

## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Analitična mehanika
<b>Course title:</b>	Analytical mechanics

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** še ni dodeljena/not assigned yet

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:** doc. dr. George Mejak

**Jeziki / Languages:**

<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

**Prerequisites:**

Vpis v letnik študija

Enrollment into the program

**Vsebina:**

**Content (Syllabus outline):**

**Lagrangeeva mehanika:** Konfiguracijski prostor. Holonomni, neholonomni sistemi vezi. Princip virtualnega dela. D'Alembertov princip. Lagrangeeve enačbe. Konstante gibanja; ciklične spremenljivke, Jacobijeva energijska funkcija, izrek Emmy Noether. Variacijski princip. Majhna nihanja okoli ravnovesne lege. Posplošen potencial.

**Hamiltonova mehanika:** Legendrova transformacija. Hamiltonova funkcija, kanonski sistem. Poissonov oklepaj; odvajanje vzdolž rešitve kanonskega sistema; konstante gibanja, Poissonov izrek. Kanonska transformacija; simplektična matrika, simplektični pogoji. Rodovne funkcije. Hamilton-Jacobijeva enačba

**Lagrangian mechanics:** Configurational space. Holonomic and nonholonomic constraints. Principle of virtual work. D'Alembert principle. Lagrangian equations. Constant of motion. Cyclic variables, Jacobi energy function, Emmy-Noether theorem. Variational principles. Small oscillations. Generalized potential.

**Hamiltonian mechanics:** Legendre transformation. Hamiltonian function, canonical system. Poisson bracket, differentiation along solution of the canonical system, integrals of motion, Poisson theorem. Canonical transformation, symplectic matrix, symplectic condition. Generating functions. Hamilton-Jacobi equation.

### Temeljni literatura in viri / Readings:

- V. I. Arnold: *Mathematical Methods of Classical Mechanics*, 2nd edition, Springer, New York, 1997.
- H. Goldstein, C. P. Poole, J. L. Safko: *Classical Mechanics*, 3rd edition, Addison-Wesley, Reading, 2002.
- A. Fasano, S. Marmi, *Analytical Mechanics: An Introduction*, Oxford University Press, Oxford, 2006
- J. V. José, E. J. Saletan: *Classical Dynamics : A Contemporary Approach*, Cambridge Univ. Press, Cambridge, 1998.

### Cilji in kompetence:

Cilj predmeta je pridobiti osnovna znanja s področja analitične mehanike. Vsebine predmeta omogočajo uspešno reševanje dinamičnih problemov in ponazarjajo uporabo različnih matematičnih področij pri reševanju problemov s področja mehanike.

### Objectives and competences:

The goal is to obtain basic knowledge of principles of analytical mechanics. Mastering them enables problem solving of dynamical problems and to understand the role of mathematics in mechanics

### Predvideni študijski rezultati:

**Znanje in razumevanje:** Poznavanje in razumevanje osnovnih metod analitične mehanike

**Uporaba:** Osnova za nadgraditev osvojenega znanja s specifičnimi modeli iz področja klasične mehanike. Temelj za nadaljnji

### Intended learning outcomes:

**Knowledge and understanding:** Knowledge and understanding of basic principles and methods of analytical mechanics.

**Application:** Application of the learnt methods in solving dynamical real word problems. First step for further graduate level study of

poglobljeni študij metod klasične in relativistične mehanike.

**Refleksija:** Povezovanje osvojenega matematičnega znanja v okviru enega predmeta in njegova uporaba na področju analitične mehanike.

**Prenosljive spretnosti – niso vezane le na en predmet:** študent razvija sposobnost predstavitve problema na jasn in logičen način. Nauči se formulirati problem, izbrati ustrezeni model, analizirati rešitev in preveriti veljavnost modela in rešitve.

methods of classical and relativistic mechanics.

**Reflection:** Crossbreeding of different mathematical subjects within a single course and their application.

**Transferable skills:** Students develop abilities to clearly and logically formulate problems. They learn to critically assess modeling by analyzing their predictions and comparing them with real examples.

#### Metode poučevanja in učenja:

predavanja, vaje, seminar, domače naloge, konzultacije

#### Learning and teaching methods:

Lectures, exercises, seminar, homeworks, consultations

Delež (v %) /

Weight (in %) **Assessment:**

#### Načini ocenjevanja:

Način (pisni izpit, ustno izpraševanje, naloge, projekt):	Delež (v %) / Weight (in %)	Assessment:
<ul style="list-style-type: none"><li>izpit iz vaj (2 kolokvija ali pisni izpit)</li><li>ustni izpit</li></ul>	50%	Type (examination, oral, coursework, project): <ul style="list-style-type: none"><li>2 midterm exams instead of written exam, written exam</li><li>oral exam</li></ul>
Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)	50%	Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

#### Reference nosilca / Lecturer's references:

doc. dr. George Mejak

MEJAK, George. On extension of functions with zero trace on a part of boundary. *J. math. anal. appl.*, 1993, let. 175, str. 305-314

MEJAK, George. Finite element solution of a model free surface problem by the optimal shape design approach. *Int. j. numer. methods eng.*, 1997, vol. 40, str. 1525-1550.

MEJAK, George. Eshelby tensors for a finite spherical domain with an axisymmetric inclusion. *Eur. j. mech. A, Solids*. [Print ed.], 2011, vol. 30, iss. 4, str. 477-490.

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

**Predmet:** Dinamični sistemi  
**Course title:** Dynamical systems

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type**

**Univerzitetna koda predmeta / University course code:**

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:**

**Jeziki / Languages:**  
**Predavanja / Lectures:** slovenski/Slovene, angleški/English  
**Vaje / Tutorial:** slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**  
potrebno predznanje linearne algebre, diferencialnih enačb, topologije v evklidskih prostorih

**Prerequisites:**  
Linear algebra, differential equations, topology in euclidean spaces.

**Vsebina:**

**Content (Syllabus outline):**

*Kvalitativna analiza sistemov nelinearnih diferencialnih enačb.* Osnovni izreki o eksistenci in enoličnosti rešitev za sisteme (ponovitev in dopolnitev).

*Fazni portret avtonomnega sistema.* Klasifikacija kritičnih točk, izrek Hartmana in Grobmana o linearizaciji, teorija stabilnosti, metoda Ljapunova.

*Periodična gibanja in cikli v ravnini.* Poincaré-Bendixsonova teorija (topološke osnove, izpeljava in uporaba), izrek Kolmogorova, Hopfova bifurkacija in nastanek ciklov, uvod v kaotično gibanje.

*Osnove diskretne dinamike.* Diferenčne enačbe. Logistična enačba. Klasifikacija fiksnih točk. Podvajanje period in kaos. Heteroklinične orbite in Smalova podkev. Polinomska iteracija v kompleksnem. Juliajeva in Fatoujeva množica. Mandelbrotova množica.

*Uporaba v fiziki, medicini, biologiji, ekonomiji, elektrotehniki.*

*Qualitative analysis of systems of nonlinear differential equations.* Basic existence and uniqueness theorems for systems (repetition and completion)

*Phase portraits of autonomous systems.* Classification of critical points, Hartman-Grobman linearization theorem, stability theory, Lyapunov method.

*Periodic motions and cycles in the real plane.* Poincaré-Bendixson theory (topological background, proof and examples), Kolmogorov theorem, Hopf bifurcation and emerging of cycles, introduction to chaotic motion.

*Basic discrete dynamics.* Difference equations. The logistic equation. Classification of fixed points. Period doubling and chaos. Heteroclinic orbits and Smale horseshoe. Polynomial iteration in the complex plane. Julia, Fatou and Mandelbrot sets.

*Examples from physics, medicine, biology, economy, electrical engineering.*

### Temeljni literatura in viri / Readings:

Gerald Teschl, *Ordinary Differential Equations*, Graduate Studies in Mathematics, Volume 140, Amer. Math. Soc., Providence, 2012.

Boris Hasselblatt, Anatole Katok, *A first course in dynamics : with a panorama of recent development*, Cambridge University Press, 2003.

L. Perko: *Differential equations and dynamical systems*, 3rd edition, Springer, New York, 2001.

C. Robinson: *Dynamical Systems, Stability, Symbolic Dynamics and Chaos*, CRC Press 1999.

D.K. Arrowsmith, C.M. Place: *Dynamical Systems: Differential Equations, Maps and Chaotic Behaviour*, Chapman & Hall, 1992.

D.W. Jordan, P. Smith: *Nonlinear Ordinary Differential Equations*, Clarendon Press, Oxford 1977.

### Cilji in kompetence:

Študent se seznani z osnovnimi metodami, ki se uporabljajo pri obravnavi dinamičnih sistemov. Pri tem uporabi znanje iz linearne algebre, diferencialnih enačb in topologije. Spozna različne zglede modeliranja pojavov v medicini, ekonomiji, biologiji in fiziki.

### Objectives and competences:

Students learn basic methods used in the theory of dynamical systems. Linear algebra, differential equations and topology are applied. Various examples of modeling from medicine, economy, biology and physics are presented.

**Predvideni študijski rezultati:****Znanje in razumevanje:**

Razumevanje pojmov, kot so: dinamični sistem, stabilnost, periodično gibanje, bifurkacija in kaos.

**Uporaba:**

Formuliranje, modeliranje in reševanje različnih problemov iz medicine, biologije, fizike in ekonomije.

**Refleksija:**

Razumevanje teorije na podlagi primerov in uporabe. Številni zgledi pomagajo spoznati vlogo matematike v naravoslovju in tehniki.

**Prenosljive spretnosti – niso vezane le na en predmet:**

Identifikacija, formulacija in reševanje problemov iz drugih strok v matematičnem jeziku. Spretnost uporabe domače in tuje literature.

**Intended learning outcomes:****Knowledge and understanding:**

Understanding concepts such as dynamical system, stability, periodic motion, bifurcation, chaos.

**Application:**

Formulation, modeling and solving various problems in medicine, biology, physics and economy.

**Reflection:**

Understanding of the theory from the applications. Examples show the role of mathematics in other sciences.

**Transferable skills:**

Understanding of the theory from the applications. Examples given explain the role of mathematics in natural sciences and engineering.

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations

Delež (v %) /

**Načini ocenjevanja:**

Weight (in %)

**Assessment:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- izpit iz vaj (2 kolokvija ali pisni izpit)
- ustni izpit
- domače naloge

33%

34%

33%

Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)

Type (examination, oral, coursework, project):

- 2 midterm exams instead of written exam, written exam
- oral exam
- homeworks

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:****prof. dr. Franc Forstnerič**

- F. Forstnerič: Actions of  $(R,+)$  and  $(C,+)$  on complex manifolds. Math. Z. 223 (1996) 123-153.
- F. Forstnerič: *Interpolation by holomorphic automorphisms and embeddings in  $C^n$* . J. Geom.

Anal. 9 (1999) 93-118.

