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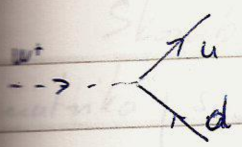
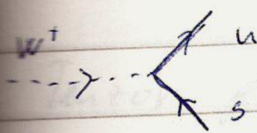


sklopitev sorazmerna $\cos\vartheta_c$



— " — — " — s $\sin\vartheta_c$

EW Lagrangian

Elektrošibki Lagrangian:

Za nevtrine vemo, da šibka interakcija sklaplja le levosučne. ~~Tudi elektronski spinor lahko razdelimo na levo in desno sučni del:~~ ~~$e(x)$:~~ Prav tako lahko operator Diracovega polja (glej kvantizac. fermionskega polja – $\psi(x,t)$) razdelimo na levo- in desno-sučno komponento:

left- and right-handed component of fermionic field

$$e(x) = e_L(x) + e_R(x)$$

pri čemer je
$$e_L(x) = \tfrac{1}{2}(1-\gamma^5)\,e(x)$$
$$e_R(x) = \tfrac{1}{2}(1+\gamma^5)\,e(x)$$

Če sedaj zanemarimo maso elektrona, tedaj ni, kar se tiče šibke interakcije, nobene razlike med nevtrinom in levo-sučno komp. elektrona. Imamo torej tri polja

$$\nu_{eL},\ e_L,\ e_R.$$

Če odmislimo interakcije lahko zapišemo

disregarding interactions the Lagrangian can be written as

Lagrangian v obliki

$$\mathcal{L}_0(x) = \left(\bar{\nu}_{eL}(x), \bar{e}_L(x)\right) i\gamma^\mu \partial_\mu \begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix} +$$

$$+ \bar{e}_R(x)\, i\gamma^\mu \partial_\mu e_R(x)$$

Tu smo poudarili simetrijo med e_L in ν_{eL}, e_R pa izpisali posebej. Z drugimi

L is written in a form to be invariant under SU(2) rotation in the (e_L, nu_L) space (weak isospin space)

besedami lahko rečemo, da je $\mathcal{L}_0$ invarian[ten] na poljubno SU(2) rotacijo v prostor[u] (e_L, ν_{eL}). (Temu prostoru lahko rečemo šib[ki] izospinski prostor in e_L pripišemo $T^3 = -\frac{1}{2}$, ν_{eL} pa $T^3 = +1/2$). $\mathcal{L}_0$ je torej inv. n[a] transf. oblike

$$\begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix} \longrightarrow U \begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix}$$

pri čemer je U kakršnakoli matrika S[U(2)] grupe, ki ni odvisna od x. $\mathcal{L}_0$ pa n[i]

L needs to be made invariant to local transformation U(x)

invarianten na lokalno transformacijo z $U(x)$. Da ga napravimo invariantnega moramo uvesti podobno kot pri EM vektorske potenciale, in sicer tri, kot j[e] število generatorjev v grupi SU(2).

similarly as with EM interaction the L can be made invariant to U(x) by introducing vector potentials (three = same as the number of SU(2) generators); corresponding fields are denoted by

Za generatorje si lahko izberemo Paulijeve matrike τ_1, τ_2 in τ_3. Ustrezna polja označimo z $W^1_\mu, W^2_\mu, W^3_\mu$.

fields are combined into Hermitian 2x2 matrix with trace =0

Skombiniramo jih v hermitsko 2×2 matriko s sledjo 0:

$$W_\lambda(x) = W_\lambda^a(x)\,\frac{\tau_a}{2}$$

Tenzor polja definiramo z

Field tensior is defined as

$$W_{\lambda\rho}(x) = \partial_\lambda W_\rho(x) - \partial_\rho W_\lambda(x) + ig\,[W_\lambda(x), W_\rho(x)] \equiv$$

$$\equiv W^a_{\lambda\rho}(x)\,\frac{\tau_a}{2}$$

pri čemer je potem

where

$$W^a_{\lambda\rho}(x) = \partial_\lambda W_\rho^a(x) - \partial_\rho W_\lambda^a(x) - g\,\varepsilon_{abc}\,W_\lambda^b(x)\,W_\rho^c(x)$$

ε_{abc} so strukturne konst. grupe $SU(2)$,

are the structure constants of SU(2)

g je gauge coupling konstanta.

is the gauge coupling constant

Lagrangian

$$\mathcal{L}(x) = \frac{1}{2}\,\mathrm{Tr}\left[W_{\lambda\rho}(x)\,W^{\lambda\rho}(x)\right] +$$

$$+ \left(\bar{\nu}_{eL}, \bar{e}_L\right) i\gamma^\lambda\left(\partial_\lambda + igW_\lambda\right)\begin{pmatrix}\nu_{eL}(x)\\ e_L(x)\end{pmatrix} +$$

$$+ \bar{e}_R(x)\, i\gamma^\lambda\partial_\lambda\, e_R(x)$$

L(x) is invariant to transformations of the form:

je invarianten na transformacije oblike

$$W_\lambda(x) \to U(x)\,W_\lambda(x)\,U^+(x) - \frac{i}{g}\,U(x)\,\partial_\lambda U^+(x)$$

$$\begin{pmatrix}\nu_{eL}(x)\\ e_L(x)\end{pmatrix} \to U(x)\begin{pmatrix}\nu_{eL}(x)\\ e_L(x)\end{pmatrix}$$

$$e_R(x) \to e_R(x)$$

Polji ν_{e_L} in e_L sestavljata šibki izospinski dublet, e_R pa singlet.

Sedaj definiramo One defines:

$$W_\lambda^{\pm} = \frac{1}{\sqrt{2}}\left(W_\lambda^1 \mp i W_\lambda^2\right)$$

~~Lagrangian~~ Člen Lagrangiana, ki nam da sklopitev med W_λ^i ter fermioni je

part of L(x) providing for couplings between W_lambda^i and fermions is

$$\mathcal{L}_{\nu e W} = -(\bar{\nu}_{e_L}, \bar{e}_L)\gamma^\mu g W_\mu \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} =$$

$$= -(\bar{\nu}_{e_L}, \bar{e}_L)\gamma^\mu g W_\mu^a \frac{\tau_a}{2} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} =$$

$$= -g(\bar{\nu}_{e_L}, \bar{e}_L)\gamma^\mu \cdot \frac{1}{2} \begin{pmatrix} W_\mu^3 & \sqrt{2}W_\mu^+ \\ \sqrt{2}W_\mu^- & -W_\mu^3 \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} =$$

$$= -\frac{g}{2}\Big[W_\mu^3 (\bar{\nu}_{e_L}\gamma^\mu \nu_{e_L} - \bar{e}_L \gamma^\mu e_L) +$$

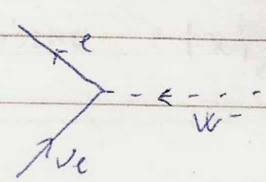
$$\boxed{+ \sqrt{2}\, W_\mu^+ \bar{\nu}_{e_L}\gamma^\mu e_L + \sqrt{2}\, W_\mu^- \bar{e}_L \gamma^\mu \nu_{e_L}} \Big]$$

Polje W_μ^- (W_μ^+) anihilira delec W^- (W^+) in kreira W^+ (W^-).

Field W_mu^- (W_mu^+) anihilates W- (W+) and creates W+ (W-).

V jeziku Feynmanovih diagramov nam te sklopitve opišejo transformacijo ν_e v e, pri čemer se absorbira en W^-; ~~tako...~~

sklopitev med e in ν že ima obliko $\gamma^\mu(1-\gamma^5)$, kakršno mora imeti



e nu coupling already has the gamma^mu (1-gamma^5) form kot tudi vse diagrame, ki jih dobimo s „križanjem" le tega

Problem opisanega Lagrangiana je v tem, da nam ne da masnih členov za W bozone. Prav tako v njem še ni vključena

This L does not provide for W masses. It also doesn't include EM interaction. W_mu^3 couples to nu_L and e_L and is hence not the photon field.

EM interakcija. W_μ^3 se namreč sklaplja z ν_L ter e_L, medtem ko se fotonsko polje sklaplja z e_L in e_R $\Rightarrow$ $W_\mu^3 \neq$ fotonsko polje. ~~EM ta~~ Lagrangian za EM interakcijo je

EM Lagrangian

$$\mathcal{L}'_{EM} = q\, \bar{e}(x) \gamma^\lambda e(x) A_\lambda(x) =$$

$$= q \left[\bar{e}_R(x) \gamma^\lambda e_R(x) + \bar{e}_L \gamma^\lambda e_L(x) \right] A_\lambda(x)$$

Za to, da vključimo EM int., si pogledamo poleg SU(2) invariantnosti za $\mathcal{L}_0$ še invarianco na dve U(1) transformaciji:

in order to include the EM interaction we make the L invariant to two U(1) transformations:

$$\begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix} \to e^{i\varphi} \begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix}$$

$$e_R(x) \to e^{i\varphi'} e_R(x)$$

Če sta φ in φ' konst. (različni) fazi, je $\mathcal{L}_0$ invariant. na to. Če bi sedaj zahtevali lokalno gauge invarianco za ti dve transf. bi dobili dve novi brezmasni vektorski polji. Potrebujemo pa le še eno (foton).

