (or density contrast)

$$\delta(\vec{r},t) = \frac{\rho_1(\vec{r},t)}{\rho_0(t)} = \frac{1}{(2\pi)^3}\underbrace{\int d^3k\, \delta_k(t)\, e^{-i\vec{k}\cdot\vec{x}}}_{} \qquad \vec{x} = \frac{\vec{r}}{a(t)}$$

expansion in co-moving Fourier modes ⇒ the wave number 1/k follows the average evolution of the Universe

↳ in this way modes with different k will be decoupled

↳ similarly also define $\vec{v}_k$ and Φ_k (Fourier transf. of $\vec{v}_1$ and Φ_1)

↳ in Fourier space (**) is

$$(***)\quad \left[\begin{array}{ll} (1) & \partial_t\delta_k - i\dfrac{\vec{k}}{a}\cdot\vec{v}_k = 0 \\ (2) & \partial_t(a\vec{v}_k) - i\vec{k}\, v_s^2\delta_k - i\vec{k}\,\phi_k = 0 \\ (3) & \phi_k = -\dfrac{4\pi G\rho_0}{k^2}a^2\delta_k \end{array}\right]$$

↳ side note: in deriving these we use $H = \frac{\dot{a}}{a}$ and $\rho_0(t) = \frac{\rho_0(t_0)}{a^3(t)}$

$$\Rightarrow \quad \partial_t\rho_0 = \partial_t\left(\frac{\rho_0(t_0)}{a^3}\right) = -3\frac{\dot{a}}{a}\frac{\rho_0(t_0)}{a^3} = -3H\rho_0(t)$$

$$\Rightarrow \quad \dot{\vec{x}} = \partial_t\left(\frac{\vec{r}}{a(t)}\right) = -\frac{\vec{r}}{a^2}\dot{a} = -H\vec{x}$$

↳ we can further simplify the 2nd equation

decompose: $\vec{v}_1 = \vec{v}_{1\parallel} + \vec{v}_{1\perp}$

where $\vec{\nabla}\cdot\vec{v}_{1\perp} = 0$ divergence free or solenoidal

$\vec{\nabla}\times\vec{v}_{1\parallel} = 0$ curl-free or irrotational component

↳ in Fourier space:

$$\vec{k}\cdot\vec{v}_{k\perp} = 0$$

$$\vec{k}\times\vec{v}_{k\parallel} = 0$$

$$\partial_t (a\vec{v}_k) - i\vec{k} v_s^2 \delta_k - i\vec{k}\,\phi_k = 0$$

↳ for $\vec{v}_{k\perp}$ this simply $\partial_t (a \vec{v}_{k\perp}) = 0$

⇒ solved by $\vec{v}_{k\perp} \propto \frac{1}{a}$

⇒ solenoidal component dies away with the expansion of the universe

↳ only $\vec{v}_{k\parallel}$ survives

$\vec{v}_{k\parallel} \propto \vec{k}$ so we can write $\vec{v}_k \to v_k \hat{k}$ $\left(\hat{k} = \frac{\vec{k}}{|\vec{k}|}\right)$

↑ only depends on $k = |\vec{k}|$

↳ can combine (***) on p. 8 to one eq. [use (1) for v_k in (2), along with (3) for ϕ_k]

$$\Rightarrow \quad \boxed{\partial_t^2 \delta_k + 2H \frac{\partial \delta_k}{\partial t} + \left(\frac{v_s^2 k^2}{a^2} - 4\pi G_N \rho_0\right)\delta_k = 0} \qquad (*) \, .$$

Jeans equation in Fourier space, and in expanding Universe

↳ Jeans wavenumber $k_J = a\sqrt{\frac{4\pi G_N \rho_0}{v_s^2}}$

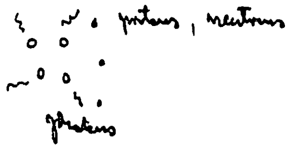
↳ for $k > k_J$ propagating waves 

↳ for $k < k_J$ collapsing solution 

↳ Note: k_J depends on $a(t)$ expansion parameter

Non-clustering of matter

↳ baryonic matter is tightly coupled to photons



⇒ unique matter fluid with relativistic sound speed

$v_s \approx \frac{c}{\sqrt{3}}$ $c=1$ speed of light

↳ since $v_s \sim \mathcal{O}(1)$ ⇒ pressure term dominates over gravitational collapse term

⇒ such fluid does not cluster

↳ for matter dominated Universe

$H = \frac{\dot{a}}{a} = \frac{2}{3} t^{-1}$ ⇒ $a(t) = \left(\frac{3}{2} t H_0\right)^{2/3}$ such that $a(t_0) = 1$ (or $t_0 = \frac{2}{3}\frac{1}{H_0}$), also $\rho_0 = \frac{3H^2}{8\pi G_N}$

↳ the solution to (*) is $\delta_k = \dfrac{\overset{\text{const.}}{c_1} \cos(\overset{\text{some fnc. of } H_0, v_s}{c_0}\, k\, t^{1/3} + \overset{\text{phase}}{\varphi})}{k\, t^{1/3}}$

⇒ <u>oscillating</u> pressure waves, <u>damped</u> in time ⇒ no clustering

↳ baryonic tightly-coupled fluid does not cluster on scales $k \gtrsim k_J(a) \sim a\frac{H}{\underset{\sim c}{v_s}} \sim \frac{1}{\text{length of the horizon}}$

⇒ does not collapse for wavelengths shorter than horizon

⇒ also [ordinary matter clusters only once it decouples from photons] !

Clustering of dark matter

cold DM is different : does not interact , described by nonrelativistic fluid with $v_s = 0$

⇒ huge difference in cosmology

↳ keeping only the gravity term in (*) on p. 9

$$\partial_t^2 \delta_k + 2H\, \partial_t \delta_k - 4\pi G_N \rho_0 \delta_k = 0$$

matter domin. universe ⇒

$$\partial_t^2 \delta_k + \frac{4}{3t} \partial_t \delta_k - \frac{2}{3t^2} \delta_k = 0 \qquad \text{using} \quad \rho_0 = \frac{3H^2}{8\pi G_N} \qquad H = \frac{2}{3t}$$

↳ This solved by

$$\delta_k = \underset{\uparrow}{c_{grow}\, t^{2/3}} + c_{decay}\, t^{-1}$$

This grows inhomogeneities from $\delta_k \sim 10^{-5}$ at $z \sim 10^3$ to large clumps now

↳ Note: the growth is power law (in static case it was exponential)

↳ expansion acts as friction and slows down the collapse

↳ side remark: if Universe radiation dominated : $H = \frac{1}{2t}$

↳ $v_s = 0$ still true for DM component

⇒ Jeans eq. $$\partial_t^2 \delta_k^{DM} + \frac{\cancel{4}1}{\cancel{3}t} \partial_t \delta_k^{DM} - \frac{2}{3t^2} \underset{\substack{\downarrow \\ \sim 0}}{\delta_k^{tot}} = 0$$

~0 since for radiation no collapse

⇒ this means that Jeans eq. for DM in radiation dominated epoch

$$\partial_t^2 \delta_k^{DM} + \frac{1}{t} \partial_t \delta_k^{DM} = 0$$

⇒ $\delta_k^{DM} \propto \ln t + \text{const.}$ so only logarithmic growth

⇒ CDM in radiation dominated epoch (early Universe) does not grow perturbations
(same for Universe dominated by vacuum eng., or by curvature)

Summary of large scale structure growth :

↳ The Universe became matter dominated at $a_{eq} \sim \frac{1}{3400}$

↳ at that point DM inhomogeneities δ_k started to grow as $\delta_k \propto t^{2/3}$ for scales $1/k$ that are ~ 3400 times smaller than present horizon

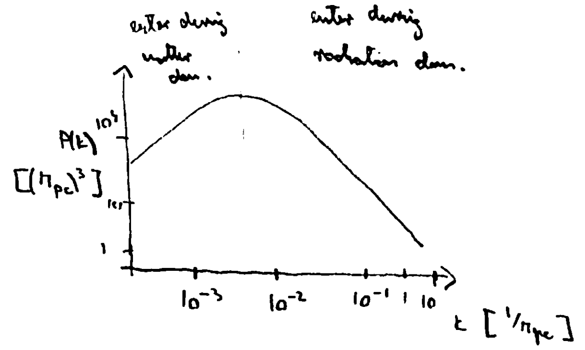
↳ normal matter still does not cluster since it is coupled tightly to photons , normal matter forms pressure waves