- F. Forstnerič: Holomorphic families of long  $C^2$ 's. Proc. Amer. Math. Soc. 140 (2012) 2383-2389.

**prof. dr. Jasna Prezelj**

- F. Forstnerič, J. Prezelj: *Oka's principle for holomorphic submersions with sprays*, Math. Ann. 322, (2002) 633-666
- J. Prezelj: *Interpolation of embeddings of Stein manifolds over discrete sets*, Math. Ann. 326 (2003) 275-296.
- J. Prezelj: *Weakly holomorphic embeddings of Stein spaces with isolated singularities*, Pac. J. Math. 220 (2005) 141-152.



## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Funkcionalna analiza
<b>Course title:</b>	Functional Analysis

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	drugi
Second cycle master study program Mathematics	none	first or second	second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2104

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
45		30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Roman Drnovšek, prof. dr. Peter Šemrl

<b>Jeziki / Languages:</b>	<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
	<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

vpis predmeta *Uvod v funkcionalno analizo*

**Prerequisites:**

enrollment into the course *Introduction to Functional Analysis*

**Vsebina:**

**Content (Syllabus outline):**

Banachovi prostori. Linearni operatorji in funkcionali na Banachovih prostorih.

Izrek o odprtih preslikavi. Izrek o zaprtem grafu. Princip enakomerne omejenosti. Drugi dual.

Adjungirani operator na Banachovem prostoru.

Šibke topologije. Banach-Alaoglujev izrek.

Krein-Milmanov izrek o ekstremnih točkah.

Banachove algebre. Ideali in kvocienti. Spekter elementa. Rieszov funkcijski račun. Gelfandova transformacija.

$C^*$ -algebre. Približne enote. Ideali in kvocienti. Komutativne  $C^*$ -algebre. Funkcijski račun v  $C^*$ -algebrah. Gelfand-Naimark-Segalova konstrukcija.

Banach spaces. Linear operators and functionals on Banach spaces.

The open mapping theorem. The closed graph theorem. The principle of uniform boundedness. The second dual.

The adjoint operator on a Banach space .

Weak topologies. The Banach-Alaoglu theorem. The Krein-Milman theorem on extreme points.

Banach algebras. Ideals and quotients. The spectrum of an element. Riesz functional calculus. The Gelfand transform.

$C^*$ -algebras. Approximate units. Ideals and quotients. Commutative  $C^*$ -algebras. The functional calculus in  $C^*$ -algebras. The Gelfand-Naimark-Segal construction.

#### Temeljni literatura in viri / Readings:

- B. Bollobás: Linear Analysis : An Introductory Course, 2nd edition, Cambridge Univ. Press, Cambridge, 1999.
- J. B. Conway: A Course in Functional Analysis, 2nd edition, Springer, New York, 1990.
- Y. Eidelman, V. Milman, A. Tsolomitis: Functional Analysis : An Introduction, AMS, Providence, 2004.
- M. Hladnik: Naloge in primeri iz funkcionalne analize in teorije mere, DMFA-založništvo, Ljubljana, 1985.
- R. Meise, D. Vogt: Introduction to Functional Analysis, Oxford Univ. Press, Oxford, 1997.
- G. K. Pedersen: Analysis Now, Springer, New York, 1996.
- W. Rudin: Functional Analysis, 2nd edition, McGraw-Hill, New York, 1991.
- I. Vidav: Linearni operatorji v Banachovih prostorih, DMFA-založništvo, Ljubljana, 1982.
- I. Vidav: Banachove algebre, DMFA-založništvo, Ljubljana, 1982.
- I. Vidav: Uvod v teorijo  $C^*$ -algeber, DMFA-založništvo, Ljubljana, 1982.

#### Cilji in kompetence:

Slušatelj spozna osnove funkcionalne analize in povezavo z drugimi področji analize.

#### Objectives and competences:

Students learn the basics of functional analysis and links with other areas of analysis.

#### Predvideni študijski rezultati:

#### Intended learning outcomes:

**Znanje in razumevanje:** Obvladanje osnovnih pojmov funkcionalne analize. Sposobnost rekonstrukcije (vsaj lažjih) dokazov. Sposobnost aplikacije pridobljenega znanja.

**Uporaba:** Uporaba funkcionalne analize sega tudi v naravoslovje in druga področja znanosti kot na primer ekonomijo.

**Refleksija:** Razumevanje teorije na podlagi uporabe.

**Prenosljive spretnosti – niso vezane le na en predmet:** Sposobnost abstraktnega razmišljanja. Spretnost uporabe domače in tuje literature.

**Knowledge and understanding:** Understanding basic concepts of functional analysis. Ability of the reconstruction (at least easier) proofs. Ability of the application of acquired knowledge.

**Application:** Functional analysis is used in natural sciences and other areas of science such as economics.

**Reflection:** Understanding of the theory on the basis of examples.

**Transferable skills:** Ability to use abstract methods to solve problems. Ability to use a wide range of references and critical thinking.

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations

**Načini ocenjevanja:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- domače naloge
- izpit iz vaj
- ustni izpit

Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)

Delež (v %) /

Weight (in %)

10%

50%

40%

**Assessment:**

Type (examination, oral, coursework, project):

- homeworks
- written exam
- oral exam

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

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**Reference nosilca / Lecturer's references:**

**prof. dr. Roman Drnovšek**

- R. Drnovšek: *Common invariant subspaces for collections of operators*, Integr. Equ. Oper. Theory 39 (2001), no. 3, 253-266.
- R. Drnovšek: *Invariant subspaces for operator semigroups with commutators of rank at most one*, J. Funct. Anal. 256 (2009), no. 12, 4187-4196.
- R. Drnovšek: *An infinite-dimensional generalization of Zenger's lemma*, J. Math. Anal. Appl. 388 (2012), no. 2, 1233-1238.

**prof. dr. Peter Šemrl**

- P. Šemrl: *Applying projective geometry to transformations on rank one idempotents*, J. Funct. Anal. 210 (2004), 248-257.
- P. Šemrl: *Similarity preserving linear maps*, J. Operator Theory 60 (2008), no. 1, 71-83.
- P. Šemrl: *Symmetries on bounded observables: a unified approach based on adjacency preserving maps*, Integral Equations Oper. Theory 72 (2012), no. 1, 7-66.

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

**Predmet:** Kompleksna analiza  
**Course title:** Complex analysis

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2100

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Miran Černe, prof. dr. Barbara Drinovec Drnovšek,  
prof. dr. Franc Forstnerič, prof. dr. Jasna Prezelj Perman

**Jeziki / Languages:**  
**Predavanja / Lectures:** slovenski/Slovene, angleški/English  
**Vaje / Tutorial:** slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija

**Prerequisites:**

Enrollment into the program

**Vsebina:**

**Content (Syllabus outline):**

Cauchyjeva integralska formula za holomorfne in neholomorfne funkcije. Rešitev nehomogene debar enačbe na ravninskih območjih s Cauchyjevim integralom.

Schwarzeva lema. Avtomorfizmi diska.

Konveksne funkcije. Hadamardov izrek o treh krožnicah. Phragmen-Lindelöfov izrek.

Kompaktnost in konvergenca v prostoru holomorfnih funkcij. Normalne družine.

Montelov izrek. Hurwitzev izrek. Riemannov upodobitveni izrek.

Koebejev izrek. Blochov izrek. Izrek Landaua, Picardovi izreki. Schottkyjev izrek.

Konvergenca produktov. Weierstrassov faktorizacijski izrek. Rungejev izrek o aproksimaciji z racionalnimi funkcijami. Mittag-Lefflerjev izrek o konstrukciji funkcije z danimi glavnimi deli. Izrek o interpolaciji s holomorfnimi funkcijami na diskretni množici.

Schwarzev princip zrcaljenja. Analitično nadaljevanje vzdolž poti. Monodromijski izrek. Kompletna analitična funkcija. Snop zarodkov holomorfnih funkcij. Pojem Riemannove ploskve.

**Druge možne vsebine:** Harmonične in subharmonične funkcije. Poissonovo jedro in rešitev Dirichletovega problema na krogu. Lastnosti Poissonovega integrala in povezava s Cauchyjevim integralom. Mergelyanov izrek. Cele funkcije. Rast in red cele funkcije. Hadamardov izrek o faktorizaciji.

Cauchy integral formula for holomorphic and non holomorphic functions. Solution to the non homogeneous debar equation on planar domains using Cauchy integral.

Schwarz lemma. Automorphisms of the unit disc.

Convex functions. Hadamard three-circle theorem. Phragmen-Lindelöf theorem.

Compactness and convergence in the space of holomorphic functions. Normal families.

Montel's theorem. Hurwitz's theorem. Riemann mapping theorem.

Koebe's theorem. Bloch's theorem. Landau's theorem, Picard's theorem. Schottky's theorem.

Product convergence. Weierstrass factorization theorem. Runge's theorem on approximation by rational functions. Mittag-Leffler's theorem on existence of holomorphic functions with prescribed principal parts. Interpolation by holomorphic functions on discrete sets.

Schwarz reflection principle. Analytic continuation along path. Monodromy theorem. Complete analytic function. Sheaf of germs of analytic functions. Riemann surface.

**Other possible topics:** Harmonic and subharmonic functions. Poisson kernel and the solution of the Dirichlet problem on the disc. Properties of Poisson integral and connection to the Cauchy integral. Mergelyan theorem. Entire functions. The genus and the order of entire function. Hadamard factorization theorem.

### Temeljna literatura in viri / Readings:

- L. Ahlfors: *Complex Analysis*, 3rd edition, McGraw-Hill, New York, 1979.
- C. A. Berenstein, R. Gay: *Complex Analysis and Special Topics in Harmonic Analysis*, Springer, New York, 1995.
- J. B. Conway: *Functions of One Complex Variable I*, 2nd edition, Springer, New York-Berlin, 1995.
- R. Narasimhan, Y. Nievergelt: *Complex Analysis in One Variable*, 2nd edition, Birkhäuser, Boston, 2001.
- W. Rudin: *Real and Complex Analysis*, 3rd edition, McGraw-Hill, New York, 1987.
- T. Gamelin: *Complex analysis*, Springer-Verlag, New York, 2001.

**Cilji in kompetence:**

Slušatelj spozna nekatere vsebine teorije holomorfnih funkcij ene kompleksne spremenljivke. Pri tem uporabi znanje iz osnovne analize in topologije.

**Objectives and competences:**

Students learn some basic concepts of theory of functions of one complex variable. Elementary methods of analysis and topology are applied.

**Predvideni študijski rezultati:**

**Znanje in razumevanje:** Razumevanje nekaterih bistvenih pojmov in rezultatov teorije holomorfnih funkcij.

**Uporaba:** V ostalih delih matematične analize in geometrije; uporaba konformnih preslikav pri reševanju problemov iz fizike in mehanike.