In order to make L gauge invariant we would need to introduce two new fileds, however, we only need one (photon).

Zato naredimo Lagrangian invarianten le na posebno kombinac. obeh transform.

Hence we make L invariant only to a special combination of the two transformations:

$$\begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix} \to e^{i y_L \chi} \begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix}$$

$$e_R(x) \to e^{i y_R \chi} e_R(x)$$

y_L and y_R are constants to be determined later.

y_L in y_R sta števili, ki ju bomo določili kasneje. Operator, ki generira to grupo transformacij, bomo imenovali šibki hipernaboj Y. Če vse tri fermione združimo v spinor $\psi(x)$ tedaj transformacijo $U(1)$ hipernabojske grupe zapišemo v obliki

Correpsonding generator for such a group of transformations is called the weak hypercharge Y.

$$\psi(x) = \begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \\ e_R(x) \end{pmatrix} \to e^{i\chi Y} \begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \\ e_R(x) \end{pmatrix} = e^{i\chi Y} \psi(x)$$

pri čemer je

$$Y = \begin{pmatrix} y_L & 0 & 0 \\ 0 & y_L & 0 \\ 0 & 0 & y_R \end{pmatrix}$$

to make L locally invariant we introduce a new field, B_mu, with gauge coupling g'

Da naredimo lagrangian inv. na lokalno transformacijo $U(1)$ postopamo analogno, kot pri QED. Upeljemo vektorsko polje B_λ in gauge coupling konstanto g'. Definiramo tenzor poljske jakosti:

$$B_{\lambda\varsigma} = \partial_\lambda B_\varsigma - \partial_\varsigma B_\lambda$$

Invarianten Lagrangian na $SU(2)$ in $U(1)$ transf. zapišemo kot

$$\mathcal{L}(x) = -\frac{1}{2} \mathrm{Tr}\left[W_{\lambda\rho}(x) W^{\lambda\rho}(x) \right] - \frac{1}{4} B_{\lambda\rho}(x) B^{\lambda\rho}(x) +$$

$$+ \bar{\Psi}(x) i \gamma^\lambda D_\lambda \Psi(x)$$

z kovariantnim odvodom

$$D_\lambda = \partial_\lambda + i g W_\lambda^a(x) T_a + i g' B_\lambda(x) Y$$

in matriko $T_a = \begin{pmatrix} \frac{1}{2}\tau_a & 0 \\ 0 & 0 \end{pmatrix}$ ← to je 3x3 matrika !

(this is 3x3 matrix!)

Matrike T_a in Y tvorijo reprezentacijo generatorjev grupe $SU(2) \times U(1)$, ker zadoščajo komutacijskim pravilom

T_a and Y matrices form a representation of SU(2) x U(1) group

$$[T_a, T_b] = i \varepsilon_{abc} T_c$$

$$[T_a, Y] = 0$$

Sedaj izpišemo sklopitvene člene iz $\mathcal{L}(x)$:

couplings from such Lagrangian:

$$\mathcal{L}'(x) = -\bar{\Psi}(x) \gamma^\lambda \left(g W_\lambda^a T_a + g' B_\lambda Y \right) \Psi(x) =$$

$$= \ldots = -\frac{g}{\sqrt{2}} \left(W_\lambda^+ \bar{\nu}_{eL} \gamma^\lambda e_L + W_\lambda^- \bar{e}_L \gamma^\lambda \nu_{eL} \right) -$$

$$- \frac{1}{2} \left(g W_\lambda^3 + 2 Y_L g' B_\lambda \right) \bar{\nu}_{eL} \gamma^\lambda \nu_{eL} +$$

$$+ \frac{1}{2} \left(g W_\lambda^3 - 2 Y_L g' B_\lambda \right) \bar{e}_L \gamma^\lambda e_L - Y_R g' B_\lambda \bar{e}_R \gamma^\lambda e_R$$

rearrangements of W_lambda^3 and B_lambda terms in order to obtain the EM interaction:

Sedaj želimo člene z W_λ^3 in B_λ preurediti tako, da nam bodo dali EM interakcijo. y_L si lahko izberemo poljubno,

we can choose y_L freely since it always enters together with g'

ker vedno nastopa skupaj z g', ki je tudi prost parameter. Izberemo si $y_L = -\frac{1}{2}$.

Iz W_λ^3 in B_λ tvorimo lin. kombinac., ki se sklaplja z nevtrini:

$$-\frac{1}{2}(g W_\lambda^3 + 2 y_L g' B_\lambda) \bar{\nu}_{e_L} \gamma^\lambda \nu_{e_L} \rightarrow$$

$$\rightarrow \quad Z_\lambda = \frac{1}{\sqrt{g^2+g'^2}} (g W_\lambda^3 - g' B_\lambda)$$

since nu's are chargless we can assume that Z_lambda is not a photon

Ker ν nima naboja lahko predvidevamo da v Z_λ ni nobene kompon. fotonskega polja. Fotonsko polje je torej ~~k~~ verjetno skrito v lin. komb. W_λ^3 in B_λ, ki je ortogonalna na Z_λ:

$$A_\lambda = \frac{1}{\sqrt{g^2+g'^2}} (g' W_\lambda^3 + g B_\lambda)$$

(v bazi W_λ^3, B_λ:

$$Z_\lambda A^\lambda = \frac{1}{g^2+g'^2} \Big(g g' W_\lambda^3 W^{\lambda 3} + g^2 W_\lambda^3 B^\lambda - g'^2 B_\lambda W^{\lambda 3} - g g' B_\lambda B^\lambda \Big) =$$

$$= \frac{1}{g^2+g'^2} (g g' - g g') = 0 \quad)$$

one defines the weak mixing angle:

Sedaj definiramo šibki mešalni kot :

$$\sin\vartheta_W = \frac{g'}{\sqrt{g^2+g'^2}} \qquad \cos\vartheta_W = \frac{g}{\sqrt{g^2+g'^2}}$$

$$\Downarrow$$

$$Z_\lambda = \cos\vartheta_W \cdot W_\lambda^3 - \sin\vartheta_W B_\lambda$$

$$A_\lambda = \sin\vartheta_W \cdot W_\lambda^3 + \cos\vartheta_W B_\lambda$$

$$\Downarrow \qquad B_\lambda = \cos\vartheta_W A_\lambda - \sin\vartheta_W Z_\lambda$$

$$W_\lambda^3 = \sin\vartheta_W A_\lambda + \cos\vartheta_W Z_\lambda$$

$$\mathcal{L}'(x) = -\frac{g}{\sqrt{2}}\left(W_\lambda^+ \bar{\nu}_{e_L}\gamma^\lambda e_L + W_\lambda \bar{e}_L \gamma^\lambda \nu_{e_L}\right) -$$

$$-\frac{1}{2}\sqrt{g^2+g'^2}\, Z_\lambda \bar{\nu}_{e_L}\gamma^\lambda \nu_{e_L} +$$

$$+\frac{1}{2} g(\sin\vartheta_W A_\lambda + \cos\vartheta_W Z_\lambda)\bar{e}_L\gamma^\lambda e_L +$$

$$+\frac{1}{2} g'(\cos\vartheta_W A_\lambda - \sin\vartheta_W Z_\lambda)\bar{e}_L\gamma^\lambda e_L -$$

$$- Y_R g'(\cos\vartheta_W A_\lambda - \sin\vartheta_W Z_\lambda)\bar{e}_R\gamma^\lambda e_R =$$

$$= -\frac{g}{\sqrt{2}}\left(W_\lambda^+ \bar{\nu}_{e_L}\gamma^\lambda e_L + W_\lambda^- \bar{e}_L\gamma^\lambda \nu_{e_L}\right) -$$

$$-\sqrt{g^2+g'^2}\, Z_\lambda\left[\frac{1}{2}\bar{\nu}_{e_L}\gamma^\lambda\nu_{e_L} - \frac{1}{2}\bar{e}_L\gamma^\lambda e_L -\right.$$

$$\left.-\sin^2\vartheta_W\left(-\bar{e}_L\gamma^\lambda e_L + Y_R\bar{e}_R\gamma^\lambda e_R\right)\right] -$$

$$-\frac{gg'}{\sqrt{g^2+g'^2}} A_\lambda\left(-\bar{e}_L\gamma^\lambda e_L + Y_R \bar{e}_R\gamma^\lambda e_R\right)$$

if we compare the above form with

To sedaj primerjamo z

$$\mathcal{L}'_{EM} = g\,[\bar{e}_R \gamma^\lambda e_R + \bar{e}_L \gamma^\lambda e_L]\, A_\lambda$$

we see that by choosing

Če postavimo $Y_R = -1$ in $\frac{gg'}{\sqrt{g^2+g'^2}} = g$ $(= e)$

from the terms with A_lambda we obtain the EM Lagrangian

dam da člen z A_λ v $\mathcal{L}'$ ravno EM Lagrangian.

