SU(3) Flavour $\leftrightarrow$ Quark Hadron Model

baryons composed from u, d and s quarks:

$$\psi_{total} = \underbrace{\psi(flavor)\psi(spin)}_{\text{SU(3)}}\underbrace{\psi(color)}_{\text{SU(3)}}\psi(space)$$

$3\otimes3\otimes3 = 10\oplus8\oplus8\oplus1$

baryons: s=1/2, 3/2 $\rightarrow$ fermions $\rightarrow$ ψ_{total} asymmetric

ψ(flavor):

$$\psi_{S1} = |uuu\rangle, \psi_{S2} = |ddd\rangle$$

$$\frac{1}{\sqrt{3}}\left[|duu\rangle + |udu\rangle + |uud\rangle\right] = \psi_{S3},$$

$$\frac{1}{\sqrt{3}}\left[|udd\rangle + |dud\rangle + |ddu\rangle\right] = \psi_{S4}$$

ψ(flavor):

$$\frac{1}{\sqrt{3}}\left[|suu\rangle+|usu\rangle+|uus\rangle\right]=\psi_{S5}$$

$$\frac{1}{\sqrt{3}}\left[|sdd\rangle+|dsd\rangle+|dds\rangle\right]=\psi_{S6}$$

$$\frac{1}{\sqrt{6}}\left[|dus\rangle+|dsu\rangle+|sdu\rangle+|uds\rangle+|sud\rangle+|usd\rangle\right]=\psi_{S7}$$

$$\frac{1}{\sqrt{3}}\left[|sus\rangle+|uss\rangle+|ssu\rangle\right]=\psi_{S8}$$

$$\frac{1}{\sqrt{3}}\left[|sds\rangle+|dss\rangle+|ssd\rangle\right]=\psi_{S9}$$

$$\psi_{S10}=|sss\rangle$$

why ψ_{Sn} ?

wave f.'s symmetric on exchange of any two quarks;

10 wave f.'s

$3\otimes3\otimes3 = 10\oplus8\oplus8\oplus1$

ψ(flavor):

$$\psi_{A1} = \frac{1}{\sqrt{6}}\left[|uds\rangle - |usd\rangle + |dsu\rangle - |dus\rangle + |sud\rangle - |sdu\rangle\right]$$

- *asymmetric on exchange of any two quarks;*

$3\otimes3\otimes3 = 10\oplus8\oplus8\oplus1$

$$\psi_{MA1} = \frac{1}{\sqrt{2}}\left[|udu\rangle - |duu\rangle\right]$$

$$\vdots$$

$$\psi_{MA8}$$

- mixed symmetry (asym. on exc. first two quarks)

$3\otimes3\otimes3 = 10\oplus8\oplus8\oplus1$

$$\psi_{MS1} = \frac{1}{\sqrt{6}}\left[|udu\rangle + |duu\rangle - 2|uud\rangle\right]$$

$$\vdots$$

$$\psi_{MS8}$$

- mixed symmetry (sym. on exc. first two quarks)

$3\otimes3\otimes3 = 10\oplus8\oplus8\oplus1$

ψ(color):

exactly same structure (SU(3))

$$\psi_{A1} = \frac{1}{\sqrt{6}}\left[|uds\rangle - |usd\rangle + |dsu\rangle - |dus\rangle + |sud\rangle - |sdu\rangle\right]$$ *(see note (1))*

$$\psi_{A1} = \frac{1}{\sqrt{6}}\left[|RGB\rangle - |RBG\rangle + |GBR\rangle - |GRB\rangle + |BRG\rangle - |BGR\rangle\right]$$

ψ(spin):

SU(2) group

$2\otimes2\otimes2 = 4\oplus2\oplus2$

S MA MS

ψ(space):

symmetry $(-1)^{\ell}(-1)^{\ell'}$ *(see note (2))*

ℓ, ℓ' relative ang. momentum between quark pairs (=0 for ground states)