**Refleksija:** Razumevanje teorije na podlagi primerov in uporabe.

**Prenosljive spretnosti – niso vezane le na en predmet:** Identifikacija, formulacija in reševanje matematičnih in nematematičnih problemov s pomočjo metod kompleksne analize. Spretnost uporabe domače in tuje literature. Privajanje na samostojno seminarsko predstavitev gradiva.

**Intended learning outcomes:**

**Knowledge and understanding:** Understanding some of the fundamental topics and techniques of complex analysis.

**Application:** Applications lie mainly in other parts of mathematical analysis and geometry. Conformal maps are applied to solving problems in physics and mechanics.

**Reflection:** Understanding the theory on the basis of examples and applications.

**Transferable skills:** The ability to identify, formulate and solve mathematical and non mathematical problems using methods of complex analysis. Acquiring skills in using domestic and foreign literature. Developing the skills of independent presentation of the material in the form of seminar lectures.

**Metode poučevanja in učenja:**

predavanja, vaje, seminar, domače naloge, konzultacije

**Learning and teaching methods:**

Lectures, exercises, seminar, homeworks, consultations

Načini ocenjevanja:	Weight (in %)	Assessment:
<p>Način (domače naloge, seminarska naloga, ustno izpraševanje):</p> <ul style="list-style-type: none"> <li>▪ domače naloge, seminarska naloga</li> <li>▪ ustni izpit</li> </ul> <p>Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)</p>	<p>50%</p> <p>50%</p>	<p>Type (homework, seminar paper, oral exam, coursework, project):</p> <ul style="list-style-type: none"> <li>▪ homework and seminar paper</li> <li>▪ oral exam</li> </ul> <p>Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)</p>

#### Reference nosilca / Lecturer's references:

**prof. dr. Miran Černe**

- M. Černe, M. Zajec, *Boundary differential relations for holomorphic functions on the disc*. Proc. Am. Math. Soc. 139 (2011), 473-484.
- M. Černe, M. Flores, *Generalized Ahlfors functions*. Trans. Am. Math. Soc. 359 (2007), 671-686.
- M. Černe, M. Flores, *Quasilinear  $\bar{\partial}$ -equation on bordered Riemann surfaces*. Math. Ann. 335 (2006), 379-403.

**prof. dr. Barbara Drinovec Drnovšek**

- B. Drinovec Drnovšek, F. Forstnerič, *The Poletsky-Rosay theorem on singular complex spaces*, Indiana Univ. Math. J. 61 (2012), 1407-1423.
- B. Drinovec Drnovšek, F. Forstnerič, *Holomorphic curves in complex spaces*, Duke Math. J., 139 (2007), 203-253
- B. Drinovec Drnovšek, *On proper discs in complex manifolds*, Ann. Inst. Fourier (Grenoble), 57 (2007), 1521-1535.

**prof. dr. Franc Forstnerič**

- F. Forstnerič, E. F. Wold, *Embeddings of infinitely connected planar domains into  $\mathbb{C}^2$* , Analysis & PDE 6 (2013) 499-514.
- F. Forstnerič: *Runge approximation on convex sets implies Oka's property*, Annals. of Math., 163 (2006), 689-707.
- F. Forstnerič: *Noncritical holomorphic functions on Stein manifolds*, Acta Math., 191 (2003), 143-189.

**prof. dr. Jasna Prezelj Perman**

- F. Forstnerič, J. Prezelj: *Oka's principle for holomorphic submersions with sprays*, Math. Ann. 322, (2002) 633-666
- J. Prezelj: *Interpolation of embeddings of Stein manifolds over discrete sets*, Math. Ann. 326 (2003) 275-296.
- J. Prezelj: *Weakly holomorphic embeddings of Stein spaces with isolated, singularities*, PJM 220 (1): 141-152 (2005).



## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Matematika v industriji
<b>Course title:</b>	Mathematics in Industry

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** še ni dodeljena/not assigned yet

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
	30				150	6

**Nosilec predmeta / Lecturer:** doc. dr. George Mejak

**Jeziki / Languages:**

<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija.  
Odobren načrt dela.

**Prerequisites:**

Enrollment into the program.  
Approved work plan

**Vsebina:**

**Content (Syllabus outline):**

Identifikacija nalog iz realnega sveta.

Matematično modeliranje.

Numerične metode.

Primerjava modelne rešitve z nalogo iz realnega sveta.

Pisanje poročila.

Identification real world problems.

Mathematical modeling.

Numerical methods.

Comparison between a model solution and real problem.

Report writing.

### Temeljna literatura in viri / Readings:

- E. Zakrajšek: *Matematično modeliranje*, DMFA-založništvo, Ljubljana, 2004.
- Capasso, *Mathematics in Industry*, Book series: Mathematics in Industry, Springer.
- C. Dym, *Principles of Mathematical Modeling*, Academic Press, 2004.
- S. Howison: *Practical Applied Mathematics: Modelling, Analysis, Approximation*, Cambridge Univ. Press, Cambridge, 2005.
- M. S. Klamkin: *Mathematical Modelling : Classroom Notes in Applied Mathematics*, SIAM, Philadelphia, 1987.

### Cilji in kompetence:

Cilj predmeta je razviti sposobnosti sodelovanja matematika z nematematikami pri reševanju problemov iz realnega sveta.

Kompetence so: razvijanje sposobnosti komuniciranja s potencialnimi uporabniki matematičnih znanj, razvijanje sposobnosti skupinskega dela, sposobnost nadgrajevanja šolskih modelov, spretnost uporabe programskih orodij, z eno besedo, vzgoja industrijskih matematikov za potrebe trga dela.

### Objectives and competences:

The aim of the course is to foster collaboration between mathematicians and non-mathematicians by solving problems from real world. The competences are: to promote communication with possible users of mathematical methods, to promote team work, to extend academic examples to a real world problems, to acquire some knowledge of mathematical software; summarizing, to educate Industrial Mathematicians to meet the growing demand for such experts.

### Predvideni študijski rezultati:

#### Znanje in razumevanje:

Sposobnost komuniciranja z uporabniki matematičnih znanj, sposobnost formuliranja problemov, razumevanje matematičnega modeliranja.

#### Uporaba:

### Intended learning outcomes:

#### Knowledge and understanding:

Knowledge how to communicate with users of mathematical methods, ability to rationally formulate problems, knowledge of mathematical modeling.

#### Application:

Reševanje problemov iz realnega sveta.  
Povezava z uporabniki matematičnih znanj.

**Refleksija:**

Refleksija lastnega razumevanja pridobljenih matematičnih znanj na problemih iz prakse, kritično ovrednotenje skladnosti med teoretičnimi načeli in dejanskim stanjem v praksi.

**Prenosljive spretnosti – niso vezane le na en predmet:**

Spretnost uporabe virov znanja, zbiranja in interpretacije podatkov, sodelovanja s strokovnjaki iz drugih področij; skupinsko delo, poročanje o rezultatih dela, pisanje poročil.

Solving real word problems. Cross breeding with users of mathematical methods.

**Reflection:**

Reflection of own understanding of mathematical knowledge by solving problems from a real world. Critical assesment of differences between theoretical and practical principles.

**Transferable skills:**

How to use knowledge bases, how to collect and interpret data, collaboration with experts from different areas; team work, how to present results, how to write reports.

**Metode poučevanja in učenja:**

Projektno delo, delo na terenu, individualen študij, seminarji, nastopi.

**Learning and teaching methods:**

Project working, field work, consultations, individual study, presentations.

**Načini ocenjevanja:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):  
▪ Projektno poročilo  
▪ Predstavitev poročila  
  
Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)

Delež (v %) /  
Weight (in %)

50%  
50%

**Assessment:**

Type (examination, oral, coursework, project):  
▪ Project  
▪ Project presentation  
  
Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

doc. dr. George Mejak:

- MEJAK, George. On extension of functions with zero trace on a part of boundary. *J. math.*

*anal. appl.*, 1993, let. 175, str. 305-314

- MEJAK, George. Finite element solution of a model free surface problem by the optimal shape design approach. *Int. j. numer. methods eng.*, 1997, vol. 40, str. 1525-1550.
- MEJAK, George. Eshelby tensors for a finite spherical domain with an axisymmetric inclusion. *Eur. j. mech. A, Solids*. [Print ed.], 2011, vol. 30, iss. 4, str. 477-490.

## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Mehanika deformabilnih teles
<b>Course title:</b>	Mechanics of deformable bodies

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** še ni dodeljena/not assigned yet

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Igor Dobovšek

**Jeziki / Languages:**

<b>Predavanja / Lectures:</b>	slovenski/Slovenian, angleški/English
<b>Vaje / Tutorial:</b>	slovenski/Slovenian, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija

**Prerequisites:**

Enrollment into the program

**Vsebina:**

Kinematika. Deformacija, deformacijski tenzor. Green-Lagrangeev in Almansi-jev tenzor. Levi in desni Cauchy-Greenov deformacijski tenzor. Geometrijska linearizacija. Kompatibilnostni pogoji. Enačbe polja. Cauchyjev, prvi in drugi Piolla-Kirchhoffov napetostni tenzor. Enačbe gibanja v prostorskem in materialnem zapisu. Energijska enačba. Termodinamika, Clausius-Duhemova neenačba. Termodinamični potenciali, funkcija disipacije.

**Content (Syllabus outline):**

Kinematics of deformation. Deformation tensor. Green-Lagrange and Almansi tensor. Left and right Cauchy-Green deformation tensor. Geometric linearization. Conditions of compatibility. Field equations. Cauchy stress tensor, first and second Piolla-Kirchhoff stress tensor. Momentum balance in material and spatial formulation. Energy balance. Thermodynamics and Clausius-Duhem inequality. Thermodynamic potentials, dissipation

Linearni modeli. Geometrijsko in materialno linearni modeli. Elastičnost. Posplošeni Hookov zakon. Princip materialne simetrije.

Anizotropni material. **Kristalografske simetrijske grupe.**

Problemi v  $\mathbf{R}^2$ . Ravninsko stanje napetosti, deformacij. Airyjeva napetostna funkcija. Flamantova rešitev za koncentrirano silo. Koncentracije napetosti.

Problemi v  $\mathbf{R}^3$ . Naviereve enačbe. Rešitve s potenciali. Beltrami-Mitchellove enačbe. Singularne rešitve. Greenova funkcija za izotropni elastični prostor. Variacijski in komplementarni variacijski princip. Ritzova in Galerkinova metoda.

Nelinearni modeli. Geometrijsko in materialno nelinearni modeli. Elastični potencial in funkcije deformacijske energije.

Hiperelastičnost. Hipoelastičnost. Primeri uporabe v biomehaniki. Ireverzibilne deformacije. Plastičnost. Vezani problemi. Termoelastičnost. Splošni termodinamični principi. Reološka transformacija. Enačba stanja. Termoviskoplastičnost.

function.