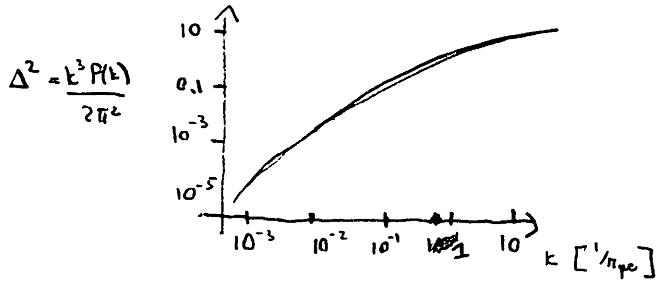
↳ at $a_{recomb} \sim \frac{1}{1100}$ Temperature low enough that e^- and p^+ bind to neutral H

⇒ normal matter decouples from radiation

⇒ starts falling into potential wells that DM already formed

⇒ DM crucial to explain present structure of the Universe





↳ if universe only had baryonic component ⇒ much smaller power in $P(k)$ (or Δ^2), i.e. Δ^2 smaller

⇒ also would see clear oscillation in $P(k)$ → in contradiction with data

↳ right mix of CDM + matter agrees with data

↳ $P(k)$ is sizeable at all scales and smooth

- change of slope at $k \gtrsim 3400\, \Omega_{tot} H_0 \sim 10^{-2}/Mpc$

↳ corresponds to modes entering the horizon at matter/radiation equality

⇒ for smaller k growth during matter domination

↳ around $k \sim 0.1\, h/Mpc$ small wiggles : "baryonic acoustic oscillations"

- imprints of pressure waves due to subdominant population of baryons

↳ Note: $N_s \simeq 0$ was crucial, DM needed to be cold not to erase small scale inhomogeneities through diffusion

Beyond linear approximation

when clustering increases to $\delta_k \sim 1$ the pert. theory breaks down

↳ gravitationally bound systems form

↳ $\delta_k(t)$ are larger to bigger k :

⇒ small structures form first

↳ at this point DM dominates the energy budget of the Universe

↳ use N-body simulations to solve for the evolution of N non-interacting particles

- at present can solve for $N \sim 10^9$ Newtonian eqs. of motion
- in our galaxy $\sim 10^{12} M_\odot$
- N-body simulation cannot resolve structures smaller than M/N

↳ N-body simulations show:

- $\sim 10^{12} M_\odot$ object form at $z \sim \mathcal{O}(1)$
- about 20% of DM remains diffuse, the rest is bound in galactic halos, etc

CMB acoustic peaks

↳ the DM vs. baryon density also leave imprint on CMB perturbations ⇐ the variance in photon temp. field

$T_0 = 2.725\,K$

$\delta T \sim 10^{-5}\, T_0$ are perturbation

↳ decomposed in spherical harmonics

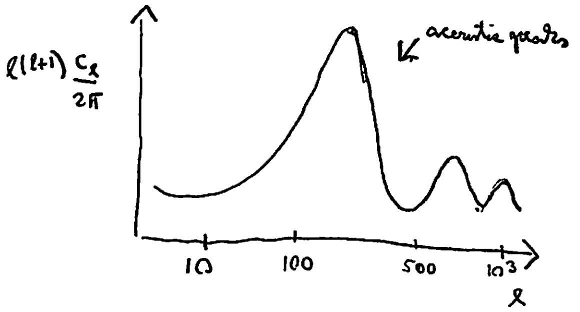
$$\frac{\delta T(\theta,\varphi)}{T_0} = \sum_{\ell,m} a_{\ell,m}\, Y_{\ell m}(\theta,\varphi)$$

– CMB anisotropy due to primordial perturbations

$$\langle a_{\ell,m} \rangle = 0$$
↑ average exp. value

↳ angular power spectrum

$$C_\ell \equiv \langle |a_{\ell m}|^2 \rangle = \frac{1}{2\ell+1} \sum_m \langle |a_{\ell m}|^2 \rangle$$



C_ℓ – the amount of anisotropy with angular size $\Theta \sim \frac{\pi}{\ell}$

angular size roughly corresponds to $k \sim \ell H_0$ (H_0 is present Hubble)

↳ The observed part means:

$$\left(\frac{\delta T}{T}\right)_{obs} \cong \underset{\Uparrow}{\frac{1}{4}\delta_\gamma} + \phi(t_{dec}, \hat{x}_{LS}) + \cdots$$

↑ – Doppler locally due to $\vec{v}$ of fluid
– integrated Sachs-Wolfe effect

since $\rho_\gamma = \frac{\pi^2}{15} T^4$

$$\delta_\gamma = \frac{\delta \rho_\gamma}{\rho_\gamma} = 4\frac{\delta T}{T}$$

$\phi(t_{dec}, \hat{x}_{LS})$ – gravit. potential at the surface of last scattering
– photons get redshifted when they climb out of the potential

using eq. (A) on p. 9 but with $\phi_k = -\frac{4\pi G_N \rho_0}{k^2} a^2 \delta_k$

$$(*) \quad \left[\partial_t^2 \delta_k^\gamma + 2H \partial_t \delta_k^\gamma + \left(\frac{v_s^2 k^2}{a^2} \delta_k^\gamma + \underset{\uparrow}{\phi_k} k^2 \right) = 0 \right]$$

δ_k^γ Fourier transf. of δ_γ

due to DM + baryon + photons

- in matter dominated universe, with DM dominating $\phi_k = $ const.

i.e., frozen

↳ for baryon-photon fluid

$$v_s^2 = \frac{c^2}{3} \frac{1}{1+R} \qquad \text{where} \quad R = \frac{3}{4} \frac{\bar{\rho}_B}{\bar{\rho}_\gamma}$$

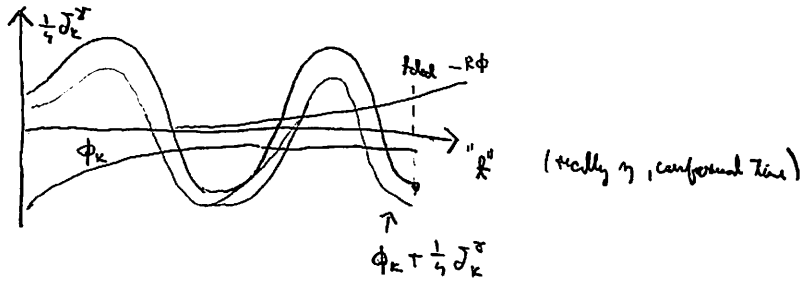
↳ taking R and ϕ_k to be constant ⇒ from here (*) is solved by

$$\underbrace{\frac{1}{4} \delta_k^\gamma + \phi_k}_{} = -R\phi_k + A_k \cos(v_s k t) + B_k \cos(v_s k t)$$

effective temperature perturbation

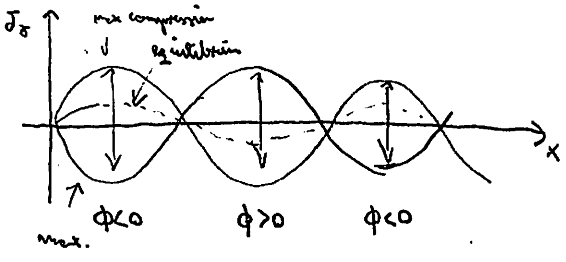
↳ gets imprinted on CMB at t_{dec} (decoupling time)

- for instance at a patch of Universe



shown: 4th acoustic peak

- spatially for one mode



↳ power spectrum

$$C_\ell \sim \left(\frac{1}{4} \delta_k^\gamma + \phi_k \Big|_{t_{dec}} \right)^2 \Bigg|_{k \sim \ell H_0} \sim \left(-R\phi_k + \underset{\uparrow}{A_k} \cos(v_s k t_{dec}) \right)^2$$

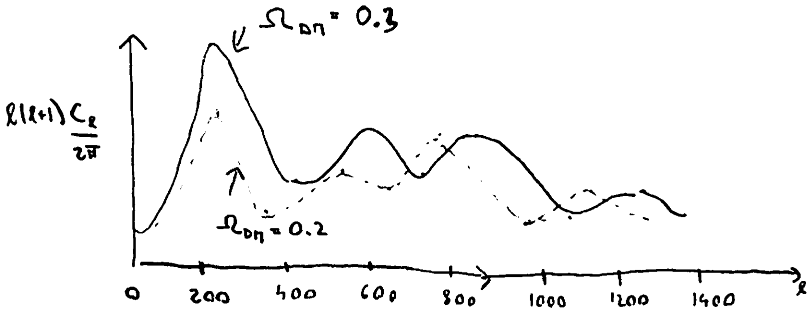
from primordial pert.