Te zveze nam dajo

$$\boxed{e = g \sin\vartheta_W = g' \cos\vartheta_W}$$

Hence we got the photon filed and an additional neutral filed Z. However, there are still no W mass terms.

Na ta način smo dobili torej pravilne člene za fotonsko polje + dodatno neutralno polje Z. Še vedno pa nimamo masnih členov za W^a_λ in Z_λ. Če bi jih dodali na roko, bi kršili gauge invarianco $\Rightarrow$ nerenormalizabilna teorija (?.). Zato uvedemo še dodatna skalarna polja (Higgs).

For this additional scalar field (Higgs) needs to be added

Dodamo dve kompleksni skalarni polji

two complex scalar fields which form a weak isospin doublet

$$\phi(x) = \begin{pmatrix} \phi_1(x) \\ \phi_2(x) \end{pmatrix}$$

ki naj tvorita šibki isospinski dublet.

Lagrangian* za to polje je

Lagrangian for this field which is invariant to SU(2) transformation is

* (invarianten na SU(2) transf. $\phi(x) \to U\phi(x)$)

$$\mathcal{L}_\phi = T - V = (\partial_\mu \phi^+)(\partial^\mu \phi) - V(\phi)$$

$$V(\phi) = -\mu^2 \phi^+ \phi + \lambda (\phi^+ \phi)^2$$

$\mu^2, \lambda > 0$ izberemo tako, da bo prišlo do spontanega zloma simetrije (pojem, ki ga lahko pojasnimo tudi klasično – osnovno stanje sistema ne izkazuje simetrijskih lastnosti osnovnih enačb sistema).

this is selected in order to obtain the spontaneous symmetry breaking

Če želimo poiskati osnovno stanje za $\phi(x)$ moramo najti minimum potenciala $V(\phi)$.

Definiramo $\frac{\rho}{\sqrt{2}} = \sqrt{\phi^+ \phi}$ $\Rightarrow$

we define

$$\Rightarrow V(\phi) = -\frac{1}{2}\mu^2 \rho^2 + \frac{1}{4}\lambda \rho^4$$

V(phi) has a minimum at

minimum je pri

$$\rho_0 = \sqrt{\frac{\mu^2}{\lambda}}$$

To nam da samo pogoj za velikost polja v minimumu, njegove orientacije v izospinskem prostoru pa še ne poznamo. Zato lahko napišemo

this is only a condition for the magnitude of the field, while the orientation in the isospin space not yet determined. Hence we can write

$$\phi = e^{i \frac{\vec{\tau}}{2} \vec{\varphi}} \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \rho_0 \end{pmatrix}$$

vec{phi} is an arbitrary vector in isospin space (this is spontaneous symmetry breaking: while L_phi is invariant to SU(2) transformation, the individual eigenstates, like for example ....

$\vec{\varphi}$ je poljuben vektor v izospinskem prostoru (spontani zlom simetrije: medtem ko je $\mathcal{L}_\phi$ inv. na SU(2) transformacijo, ~~lastna~~ posamezna lastna stanja, npr. $\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \rho_0 \end{pmatrix}$, niso).

.... are not).

Sedaj skušamo napisati interakcijo Higgsovega polja z bozoni in fermioni. Struktura le-te mora biti taka, da bo invariantna na $SU(2) \times U(1)$ transf. v izospinskem in hiperšibkem prostoru $\Rightarrow$

we try to write interaction among Higgs, fermions and bosons; it has to be SU(2) x U(1) invariant

$$\Rightarrow \mathcal{L}_{Yuk} = -c_e \bar{e}_R \phi^+ \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} + h.c.$$

SU(2) transf.:

$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \to U(x) \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$$

$$e_R \to e_R$$

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \to U(x) \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

$$\mathcal{L}_{Yuk} \to -c_e \bar{e}_R (\phi_1^+, \phi_2^+) U^+ U \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} + h.c. = \mathcal{L}_{Yuk}$$

U(1) transf.:

$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \to e^{iY_L \chi(x)} \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$$

$$e_R \to e^{iY_R \chi(x)} e_R$$

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \to e^{iY_H \chi(x)} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

$$\mathcal{L}_{Yuk} \to -c_e e^{-iY_R \chi(x)} \bar{e}_R (\phi_1^+, \phi_2^+) e^{-iY_H \chi(x)} \cdot e^{iY_L \chi(x)} \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} + h.c. = \mathcal{L}_{Yuk}$$

if y_H = y_L - y_R

če $Y_H = Y_L - Y_R = \frac{1}{2}$

Sedaj še v $\mathcal{L}_\phi$ odvode ∂_μ nadomestimo s kovariantnimi $D_\mu = \partial_\mu + ig W_\lambda^a \frac{\tau_a}{2} + ig' B_\lambda Y_H$ in napišemo celotni $\mathcal{L}$:

$$\boxed{\mathcal{L} = -\frac{1}{2} Tr(W_{\lambda\rho} W^{\lambda\rho}) - \frac{1}{4} B_{\lambda\rho} B^{\lambda\rho} + (\bar{\nu}_{eL}, \bar{e}_L) i\gamma^\lambda D_\lambda \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} + \bar{e}_R i\gamma^\lambda D_\lambda e_R - c_e \bar{e}_R \phi^+ \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} - c_e^* (\bar{\nu}_{eL}, \bar{e}_L) \phi e_R + (D_\lambda \phi)^+ (D^\lambda \phi) - V(\phi)}$$

such L is invariant to SU(2) x U(1) transfrom. of the form

$\mathcal{L}$ je invarianten na transf. $SU(2) \times U(1)$, ki jih napišemo:

SU(2):

$$W_\lambda(x) \to U(x) W_\lambda(x) U^+(x) - \frac{i}{g} U(x) \partial_\lambda U^+(x)$$

$$B_\lambda(x) \to B_\lambda(x)$$

$$\begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix} \to U(x) \begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix}$$

$$e_R(x) \to e_R(x)$$

$$\begin{pmatrix} \phi_1(x) \\ \phi_2(x) \end{pmatrix} \to U(x) \begin{pmatrix} \phi_1(x) \\ \phi_2(x) \end{pmatrix}$$

~~Ker~~ Tu je $U(x) = e^{i \frac{\tau_a}{2} \varphi^a(x)}$

$\varphi^a(x)$ - poljubna funkc. x

$u(1)$:

$$W_\lambda(x) \to W_\lambda(x)$$

$$B_\lambda(x) \to B_\lambda(x) - \frac{1}{g'}\partial_\lambda \chi(x)$$

$$\begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix} \to e^{iY_L \chi(x)} \begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix}$$

$$e_R(x) \to e^{iY_R \chi(x)} e_R(x)$$

$$\begin{pmatrix} \phi_1(x) \\ \phi_2(x) \end{pmatrix} \to e^{iY_H \chi(x)} \begin{pmatrix} \phi_1(x) \\ \phi_2(x) \end{pmatrix}$$

$\chi(x)$ je poljubna funkc. x.

~~Kasneje bomo videli, da lahko Higgsovo polje napisano v obliki~~ Vedno obstaja $SU(2)$ transf., ki nam Higgsovo polje transf.:

we can always find a SU(2) transform. which:

$$U(x)\,\phi(x) = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\rho(x) \end{pmatrix}$$

Z drugimi besedami: vedno lahko zahtevamo, da je Higgsovo polje oblike

hence we can always require the Higgs field to be of the form:

$$\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\rho(x) \end{pmatrix},$$

Pričakovana vrednost operatorja polja za vakuumsko stanje (osnovno) je

vacuum expectation value

$$\langle 0|\phi(x)|0\rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\rho_0 \end{pmatrix}$$

torej:

$$\langle 0|\rho(x)|0\rangle = \rho_0$$

Sedaj uvedemo novo polje we introduce a new field:

$$\rho'(x) = \rho(x) - \rho_0,$$

with expectation value

ki ima pričakovano vrednost

$$\langle 0|\rho'(x)|0\rangle = 0$$ this enables a perturbative calculations

To omogoča perturbativni račun. Sedaj napišemo Lagrangian s tem novim poljem ρ':

EW Lagrangian

Elektro-šibki Lagrangian

$$\boxed{\begin{aligned}\mathcal{L} = & -\frac{1}{2}\mathrm{Tr}(W_{\lambda\rho}W^{\lambda\rho}) - \frac{1}{4}B_{\lambda\rho}B^{\lambda\rho} + (\bar\nu_{eL}, \bar e_L)\, i\gamma^\lambda D_\lambda \begin{pmatrix}\nu_{eL}\\ e_L\end{pmatrix} + \\ & + \bar e_R\, i\gamma^\lambda D_\lambda e_R - c_e \bar e_R \phi^+ \begin{pmatrix}\nu_{eL}\\ e_L\end{pmatrix} - c_e^* (\bar\nu_{eL}, \bar e_L)\phi e_R + \\ & + (D_\lambda\phi)^+(D^\lambda\phi) - V(\phi)\end{aligned}}$$

this should be written out, B_lambda and W_lambda^3 replaced by A_lambda and Z_lambda, and also Phi should be written out

to bi morali izpisati, B_λ in W_λ^3 nadomestiti z A_λ in Z_λ, namesto $\phi = \begin{pmatrix}0\\ \frac{1}{\sqrt2}\rho(x)\end{pmatrix}$ pisati $\begin{pmatrix}0\\ \frac{1}{\sqrt2}(\rho'(x)+\rho_0)\end{pmatrix}$ in upoštevati $D_\lambda = \partial_\lambda + ig W_\lambda^a \frac{\tau_a}{2} + ig' B_\lambda$.