Linear models. Geometrically and materially linear models. Elasticity. Generalized Hooke's law. Principle of material symmetry. Anisotropic material. Crystal symmetry groups.

Problems in  $\mathbf{R}^2$ . Plane stress and plane strain. Airy stress function. Flamant's solution for concentrated force. Stress concentration. Problems in  $\mathbf{R}^3$ . Navier's equations. Solutions with potentials. Beltrami-Mitchell's equations. Singular solutions. Green's function for isotropic elastic space.

Variational and complementary variational principle. Method of Ritz and Galerkin.

Nonlinear models. Geometrically and materially nonlinear models. Elastic potential and deformation energy functions. Hyperelasticity. Hypoelasticity. Applications in biomechanics. Irreversible deformations. Plasticity. Coupled problems. Thermoelasticity. Generalized thermodynamical principles. Rheological transformation. Equations of state. Thermoviscoplasticity.

#### **Temeljna literatura in viri / Readings:**

- R. W. Ogden: *Non-Linear Elastic Deformations*, Prentice Hall, Dover, 1997.
- Y. C. Fung: *Biomechanics, Mechanical Properties of Living Tissues*, Springer, 1993.
- P. Haupt: *Continuum Mechanics and Theory of Materials*, Springer, 2002.
- R. W. Soutas-Little: *Elasticity*, Dover Publications, Dover, 1999.
- **R. J. Asaro, V. A. Lubarda: *Mechanics of Solids and Materials*, Cambridge University Press, New York, 2006.**

#### **Cilji in kompetence:**

Predstavitev osnovnih pojmov in vsebin mehanike deformabilnih teles s poudarkom na korektni matematični formulaciji in povezovanju predhodno osvojenih matematičnih znanj.

#### **Objectives and competences:**

An overview of fundamental facts and ingredients of mechanics of deformable bodies with emphasis on strict mathematical formulation based on previously mastered mathematical knowledge.

#### **Predvideni študijski rezultati:**

#### **Intended learning outcomes:**

**Znanje in razumevanje:** Poznavanje in razumevanje osnovnih pojmov in principov mehanike deformabilnih teles.

**Uporaba:** Osnova za nadaljnje raziskovalno delo in specialistični študij na področju mehanike.

**Refleksija:** Povezovanje osvojenega matematičnega znanja v okviru enega predmeta in njegova uporaba na področju mehanike.

**Prenosljive spretnosti – niso vezane le na en predmet:** Celovit pogled na mehaniko deformabilnih teles v okviru mehanike kontinuuma. Reševanje problemov iz sorodnih področij mehanike materialov.

**Knowledge and understanding:**

To establish knowledge and understanding of fundamental principles of mechanics of deformable bodies.

**Application:** Mastered coursework represents a foundation for specialized research in the field of mechanics.

**Reflection:** Connecting acquired mathematical knowledge within the course with application of that knowledge in a general field of mechanics.

**Transferable skills:**

An overview of mechanics of deformable bodies within a general framework of continuum mechanics. Solving problems from related areas of mechanics of materials.

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije, seminar

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations, seminar

**Načini ocenjevanja:**

Delež (v %) /

Weight (in %)

**Assessment:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

Ustni in pisni zagovor teoretičnega dela vključno s seminarskimi nalogami. Končna ocena je kombinacija navedenega zgoraj.

Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)

100%

Type (examination, oral, coursework, project):

Oral and written defense of theoretical part including seminar assignments. Grade is combination of the above.

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

DOBOVŠEK, Igor. The influence of dislocation distribution density on curvature and interface stress in epitaxial thin films on a flexible substrate. Int. j. mech. sci.. [Print ed.], 2010, issue 2, vol. 52, str. 212-218.

DOBOVŠEK, Igor. A theoretical model of the interaction between plastic distortion and configurational stress on the phase transformation front. Mater. sci. eng., A Struct. mater. : prop. microstruct. process.. [Print ed.], 2008, vol. 481-482, str. 956-361.

DOBOVŠEK, Igor. Problem of a point defect, spatial regularization and intrinsic length scale in second gradient elasticity. Mater. sci. eng., A Struct. mater. : prop. microstruct. process.. [Print ed.], 2006, vol. 423, str. 92-96.

DOBOVŠEK, Igor. Micromechanical modeling of nanostructured materials by poly-clustering techniques. International journal of nanoscience, 2005, vol. 4, no. 4, str. 623-629.



**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

**Predmet:** Mehanika fluidov  
**Course title:** Fluid mechanics

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** še ni dodeljena/not yet assigned

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:** doc. dr. George Mejak

**Jeziki / Languages:**  
**Predavanja / Lectures:** slovenski/Slovene, angleški/English  
**Vaje / Tutorial:** slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija

**Prerequisites:**

Enrollment into the program

**Vsebina:**

**Content (Syllabus outline):**

**Kinematika mehanike fluidov:**

Eulerjev opis gibanja. Tenzor deformacijskih hitrosti. Materialni odvod in transportni izrek. Tokovnice, tirnice, slednice in vrtničnice.

**Fizikalno mehanske osnove:**

Pojem površinske sile in napetostnega tenzorja. Zakon o ohranitvi mase. Cauchyjeva momentna enačba. Termodinamični principi. Konstitutivna zveza med napetostjo in tenzorjem deformacijskih hitrosti. Hidrostatika.

**Newtonovi fluidi:**

Pojem viskoznosti. Navier-Stokesova enačba. Primeri laminarnega viskoznega toka; ravninski Coettov tok, Poiseuillev tok, Stokesova naloga. Difuzija in konvekcija vrtničnosti. Turbulenca.

**Idealen fluid:**

Eulerjeva enačba. Bernoullijev izrek. Potencialni tok nestisljivega fluida. Reševanje ravninskega potencialnega toka z metodo kompleksnih potencialov. Potencialni tok stisljivega fluida, akustična aproksimacija.

**Pregled numeričnih metod reševanja enačb mehanike fluidov:**

Ohranitveni zapis enačb gibanja. Metoda končnih volumnov. Pregled osnovnih modelnih primerov.

**Kinematics of the fluid flow:**

Eulerian description. Rate of deformation tensor. Material derivative and transport theorems. Stream lines, pathlines, streak lines, vortex lines.

**Physical properties of fluids:**

Stress vector and tensor. Mass conservation law. Momentum equation. Thermodynamical principles. Constitutive relation. Hydrostatics.

**Newtonian fluids:**

Viscosity. Navier-Stokes equation. Examples of laminar flow, plane Coette flow, Poiseuille flow, Stokes problem. Diffusion and convection of the vorticity. Turbulence.

**Ideal fluids:**

Eulerian equation. Bernoulli's theorem. Potential flow of incompressible fluid. Complex variable methods. Compressible fluid. Acoustic approximation.

**Review of numerical methods in fluid mechanics:**

Equations in conservative forms. Finite volume method. Benchmark problems.

**Temeljna literatura in viri / Readings:**

- L. Škerget: *Mehanika tekočin*, Fakulteta za strojništvo, Ljubljana, 1994.
- G.K. Batchelor, *An introduction to Fluid Dynamics*, Cambridge University Press, 1967.
- A. J. Chorin, J. E. Marsden: *A Mathematical Introduction to Fluid Mechanics*, 3rd edition, Springer, New York, 2000.
- J. H. Spurk: *Fluid Mechanics : Problems and Solutions*, Springer, Berlin, 1997.

**Cilji in kompetence:****Objectives and competences:**

Cilj predmeta je pridobiti osnovna znanja s področja mehanike fluidov. Pridobljeno znanje omogoča nadaljni samostojni študij mehanike fluidov.

The goal is to obtain basic knowledge of fluid mechanics. Acquired knowledge allows further individual study of fluid mechanics.

**Predvideni študijski rezultati:**

**Znanje in razumevanje:**

Poznavanje in razumevanje osnovnih pojmov in principov iz mehanike fluidov

**Uporaba:**

Temelj za nadgraditev osvojenega znanja s specifičnimi znanji iz prakse s področja mehanike fluidov. Osnova za nadaljnji specialistični študij mehanike fluidov.

**Refleksija:**

Povezovanje osvojenega matematičnega znanja v okviru enega predmeta in njihova uporaba na področju mehanike fluidov.

**Prenosljive spretnosti – niso vezane le na en predmet:**

Celovit pogled na mehaniko fluida v okviru mehanike kontinuuma. Sposobnost reševanja nalog in problemov iz sorodnih področij uporabne matematike.

**Intended learning outcomes:**

**Knowledge and understanding:**

Knowledge and understanding of basic principles of fluid mechanics.

**Application:**

Application of the acquired knowledge in solving real-life problems of fluid mechanics. First step for further graduate level study of fluid mechanics.

**Reflection:**

Crossbreeding of different mathematical subjects within a single course and their application in the field of fluid mechanics.

**Transferable skills:**

Understanding of fluid mechanics in the context of the continuum mechanics. Ability of solving related problems from the applied mathematics.

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations

**Načini ocenjevanja:**

Delež (v %) /

Weight (in %) **Assessment:**

<p>Način (pisni izpit, ustno izpraševanje, naloge, projekt):</p> <ul style="list-style-type: none"> <li>▪ izpit iz vaj (2 kolokvija ali pisni izpit)</li> <li>▪ ustni izpit</li> </ul> <p>Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)</p>	<p>50%</p> <p>50%</p>	<p>Type (examination, oral, coursework, project):</p> <ul style="list-style-type: none"> <li>▪ 2 midterm exams instead of written exam, written exam</li> <li>▪ oral exam</li> </ul> <p>Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)</p>
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**Reference nosilca / Lecturer's references:**

- G. Mejak: Finite element solution of a model free surface problem by the optimal shape design approach, *Int. J. Numer. Methods Eng.*, 1997, vol. 40, str. 1525-1550.
- G. Mejak: Numerical solution of Bernoulli-type free boundary value problems by variable domain method, *Int. J. Numer. Methods Eng.*, 1994, let. 37, št. 24, str. 4219-4245.
- MEJAK, George. Finite element analysis of axisymmetric free jet impingement. *Int. j. numer. methods fluids*, 1991, let. 13, št. 4, str. 491-505.

## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Mehanika kontinuuma
<b>Course title:</b>	Continuum Mechanics

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2109

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Igor Dobovšek

<b>Jeziki / Languages:</b>	<b>Predavanja / Lectures:</b>	slovenski/Slovenian, angleški/English
	<b>Vaje / Tutorial:</b>	slovenski/Slovenian, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija

**Prerequisites:**

Enrollment into the program

**Vsebina:**

Osnove tenzorske analize. Krivuljne koordinate, metrični tenzor, kovariantne in kontravariantne komponente. Christoffelovi simboli. Diferencialni operatorji v krivuljnih koordinatah. Odvodi tenzorskih funkcij. Kinematika kontinuuma. Deformacijski gradient. Polarni razcep deformacijskega gradienta. Mere deformacije, deformacijski tenzor. Homogena deformacija, razteg, strig. Deformacija ločnega, površinskega in volumnskega elementa. Lagrangeev in Eulerjev

**Content (Syllabus outline):**

Introduction to tensor analysis. Convected coordinates, metric tensor, covariant and contravariant components. Christoffel symbols. Differential operators in convected coordinates. Derivatives of tensor functions. Kinematics of continuum. Deformation gradient. Polar decomposition of deformation gradient. Deformation measures. Strain tensor. Homogeneous deformation. Stretch and shear. Deformation of arc, surface and volume element. Motion. Lagrangian and Eulerian

opis gibanja. Materialni odvod. Transportni izreki.

Ohranitveni zakoni. Zakon o ohranitvi mase. Napetostni tenzor. Enačba gibanja. Zakon o ohranitvi energije.

Osnovni konstitutivni principi. Konstitutivne zveze. Princip materialne objektivnosti. Materialne simetrije, izotropija. Reprezentacija konstitucijskih funkcij. Pregled osnovnih modelov. Definicije elastičnosti, viskoelastičnosti in fluidov.

description. Material time derivative. Transport theorems.

Balance laws. Conservation of mass. The stress tensor. Balance of momentum. Conservation of energy.

Basic principles of constitutive theories. Constitutive relation. Principle of material objectivity. Material symmetry. Representation of constitutive functions. Overview of basic models. Definitions of elasticity, viscoelasticity and fluids.

#### Temeljna literatura in viri / Readings:

- P. Chadwick: *Continuum Mechanics : Concise Theory and Problems*, 2nd edition, Dover Publications, Mineola, 1999.
- M. E. Gurtin: *An Introduction to Continuum Mechanics*, Academic Press, New York-London, 1981.
- I-S.Liu: *Continuum Mechanics*, Springer, New York, 2002.
- J.L. Wegner, J. B. Haddow: *Elements of Continuum Mechanics and Thermodynamics*, Cambridge University Press, New York, 2009.

#### Cilji in kompetence:

Predstavitev osnovnih pojmov in vsebin mehanike kontinuuma s poudarkom na korektni matematični formulaciji in povezovanju predhodno osvojenih matematičnih znanj.

#### Objectives and competences:

An overview of fundamental facts and ingredients of continuum mechanics with emphasis on strict mathematical formulation based on previously mastered mathematical knowledge.

#### Predvideni študijski rezultati:

#### Intended learning outcomes:

**Znanje in razumevanje:** Poznavanje in razumevanje osnovnih pojmov in principov mehanike kontinuuma.

**Uporaba:** Osnova za nadaljnje raziskovalno delo in specialistični študij na področju mehanike.

**Refleksija:** Povezovanje osvojenega matematičnega znanja v okviru enega predmeta in njegova uporaba na področju mehanike.

**Prenosljive spretnosti – niso vezane le na en predmet:** Celovit pogled na mehaniko kontinuuma v okviru matematičnih sredstev, ki jih študent spozna tokom študija pri tem in ostalih predmetih.  
Reševanje problemov iz sorodnih področij uporabne matematike.

**Knowledge and understanding:**

To establish knowledge and understanding of fundamental principles of continuum mechanics.

**Application:** Mastered coursework represents a foundation for specialized research in the field of mechanics.

**Reflection:** Connecting acquired mathematical knowledge within the course with application of that knowledge in a general field of mechanics.

**Transferable skills:**

An overview of continuum mechanics within the realm of mathematical apparatus mastered by student during this and other related courses.

Solving problems from related areas of applied mathematics.

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije, seminar

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations, seminar

**Načini ocenjevanja:**

Delež (v %) /

Weight (in %)

**Assessment:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

Ustni in pisni zagovor teoretičnega dela vključno s seminarskimi nalogami.  
Končna ocena je kombinacija navedenega zgoraj.

Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)

100%

Type (examination, oral, coursework, project):

Oral and written defense of theoretical part including seminar assignments.  
Grade is combination of the above.

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

DOBOVŠEK, Igor. The influence of dislocation distribution density on curvature and interface stress in epitaxial thin films on a flexible substrate. Int. j. mech. sci.. [Print ed.], 2010, issue 2, vol. 52, str. 212-218.

DOBOVŠEK, Igor. A theoretical model of the interaction between plastic distortion and configurational stress on the phase transformation front. Mater. sci. eng., A Struct. mater. : prop. microstruct. process.. [Print ed.], 2008, vol. 481-482, str. 956-361.

DOBOVŠEK, Igor. Problem of a point defect, spatial regularization and intrinsic length scale in second gradient elasticity. Mater. sci. eng., A Struct. mater. : prop. microstruct. process.. [Print ed.], 2006, vol. 423, str. 92-96.

DOBOVŠEK, Igor. Micromechanical modeling of nanostructured materials by poly-clustering techniques. International journal of nanoscience, 2005, vol. 4, no. 4, str. 623-629.



**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

**Predmet:** Parcialne diferencialne enačbe  
**Course title:** Partial differential equations

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type**

**Univerzitetna koda predmeta / University course code:**

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:**

**Jeziki / Languages:**  
**Predavanja / Lectures:**   
**Vaje / Tutorial:**

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

**Prerequisites:**

**Vsebina:**

**Content (Syllabus outline):**

Prostori odvedljivih funkcij. Hölderjevi prostori. Schwartzov razred hitro padajočih funkcij. Testne funkcije. Distribucije. Prostori Soboljeva. Osnovna rešitev. Karakteristike linearnega parcialnega diferencialnega operatorja. Cauchyeva naloga. Izrek Cauchy-Kowalevski. Lewyjev primer.

Laplaceva enačba. Newtonov potencial kot osnovna rešitev Laplaceve enačbe. Dirichletov problem. Subharmonične funkcije. Perronova metoda. Šibke rešitve Dirichletevega problema. Lastne funkcije in lastne vrednosti Laplaceovega operatorja. Regularnost rešitev.

Toplotna enačba v višjih dimenzijah. Gaussovo jedro. Fundamentalna rešitev toplotne enačbe. Nehomogena toplotna enačba. Weierstrassov izrek. Toplotna enačba na omejenih območjih. Princip maksima. Fourierova metoda separacij spremenljivk.

Valovna enačba v višjih dimenzijah. Sferična povprečja. Valovna enačba v ravnini in prostoru. Fundamentalna rešitev valovne enačbe. Rešitve nehomogene valovne enačbe. Valovna enačba na omejenih območjih. Fourierova metoda separacij spremenljivk.

Spaces of differentiable functions. Hölder spaces. Schwarz class of rapidly decreasing functions. Test functions. Distributions. Sobolev spaces. Fundamental solution. Characteristics of linear partial differential operator. Cauchy problem. Cauchy-Kowalevski theorem. Lewy's example.

Laplace equation. Newton potential as fundamental solution of the Laplace equation. Dirichlet problem. Subharmonic functions. Perron method. Weak solutions of the Dirichlet problem. Eigenfunctions and eigenvalues of the Laplace operator. Regularity of solutions.

The heat equation in higher dimensions. Gauss kernel. Fundamental solution of the heat equation. Inhomogeneous heat equation. Weierstrass theorem. The heat equation on bounded domains. Maximum principle. Fourier method of separation of variables.

The wave equation in higher dimensions. Spherical means. The wave equation in the space and in the plane. Fundamental solution of the wave equation. Inhomogeneous wave equation. The wave equation on bounded domains. Fourier method of separation of variables.

### Temeljni literatura in viri / Readings:

- L. C. Evans: *Partial Differential Equations*, 2nd edition, AMS, Providence, 2010.
- G. B. Folland: *Introduction to Partial Differential Equations*, 2nd edition, Princeton Univ. Press, Princeton, 1995.
- L. Hörmander: *The Analysis of Linear Partial Differential Operators I: Distribution Theory and Fourier Analysis*, 2nd edition, Springer, Berlin, 2003.
- F. John: *Partial Differential Equations*, 4th edition, Springer, New York, 1991.
- F. Križanič: *Parcialne diferencialne enačbe*, DMFA-založništvo, Ljubljana, 2004.
- E. H. Lieb, M. Loss: *Analysis*, 2nd edition, AMS, Providence, 2001.
- Y. Pinchover, J. Rubinstein: *An Introduction to Partial Differential Equations*, CUP, Cambridge, 2005
- A. Suhadolc: *Integralske transformacije/Integralske enačbe*, DMFA-založništvo, Ljubljana, 1994.
- M. E. Taylor: *Partial differential equations I: Basic theory*, 2nd edition, Springer, New York, 2011

**Cilji in kompetence:**

Slušatelj se seznani s parcialnimi diferencialnimi enačbami v poljubni dimenziji. Predstavljene so distribucije kot posplošene rešitve linearnih parcialnih diferencialnih enačb. Dokazani so eksistenčni izreki za Laplaceovo, toplotno in valovno enačbo ter osnovne regularnostne lastnosti njihovih rešitev.

**Objectives and competences:**

Student becomes familiar with partial differential equations in arbitrary dimensions. Introduced are distributions as generalized solutions of linear partial differential equations. Proved are existence and basic regularity theorems for solutions of the Laplace equation, the heat equation and the wave equation.

**Predvideni študijski rezultati:**

**Znanje in razumevanje:** Razumevanje pojma posplošene rešitve parcialne diferencialne enačbe. Obvladanje postopkov za analitično reševanje nekaterih tipov parcialnih diferencialnih enačb v poljubni dimenziji. Razumevanje lastnosti rešitev različnih parcialnih diferencialnih enačb drugega reda.

**Uporaba:** Formulacija nekaterih matematičnih in nematematičnih problemov v obliki parcialnih diferencialnih enačb. Reševanje dobljenih parcialnih diferencialnih enačb.

**Refleksija:** Razumevanje teorije na podlagi uporabe.

**Prenosljive spretnosti – niso vezane le na en predmet:** Identifikacija, formulacija in reševanje matematičnih in nematematičnih problemov s pomočjo parcialnih diferencialnih enačb. Spretnost uporabe domače in tuje literature.

**Intended learning outcomes:**

**Knowledge and understanding:** Understanding the notion of a generalized solution of a partial differential equation. Skills to analytically find solutions of certain types of partial differential equation in higher dimensions. Understanding the properties of solutions of different types of second order partial differential equations.

**Application:** Formulation of certain mathematical and non-mathematical problems in the form of partial differential equations. Solving these partial differential equations.

**Reflection:** Understanding of the theory from the applications.

**Transferable skills:** The ability to identify, formulate, analyze and solve mathematical and non-mathematical problems with the help of partial differential equations. Skills in using the domestic and foreign literature.