⇒ maximal power in C_ℓ for $\cos(v_s k t_{dec}) = 1$

so $v_s k t_{dec} = m\pi$ $m = 1, 2, \dots$ with $k \sim \ell H_0$

↳ which gives the first acoustic peak at $\ell \sim 200$

↳ even more importantly: the heights of the peaks are sensitive to ϕ_K



↳ can distinguish between CDM and baryons

– a fluid that has charged particles behaves differently in photon bath than neutral one

↳ from Planck   1807.06209

$\Omega_b h^2 = 0.0224 \pm 0.0001$ $h = 0.674 \pm 0.005$

$\Omega_{DM} h^2 = 0.120 \pm 0.001$

2. WHERE IS DM?

↳ we already know something about DM

↳ for direct and indirect detection need:

- DM density (locally, in our galaxy, and in the neighborhood)
- DM velocity distrib.

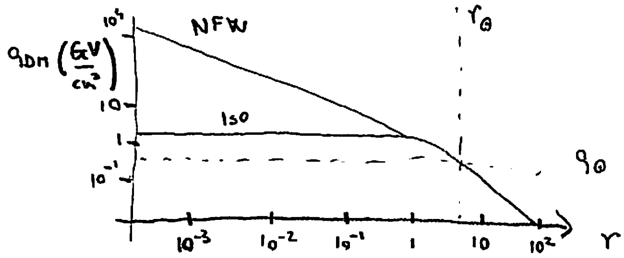
↳ certain things have to be true:

- DM in our galaxy is non-relativistic : since it is bound in our galaxy

$$\Rightarrow \quad v < \underset{\substack{\uparrow \\ \text{escape velocity}}}{v_{esc}} \simeq 500\,\text{km/s}$$

- DM is almost spherically distributed
  - when normal matter collapses to gravit. potential well, it interacts and cools ⇒ creates spiral disks
  - DM does not interact with each other. Neglecting gravit. int with visible matter gives spherical distrib.

↳ DM density profile quite uncertain toward the center



usually DM density at solar $r_\odot \simeq 8$ kpc usually taken

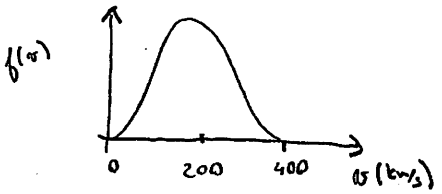
$$\rho_\odot = \rho(r_\odot) = 0.3\ GeV/cm^3$$

↳ from observations large uncertainty $\rho_\odot \in [0.2, 0.8]\ GeV/cm^3$

↳ DM velocity distribution:

- mainly come from N-body simulations

DM halo rest-frame:



- need to transform to solar + earth system

$$\vec{v}_\odot = (0,220,0)\frac{km}{s} + (10,13,7)\frac{km}{s} \qquad \text{in galactic coordinates}$$

several properties of particle DM

↳ DM must be stable $\qquad \tau \gg t_{universe} \simeq 13.8 \cdot 10^9$ years

↳ electric charge of DM needs to be very small : otherwise problems with cosmology

$$q \lesssim 10^{-4}\,(M/TeV)$$

↳ the cross section for scattering between DM and visible matter needs to be small due to direct detection experiments

- much smaller than weak xsec

$$\sigma \ll \frac{1}{m_Z^2}$$

↳ cross section between two DM particles needs to smaller than a typical QCD xsec

$$\frac{\sigma}{M} \ll \left(\frac{GeV}{M}\right)\frac{1}{m_\pi^2}$$

↳ DM must be cold

- need to be nonrelativistic at the time of CMB formation , z~1100

3 WHEN WAS DM PRODUCED?

thermal relic

↳ a simple assumption that gives the right DM density $\Omega_{DM} \sim 0.2$ is that DM is a thermal relic

↳ assume that : - DM a stable particle with mass m_{DM}

- in early universe in thermal equilibrium with the SM thermal bath

↳ typical eng : $E \sim T$

typical Compton wavelength : $\lambda \sim \frac{1}{T}$

number density : $n \sim T^3$

eng. density : $\rho \sim T^4$

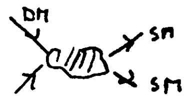
for $T \gg M$

↳ when T drops below m_{DM}

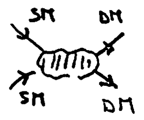
$$n_{DM} \propto e^{-m_{DM}/T} \quad \text{Boltzmann suppressed}$$

- annihilations try to keep thermal equil.

DM+DM <-> SM + SM



or



- at some point n_{DM} so small that annihilation rate slower than the expansion rate of the universe

⇒ DM "freezes out" (stops annihilating)

↳ this happens when

$$\Gamma \sim \langle n_{DM} \sigma \rangle \lesssim H \sim \frac{T^2}{M_{Pl}} \qquad M_{Pl} = \frac{1}{\sqrt{G_N 8\pi}}$$

↳ from here we can estimate

$$\frac{n_{DM}}{n_\gamma} \sim \frac{T^2/M_{Pl}\sigma}{T^3} \sim \frac{1}{M_{Pl}\sigma m_{DM}}$$

since the expon. suppression happens at $T \lesssim m_{DM}$

↖ inversely proportional to xsec !

↳ if we very roughly approximate $\rho_{DM} \sim \rho_\gamma$

and use rough approximation $\sigma \sim \frac{g^4}{m_{DM}^2}$

$$\frac{\rho_{DM}}{\rho_\gamma} \sim \frac{m_{DM}}{T_0}\frac{n_{DM}}{n_\gamma} \sim \frac{1}{M_{Pl}\sigma T_0} \Rightarrow \sigma(1) \sim \frac{1}{M_{Pl}T_0} \frac{m_{DM}^2}{g^4} \Rightarrow \frac{m_{DM}}{g^2} \sim \sqrt{T_0 M_{Pl}} \sim \text{TeV}$$

↳ a thermal relic DM (without any hierarchies) has <u>EW scale mass</u>

⇒ huge experimental effort to test and search for this EW relic

↳ also : $\frac{n_{DM}}{n_\gamma} \sim \frac{T_0}{m_{DM}} \sim 10^{-12}$ ⇒ freeze out happened at $T \sim m_{DM}/\ln\left(\frac{m_{DM}}{T_0}\right) \sim \frac{m_{DM}}{25}$

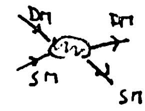
so happens when DM is nonrelativistic

precise calculation of relic abundance

↳ using classical Boltzmann equations for DM number density $\frac{dn_i(t,\vec{x},\vec{p})}{d^3\vec{x}\, d^3\vec{p}}$

– inhomogeneities in early cosmology $\sim 10^{-5}$ ⇒ can neglect them (no $\vec{x}$ dep.)