Če pogledamo samo člene, ki prispevajo k Lagrangianu za proste elektrone: looking only at the terms for free electrons:

$$(\bar\nu_{eL}, \bar e_L)\, i\gamma^\lambda D_\lambda \begin{pmatrix}\nu_{eL}\\ e_L\end{pmatrix} + \bar e_R i\gamma^\lambda D_\lambda e_R - \cancel{c_e^*(\bar\nu_{eL}, \bar e_L)\phi}$$

$$- c_e \bar e_R \phi^+ \begin{pmatrix}\nu_{eL}\\ e_L\end{pmatrix} - c_e^* (\bar\nu_{eL}, \bar e_L)\phi e_R$$

Vsi ti členi prispevajo k $\mathcal{L}$ za prost elektron.

c_e always appears together with e_R;

Konst. c_e vedno nastopa skupaj z e_R;

since we have invarinace to transformations

ker je Lagrangian inv. na transf.

$e_R \to e^{i\varphi} e_R$ lahko tako transf. vedno naredimo. To je enakovredno temu, da c_e

we can always multiply c_e by ....

pomnožimo z $e^{-i\varphi}$. To pa pomeni, da

meaning that we can choose c_e real.

je faza c_e irelevantna in si lahko c_e izberemo realen. Sedaj zgornje člene izpišemo:

$$(\bar{\nu}_{eL}, \bar{e}_L)\, i\gamma^\lambda \big(\partial_\lambda + \underbrace{ig\, W_\lambda^a \tfrac{\tau_a}{2} + ig' B_\lambda}\big) \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} + \bar{e}_R\, i\gamma^\lambda (\;)\, e_R -$$

$$- c_e\, \bar{e}_R\, \Phi^+ \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} - c_e (\bar{\nu}_{eL}, \bar{e}_L)\, \Phi\, e_R$$

ta dva člena data sklopitev e^- z W_λ in B_λ $\Rightarrow$ ju izpustimo;

prav tako izpustimo člene z ν_{eL} $\Rightarrow$

$$\Rightarrow \bar{e}_L\, i\gamma^\lambda (\partial_\lambda e_L) + \bar{e}_R\, i\gamma^\lambda (\partial_\lambda e_R) -$$

$$- c_e\, \bar{e}_R \left(0, \frac{\rho' + \rho_0}{\sqrt{2}}\right) \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} - c_e (\bar{\nu}_{eL}, \bar{e}_L) \begin{pmatrix} 0 \\ \frac{\rho' + \rho_0}{\sqrt{2}} \end{pmatrix} e_R \Rightarrow$$

$$\Rightarrow \bar{e}_L\, i\gamma^\lambda (\partial_\lambda e_L) + \bar{e}_R\, i\gamma^\lambda (\partial_\lambda e_R) -$$

$$- \frac{c_e}{\sqrt{2}} (\rho' + \rho_0)\, \bar{e}_R e_L - \frac{c_e}{\sqrt{2}} (\rho' + \rho_0)\, \bar{e}_L e_R$$

členi z ρ' so sklopitev z Higgsom $\Rightarrow$ izpustimo $\Rightarrow$

$$\Rightarrow \bar{e}_L\, i\gamma^\lambda (\partial_\lambda e_L) + \bar{e}_R\, i\gamma^\lambda (\partial_\lambda e_R) -$$

$$- c_e \rho_0 \frac{1}{\sqrt{2}}\, \bar{e}_R e_L - c_e \rho_0 \frac{1}{\sqrt{2}}\, \bar{e}_L e_R$$

$e = e_R + e_L$

$e_L = \frac{1}{2}(1-\gamma^5)e \qquad \bar{e}_L = e_L^+\gamma^0 = e^+\frac{1}{2}(1-\gamma^{5+})\gamma^{0+} =$

$= \frac{1}{2}e^+\gamma^0(1+\gamma^5) = \frac{1}{2}\bar{e}(1+\gamma^5)$

$\bar{e}_L e_L = \frac{1}{4}\bar{e}(1+\gamma^5)(1-\gamma^5)e$

$(1+\gamma^5)(1-\gamma^5) = \begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix} = \phi \Rightarrow$

$\Rightarrow \bar{e}_L e_L = 0 \Rightarrow \quad \underline{\bar{e}e = \bar{e}_R e_L + \bar{e}_L e_R}$

$\bar{e}_L \gamma^\lambda(\partial_\lambda e_R) = \frac{1}{4}\bar{e}(1+\gamma^5)\gamma^\lambda(\partial_\lambda(1+\gamma^5)e) =$

$= \frac{1}{4}\bar{e}\,(1+\gamma^5)(1-\gamma^5)\gamma^\lambda(\partial_\lambda e) =$

$= 0 \Rightarrow \quad \bar{e}\gamma^\lambda(\partial_\lambda e) =$

$= \underline{\bar{e}_L\gamma^\lambda(\partial_\lambda e_L) + \bar{e}_R\gamma^\lambda(\partial_\lambda e_R)}$

$\Rightarrow \mathcal{L}_{\text{prost } e^-} = \bar{e}i\gamma^\lambda(\partial_\lambda e) - \frac{c_e g_0}{\sqrt{2}}\bar{e}e$

Lagrangian iz katerega z Euler-Lagrangeovimi enačbami dobimo Diracovo enačbo za prost e^-:

Lagrangian from which using the Euler-Lagrange equations
we get the Dirac equation for the free electron:

$$\mathcal{L} = \bar{\psi}(i\gamma^\lambda\partial_\lambda - m_e)\psi$$

hence

$$\Downarrow$$

$$\boxed{m_e = \frac{c_e g_0}{\sqrt{2}}}$$

similar could be done for Z_lambda and W_lambda

Podobno bi lahko naredili za W_λ in Z_λ; če izpišemo samo člen

$$(D_\lambda \phi)^+ (D^\lambda \phi) =$$

$$= (\partial_\lambda \Phi^+)(\partial^\lambda \Phi) + (\partial_\lambda \Phi^+)\, ig \begin{pmatrix} \frac{1}{2} W^{\lambda 3} & \frac{1}{\sqrt{2}} W^{\lambda -} \\ \frac{1}{\sqrt{2}} W^{\lambda +} & -\frac{1}{2} W^{\lambda 3} \end{pmatrix} \begin{pmatrix} 0 \\ \rho' + \rho_0 \end{pmatrix} +$$

$$+ (\partial_\lambda \Phi^+)\, ig' B^\lambda Y_H \Phi - ig \left[\begin{pmatrix} \frac{1}{2} W^3_\lambda & \frac{1}{\sqrt{2}} W^-_\lambda \\ \frac{1}{\sqrt{2}} W^+_\lambda & -\frac{1}{2} W^3_\lambda \end{pmatrix} \begin{pmatrix} 0 \\ \rho' + \rho_0 \end{pmatrix} \right]^+ (\partial^\lambda \Phi) -$$

$$- ig \left[\quad \right]^+ ig \begin{pmatrix} \frac{1}{2} W^3_\lambda & \frac{1}{\sqrt{2}} W^{\lambda -} \\ \frac{1}{\sqrt{2}} W^{\lambda +} & -\frac{1}{2} W^{\lambda 3} \end{pmatrix} \Phi -$$

$$- ig \left[\quad \right]^+ ig' B^\lambda Y_H \Phi - ig' B_\lambda Y_H \Phi^+ (\partial^\lambda \Phi) -$$