**Metode poučevanja in učenja:****Learning and teaching methods:**

predavanja, vaje, domače naloge, seminar, konzultacije	Lectures, exercises, homeworks, seminar, consultations
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Delež (v %) /

**Načini ocenjevanja:**

Weight (in %)

**Assessment:**

Način (pisni izpit, ustno izpraševanje, seminarska naloga):		Type (examination, oral, coursework, seminar paper):
<ul style="list-style-type: none"> <li>▪ izpit iz vaj (2 kolokvija ali pisni izpit), seminarska naloga</li> <li>▪ ustni izpit</li> </ul>	50%	<ul style="list-style-type: none"> <li>▪ 2 midterm exams instead of written exam, written exam, seminar paper</li> <li>▪ oral exam</li> </ul>
Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)	50%	Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

**prof. dr. Miran Černe**

- M. Černe, M. Zajec, Boundary differential relations for holomorphic functions on the disc. *Proc. Am. Math. Soc.* 139 (2011), 473-484.
- M. Černe, M. Flores, Generalized Ahlfors functions. *Trans. Am. Math. Soc.* 359 (2007), 671-686.
- M. Černe, M. Flores, Quasilinear  $\bar{\partial}$ -equation on bordered Riemann surfaces. *Math. Ann.* 335 (2006), 379-403.

**prof. dr. Franc Forstnerič**

- F. Forstnerič: Runge approximation on convex sets implies Oka's property, *Annals of Math.* 163 (2006), 689-707.
- F. Forstnerič: Noncritical holomorphic functions on Stein manifolds, *Acta Math.* 191 (2003), 143-189.
- F. Forstnerič, J.-P. Rosay: Approximation of biholomorphic mappings by automorphisms of  $\mathbb{C}^n$ , *Invent. Math.* 112 (1993), 323-349.

**prof. dr. Pavle Saksida**

- P. Saksida: Lattices of Neumann oscillators and Maxwell-Bloch equations, *Nonlinearity* 19 (2006), no. 3, 747-768.
- P. Saksida: Maxwell-Bloch equations, C Neumann system and Kaluza-Klein theory, *J. Phys A* 38 (2005), no. 48, 10321-10344.
- P. Saksida: *Nahm's equations and generalizations of the Neumann system*, *Proc. London Math. Soc.* 78 (1999), no.3, 701-720.

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

**Predmet:** Specialne funkcije  
**Course title:** Special functions

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type**

**Univerzitetna koda predmeta / University course code:**

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:**

**Jeziki / Languages:**  
**Predavanja / Lectures:**   
**Vaje / Tutorial:**

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

**Prerequisites:**

**Vsebina:**

**Content (Syllabus outline):**

Osnovni pojmi Liejeve teorije, predstavljeni na primerih matričnih grup in algeber. Osnovni pojmi teorije upodobitev kompaktnih Liejevih grup. Upodobitve grupe  $SU(2)$ .

Splošni pojem specialne funkcije na kompaktni Liejevi grupi. **Karakteristične funkcije upodobitev**. **Ortogonalnostne relacije**. Peter-Weylov izrek.

Sferne funkcije kot **reprezentacijske funkcije**, pripadajoče upodobitvam grupe  $SU(2)$ . Legendrovi polinomi in njihove lastnosti.

Laplaceov operator v različnih koordinatah. Laplaceova enačba in sferne funkcije. Besselove funkcije.

Diferencialne enačbe v kompleksnem. Riemannova in hipergeometrična enačba. Hipergeometrična funkcija. Zveza med hipergeometrično funkcijo in sfernimi funkcijami.

Linearni diferencialni operatorji. **Posplošene** Fourierove vrste in pojem šibke rešitve. Diferencialni operatorji drugega reda in sistemi njihovih lastnih vektorjev.

Elementary Lie theory of matrix groups and algebras. Fundamental concepts of the theory of representations of compact Lie groups. Representations of the group  $SU(2)$ . Special functions as representative functions on compact matrix groups. Characters of representations. Orthogonality relations. Peter-Weyl theorem. Spherical harmonics as the representative functions of the group  $SU(2)$ . Legendre polynomials and their properties. Laplace operator in various coordinate systems. Laplace equation and spherical harmonics. Bessel functions. Complex differential equations. Riemann and hypergeometric equations. Hypergeometric function. Relation between the hypergeometric function and spherical harmonics. Linear differential operators. Generalized Fourier series and weak solutions. Differential operators of the Sturm-Liouville type and the associated eigenproblems.

### Temeljni literatura in viri / Readings:

- J. Dieudonné: *Special Functions and Linear Representations of Lie Groups*, AMS, Providence, 1979.
- T. Bröcker, T. T. Dieck: *Representations of Compact Lie Groups*, Springer, New York, 1985.
- E. Zakrajšek: *Analiza III*, DMFA-založništvo, Ljubljana, 2002.
- F. Križanič: *Navadne diferencialne enačbe in variacijski račun*, DZS, Ljubljana, 1974.
- S. Helgason: *Invariant Differential Operators and Eigenvalue Representations*, v *Representation Theory of Lie Groups*, Cambridge Univ. Press, Cambridge, 1980.

### Cilji in kompetence:

Študent spozna na poenoten način nekatere pomembne razrede specialnih funkcij. Seznan se z nekaterimi pomembnimi uporabami teh funkcij v matematiki in fiziki. Predstavljena je povezava teorije specialnih funkcij s tremi matematičnimi področji: s teorijo upodobitev Liejevih grup, s parcialnimi diferencialnimi enačbami in s teorijo linearnih diferencialnih operatorjev. Opisane so tudi osnove teorije

### Objectives and competences:

In the course some important classes of special functions are introduced. Some important applications of these functions in mathematics and physics are described. Special functions are considered from three different viewpoints: from the viewpoint of the representation theory of Lie groups, through the theory of differential equations and by means of the theory of differential operators and their

diferencialnih enačb v kompleksnem s poudarkom na hipergeometrični enačbi in hipergeometrični funkciji.

eigenproblems. Fundamental concepts of the theory of complex differential equations with the emphasis on the hypergeometric equation are presented.

#### **Predvideni študijski rezultati:**

**Znanje in razumevanje:** Poznavanje najpomembnejših razredov specialnih funkcij in njihovih lastnosti. Poznavanje najpomembnejših uporab teh funkcij. Na primeru specialnih funkcij študent vidi enotnost matematike, oziroma tesno povezanost različnih matematičnih področij.

**Poudarjena je pomembnost pojma simetrije v teoriji diferencialnih enačb.**

**Uporaba:** Reševanje nekaterih težjih matematičnih in fizikalnih problemov, katerih rešitve niso izrazljive z elementarnimi funkcijami.

**Refleksija:** Razumevanje teorije na podlagi uporabe. Razumevanje povezav med različnimi področji matematike na konkretnem primeru..

#### **Prenosljive spretnosti – niso vezane le na en**

**predmet:** Sposobnost uporabe širokega spektra različnih funkcij in z njimi povezanih diferencialnih enačb pri reševanju matematičnih in nematematičnih problemov. Študentovo znanje sega izven relativno omejenega sveta elementarnih funkcij.

#### **Intended learning outcomes:**

**Knowledge and understanding:** Familiarity with the most important classes of special functions. Understanding some important applications of these functions. Special functions provide a setting where elements of various mathematical fields merge into a unique theory.

The fundamental importance of the notion of symmetry in the theory of differential equations is discussed.

**Application:** Solving of some advanced mathematical and physical problems whose solutions cannot be expressed in terms of the elementary functions.

**Reflection:** Mastering the theory through its applications. Understanding various connections among different mathematical theories.

**Transferable skills:** Ability to use a vast variety of special functions and of the related differential equations in solving mathematical and non-mathematical problems. Students extend their horizon beyond the relatively limited realm of the elementary functions.

#### **Metode poučevanja in učenja:**

#### **Learning and teaching methods:**

Predavanja, vaje, <b>seminarski projekti</b> , domače naloge, konzultacije.	Lectures, classes, seminar projects, homework, consultations
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Delež (v %) /

**Načini ocenjevanja:**

Weight (in %)

**Assessment:**

<p>Način (domače naloge, pisni izpit, ustno izpraševanje, naloge, projekt):</p> <ul style="list-style-type: none"> <li>▪ <b>seminarski projekt</b></li> <li>▪ pisni izpit</li> <li>▪ ustni izpit</li> </ul> <p>Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)</p>	<p>20%</p> <p>40%</p> <p>40%</p>	<p>Type (written examination, oral examination, seminar project):</p> <ul style="list-style-type: none"> <li>▪ seminar project</li> <li>▪ written exam</li> <li>▪ oral exam</li> </ul> <p>Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)</p>
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**Reference nosilca / Lecturer's references:**

**prof. dr. Miran Černe**

- M. Černe, M. Zajec, Boundary differential relations for holomorphic functions on the disc. Proc. Am. Math. Soc. 139 (2011), 473-484.
- M. Černe, M. Flores, Generalized Ahlfors functions. Trans. Am. Math. Soc. 359 (2007), 671-686.
- M. Černe, M. Flores, Quasilinear  $\bar{\partial}$ -equation on bordered Riemann surfaces. Math. Ann. 335 (2006), 379-403.

**prof. dr. Janez Mrčun**

- J. Mrčun: *Functoriality of the bimodule associated to a Hilsum-Skandalis map*, K-Theory 18 (1999), 235-253.
- I. Moerdijk, J. Mrčun: *Introduction to Foliations and Lie Groupoids*, Cambridge Studies in Advanced Mathematics, 91. Cambridge University Press, Cambridge, 2003.
- I. Moerdijk, J. Mrčun: *Lie groupoids, sheaves and cohomology*, Poisson Geometry, Deformation Quantisation and Group Representations, 145-272, London Math. Soc. Lecture Note Ser. 323, Cambridge University Press, Cambridge, 2005.

**prof. dr. Pavle Saksida**

- P. Saksida: *Lattices of Neumann oscillators and Maxwell-Bloch equations*, Nonlinearity 19 (2006), 747-768.
- P. Saksida: *Integrable oscillators on spheres and hyperbolic spaces*, Nonlinearity 14 (2001), 977-994.
- P. Saksida: *On zero-curvature condition and Fourier analysis*, J. Phys. A: Math. Theor. 44 (2011), 85203-85222



## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Teorija mere
<b>Course title:</b>	Measure Theory

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2101

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
45		30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Roman Drnovšek, prof. dr. Bojan Magajna

**Jeziki / Languages:**

<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

**Prerequisites:**

Vpis v letnik študija

Enrollment into the program

**Vsebina:**

**Content (Syllabus outline):**

**Mere:**  $\sigma$ -algebre, pozitivne mere, zunanje mere, Caratheodoryjev izrek, razširitev mere iz algebre na sigma algebro, Borelove mere na  $\mathbf{R}$ , Lebesguova mera na  $\mathbf{R}$ .