- *asymmetric on exchange of any two quarks;*

$3\otimes3\otimes3 = 10\oplus8\oplus8\oplus1$

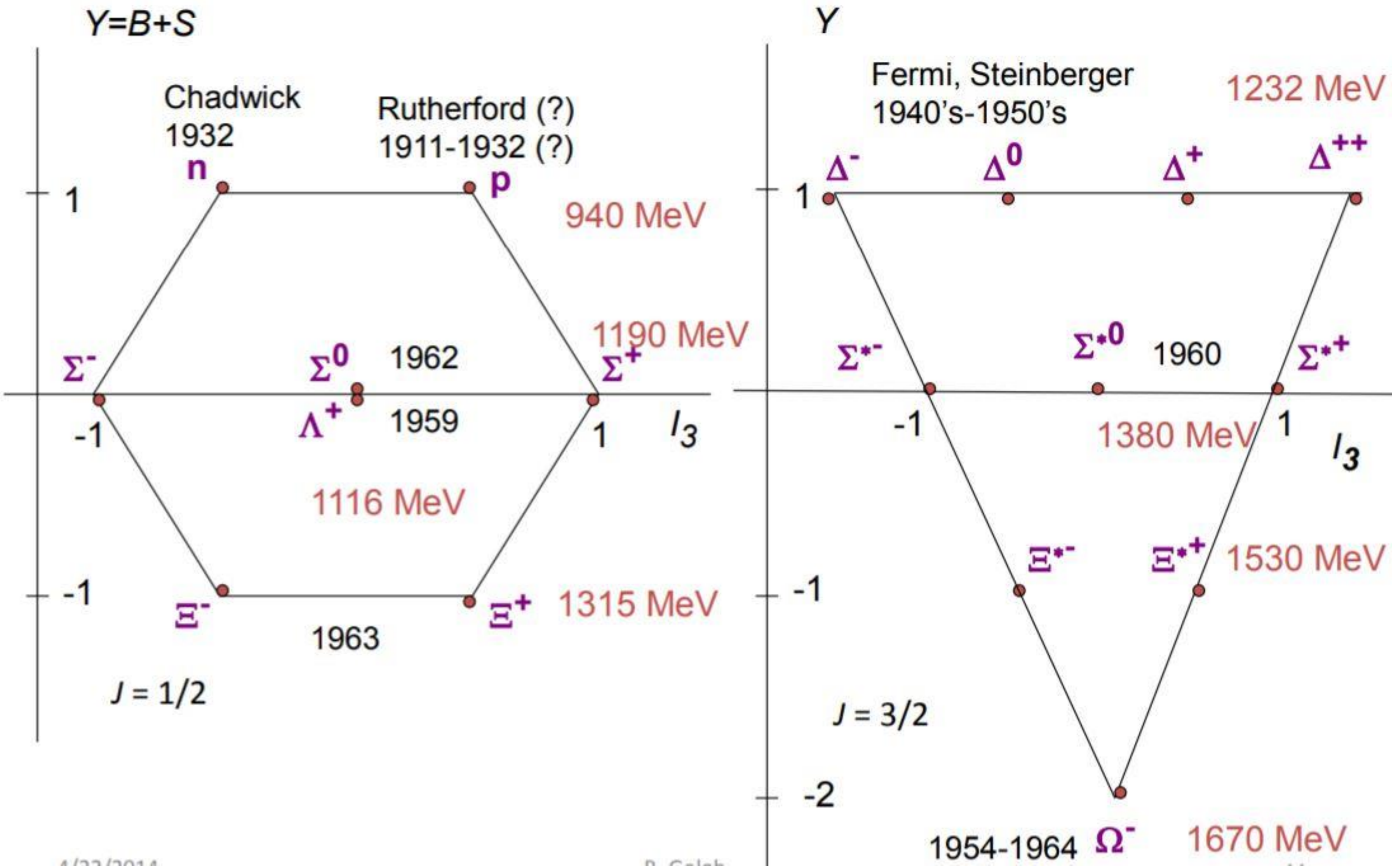
- hadrons are color neutral (carry no color charge); only singlet wave f. is appropriate

$$\psi_{total} = \psi_{S1-10}(flavor)\psi_{S1-4}(spin)\psi_{A1}(color)\psi_{S}(space) \rightarrow antisym.$$

m=-3/2, -1/2, 1/2, 3/2

$$\psi_{total} = a\psi_{MS1-8}(flavor)\psi_{MS1-2}(spin)\psi_{A1}(color)\psi_{S}(space) + b\psi_{MA1-8}(flavor)\psi_{MA1-2}(spin)\psi_{A1}(color)\psi_{S}(space) \rightarrow antisym.$$

m=-1/2, ½



Y=B+S
Chadwick
1932
Rutherford (?)
1911-1932 (?)
n
p
1
940 MeV
1190 MeV
Σ-
Σ0
1962
Σ+
Λ+
1959
-1
1
I3
1116 MeV
-1
Ξ-
Ξ+
1315 MeV
1963
J = 1/2
Y
Fermi, Steinberger
1940's-1950's
1232 MeV
Δ-
Δ0
Δ+
Δ++
1
Σ*-
Σ*0
1960
Σ*+
-1
1380 MeV
1
I3
Ξ*-
Ξ*+
1530 MeV
-1
J = 3/2
-2
1954-1964
Ω-
1670 MeV

note (1) Slater determinant (anti-symmetrization)

$$\begin{vmatrix} R(1) & G(1) & B(1) \\ R(2) & G(2) & B(2) \\ R(3) & G(3) & B(3) \end{vmatrix} = R(1)\big(G(2)B(3) - B(2)G(3)\big) + \ldots =$$

$$= \left| RGB \right\rangle - \left| RBG \right\rangle + \ldots$$

note (2) Symmetry of wave f.'s with orbital ang. momentum >0 (excited baryons) is actually a bit more complex

$$\vec{\rho} = \frac{1}{\sqrt{2}}\left(\vec{r}_1 - \vec{r}_2\right), \quad \vec{\lambda} = \frac{1}{\sqrt{6}}\left(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3\right)$$

ρ and λ symmetric upon $1 \leftrightarrow 2$, otherwise mixed symmetry

$$\psi(spatial; \ell = 0) \propto e^{-\kappa(\rho^2 + \lambda^2)}$$ symmetric

$\rho^2 + \lambda^2$: completely symmetric upon particle exchange ($\vec{r}_1 \leftrightarrow \vec{r}_2 \leftrightarrow \vec{r}_3$)

$$\psi(spatial; \ell = 1) \propto \rho e^{-(\kappa_1 \rho^2 + \kappa_2 \lambda^2)}$$ mixed symmetry, symmetric upon $1 \leftrightarrow 2$

excited baryons:

$$\psi_{total} = a\psi_{MS1-8}(flavor)\psi_{MS1-2}(spin)\psi_{A1}(color)\psi_{M}(space) + b\psi_{MA1-8}(flavor)\psi_{MA1-2}(spin)\psi_{A1}(color)\psi_{M}(space)$$

with $\psi(flavor)\psi(spin)\psi(spatial)$ totally symmetric