– scatterings that maintain kinetic equilibrium are fast enough



⇒ DM follows Fermi-Dirac or Bose-Einstein distrib in $\vec{p}$

⇒ we can just write a single eq. for total DM number density $n(t)$

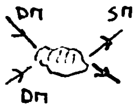
$$\frac{1}{a^3}\frac{d(na^3)}{dt} = \left[\frac{dn}{dt} + 3Hn = \langle\sigma v\rangle\left(n_{eq}^2 - n^2\right)\right]$$

$H = \frac{\dot{a}}{a}$ - Hubble rate

$n_{eq}(t)$ - DM number density that it would have in thermal equilibrium

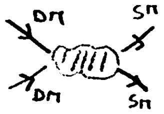
v - relative velocity between annihilating DM particles

σ - annih. Xsec

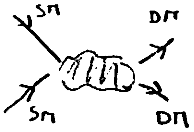


$\langle \ldots \rangle$ - thermal average

↳ the term $-\langle\sigma v\rangle n^2$ describes DM depletion due to annihilation



↳ the term $+\langle\sigma v\rangle n_{eq}^2$ describes DM creation via inverse annihilation



↳ for large enough $\langle\sigma v\rangle$ DM abundance stays in thermal equilibrium, $n = n_{eq}$, until DM nonrel.

then: $n_{eq} = g_{DM}\left(\frac{m_{DM}T}{2\pi}\right)^{3/2} e^{-\frac{m_{DM}}{T}}$

⇒ receives exponential suppression

g_{DM} - no. of d.o.f. for the DM particle

↳ observed cosmological abundance of DM reproduced if thermal eq. fails at $T_f \sim \frac{m_{DM}}{25}$

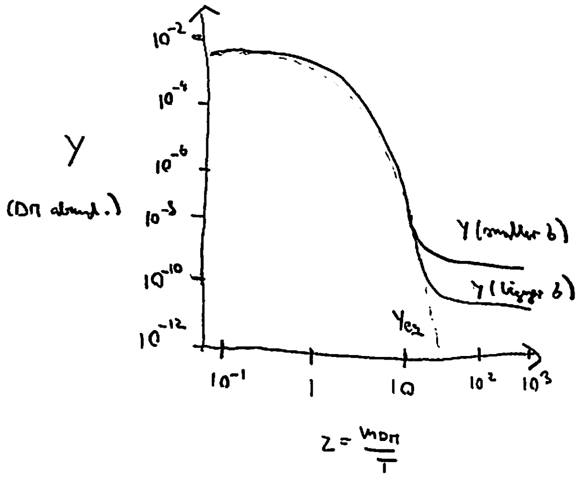
⇒ $n_{DM} \simeq n_{eq}(T=T_f)$

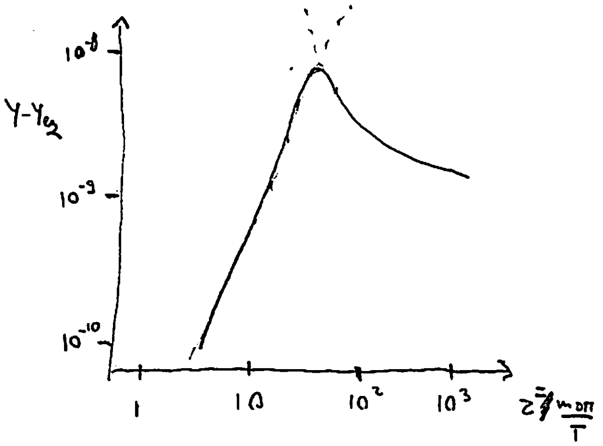
↳ easiest to solve using a new set of variables

effective number of rel. d.o.f.

$Y_{DM}(T) \equiv \frac{n_{DM}(T)}{s(T)}$ $\qquad$ $s(T) = \frac{2\pi^2 g_*(T)\, T^3}{45}$ total entropy density

because total entropy in comoving volume V, $S = s(T)V$, is conserved during cosmological evolution as long as thermal equil. holds





this gives numerically

$$\frac{\Omega_{DM} h^2}{0.110} \simeq \frac{\sigma v_{canon}}{\langle \sigma v \rangle_{T \simeq \frac{m_{DM}}{25}}}$$

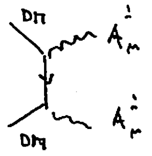
$$\sigma v_{canon} \simeq 2.2 \cdot 10^{-26} \frac{cm^3}{s} = \frac{1}{(23.0\,\text{TeV})^2} \simeq 0.73\,\text{pb}$$ ↖ typical weak cross section

h is H_0 in units: $H_0 = 100\, h \,\frac{km}{s}\, Mpc^{-1}$ $\Rightarrow$ $h = 0.678(9)$ (PDG 2018)

sample annihilation cross sections

↳ for instance DM particle in a repr. R of unbroken gauge group G

– annihilation cross section into massless gauge bosons



$$\sigma_0 = \frac{1}{g_{DM}} \, 2 C_R \left(C_R - \frac{C_G}{4} \right) \frac{\pi \alpha^2}{M_{DM}^2}$$

$\alpha = \frac{g^2}{4\pi}$ – gauge coupling

C_R – quadratic Casimir of representation R : $\delta_{ij} C_R = (T^a T^a)_{ij}$

C_G – is C_R for adjoint repr.

g_{DM} – number of DM d.o.f. $g_{DM} = d_R$ real scalar $d_R = \dim(R)$
$2 d_R$ complex scalar, or Majorana fermion
$4 d_R$ Dirac fermion

↳ for SU(2): $d_G = 3$ $C_R = (d_R^2 - 1)/4$
SU(N): $d_G = N^2 - 1$ $C_N = (N^2-1)/2N$ $C_G = N$

↳ a general estimate for 2 → 2 DM annihilation in s-wave

$$\langle \sigma v \rangle = \sigma_0 \simeq \frac{g^4}{4\pi\, m_{DM}^2} \qquad g \text{ - dimensionless coupling}$$

⇒ DM mass that reproduces Ω_{DM} observed is

$$m_{DM} \simeq g^2 \cdot 6.5\,\text{TeV}$$

↳ perturbativity upper bound $g^2 \lesssim 4\pi$ implies

$$\left[\; m_{DM} \lesssim 100\,\text{TeV} \;\right]$$

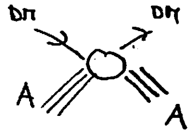
2) SEARCHES FOR DM

three main avenues are pursued :

- direct detection : detect a recoil due to DM hitting a target (nucleus or electron)
- indirect detection : looks for DM annihilation signals
- accelerator based searches : produce DM at colliders (e^+e^- or pp)

2.1) Direct detection

↳ galactic DM scattering on nuclei or electrons



↳ the kinematics :

- energy transferred ~ comparable to kin. eng. of DM-nucleus system $K = \mu_A v^2/2$

  $v \sim 10^{-3}$ from galactic kinematics $\qquad \mu_A = \dfrac{m_{DM}\, m_A}{(m_{DM} + m_A)}$

- for $m_{DM} \sim m_A \sim 100\,\text{GeV}$ ⇒ $K_A \simeq 20\,\text{keV}$

⇒ scattering is elastic since eng. so small ⇒ nucleus stays in the same state as initially

↳ eng. transfer also large enough that it can be detected

↳ typical momentum transfer

$$q \sim \sqrt{m_{DM} K} \quad \Rightarrow^{\text{typical:}} \quad q \sim 20\,\text{MeV} \quad \text{for scattering on light nuclei (e.g. F)}$$

$$q \sim 60\,\text{MeV} \quad \text{for scattering on heavy nuclei (e.g. Xe)}$$

↳ The experiments can tag the scattering events by observing one of the three end-products:

↳ heat (phonons)

↳ ionization

↳ scintillation

↳ expected number of events per unit of time

$$\text{event rate} = N_T \frac{\rho_\odot}{m_{DM}} v \sigma_A \simeq \frac{1}{yr} \cdot \frac{M_T/A}{kg} \cdot \frac{\sigma_A}{10^{-39} cm^2} \cdot \frac{\rho_\odot}{0.3\ GeV/cm^3} \cdot \frac{v}{200\ km/s} \cdot \frac{100\ GeV}{m_{DM}}$$

$N_T = N_A M_A$ mass of the detector composed of N_A nuclei (atomic number A, mass $M_A \simeq A m_N$)

↑ nucleon mass

σ_A DM cross section for scattering on the <u>nucleus</u>

↳ quite often the bounds quoted for cross section for scattering on <u>nucleon</u>, σ_N