**Merljive funkcije:** aproksimacija s stopničastimi funkcijami, načini konvergence funkcijskih zaporedij, izrek Jegorova.

**Integracija:** integral nenegativne funkcije, izrek o monotoni konvergenci, Fatoujeva lema, integral kompleksne funkcije, izrek o dominirani konvergenci, primerjava Riemannovega in Lebesguovega integrala, izrek Jegorova.

**Produktne mere:** konstrukcija produktnih mer, monotoni razredi, Tonellijev in Fubinijev izrek, Lebesguov integral na  $\mathbf{R}^n$ .

**Kompleksne mere:** predznačene mere, Hahnov in Jordanov razcep, kompleksne mere, variacija mere, absolutna zveznost in vzajemna singularnost, Lebesgue-Radon-Nikodymov izrek.

**$L^p$ -prostor:** neenakosti Jensena, Hölderja in Minkovskega, omejeni linearni funkcionali, dualni prostor.

**Integriranje na lokalno kompaktnih prostorih:** pozitivni linearni funkcionali na  $C_c(X)$ , Radonove mere, Rieszov izrek, Lusinov izrek, gostost prostora  $C_c(X)$  v  $L^p$ -prostorih.

**Odvajanje mer na  $\mathbf{R}^n$ :** odvajanje mer, absolutno zvezne in funkcije z omejeno totalno variacijo.

**Measures:**  $\sigma$ -algebras, positive measures, outer measures, Caratheodory's theorem, extension of measures from algebras to  $\sigma$ -algebras, Borel measures on  $\mathbf{R}$ , Lebesgue measure on  $\mathbf{R}$ .

**Measurable functions:** approximation by step functions, modes of convergence of sequences of functions, Egoroff's theorem.

**Integration:** integration of nonnegative functions, Lebesgue monotone convergence theorem, Fatou's lemma, integration of complex functions, Lebesgue dominated convergence theorem, comparison with Riemann's integral.

**Product measures:** construction of product measures, monotone classes, Tonelli's and Fubini's theorem, the Lebesgue integral on  $\mathbf{R}^n$ .

**Complex measures:** signed measures, the Hahn and the Jordan decomposition, complex measures, variation of a measure, absolute continuity and mutual singularity, the Lebesgue-Radon-Nikodym theorem.

**$L^p$ -spaces:** inequalities of Jensen, Hölder and Minkovski, bounded linear functionals, dual spaces.

**Integration on locally compact spaces:** positive linear functionals on  $C_c(X)$ , Radon measures, Riesz representation theorem, Lusin's theorem, density of  $C_c(X)$  in  $L^p$ -spaces.

**Differentiation of measures on  $\mathbf{R}^n$ :** differentiation of measures, absolutely continuous and functions of bounded variation,

### Temeljni literatura in viri / Readings:

- C. D. Aliprantis, O. Burkinshaw: *Principles of Real Analysis*, 3rd edition, Academic Press, San Diego, 1998.
- R. Drnovšek: *Rešene naloge iz teorije mere*, DMFA-založništvo, Ljubljana, 2001.
- G. B. Folland: *Real Analysis : Modern Techniques and Their Applications*, 2nd edition, John Wiley & Sons, New York, 1999.
- M. Hladnik: *Naloge in primeri iz funkcionalne analize in teorije mere*, DMFA-založništvo, Ljubljana, 1985.
- S. Kantorovitz: *Introduction to Modern Analysis*, Oxford Univ. Press, 2003.

- B. Magajna: *Osnove teorije mere*, DMFA-založništvo, Ljubljana, 2011.
- W. Rudin: *Real and Complex Analysis*, 3rd edition, McGraw-Hill, New York, 1987.

#### Cilji in kompetence:

Študent pridobi znanje osnov teorije mere, ki jih potrebuje za razumevanje osnov sodobnega verjetnostnega računa, statistike in funkcionalne analize.

#### Objectives and competences:

Students acquire basic knowledge of measure theory needed to understand probability theory, statistics and functional analysis.

#### Predvideni študijski rezultati:

**Znanje in razumevanje:** Razumevanje osnovnih pojmov teorije mere.

**Uporaba:** Teorija mere sodi med temeljne predmete na 2. stopnji študija matematike, saj je nujno potrebna za razumevanje osnov sodobnega verjetnostnega računa, statistike in funkcionalne analize. Poleg tega njena uporaba sega tudi v naravoslovje in druga področja znanosti kot na primer ekonomijo.

**Refleksija:** Razumevanje teorije na podlagi primerov uporabe.

**Prenosljive spretnosti – niso vezane le na en predmet:** Sposobnost abstraktnega razmišljanja. Spretnost uporabe domače in tuje literature.

#### Intended learning outcomes:

**Knowledge and understanding:** understanding basic concepts of measure and integration theory.

**Application:** measure theory is a part of the basic curriculum of the graduate study of mathematics since it is needed in other areas, for example, in probability calculus, statistics and functional analysis. It is useful also in other sciences, for example in economy.

**Reflection:** understanding of the theory on the basis of examples of application.

**Transferable skills:** Ability to use abstract methods to solve problems. Ability to use a wide range of references and critical thinking.

#### Metode poučevanja in učenja:

#### Learning and teaching methods:

predavanja, vaje, domače naloge, konzultacije	Lectures, exercises, homeworks, consultations
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Delež (v %) /

Weight (in %) **Assessment:**

**Načini ocenjevanja:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):		Type (examination, oral, coursework, project):
<ul style="list-style-type: none"> <li>▪ izpit iz vaj (2 kolokvija ali pisni izpit)</li> <li>▪ ustni izpit</li> </ul>	50%	<ul style="list-style-type: none"> <li>▪ 2 midterm exams instead of written exam, written exam</li> <li>▪ oral exam</li> </ul>
Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)	50%	Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

<p><b>prof. dr. Roman Drnovšek</b></p> <ol style="list-style-type: none"> <li>1. R. Drnovšek: <i>Spectral inequalities for compact integral operators on Banach function spaces</i>, Math. Proc. Camb. Philos. Soc. 112 (1992), 589-598.</li> <li>2. R. Drnovšek: <i>On invariant subspaces of Volterra-type operators</i>, Integr. Equ. Oper. Theory 27 (1997), 1-9.</li> <li>3. R. Drnovšek: <i>A generalization of Levinger's theorem to positive kernel operators</i>, Glasg. Math. J. 45 (2003), 545-555.</li> </ol> <p><b>Prof. dr. Bojan Magajna</b></p> <ol style="list-style-type: none"> <li>1. B. Magajna: <i>Sums of products of positive operators and spectra of Lüders operators</i>, Proc. Am. Math. Soc. 141 (2013), 1349-1360.</li> <li>2. B. Magajna: <i>Fixed points of normal completely positive maps on <math>B(H)</math></i>, J. Math. Anal. Appl. 389 (2012), 1291-1302.</li> <li>3. B. Magajna: <i>Uniform approximation by elementary operators</i>, Proc. Edinb. Math. Soc. 52 (2009) 731-749.</li> </ol>
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## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Teorija operatorjev
<b>Course title:</b>	Operator theory

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	drugi
Second cycle master study program Mathematics	none	first or second	second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2108

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
45		30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Roman Drnovšek, prof. dr. Peter Šemrl

**Jeziki / Languages:**

<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

vpis predmeta *Uvod v funkcionalno analizo*

**Prerequisites:**

enrollment into the course *Introduction to Functional Analysis*

**Vsebina:**

**Content (Syllabus outline):**

Kompaktni operatorji na Banachovih prostorih.

Schauderjev izrek o negibni točki.

Invariantni podprostori. Izrek Lomonosova.

Rieszov razcep kompaktnega operatorja.

Fredholmovi operatorji. Calkinova algebra.

Bistveni spekter.

Parcialne izometrije in unitarni operatorji.

Schmidtova reprezentacija kompaktnih operatorjev.

Hilbert-Schmidtovi operatorji. Dualnost med algebrami vseh omejenih operatorjev, vseh operatorjev s sledjo in vseh kompaktnih operatorjev.

Spekter normalnih operatorjev.

Spektralni izrek za normalne operatorje (v obliki operatorja množenja in v integralški obliki).

Fuglede-Putnamov izrek.

Compact operators on Banach spaces.

The Schauder fixed point theorem.

Invariant subspaces. Lomonosov's theorem. The

Riesz decomposition of a compact operator.

Fredholm operators. The Calkin algebra. The essential spectrum.

Partial isometries and unitary operators.

The Schmidt representation of a compact operator.

Hilbert-Schmidt operators. Duality between the algebra of all bounded operators, the algebra of all trace-class operators and the algebra of all compact operators.

The spectrum of normal operators.

The spectral theorem for normal operators (in the multiplication operator form and in the integral form).

The Fuglede-Putnam theorem.

### Temeljni literatura in viri / Readings:

- R. Bhatia: Notes on Functional Analysis, Texts and Readings in Mathematics 50, Hindustan Book Agency, New Delhi, 2009.
- J. B. Conway: A Course in Functional Analysis, 2nd edition, Springer, New York, 1990.
- I. Gohberg, S. Goldberg, M. A. Kaashoek: Classes of Linear Operators I, Birkhäuser, Basel, 1990.
- G. K. Pedersen: Analysis Now, Springer, New York, 1996.
- I. Vidav: Linearni operatorji v Banachovih prostorih, DMFA-založništvo, Ljubljana, 1982.

### Cilji in kompetence:

Obravnava nekaterih razredov omejenih linearnih operatorjev na Hilbertovih in Banachovih prostorih.

### Objectives and competences:

Treatment of some classes of bounded linear operators on Hilbert and Banach spaces.

### Predvideni študijski rezultati:

**Znanje in razumevanje:** Poznavanje osnovnih razredov linearnih operatorjev, sposobnost aplikacije pridobljenega znanja.

**Uporaba:** Uporaba teorije operatorjev sega tudi v naravoslovje in druga področja znanosti

### Intended learning outcomes:

**Knowledge and understanding:** Knowledge of some classes of linear operators, the ability to apply the acquired knowledge.

**Application:** Operator theory is used in natural sciences and other areas of science such as

kot na primer ekonomijo.

**Refleksija:** Razumevanje teorije, utrjeno s primeri uporabe.

**Prenosljive spretnosti – niso vezane le na en predmet:** Identifikacija in reševanje problemov. Spretnost uporabe domače in tuje literature.

economics.

**Reflection:** Understanding of the theory, strengthened by examples.

**Transferable skills:** Identifying and solving problems. Ability to use a wide range of references.

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations

**Načini ocenjevanja:**

Delež (v %) /  
Weight (in %)

**Assessment:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- domače naloge
- izpit iz vaj
- ustni izpit

10%

50%

40%

Type (examination, oral, coursework, project):

- homeworks
- written exam
- oral exam

Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

**prof. dr. Roman Drnovšek**

- R. Drnovšek: *Common invariant subspaces for collections of operators*, Integr. Equ. Oper. Theory 39 (2001), no. 3, 253-266.
- R. Drnovšek: *A generalization of Levinger's theorem to positive kernel operators*, Glasg. Math. J. 45 (2003), no. 3, 545-555.
- R. Drnovšek: *Invariant subspaces for operator semigroups with commutators of rank at most one*, J. Funct. Anal. 256 (2009), no. 12, 4187-4196.