$$- ig' B_\lambda Y_H \Phi^+ ig \begin{pmatrix} \frac{1}{2} W^{\lambda 3} & \frac{1}{\sqrt{2}} W^{\lambda -} \\ \frac{1}{\sqrt{2}} W^{\lambda +} & -\frac{1}{2} W^{\lambda 3} \end{pmatrix} \Phi - ig' B_\lambda Y_H \Phi^+ ig' B^\lambda Y_H \Phi$$

$$= (\partial_\lambda \rho')(\partial^\lambda \rho') - ig \frac{1}{2} (\partial_\lambda \rho') W^{\lambda 3} (\rho' + \rho_0) +$$

$$+ \frac{1}{2} ig' (\partial_\lambda \rho') B^\lambda (\rho' + \rho_0) + ig \frac{1}{2} (\rho' + \rho_0) W^3_\lambda (\partial^\lambda \rho') +$$

$$+ g^2 (\rho' + \rho_0)^2 W^{\lambda +} W^-_\lambda + g^2 \cdot \frac{1}{4} (\rho' + \rho_0)^2 W^{\lambda 3} W^3_\lambda -$$

$$- \frac{1}{4} g g' (\rho' + \rho_0)^2 W^3_\lambda B^\lambda - \frac{1}{2} i g' B_\lambda (\rho' + \rho_0)(\partial^\lambda \rho') -$$

$$- \frac{1}{4} g g' B_\lambda W^3_\lambda (\rho' + \rho_0)^2 + \frac{1}{4} g'^2 (\rho' + \rho_0)^2 B_\lambda B^\lambda =$$

tu še treba vsakemu ρ' oz ρ_0 dodati še faktor $\frac{1}{\sqrt{2}}$ (narobe!) in vsakemu W^+_λ in W^-_λ prav tako

↓ tu je že prav

$$= \frac{1}{2}(\partial_\lambda \rho')(\partial^\lambda \rho') + \frac{g^2}{4}\rho'^2 W^{\lambda +} W_\lambda^- +$$

$$+ \frac{g^2}{2}\rho'\rho_0 W_\lambda^+ W_\lambda^- + \frac{g^2}{4}\rho_0^2 W_\lambda^+ W_\lambda^- +$$

$$+ \frac{1}{8}\rho'^2(g^2+g'^2) Z_\lambda Z^\lambda +$$

$$+ \frac{1}{4}\rho'\rho_0(g^2+g'^2) Z_\lambda Z^\lambda +$$

$$+ \frac{1}{8}\rho_0^2(g^2+g'^2) Z_\lambda Z^\lambda$$

$$Z_\lambda Z^\lambda = \frac{1}{g^2+g'^2}\left(g^2 W_\lambda^3 W^{\lambda 3} - 2gg' W_\lambda^3 B^\lambda + g'^2 B_\lambda B^\lambda\right)$$

if we would now take into account also the kinetic terms with W_lambda and Z_lambda
we would obtain the Lagrangian from which the Klein-Gordon equation
can be obtained.
From this we would be able to read: ....

Če bi sedaj upoštevali še kinetične člene (z odvodi) v katerih odstopajo W_λ in Z_λ (to so $\mathrm{Tr}(W_{\lambda\rho} W^{\lambda\rho})$ in $B_{\lambda\rho} B^{\lambda\rho}$) bi dobili Lagrangian, ki nam da Klein-Gordonovo enačbo. Iz le tega bi lahko razbrali:

....

$$\boxed{\begin{aligned} m_W^2 &= \frac{g^2\rho_0^2}{4} = \frac{e^2\rho_0^2}{4\sin^2\vartheta_W} \\ m_Z^2 &= \frac{(g^2+g'^2)\rho_0^2}{4} = \frac{e^2\rho_0^2}{4\sin^2\vartheta_W\cos^2\vartheta_W} \end{aligned}}$$

Podobno dobimo še

$$\boxed{m_{\rho'}^2 = 2\lambda\rho_0^2}$$

Za proste parametre modela lahko torej vzamemo

free parameters of the model:

$$\boxed{e,\ \sin\vartheta_W,\ m_e,\ m_W^2,\ m_{\rho'}^2}$$

iz njih izračunamo

$$\boxed{m_Z^2 = \frac{m_W^2}{\cos^2\vartheta_W} \qquad \rho_0 = 2m_W\sqrt{\frac{\sin^2\vartheta_W}{e^2}}}$$

Extension to other fermions

Razširitev na ostale fermione:

Vse fermione podobno kot elektr. razdelimo na levosučne izospinske dublete, ter desnosučne singlete:

			T	T^3	Y	Q
$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	$\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$ $-\frac{1}{2}$	$-\frac{1}{2}$ $-\frac{1}{2}$	0 -1
e_R	μ_R	τ_R	0	0	-1	-1
$\begin{pmatrix} u_L \\ d'_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s'_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b'_L \end{pmatrix}$	$\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$ $-\frac{1}{2}$	$1/6$ $1/6$	$2/3$ $-1/3$
u_R	c_R	t_R	0	0	$2/3$	$2/3$
d_R	s_R	b_R	0	0	$-1/3$	$-1/3$

Da vse fermione vključimo v $\mathcal{L}$ jih zapišemo v skupni spinor.

$$\psi = \begin{pmatrix} \nu_{eL} \\ e_L \\ e_R \\ \nu_{\mu L} \\ \vdots \\ b_R \end{pmatrix}$$

Še vedno se zadovoljimo s Higgsovim dubletom. Sklopitev le tega z bozoni in s samim seboj ostanejo enake, spremeniti pa moramo Yukawa sklopitve med fermioni in Higgsom.

$$\mathcal{L}_{Yuk} = \bar{\psi} \phi_i C_i \psi + h.c.$$

pri čemer je $i = 1,2$ (ϕ_i - komp. izosp. dubl.)

~~[illegible]~~ C_i sta zaenkrat poljubni matriki.

$\mathcal{L}$ je sedaj

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}(W_{\lambda\varsigma}W^{\lambda\varsigma}) - \frac{1}{4}B_{\lambda\varsigma}B^{\lambda\varsigma} + \bar{\Psi} i\gamma^\lambda D_\lambda \Psi +$$

$$+ \mathcal{L}_{Yuk} + (D_\lambda\phi)^+(D^\lambda\phi) + V(\phi)$$

Če sedaj pišemo $\phi = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(\varsigma' + \varsigma_0)\end{pmatrix} \Rightarrow$

$\Rightarrow$

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}(\quad) - \frac{1}{4}B_{\lambda\varsigma}B^{\lambda\varsigma} + W_\lambda^+ W^{-\lambda} m_W^2\left(1 + \frac{\varsigma'}{\varsigma_0}\right)^2 +$$

$$+ \frac{1}{2}Z_\lambda Z^\lambda m_Z^2\left(1 + \frac{\varsigma'}{\varsigma_0}\right)^2 + \bar{\Psi} i\gamma^\lambda D_\lambda \Psi - \bar{\Psi} M \Psi\left(1 + \frac{\varsigma'}{\varsigma_0}\right) +$$

$$+ \frac{1}{2}(\partial_\lambda \varsigma')(\partial^\lambda \varsigma') - \frac{1}{2}m_{\varsigma'}^2 \varsigma'^2\left[1 + \frac{\varsigma'}{\varsigma_0} + \frac{1}{4}\left(\frac{\varsigma'}{\varsigma_0}\right)^2\right]$$

To je posplošitev $\mathcal{L}$ za samo e; če bi slednjega napisali v tej celotni obliki, bi imeli

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}(\quad) - \frac{1}{4}B_{\lambda\varsigma}B^{\lambda\varsigma} + W_\lambda^+ W^{-\lambda} m_W^2\left(1 + \frac{\varsigma'}{\varsigma_0}\right)^2 +$$

$$+ \frac{1}{2}Z_\lambda Z^\lambda m_Z^2\left(1 + \frac{\varsigma'}{\varsigma_0}\right)^2 + \bar{e}_R i\gamma^\lambda D_\lambda e_R + (\bar{\nu}_{eL}, \bar{e}_L) i\gamma^\lambda D_\lambda \begin{pmatrix}\nu_{eL} \\ e_L\end{pmatrix}$$

$$- m_e \bar{e}e\left(1 + \frac{\varsigma'}{\varsigma_0}\right) + \frac{1}{2}(\partial_\lambda \varsigma')(\partial^\lambda \varsigma') -$$

$$- \frac{1}{2}m_{\varsigma'}^2 \varsigma'^2\left[1 + \frac{\varsigma'}{\varsigma_0} + \frac{1}{4}\left(\frac{\varsigma'}{\varsigma_0}\right)^2\right]$$

Matrika M v zgornjem $\mathcal{L}$ je $M = -(C_2 + C_2^+)\frac{\varsigma_0}{\sqrt{2}}$ in je torej posplošitev mase e : $m_e = \frac{c_e \varsigma_0}{\sqrt{2}}$