- often scattering does not depend on spin $\sigma_N = \sigma_{SI}$

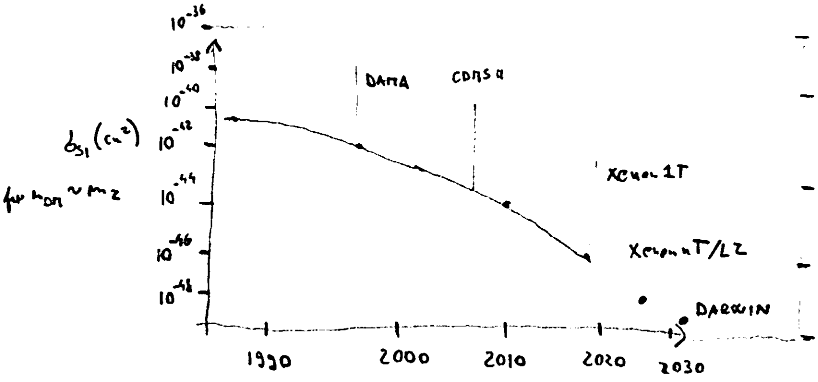
↳ spin independent cross section coherently enhanced

$$\sigma_A \simeq \underset{\sigma_{SI}}{\overset{\shortparallel}{\sigma_N}} A^2$$

↳ historic progress of the bounds :

- assume SI scattering, $m_{DM} \sim m_Z$ (roughly maximizes bounds)
  - presently most precise XENON 1T

$$\sigma_{SI} \lesssim 10^{-46} cm^2$$



- cosmology $\sigma \simeq \frac{1}{T_0 M_{Pl}}$
- vector $\sigma \sim \frac{\alpha^2 m_N^2}{m_Z^4}$
- Higgs $\sigma \sim \frac{\alpha^8 m_N^4}{m_h^6}$
- loop $\sigma \sim \frac{\alpha^4 m_N^4}{(4\pi)^2 m_W^6}$
- atmospheric ν background

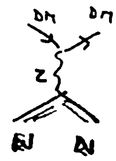
↳ some typical cross sections

↳ strongly interacting massive particles (SIMP), i.e., particles with QCD interactions

$$\sigma_{SI} \sim \frac{1}{\Lambda_{QCD}^2} \sim 10^{-26} cm^2$$

would be stopped by collisions in the upper atmosphere, excluded by balloon experiments

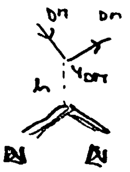
↳ tree level Z exchange



$$\Rightarrow \quad \sigma_{SI} \sim \alpha_Y^2 Y_{DM}^2 \frac{m_N^2}{m_Z^4} \sim 10^{-38} cm^2$$

excluded unless DM has small effective hypercharge $|Y_{DM}| \lesssim 10^{-4}$

↳ tree level Higgs coupling (leading to one loop coupling to gluons)

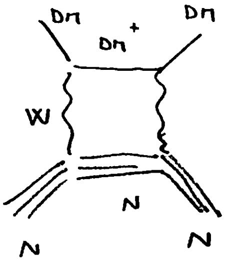


$$\Rightarrow \quad \sigma_{SI} \sim \frac{\alpha_{DM} m_N^4}{m_h^2} \sim 10^{-43} cm^2 \quad \text{with} \quad \alpha_{DM} \sim \frac{Y_{DM}^2}{4\pi}$$

Y_{DM} – DM Yukawa coupling

$\Rightarrow$ constrained to be $Y_{DM} \lesssim 0.1$

↳ DM with $SU(2)_L$ interactions but $Y=0$ ⇒ no tree level Z exchange ⇒ scatters through a loop



gives $\sigma_{SI} \sim \frac{\alpha_2^4 m_N^4}{(4\pi M_W^3)^2} \sim 10^{-48}\,\text{cm}^2$ still allowed experimentally

Computing the event rates

DM + A → DM + A event rate (# of events / time interval)

(*) $$\frac{dN_{ev}}{dE_R} = N_T \int_{v>v_{min}}^{\infty} \frac{d\sigma_A}{dE_R}\, v\, dn_{DM}(\vec{v}) \qquad \text{where } dn_{DM} = \frac{\rho_\oplus}{m_{DM}} f_\oplus(\vec{v})\, d^3\vec{v}$$

N_T number of nuclei in the target

$E_R \sim O(\text{keV})$ the recoil energy of nucleus

$\rho_\oplus$ DM density at Earth's location (usually the same as $\rho_\odot = 0.3\,\text{GeV/cm}^3$)

$f_\oplus(\vec{v})$ DM velocity distribution on Earth

$$\int d^3\vec{v}\, f_\oplus(\vec{v}) = 1$$

↳ velocity distribution obtained from galactic DM velocity distribution

$$f_\oplus(\vec{v},t) = f(\vec{v} + \vec{v}_\oplus(t))$$

$$\vec{v}_\oplus(t) = \vec{v}_\odot + \vec{V}_\oplus\left[\hat{\varepsilon}_1 \cos\omega(t-t_1) + \hat{\varepsilon}_2 \sin\omega(t-t_1)\right]$$

$\vec{v}_\odot$ - velocity of the sun

$V_\oplus = 29.8$ km/s Earth's orbital speed

$t_1 = 0.218$ yr = March 21 time of spring equinox

$\omega = 2\pi/\text{yr}$ $\hat{\varepsilon}_1 = (0.9931, 0.1170, -0.0103)$ $\hat{\varepsilon}_2 = (-0.0670, 0.4927, -0.8676)$ in galactic coordinates

↳ in order to transfer a kinetic eng. E_R to the nucleus the incoming DM needs to have a minimal velocity

$$v_{min} = \sqrt{\frac{m_A E_R}{2\mu_A^2}} \qquad \frac{1}{\mu_A} = \frac{1}{M_A} + \frac{1}{m_{DM}}$$

↳ in (*) the integral over DM velocity is from v_{min} to escape velocity from galaxy ~550 km/s

Typical DM velocity is ~200 to 300 km/s

↳ particle physics in (*) enters through $d\sigma_A/dE_R$

↳ ignoring velocity suppressed interactions there are only two possibilities

- spin-independent (SI) scattering (interactions do not involve the spin)
- spin-dependent (SD) scattering (DM couples to nuclear spin)

↳ SI cross section

$$\frac{d\sigma_A}{dE_R} = \frac{m_A \hat{\sigma}_N}{2v^2 \mu_N^2} A^2 F_A^2(q)$$

$\hat{\sigma}_N$ – average cross section on nucleons

$q = \sqrt{2 m_A E_R}$ momentum exchange

$\mu_N = \frac{m_N m_{DM}}{(m_N + m_{DM})}$ reduced mass of nucleon-DM system

↳ for $q \to 0$ long wavelength limit ⇒ DM couples coherently to all nucleons inside a nucleus

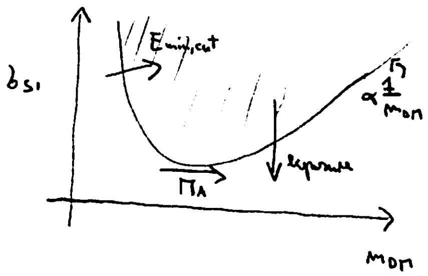
$F_A(0) = 1$ for nuclear form factor, then exponential fall off with $q \sim \frac{1}{1\,fm}$

e.g. $F_{19}(100\,MeV) \simeq 0.77$ Fluorine

$F_{132}(100\,MeV) \simeq 0.35$ Xenon

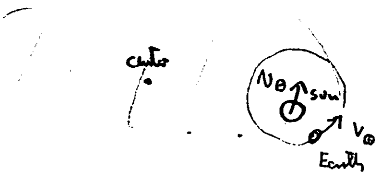
↳ experimental bound on N_W ⇒ bound on σ_{SI}

Typical shape of the bound



↳ annual modulation

– sun orbits the center of the galaxy, moves through gas of DM particles with $v_\odot \sim 230\,km/s$



