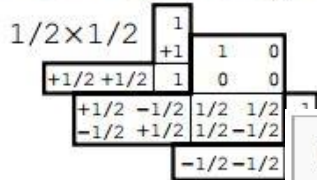
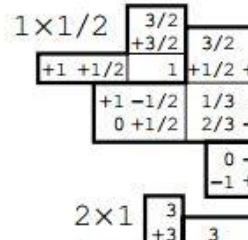
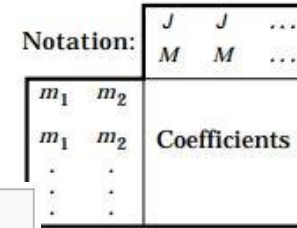
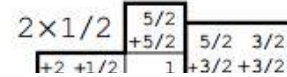


# 34. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND $d$ FUNCTIONS

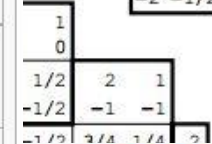
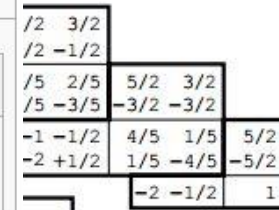
Note: A square-root sign is to be understood over *every* coefficient, e.g., for  $-8/15$  read  $-\sqrt{8/15}$ .



$$Y_1^0 = \sqrt{\frac{3}{4\pi}} \cos \theta$$



$m = \frac{1}{2}$	$j$	$(j_1=5/2, j_2=2)$				
$m_1, m_2$		$\frac{9}{2}$	$\frac{7}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{1}{2}$
$\frac{5}{2}, -2$		$\sqrt{\frac{1}{126}}$	$\sqrt{\frac{4}{63}}$	$\sqrt{\frac{3}{14}}$	$\sqrt{\frac{8}{21}}$	$\sqrt{\frac{1}{3}}$
$\frac{3}{2}, -1$		$\sqrt{\frac{10}{63}}$	$\sqrt{\frac{121}{315}}$	$\sqrt{\frac{6}{35}}$	$-\sqrt{\frac{2}{105}}$	$-\sqrt{\frac{4}{15}}$
$\frac{1}{2}, 0$		$\sqrt{\frac{10}{21}}$	$\sqrt{\frac{4}{105}}$	$-\sqrt{\frac{8}{35}}$	$-\sqrt{\frac{2}{35}}$	$\sqrt{\frac{1}{5}}$
$-\frac{1}{2}, 1$		$\sqrt{\frac{20}{63}}$	$-\sqrt{\frac{14}{45}}$	$0$	$\sqrt{\frac{5}{21}}$	$-\sqrt{\frac{2}{15}}$
$-\frac{3}{2}, 2$		$\sqrt{\frac{5}{126}}$	$-\sqrt{\frac{64}{315}}$	$\sqrt{\frac{27}{70}}$	$-\sqrt{\frac{32}{105}}$	$\sqrt{\frac{1}{15}}$



**Alfred Clebsch**



Rudolf Friedrich Alfred Clebsch

**Paul Gordan**



Paul Gordan

$$d_{m,0}^\ell = \sqrt{\frac{4\pi}{2\ell+1}} Y_\ell^m e^{-im\phi}$$

nemški matematik, 1833 - 1872

nemški matematik, 1837 - 1912