Sklopljanje fermionov z bozoni je skrito v kovariantnem odvodu

$$\bar{\psi} i\gamma^\lambda D_\lambda \psi = \bar{\psi} i\gamma^\lambda \partial_\lambda \psi + \mathcal{L}_{int}$$

$$\mathcal{L}_{int} = -\bar{\psi} i\gamma^\lambda \left(g W_\lambda^a T_a + g' B_\lambda Y\right)\psi$$

Če sedaj to razvijemo in izrazimo z Z_λ in A_λ, dobimo

$$\mathcal{L}_{int} = -e\left[A_\lambda J^\lambda_{em} + \frac{1}{\sin\vartheta_W \cos\vartheta_W} Z_\lambda J^\lambda_{NC} + \boxed{\frac{1}{\sqrt{2}\sin\vartheta_W}\left(W^+_\lambda J^\lambda_{CC} + W^-_\lambda J^{\lambda +}_{CC}\right)}\right].$$

kjer smo označili

$$J^\lambda_{em} = \bar{\psi}\gamma^\lambda (T_3 + Y)\psi = \bar{\psi}\gamma^\lambda Q \psi$$

$$J^\lambda_{NC} = \bar{\psi}\gamma^\lambda \left(T_3 - \sin^2\vartheta_W (T_3 + Y)\right)\psi = \bar{\psi}\gamma^\lambda T_3 \psi - \sin^2\vartheta_W J^\lambda_{em}$$

$$\boxed{J^\lambda_{CC} = \bar{\psi}\gamma^\lambda (T_1 + iT_2)\psi}$$

<u>CKM Matrika</u> : CKM matrix

Sedaj nas zanimajo Yukawa sklopitve, sklopitev med poljubnim in levosučnim fermionskim dubletom ter Higgsom lahko zapišemo na dva različna načina:

$$\Phi^+ \begin{pmatrix} \psi_{1L} \\ \psi_{2L} \end{pmatrix} = \phi_1^+ \psi_{1L} + \Phi_2^+ \psi_{2L}$$

ali

$$\Phi^T \varepsilon \begin{pmatrix} \psi_{1L} \\ \psi_{2L} \end{pmatrix} = \Phi_1 \psi_{2L} - \Phi_2 \psi_{1L}$$

ε - antisim. tenzor $= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Ti dve invarianti moramo kombinirati na vse ~~možne~~ ~~na~~ z desnosučnimi singleti.

$$\mathcal{L}_{Yuk} = -(\bar{e}_R, \bar{\mu}_R, \bar{\tau}_R)\, C_e \begin{pmatrix} \phi^+ \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \\ \phi^+ \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \\ \phi^+ \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} \end{pmatrix} +$$

$$+ (\bar{u}_R, \bar{c}_R, \bar{t}_R)\, C'_{\mathcal{Q}} \begin{pmatrix} \Phi^T \varepsilon \begin{pmatrix} u_L \\ d'_L \end{pmatrix} \\ \phi^T \varepsilon \begin{pmatrix} c_L \\ s'_L \end{pmatrix} \\ \phi^T \varepsilon \begin{pmatrix} t_L \\ b'_L \end{pmatrix} \end{pmatrix} -$$

$$- (\bar{d}'_R, \bar{s}'_R, \bar{b}'_R)\, C_{\mathcal{Q}} \begin{pmatrix} \phi^+ \begin{pmatrix} u_L \\ d'_L \end{pmatrix} \\ \phi^+ \begin{pmatrix} c_L \\ s'_L \end{pmatrix} \\ \phi^+ \begin{pmatrix} t_L \\ b'_L \end{pmatrix} \end{pmatrix} + \text{h.c.}$$

C_e, $C'_{\mathcal{Q}}$ in $C_{\mathcal{Q}}$ so poljubne matrike. Člena z C'_e ni, ker nimamo desnosučnih nevtrinov.

z unitarno transform. lahko spremenimo bazo desnosučnih leptonov:

$$\begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \to U_1 \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$$

Lagrangian se pri tem ne spremeni, razen Yukawinega člena. V slednjem je ~~pa~~ taka transformacija ekvivalentna spremembi

$$C_\ell \to U_1^+ C_\ell$$

Na analogen način lahko zamenjamo bazo leptonskim in kvarkovskim dubletom ter kvark singletom. Pri tem se matrike transformirajo:

$$C_\ell \to U_1^+ C_\ell V_1$$
$$C_q' \to U_2^+ C_q' V_2$$
$$C_q \to U_3^+ C_q V_2$$

← z desne moramo pomnožiti z enako matriko, ker v $\mathcal{L}_{yuk}$ desno od teh matrik stojijo ista polja.

To pomeni, da lahko matrike C pomnožimo z leve in desne s poljubnimi unitarnimi 3x3 matrikami. Pri tem se nič ne spremeni, saj samo menjamo bazo.

Če na omenjen način transformiramo $C_\ell C_\ell^+$ dobimo

$$c_\ell c_\ell^+ \to U_1^+ c_\ell c_\ell^+ U_1$$

Z ustrezno izbiro U_1 dosežemo diagonalizacijo

$$c_\ell c_\ell^+ = \begin{pmatrix} c_e^2 & & \phi \\ & c_\mu^2 & \\ \phi & & c_\tau^2 \end{pmatrix}$$

To pomeni, da lahko zapišemo c_ℓ v obliki

$$c_\ell = \begin{pmatrix} c_e & & \phi \\ & c_\mu & \\ \phi & & c_\tau \end{pmatrix} W, \text{ pri čemer je } W \text{ unitarna.}$$

Če sedaj še enkrat transformiramo bazo leptonskih dubletov (še enkrat množimo c_ℓ z desne z V_1) in si pri tem izberemo $V_1 = W^+$ dobimo

$$c_\ell = \begin{pmatrix} c_e & & \phi \\ & c_\mu & \\ \phi & & c_\tau \end{pmatrix}$$

Podobno lahko diagonaliziramo tudi c_2':

$$c_2' = \begin{pmatrix} c_u & & \phi \\ & c_c & \\ \phi & & c_t \end{pmatrix}$$

Pri c_2 je nekoliko drugače. Z desne jo moramo množiti z enako matriko kot c_2'. To pomeni, da jo lahko spravimo v obliko

$$c_2 = \begin{pmatrix} c_d & & \phi \\ & c_s & \\ \phi & & c_b \end{pmatrix} V^+,$$

pri čemer je V^+ matrika, s katero smo pri drugi transformac. c_2' le-to dokončno diagonalizirali. Lahko pa sedaj c_2 pomnožimo z leve z $U_3 = V^+$, da dobimo kanonično obliko:

$$c_2 = V \begin{pmatrix} c_d & & \phi \\ & c_s & \\ \phi & & c_b \end{pmatrix} V^+$$

Matriko V imenujemo CKM matrika.

Sedaj smo izkoristili še skoraj vso svobodo pri izbiri C matrik. C_2 in C_2' ~~tudi~~ ostaneta nespremenjeni le še pri transformac. z diagonalno unitarno matriko. Če postavimo

$$V_2 = U_2 = U_3 = U_\varphi = \begin{pmatrix} e^{i\varphi_1} & & \phi \\ & e^{i\varphi_2} & \\ \phi & & e^{i\varphi_3} \end{pmatrix}$$

je

$$C_2' \to U_\varphi^+ C_2' U_\varphi = C_2'$$

$$C_2 = V \begin{pmatrix} c_d & & \phi \\ & c_s & \\ \phi & & c_b \end{pmatrix} V^+ \to V' \begin{pmatrix} & & \end{pmatrix} V'^+$$

$$V' = U_\varphi^+ V$$

C_2 ostane nespremenjena tudi, če V pomnožimo z desne z $U_\chi = \begin{pmatrix} e^{i\chi_1} & & \phi \\ & e^{i\chi_2} & \\ \phi & & e^{i\chi_3} \end{pmatrix}$

Se pravi, da bosta c_2 in C_2' še vedno imeli zahtevano obliko, če naredimo

$$V \to U_\varphi^+ V U_\chi \; ;$$

Kaj to pomeni za V? Za primer poglejmo matrike 2x2 (samo dve družini fermionov).

$$V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} \to U_\varphi^+ V U_\chi = \begin{pmatrix} e^{-i(\varphi_1-\chi_1)} V_{11} & e^{-i(\varphi_1-\chi_2)} V_{12} \\ e^{-i(\varphi_2-\chi_1)} V_{21} & e^{-i(\varphi_2-\chi_2)} V_{22} \end{pmatrix}$$

Tri razlike faz lahko poljubno izberemo, četrta je fiksirana, ker $\varphi_2 - \chi_2 = (\varphi_2 - \chi_1) - (\varphi_1 - \chi_1) + (\varphi_1 - \chi_2)$

Tako lahko izberemo

$$V_{11} \geq 0$$
$$V_{12} \geq 0$$
$$V_{21} \leq 0$$

Unitarnost V-ja da dodatne zahteve:

$$|V_{11}|^2 + |V_{12}|^2 = 1$$

$$V_{11}^* V_{21} + V_{12}^* V_{22} = 0$$

$$|V_{21}|^2 + |V_{22}|^2 = 1$$

Vsem tem zahtevam lahko zadostimo z izbiro

$V_{11} = \cos\vartheta_C$; $V_{12} = \sin\vartheta_C$, $0 \leq \vartheta_C \leq \frac{\pi}{2}$ $\Rightarrow$

$$\Rightarrow \quad V = \begin{pmatrix} \cos\vartheta_C & \sin\vartheta_C \\ -\sin\vartheta_C & \cos\vartheta_C \end{pmatrix} \rightarrow$$ izkaže se, da je ϑ_C ravno Cabbibov kot.