⇒ induces a 230 km/s wind of DM particles in the solar system

↳ modulated by $v_\oplus \approx 30\,km/s$

⇒ $\frac{dN_W}{dE_R}$ changes slightly

$$\frac{dN_W}{dE_R} = \overline{\frac{dN_W}{dE_R}} \left[1 + A_R(E_R) \cos(\omega(t - t_c)) \right]$$

$$A_R \sim \mathcal{O}\left(\frac{v_\oplus}{v_\odot}\right) \sim 0.1$$

maximal rate expected at t_c = June 2

↳ note: there is also diurnal modulation due to rotation of the Earth

The irreducible neutrino background

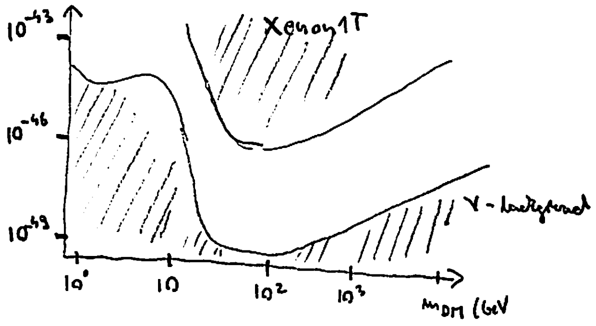
↳ coherent neutrino/nucleus scattering an irreducible background to the searches

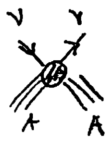
$$\sigma(\nu A) \simeq \frac{G_F}{4\pi} \underset{\uparrow}{N^2} E_\nu^2 F_A^2(q)$$

\# of neutrons

↳ solar neutrinos are the dominant bkg for light DM, $m_{DM} \lesssim 10$ GeV

↳ atmospheric & diffuse supernova neutrinos important for heavier DM masses





same signal as SI DM scattering

Astrophysical uncertainties

↳ event rate $\propto \rho_\odot \sigma_A$ ⇒ need to know $\rho_\odot$ to extract σ_A

↳ even more important: $f(\vec{v})$ is quite uncertain

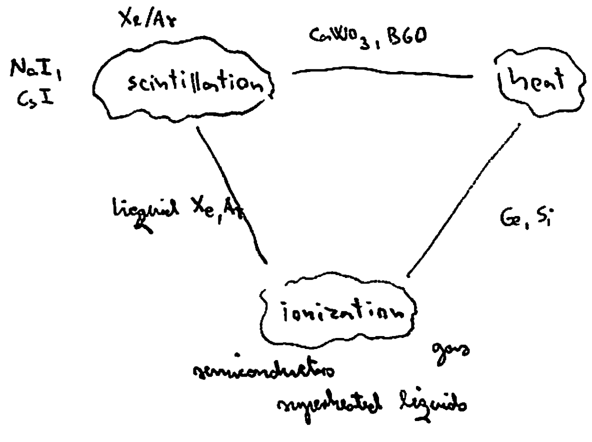
↳ enters when comparing different experiments

↳ possible to compare experiments independently of $f(\vec{v})$: compare diff. distrib. that arise from the same v

Experimental techniques

↳ different ways to measure deposited energy

↳ different materials



↳ many experiments (so far more than 50)

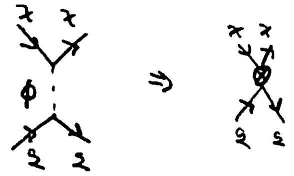
Effective operators

↳ in many theories the mediators between the DM sector and visible sector heavier than typical momentum exchange in direct detection $q \lesssim 200\,\mathrm{MeV}$

$\Rightarrow$ can "integrate out" the mediators $\Rightarrow$ EFT

↳ example : scalar mediator ϕ , DM χ

$$\mathcal{L}_\phi \supset y_\chi \phi \bar{\chi}\chi + y_q \phi \bar{q} q$$



$$-i\,(\bar{u}_\chi u_\chi)\,\frac{y_\chi y_q}{q^2 - m_\phi^2}\,(\bar{u}_q u_q) \overset{s \ll m_\phi}{\simeq} i\,(\bar{u}_\chi u_\chi)\,\frac{y_\chi y_q}{m_\phi^2} + \mathcal{O}\left(\frac{q^2}{m_\phi^2}\right)$$

↳ so up to power suppressed corrections DM equally well described by

$$\mathcal{L}_{EFT} \supset C\,(\bar{\chi}\chi)(\bar{q}q)$$

↳ all the relevant physics of the mediator in the value of the Wilson coefficient

$$C = \frac{y_\chi y_q}{m_\phi^2}$$

↳ as long as mediator heavy, any NP described by

$$\mathcal{L}_{EFT} = \sum_{a,d} \frac{C_a^{(d)}}{\Lambda^{d-4}}\, Q_a^{(d)}$$

$C_a^{(d)}$ dimensionless

Λ ~ mass of the mediators

d - dimension of the operator

↳ dimensions of QFT fields (in powers of mass)

$\hbar = c = 1$

$[m] = 1$

$[\partial] = 1$

$[x] = -1$

action is dimensionless

fermion $\quad S = \int d^4x\, \left(i\bar{\Psi}\not{\partial}\Psi - m\bar{\Psi}\Psi\right) \qquad [S] = 0 \Rightarrow [\Psi] = 3/2$

$\downarrow$

$D_\mu + ie A_\mu \quad \Rightarrow \quad [A_\mu] = 1$

scalar $\quad S = \int d^4x \left[\frac{1}{2}(\partial^\mu \phi)^2 - \frac{1}{2}m^2\phi^2\right] \quad \Rightarrow \quad [\phi] = 1$

↳ example : Dirac fermion χ, EFT at quark level at $\mu = 2\,\text{GeV}$

↳ dimension 5

$$Q_1^{(5)} = \frac{e}{8\pi^2}\,(\bar\chi\,\sigma^{\mu\nu}\chi)\,F_{\mu\nu} \qquad\qquad Q_2^{(5)} = \frac{e}{8\pi^2}\,(\bar\chi\,\sigma^{\mu\nu} i\gamma_5\chi)\,F_{\mu\nu}$$

magnetic dipole   electric dipole

↳ dimension 6

$$Q_{1q}^{(6)} = (\bar\chi\gamma_\mu\chi)(\bar q\gamma^\mu q) \qquad\qquad Q_{2q}^{(6)} = (\bar\chi\gamma_\mu\gamma_5\chi)(\bar q\gamma^\mu q)$$

$$Q_{3q}^{(6)} = (\bar\chi\gamma_\mu\chi)(\bar q\gamma^\mu\gamma_5 q) \qquad\qquad Q_{4q}^{(6)} = (\bar\chi\gamma_\mu\gamma_5\chi)(\bar q\gamma^\mu\gamma_5 q)$$

↳ dimension 7

$$Q_{1G}^{(7)} = \frac{\alpha_s}{12\pi}\,(\bar\chi\chi)\,G^{a\mu\nu}G^a_{\mu\nu} \qquad\qquad Q_{2G}^{(7)} = \frac{\alpha_s}{12\pi}\,(\bar\chi i\gamma_5\chi)\,G^{a\mu\nu}G^a_{\mu\nu}$$

$$\vdots \qquad\qquad\qquad\qquad \vdots$$

+ 8 more ops.

↳ to get to cross sections :

↳ need hadronic matrix elements, e.g.

$$\langle N'|\bar q\gamma^\mu q|N\rangle = \bar u_N' \left[F_1^{q/N}(q^2)\,\gamma^\mu + \frac{i}{2m_N}\,F_2^{q/N}(q^2)\,\sigma^{\mu\nu}q_\nu \right] u_N$$

$$\vdots$$

↳ need to go to non-relativistic limit for χ and also nucleons

e.g.:

$$(\bar\chi\sigma^{\mu\nu}\chi)\,F_{\mu\nu} \rightarrow \vec S_\chi\cdot\vec B$$

$$(\bar\chi\gamma_\mu\chi)(\bar q\gamma^\mu q) \rightarrow \mathbb{1}_\chi\cdot\mathbb{1}_N$$

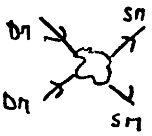
$$\vdots$$

↳ finally need response functions for $\overset{\text{nonrel.}}{\downarrow}$ nucleon currents inside nuclei (chiral EFT can be used for counting, the response functions from nuclear physics models, e.g. shell models)

↳ the big gain : can model independently compare results of direct detection experiments

↳ above steps also needed within UV complete models to get predictions for scattering rates

2.2 Indirect detection



↳ searches for: DM+DM → SM+SM annihilations

↳ search for excess in cosmic rays (collected on Earth)

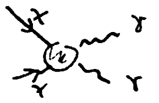
↳ promising sources: wherever DM densest

- center of our Galaxy
- inner halo of our Galaxy
- nearby galaxies dominated by DM
- center of the Sun, Earth

↳ some regions complicated by astrophysics (e.g. center of our Galaxy)

↳ many different probes

- high energy photons (γ-rays)

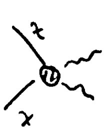


- freely propagate in the galactic environment ⇒ so info in eng. and angular distrib.
- however, signal expected to be smaller (needs to be loop induced)

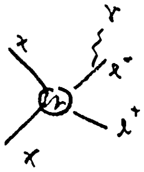
e.g.