V primeru dveh družin se lahko znebimo vseh kompleksnih faz. V primeru treh družin to ni mogoče. V parametriziramo s tremi koti in eno fazo:

$$V = \begin{pmatrix} c_1 & , s_1 c_3 & , s_1 s_3 \\ -s_1 c_2 & , c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & , c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ -s_1 s_2 & , c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & , c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix}$$

Sedaj napišimo Yukawa sklopitve ob upoštevanju

$\phi = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + s_0) \end{pmatrix}$ $\downarrow$

$$\mathcal{L}_{Yuk} = \Bigg[-(\bar{e}_R, \bar{\mu}_R, \bar{\tau}_R) \begin{pmatrix} c_e & & \phi \\ & c_\mu & \\ \phi & & c_\tau \end{pmatrix} \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} -$$

$$- (\bar{u}_R, \bar{c}_R, \bar{t}_R) \begin{pmatrix} c_u & & \phi \\ & c_c & \\ \phi & & c_t \end{pmatrix} \begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix} -$$

$$- (\bar{d}'_R, \bar{s}'_R, \bar{b}'_R) V \begin{pmatrix} c_d & & \phi \\ & c_s & \\ \phi & & c_b \end{pmatrix} V^+ \begin{pmatrix} d'_L \\ s'_L \\ b'_L \end{pmatrix} + h.c. \Bigg] \cdot \frac{\rho_0}{\sqrt{2}} \left(1 + \frac{\rho'}{\rho_0}\right)$$

$$= \Bigg[-(\bar{e}, \bar{\mu}, \bar{\tau}) \begin{pmatrix} m_e & & \phi \\ & m_\mu & \\ \phi & & m_\tau \end{pmatrix} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} - (\bar{u}, \bar{c}, \bar{t}) \begin{pmatrix} m_u & & \phi \\ & m_c & \\ \phi & & m_t \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix} -$$

$$\boxed{- (\bar{d}', \bar{s}', \bar{b}') V \begin{pmatrix} m_d & & \phi \\ & m_s & \\ \phi & & m_b \end{pmatrix} V^+ \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} \Bigg] \left(1 + \frac{\rho'}{\rho_0}\right)}$$

pri tem smo pisali $m_i = \frac{c_i \rho_0}{\sqrt{2}}$; to konst. lahko, kot smo to naredili že za e^-, identificiramo z masami posameznih fermionov; upoštevali smo še $\bar{e}e = \bar{e}_R e_L + \bar{e}_L e_R$ (smo že dokazali).

Vidimo pa, da ker V ni enotska matrika $\Rightarrow$ $\Rightarrow$ polja d', s' in b' ~~niso~~ ne predstavljajo delcev z dobro definirano maso (ko zmnožimo $(\bar{d}', \bar{s}', \bar{b}') V (\quad) V^+ \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}$ dobimo diagonalne + druge člene, npr. $\bar{d}'s'$ itd.; iz teh členov ne bi dobili Diracove enačbe za posamezen fermion).

d', s' in b' so le izospinski partnerji polj u, c, t; dobro definirano mešanje pa imajo

$$\begin{pmatrix}d\\s\\b\end{pmatrix} = V^{+}\begin{pmatrix}d'\\s'\\b'\end{pmatrix}$$

Zadnji člen v $\mathcal{L}_{Yuk}$ je potem

$$(\bar d, \bar s, \bar b)\begin{pmatrix} m_d & & 0\\ & m_s & \\ 0 & & m_b\end{pmatrix}\begin{pmatrix}d\\s\\b\end{pmatrix} \Rightarrow$$

$\Rightarrow$ dobimo samo diag. člene, npr. $\bar d d$.

Sedaj poglejmo, kako izgleda nabiti tok:

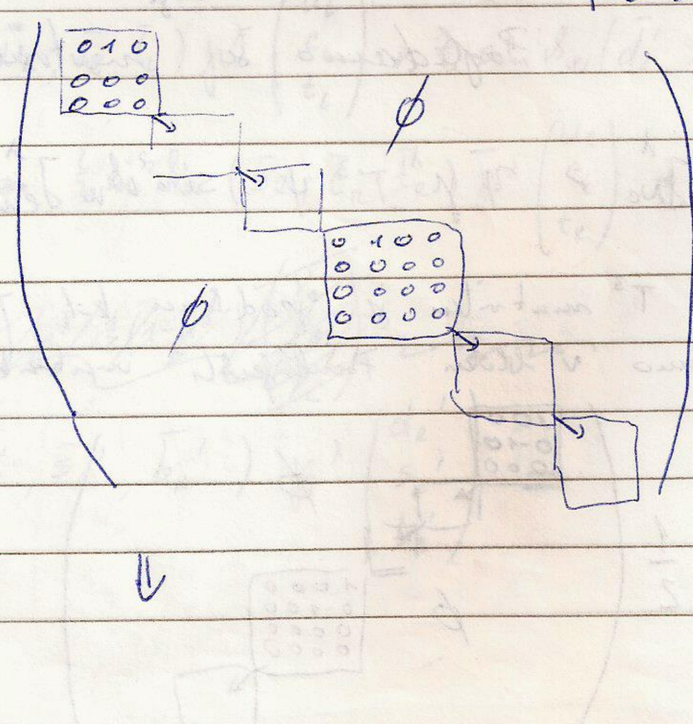
$$J^{\lambda}_{cc} = \bar\Psi \gamma^{\lambda}(T_1 + iT_2)\Psi$$

$T_1 + iT_2$ matrika za 3 člane $\begin{pmatrix}\nu_{eL}\\e_L\\e_R\end{pmatrix}$: $\begin{pmatrix}0&1&0\\0&0&0\\0&0&0\end{pmatrix}$

za $\begin{pmatrix}u_L\\d'_L\\u_R\\d'_R\end{pmatrix}$ bo ta matrika izgledala

$$\begin{pmatrix}\frac{1}{2}(\tau_1 + i\tau_2) & 0\\ 0 & 0\end{pmatrix} = \begin{pmatrix}0&1&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}$$

Za celoten Ψ:



$\Downarrow$

$$J_{cc}^{\lambda} = \bar{\Psi}\gamma^{\lambda}\begin{pmatrix} e_L \\ 0 \\ 0 \\ \mu_L \\ \vdots \\ d'_L \\ 0 \\ 0 \\ 0 \\ \vdots \end{pmatrix} = \bar{\nu}_{eL}\gamma^{\lambda}e_L + \bar{\nu}_{\mu L}\gamma^{\lambda}\mu_L + \ldots + \bar{u}_L\gamma^{\lambda}d'_L + \ldots + \bar{t}_L\gamma^{\lambda}b'_L =$$

$$= (\bar{\nu}_{eL}, \bar{\nu}_{\mu L}, \bar{\nu}_{\tau L})\gamma^{\lambda}\begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} + (\bar{u}_L, \bar{c}_L, \bar{t}_L)\gamma^{\lambda}\begin{pmatrix} d'_L \\ s'_L \\ b'_L \end{pmatrix} =$$

$$\boxed{= (\bar{\nu}_{eL}, \bar{\nu}_{\mu L}, \bar{\nu}_{\tau L})\gamma^{\lambda}\begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} + (\bar{u}_L, \bar{c}_L, \bar{t}_L)\gamma^{\lambda} V\begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}}$$

$\Downarrow$

u se sklaplja z d sorazmerno z V_{11}, itd.

V leptonskem delu nabitega toka ni mixing matrike, kar je posledica dejstva, da ν nimajo mase. Če bi jo imeli, bi imeli podobno mixing matriko tudi tam. To bi med drugim povzročilo tudi nevtrinske oscilacije.

Poglejamo še nevtralni tok:

$$J_{NC}^{\lambda} = \bar{\Psi}\gamma^{\lambda}T^3\Psi - \sin^2\theta_W J_{em}^{\lambda}$$

T^3 matrika je podobna kot T_+ ali T_-, le da imamo v blokih Paulijevih matrik $\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$\frac{1}{2}\begin{pmatrix} \begin{smallmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{smallmatrix} & & & \emptyset \\ & \ddots & & \\ & & \begin{smallmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{smallmatrix} & \\ \emptyset & & & \ddots \end{pmatrix}$$

$$J_{em}^{\lambda} = \bar{\psi}\,\gamma^{\lambda} Q\,\psi$$

$$\Downarrow$$

$$J_{NC}^{\lambda} = \frac{1}{2}\bar{\psi}\,\gamma^{\lambda}\begin{pmatrix} \nu_{eL} \\ -e_L \\ 0 \\ \nu_{\mu L} \\ -\mu_L \\ 0 \\ \vdots \\ u_L \\ -d_L' \\ 0 \\ 0 \\ \vdots \end{pmatrix} - \sin^2\vartheta_W\, \cancel{J_{em}^{\lambda}}\, \bar{\psi}\,\gamma^{\lambda} Q\,\psi =$$

$$= \frac{1}{2}\Big(\bar{\nu}_{eL}\gamma^{\lambda}\nu_{eL} - \bar{e}_L\gamma^{\lambda}e_L + \bar{\nu}_{\mu L}\gamma^{\lambda}\nu_{\mu L} - \bar{\mu}_L\gamma^{\lambda}\mu_L + \ldots +$$

$$+ \bar{u}_L\gamma^{\lambda}u_L - \bar{d}_L'\gamma^{\lambda}d_L' + \ldots\Big) - \sin^2\vartheta_W\Big[-\bar{e}_L\gamma^{\lambda}e_L - \bar{\mu}_L\gamma^{\lambda}\mu_L -$$

$$- \ldots + \frac{2}{3}\bar{u}_L\gamma^{\lambda}u_L - \frac{1}{3}\bar{d}_L'\gamma^{\lambda}d_L' + \ldots + \frac{2}{3}\bar{u}_R\gamma^{\lambda}u_R + \ldots\Big] =$$

$$= \frac{1}{2}\left(\bar{\nu}_{eL}, \bar{\nu}_{\mu L}, \bar{\nu}_{\tau L}\right)\gamma^{\lambda}\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L}\end{pmatrix} - \frac{1}{2}\left(\bar{e}_L, \bar{\mu}_L, \bar{\tau}_L\right)\gamma^{\lambda}\begin{pmatrix} e_L \\ \mu_L \\ \tau_L\end{pmatrix} +$$

$$\cancel{+\sin^2\vartheta_W\left(\bar{e}_L, \bar{\mu}_L, \bar{\tau}_L\right)\gamma^{\lambda}\begin{pmatrix} e_L \\ \mu_L \\ \tau_L\end{pmatrix}} + \frac{1}{2}\left(\bar{u}_L, \bar{c}_L, \bar{t}_L\right)\gamma^{\lambda}\begin{pmatrix} u_L \\ c_L \\ t_L\end{pmatrix} -$$

$$- \frac{1}{2}\left(\bar{d}_L', \bar{s}_L', \bar{b}_L'\right)\gamma^{\lambda}\begin{pmatrix} d_L' \\ s_L' \\ b_L'\end{pmatrix} + \sin^2\vartheta_W\left(\bar{e}_L, \bar{\mu}_L, \bar{\tau}_L\right)\gamma^{\lambda}\begin{pmatrix} e_L \\ \mu_L \\ \tau_L\end{pmatrix} -$$

$$- \frac{2}{3}\sin^2\vartheta_W\left(\bar{u}_L, \bar{c}_L, \bar{t}_L\right)\gamma^{\lambda}\begin{pmatrix} u_L \\ c_L \\ t_L\end{pmatrix} + \frac{1}{3}\sin^2\vartheta_W\left(\bar{d}_L', \bar{s}_L', \bar{b}_L'\right)\cdot$$

$$\gamma^{\lambda}\begin{pmatrix} d_L' \\ s_L' \\ b_L'\end{pmatrix} - \frac{2}{3}\sin^2\vartheta_W\left(\bar{u}_R, \bar{c}_R, \bar{t}_R\right)\gamma^{\lambda}\begin{pmatrix} u_R \\ c_R \\ t_R\end{pmatrix} +$$

$$\cancel{= \frac{1}{2}\left(\bar{\nu}_{eL}, \bar{\nu}_{\mu L}, \bar{\nu}_{\tau L}\right)\gamma^{\lambda}\frac{1}{2}(1-\gamma^5)\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau\end{pmatrix}}$$

$$+ \frac{1}{3}\sin^2\vartheta_W\left(\bar{d}_R', \bar{s}_R', \bar{b}_R'\right)\gamma^{\lambda}\begin{pmatrix} d_R' \\ s_R' \\ b_R'\end{pmatrix} =$$

$\leftarrow \frac{1}{2}(\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau)\,\frac{1}{2}(1-\gamma^5)\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \frac{1}{...}$

$\bar{\nu}_L \gamma^\lambda \nu_L =$

$= \frac{1}{2}\bar{\nu}(1+\gamma^5)\gamma^\lambda \frac{1}{2}(1-\gamma^5)\nu =$

$= \frac{1}{4}\bar{\nu}(1+\gamma^5)^2\gamma^\lambda \nu =$

$= \frac{1}{4}\bar{\nu}(1+2\gamma^5+\gamma^{5\,2})\gamma^\lambda \nu =$

$= \frac{1}{2}\bar{\nu}(1+\gamma^5)\gamma^\lambda \nu =$

$= \frac{1}{2}\bar{\nu}\gamma^\lambda(1-\gamma^5)\nu$

$\bar{e}\gamma^\lambda e = \bar{e}_L\gamma^\lambda e_L + \bar{e}_R\gamma^\lambda e_R$

$(\bar{d}'\bar{s}'\bar{b}')\gamma^\lambda\begin{pmatrix}d'\\ s'\\ b'\end{pmatrix} =$

$= (\bar{d},\bar{s},\bar{b})V^+\gamma^\lambda V\begin{pmatrix}d\\ s\\ b\end{pmatrix} =$

$= (\bar{d},\bar{s},\bar{b})\gamma^\lambda\begin{pmatrix}d\\ s\\ b\end{pmatrix}$

$= \frac{1}{2}(\bar{\nu}_e,\bar{\nu}_\mu,\bar{\nu}_\tau)\frac{1}{2}\gamma^\lambda(1-\gamma^5)\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} -$

$- \frac{1}{2}(\bar{e},\bar{\mu},\bar{\tau})\frac{1}{2}\gamma^\lambda(1-\gamma^5)\begin{pmatrix}e\\ \mu\\ \tau\end{pmatrix} +$

$+ \frac{1}{2}(\bar{u},\bar{c},\bar{t})\frac{1}{2}\gamma^\lambda(1-\gamma^5)\begin{pmatrix}u\\ c\\ t\end{pmatrix} -$

$- \frac{1}{2}(\bar{d}',\bar{s}',\bar{t}')\frac{1}{2}\gamma^\lambda(1-\gamma^5)\begin{pmatrix}d'\\ s'\\ t'\end{pmatrix} +$

$+ \sin^2\vartheta_W(\bar{e},\bar{\mu},\bar{\tau})\gamma^\lambda\begin{pmatrix}e\\ \mu\\ \tau\end{pmatrix} -$

$- \frac{2}{3}\sin^2\vartheta_W(\bar{u},\bar{c},\bar{t})\gamma^\lambda\begin{pmatrix}u\\ c\\ t\end{pmatrix} +$

$+ \frac{1}{3}\sin^2\vartheta_W(\bar{d}',\bar{s}',\bar{b}')\gamma^\lambda\begin{pmatrix}d'\\ s'\\ t'\end{pmatrix} \Rightarrow$

$= \cancel{\nu_L\alpha}$

$$\Rightarrow J^\lambda_{NC} = (\bar{\nu}_e,\bar{\nu}_\mu,\bar{\nu}_\tau)\frac{1}{2}\gamma^\lambda\frac{1}{2}(1-\gamma^5)\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} +$$

$$+ (\bar{e},\bar{\mu},\bar{\tau})\gamma^\lambda\left[-\frac{1}{2}\cdot\frac{1}{2}(1-\gamma^5)+\sin^2\vartheta_W\right]\begin{pmatrix}e\\ \mu\\ \tau\end{pmatrix} +$$

$$+ (\bar{u},\bar{c},\bar{t})\gamma^\lambda\left[\frac{1}{2}\cdot\frac{1}{2}(1-\gamma^5)-\frac{2}{3}\sin^2\vartheta_W\right]\begin{pmatrix}u\\ c\\ t\end{pmatrix} +$$

$$+ (\bar{d},\bar{s},\bar{b})\gamma^\lambda\left[-\frac{1}{2}\cdot\frac{1}{2}(1-\gamma^5)+\frac{1}{3}\sin^2\vartheta_W\right]\begin{pmatrix}d\\ s\\ b\end{pmatrix}$$