- low energy photons (X-rays, radio waves)

either from



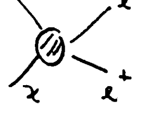
for light DM or



for heavy DM

⇑ have more uncertainties in DM predictions as "doubly indirect", depend on astrophysical env. (for synchrotron radiation on magnetic fields)

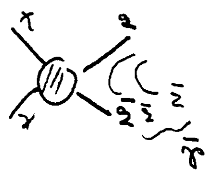
- positrons



- diffuse in galactic magnetic fields ⇒ randomises their directions
- lose eng. via synchrotron emission, Coulomb scattering, ionization, bremsstrahlung, inverse Compton scattering
- info on DM only in eng. spectrum

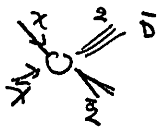
- electrons - similar comments as for positrons + astrophysical backgrounds are bigger

- antiprotons



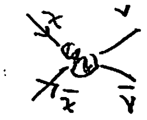
- diffuse in galactic magnetic fields, so random direction
- but negligible eng.-losses ⇒ even far away regions contribute to the flux
- DM info is in eng. spectr.

↳ anti-deuterons



- small yields, but also small astrophysical backgrounds
- propagation similar to anti-protons

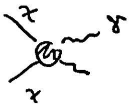
- neutrinos



- propagate freely in the Galaxy, even through dense matter such as Earth and the Sun (up to multi-TeV energies)
- small xsec makes them more difficult to detect

↳ we will look in more detail in one probe: Gamma rays

- differential flux of photons from annihilation



$$\frac{d\phi_\gamma}{d\Omega\, dE} = \text{[illegible]} \left(\frac{r_\odot}{4\pi}\right)\left(\frac{\rho_\odot}{m_{DM}}\right)^2 \underset{\uparrow}{J_a} \frac{dN_\gamma}{dE}$$

integral over sources

↑ the flux of photons at source

$$J_a = \int_{l.o.s.} \frac{ds}{r_\odot} \left(\frac{\rho(r(s,\theta))}{\rho_\odot}\right)^2$$

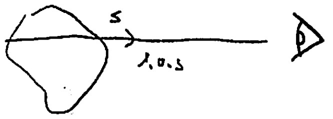
– depends only on astrophysics

- for decaying DM

$$\frac{d\phi_\gamma}{d\Omega\, dE} = \frac{r_\odot}{4\pi} \frac{\rho_\odot}{m_{DM}} J_d \frac{dN_\gamma}{dE}$$

$$J_d = \int_{l.o.s} \frac{ds}{r_\odot} \left(\frac{\rho(r(s,\theta))}{\rho_\odot}\right)$$

J-factors encode the power of a source to emit DM (purely astroph. quantity)



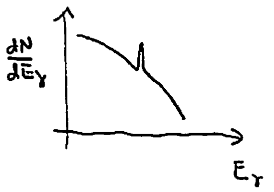
↳ Note: $J_a \propto \rho^2$ } they are very sensitive to DM distributions, esp. toward the center

$J_d \propto \rho$ large uncertainties

↳ a clear signal: if it is a two body annihilation or decay

$\chi + \chi \to 2\gamma \quad \Rightarrow \quad E_\gamma = m_\chi$

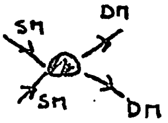
$\chi \to \gamma + \chi' \quad \Rightarrow \quad E_\gamma$ also a line



a line not corresponding to any known transition ⇒ DM

2.3 SEARCHES FOR DM AT COLLIDERS

↳ can produce DM in

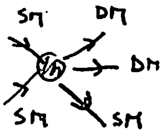


↳ since DM is stable ⇒ can only be produced in pairs

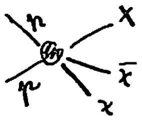
⇒ the eng. of a collider needs to be $\sqrt{s} > 2 m_{DM}$

↳ once produced DM escapes without interacting with the detectors

⇒ can detect these processes only if other visible particles are produced in collision



↳ at the LHC : mono-X searches



$X = j, \gamma, Z, W, H, t, t\bar{t}$

↳ monojet : $pp \to j + MET$ (cuts : $p_T > 250$ GeV on leading jet , $|\eta| < 2.4$)

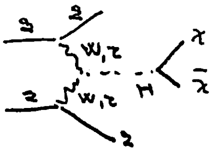
↑ pseudorapidity $\eta = -\ln\left[\tan\frac{\theta}{2}\right]$

- the challenge is controlling the bkg
- irreducible bkg from $pp \to Z(\to \bar{\nu}\nu) + \text{jets}$

↳ invisible Higgs decays : $H \to \bar{\chi}\chi$ possible , if $m_\chi \lesssim \frac{M_h}{2}$

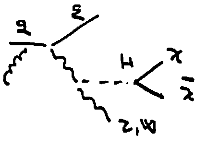
↳ can search in two production modes

- vector boson fusion



2 forward jets + MET

- associated production WH, ZH



this signature very similar to mono-V

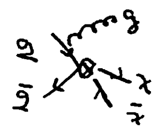
↳ predicting the signal at LHC

↳ need a theory that is valid at LHC energies (given by kinematics, usually few 100 GeV)

↳ one option EFT : higher dim operators

for instance :

$$O = \frac{1}{\Lambda^2} (\bar{q}\gamma^\mu\gamma_5 q)(\bar{\chi}\gamma_\mu\gamma_5\chi) \Rightarrow$$



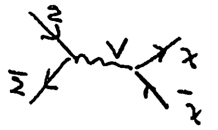
⇒ the bound on this operator great, however Λ bounded below typical p_T of the jet

⇒ EFT breaks down

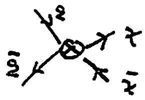
↳ simplified models : include only relevant states for LHC

↳ in the above example : vector mediator

$$\mathcal{L} \supset \frac{m_V^2}{2} V^\mu V_\mu + V^\mu g_q \bar{q}\gamma_\mu\gamma_5 q + V^\mu g_{DM} \bar{\chi}\gamma_\mu\gamma_5\chi$$

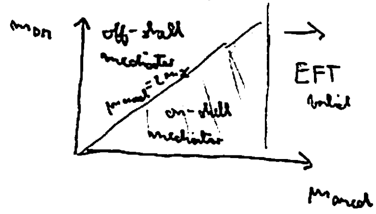


⇒



$$\mathcal{M} \propto \frac{g_q g_{DM}}{m_V^2 - s} \quad \overset{m_V^2 \gg s}{\Longrightarrow} \quad \frac{1}{\Lambda^2} = \frac{g_q g_{DM}}{m_V^2}$$

↳ typical plane of validity

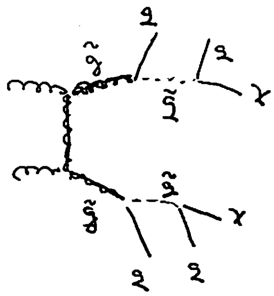


↳ complete models : example supersymmetric DM

↳ can lead to complicated decay chains to be searched for

e.g. for gluino decays

$$\tilde{g} \to \bar{q}\tilde{q} \to \bar{q}q\chi$$



3. DARK MATTER THEORIES

↳ very easy to write down DM models, main motivation usually if they also solve "something else".

- solve naturalness of the Higgs mass
- solve QCD Θ problem
- dark energy

↳ quick overview next

Standard model DM candidates : several attempts in the past

↳ neutrinos : Big bang also produced relic neutrinos, however $m_\nu \lesssim 0.1\,\text{eV}$ (from CMB + structure formation as warm DM) implies that $\Omega_\nu \lesssim 10^{-2}\,\Omega_{DM}$ so too small

↳ strangelets : spin-0 sexaquark uuddss could potentially be a bound state, would be stable if its mass is below $2(m_p + m_e) \simeq 1.88\,\text{GeV}$

↳ excluded by indirect detection experiment XQC in the upper atmosphere

Scalar singlet

↳ the most minimal DM model : only add a real neutral singlet scalar S to the SM

- impose Z_2 symmetry by hand

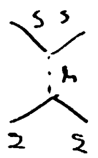
$$S \to -S$$

↳ the Lagrangian

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2} m_S^2 S^2 - \lambda_{HS} S^2 |H|^2 - \frac{1}{4}\lambda_S S^4$$

↳ after EWSB $\quad H \to \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+h) \end{pmatrix} \qquad v = 246\,\text{GeV}$

↳ spin indep. Xsec from Higgs exchange

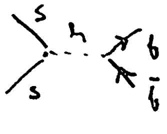


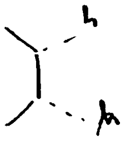
$$\Rightarrow \sigma_{SI} = \frac{\lambda_{HS}\, m_N^4\, f^2}{\pi\, m_{DM}^2\, m_h^4}$$

nucleon matrix element ↙ f

↳ DM relic abundance : s-wave annihilation

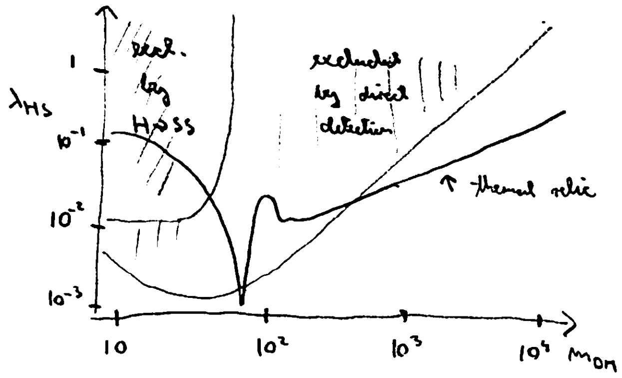
$SS \to h \to b\bar{b}$ or $SS \to hh$





$$\sigma v = \frac{16\lambda_{HS}^2 v^2}{(4m_{DM}^2 - m_h^2)^2 + m_h^2(\Gamma_h)^2} \frac{\Gamma_h}{2m_{DM}} + \frac{\lambda_{HS}^2}{64\pi m_{DM}^2} \operatorname{Re}\sqrt{1 - \frac{m_h^2}{m_{DM}^2}}$$

for $m_h < m_{DM}$



↳ S model allowed for

$$m_{DM} \gtrsim 700 \text{ GeV}$$

↳ for $m_{DM} \gtrsim 7$ TeV λ_{HS} becomes large $(\lambda_{HS} \lesssim 1)$

- potential problems with perturbativity

Fermionic singlet: "sterile neutrino"

↳ add an extra Weyl fermion to the SM: N

- impose ad hoc Z_2 symmetry $N \to -N$ so that stable $\Rightarrow$ then no couplings to the SM

↳ so drop $N \to -N$ symmetry

↳ right-handed neutrino with mass $m_{DM} \gtrsim$ keV

- characterised by mixing angle between sterile and active neutrinos $\theta = \frac{yv}{m} \ll 1$
- decays to active neutrinos

$$\Gamma(N\to 3\nu) = \frac{G_F^2 M^5}{96\pi^3}\theta^2 \approx \frac{\theta^2}{33\,T_U}\left(\frac{M}{\text{keV}}\right)^5$$

↑ life-time of the universe

$$\Gamma(N\to\nu\gamma) = \frac{9\alpha_{em}}{256\pi^4} G_F^2 M^5 \theta^2 \approx \frac{\theta^2}{4250\,T_U}\left(\frac{M}{\text{keV}}\right)^5$$

$\Downarrow$

bound is $< \frac{1}{10^8 T_U}$

↳ the same mixing θ also in production of sterile neutrinos in the early universe

allowed ranges are $\theta \sim 10^{-6}$

$m_{DM} \sim 10 - 100$ keV

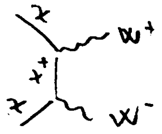
so that the right relic abundance

Minimal DM

↳ an interesting option that DM stable because Z_2 is accidental:

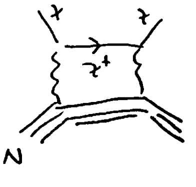
- simply cannot write operators that would decay it
- happens for fermion: 5 of $SU(2)_L$, singlet of $U(1)_Y$ and $SU(3)_c$

↳ relic abundance from



$+ \dots \Rightarrow m_{DM} \approx 11.5$ TeV to obtain the right relic abundance

↳ direct detection



$$\Rightarrow \sigma_{SI} \sim 2.0 \cdot 10^{-46} \text{ cm}^2$$

Supersymmetric DM

↳ the Z_2 here is R-parity

- all SUSY particles of MSSM are R-parity odd (gauginos, higgsinos, sleptons, squarks)
- R-parity postulated so that proton does not decay

↳ neutral candidates in MSSM

Neutralino / Bino

$$\sigma v_{rel} (\tilde{B}\tilde{B} \to l^+ l^-) \sim v_{rel}^2 \frac{g_Y^4}{4\pi M^2}$$ p-wave cross section

cosmological Ω_{DM} reproduced for $M \sim 120\,\text{GeV}$, problem with LHC searches

Wino

$\tilde{W}_3$ is SUSY partner of W_3

DM relic abundance reproduced for $M_2 \approx 2.5\,\text{TeV}$, out of reach of LHC, possibly at 100 TeV

Higgsino

$\tilde{h}$ is SUSY partner of the Higgs doublet, has $Y = \pm 1/2$

⇒ direct detection excludes it (up to "blind spots")

Gravitino

$\tilde{G}$: neutral spin 3/2 particle, SUSY partner of the graviton

has gravitational interaction - cannot detect directly

↳ can search for it in decays of other SUSY particles

Axion DM

↳ axion is a solution of a strong CP problem

$$\mathcal{L} = \mathcal{L}_{SM} - \theta \frac{g_s^2}{64\pi^2} G^a_{\mu\nu} \underset{\overset{\uparrow}{\frac{1}{2}\varepsilon_{\mu\nu\alpha\beta} G^{a\alpha\beta}}}{\tilde{G}^a_{\mu\nu}}$$

↳ this term violates CP, however from neutron EDM: $\theta \lesssim 10^{-10}$

↳ can solve this, if θ is a field $\theta \to \frac{a(x)}{f}$, axionic solution

- The axion is the phase of U(1) spontaneously broken, has to be anomalous under the SM

↳ effective Lagrangian

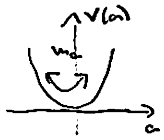
$$\mathcal{L} = \frac{1}{2}(\partial_\mu a)^2 - \frac{a}{f}\left[\frac{\alpha_s}{8\pi} G^a_{\mu\nu}\tilde{G}^{a\mu\nu} + c_2 \frac{\alpha_2}{8\pi} W^a_{\mu\nu}\tilde{W}^{a\mu\nu} + c_1 \frac{\alpha_Y}{8\pi} B_{\mu\nu}\tilde{B}^{\mu\nu}\right]$$
$$+ \frac{\partial_\mu a}{f}\left[c_H (H^\dagger i D_\mu H) + \sum_i c_i (\bar{\Psi}_i \gamma^\mu \Psi) + h.c.\right]$$

↳ mass of the axion comes from QCD condensates

$$m_a \sim \frac{\Lambda_{QCD}^2}{f} \sim 0.6\,\text{meV}\,\frac{10^{10}\,\text{GeV}}{f}$$

⇒ so very light, but still a cold DM candidate

↳ the reason is that during QCD phase transition it obtains mass, which results in coherent oscillations ⇒ act as CDM (the frequency does not shift)



↳ search for it mainly through the couplings to photons.

$$-\frac{a}{f}\frac{\alpha}{8\pi} c_\gamma F_{\mu\nu}\tilde{F}^{\mu\nu}$$