**prof. dr. Peter Šemrl**

- P. Šemrl: *Similarity preserving linear maps*, J. Operator Theory 60 (2008), no. 1, 71-83.
- P. Šemrl: *Local automorphisms of standard operator algebras*, J. Math. Anal. Appl. 371 (2010), no. 2, 403-406.
- P. Šemrl: *Symmetries on bounded observables: a unified approach based on adjacency preserving maps*, Integral Equations Oper. Theory 72 (2012), no. 1, 7-66.



## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Uvod v C*-algebre
<b>Course title:</b>	Introduction to C*-algebras

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** še ni dodeljena/not assigned yet

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
45		30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Matej Brešar, prof. dr. Bojan Magajna

<b>Jeziki / Languages:</b>	<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
	<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija in poznavanje osnov funkcionalne analize.

**Prerequisites:**

Enrollment into the program and basic knowledge of functional analysis.

**Vsebina:**

**Content (Syllabus outline):**

**Banachove algebre:** ideali, kvocienti, holomorfen funkcijski račun, šibka\* topologija in Banach Alaoglujev izrek, Gelfandova transformacija.

**C\*-algebre:** urejenost, približna enota, ideali in kvocienti, karakterizacija komutativnih C\*-algeber, zvezen funkcijski račun, stanja in upodobitve, univerzalna upodobitev.

**Operatorske topologije in aproksimacijski izreki:** von Neumannov o bikomutantu, Kaplanskega o gostosti in Kadisonov o tranzitivnosti.

**Spektralni izrek za omejene normalne operatorje:** Borelov funkcijski račun, komutativne von Neumannove algebre, grupna algebra  $L^1(G)$ .

**Banach algebras:** ideals, quotients, holomorphic functional calculus, weak\* topology, Banach Alaoglu's theorem, Gelfand's transform.

**C\*-algebras:** order, approximate units, ideals, quotients, the characterization of commutative C\*-algebras, continuous functional calculus, states and representations, the universal representation.

**Operator topologies and approximation theorems:** von Neumann's bicommutation theorem, Kaplansky's density theorem and Kadison's transitivity theorem.

**The spectral theorem for bounded normal operators:** the Borel functional calculus, commutative von Neumann algebras, the group algebra  $L^1(G)$ .

#### Temeljni literatura in viri / Readings:

- G. K. Pedersen: *Analysis Now*, Springer, Berlin, 1989.
- J. B. Conway: *A Course in Functional Analysis*, Springer, Berlin, 1978.
- J. B. Conway: *A Course in Operator Theory*, GSM 91, Amer. Math. Soc., 2000.
- R. V. Kadison in J. R. Ringrose: *Fundamentals of the Theory of Operator Algebras I, II*, Graduate Studies in Math. 15, 16, Amer. Math. Soc., 1997.
- I. Vidav: *Banachove algebre*, DMFA-založništvo, Ljubljana, 1982.
- I. Vidav: *Uvod v teorijo C\*-algeber*, DMFA-založništvo, Ljubljana, 1982.
- N. Weaver: *Mathematical Quantization*, Chapman & Hall/CRC, London, 2001.

#### Cilji in kompetence:

Spoznati osnovna orodja spektralne teorije in njihovo uporabo v C\*-algebrah.

#### Objectives and competences:

To master basic tools of spectral theory and their use in C\*-algebras.

#### Predvideni študijski rezultati:

#### Intended learning outcomes:

**Znanje in razumevanje:** pridobljeno osnovno znanje o C\*-algebrah bo koristilo tudi izven matematike, npr. pri razumevanju kvantne fizike.

**Uporaba:** Pridobljeno znanje bo uporabno tudi drugod v matematični analizi in matematični fiziki.

**Refleksija:** C\*-algebre so eno temeljnih aktivnih področij sodobne matematike.

**Prenosljive spretnosti – niso vezane le na en predmet:**

Formulacija in reševanje problemov z abstraktnimi metodami.

**Knowledge and understanding:** the basic knowledge on C\*-algebras may be useful also outside of mathematics, for example, it may facilitate the understanding of quantum physics.

**Application:** The acquired knowledge is applicable elsewhere in mathematics and mathematical physics.

**Reflection:** C\*-algebras are one of the basic active fields of modern mathematics.

**Transferable skills:**

An approach to problems using abstract methods.

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations

**Načini ocenjevanja:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- izpit iz vaj (2 kolokvija ali pisni izpit)
- ustni izpit

Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)

Delež (v %) /

Weight (in %)

50%

50%

**Assessment:**

Type (examination, oral, coursework, project):

- 2 midterm exams instead of written exam, written exam
- oral exam

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

**Prof. dr. Matej Brešar**

1. M. Brešar, E. Kissin, V. Shulman, *Lie ideals: from pure algebra to C\*-algebras*, J. Reine

Angew. Math. 623 (2008), 73-121.

2. M. Brešar, Š. Špenko, *Determining elements in Banach algebras through spectral properties*, J. Math. Anal. Appl. 393 (2012), 144-150.
3. M. Brešar, B. Magajna, Š. Špenko, *Identifying derivations through the spectra of their values*, Integr. Eq. Oper. Theory 73 (2012), 395-411.

**Prof. dr. Bojan Magajna**

1. B. Magajna: *The Haagerup norm on the tensor product of operator modules*, J. Funct. Anal. 129 (1995), 325-348.
2. D. Blecher, B. Magajna: *Duality and operator algebras: automatic weak\* continuity and applications*, J. Funct. Anal. 224 (2005), 386-407.
3. B. Magajna: *Fixed points of normal completely positive maps on  $B(H)$* , J. Math. Anal. Appl. 389 (2012), 1291-1302.

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

<b>Predmet:</b>	Uvod v funkcionalno analizo
<b>Course title:</b>	Introduction to Functional Analysis

<b>Študijski program in stopnja</b> Study programme and level	<b>Študijska smer</b> Study field	<b>Letnik</b> Academic year	<b>Semester</b> Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2102

<b>Predavanja</b> Lectures	<b>Seminar</b> Seminar	<b>Vaje</b> Tutorial	<b>Klinične vaje</b> work	<b>Druge oblike</b> študija	<b>Samost. delo</b> Individ. work	<b>ECTS</b>
<b>45</b>		<b>30</b>			<b>105</b>	<b>6</b>

**Nosilec predmeta / Lecturer:** prof. dr. Roman Drnovšek, prof. dr. Bojan Magajna,  
prof. dr. Peter Šemrl

**Jeziki / Languages:** **Predavanja / Lectures:** slovenski/Slovene, angleški/English  
**Vaje / Tutorial:** slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija

**Prerequisites:**

Enrollment into the program

**Vsebina:**

**Content (Syllabus outline):**

Hilbertovi prostori. Ortonormirani sistemi. Besslova neenakost. Kompletnost. Fourierove vrste. Parsevalova enakost.

Linearni operatorji in funkcionali na Hilbertovih prostorih.

Reprezentacija zveznega linearnega funkcionala.

Adjungirani operator. Sebiadjungirani in normalni operatorji.

Projektorji in idempotenti. Invariantni podprostori.

Kompaktni operatorji. Spekter kompaktne operatorja.

Diagonalizacija kompaktne sebiadjungiranega operatorja.

Uporaba: Sturm-Liouillovi sistemi.

Banachovi prostori. Primeri.

Linearni operatorji in funkcionali na Banachovih prostorih.

Končnorazsežni normirani prostori. Kvocienti in produkti normiranih prostorov.

Hahn-Banachov izrek in posledice. Separacija konveksnih množic.

Hilbert spaces. Orthonormal systems. Bessel's inequality. Completeness. Fourier series. Parseval's identity.

Linear operators and functionals on Hilbert spaces.

The representation of a continuous linear functional.

Adjoint operator. Selfadjoint and normal operators.

Projectors and idempotents. Invariant subspaces.

Compact operators. The spectrum of a compact operator.

Diagonalization of a selfadjoint compact operator.

An application: Sturm-Liouville systems.

Banach spaces. Examples.

Linear operators and functionals on Banach spaces.

Finite dimensional normed spaces. Quotients and products of normed spaces.

The Hahn-Banach theorem and consequences.

Separation of convex sets.

#### Temeljni literatura in viri / Readings:

- B. Bollobás: *Linear Analysis : An Introductory Course*, 2nd edition, Cambridge Univ. Press, Cambridge, 1999.
- J. B. Conway: *A Course in Functional Analysis*, 2nd edition, Springer, New York, 1990.
- Y. Eidelman, V. Milman, A. Tzolomitis: *Functional Analysis : An Introduction*, AMS, Providence, 2004.
- D. H. Griffel: *Applied Functional Analysis*, Dover Publications, Mineola, 2002.
- M. Hladnik: *Naloge in primeri iz funkcionalne analize in teorije mere*, DMFA-založništvo, Ljubljana, 1985.
- E. Zeidler: *Applied Functional Analysis : Main Principles and Their Applications*, Springer, New York, 1995.

**Cilji in kompetence:**

**Objectives and competences:**

Študent spozna osnovne pojme teorije Hilbertovih prostorov in linearnih operatorjev med njimi. Z njeno uporabo se seznanijo pri reševanju Sturm-Liouville problema.

Nekoliko spozna tudi teorijo Banachovih prostorov, ki so posplošitev Hilbertovih prostorov.

Students acquire basic knowledge of the theory of Hilbert spaces and linear operators between them. The theory is applied for solving simple Sturm-Liouville problems. Students also learn some basic concepts from the theory of Banach spaces, which are a generalization of Hilbert spaces.

#### Predvideni študijski rezultati:

**Znanje in razumevanje:** Razumevanje teorije Hilbertovih prostorov s teoretičnega in uporabnega vidika.

**Uporaba:** Uporaba funkcionalne analize sega tudi v naravoslovje in druga področja znanosti kot na primer ekonomijo.

**Refleksija:** Razumevanje teorije na podlagi uporabe.

**Prenosljive spretnosti – niso vezane le na en predmet:** Sposobnost abstraktnega razmišljanja. Spretnost uporabe domače in tuje literature.

#### Intended learning outcomes:

**Knowledge and understanding:** Understanding of the theory of Hilbert spaces.

**Application:** Functional analysis is used in natural sciences and other areas of science such as economics.

**Reflection:** Understanding of the theory on the basis of examples.

**Transferable skills:** Ability to use abstract methods to solve problems. Ability to use a wide range of references and critical thinking.

#### Metode poučevanja in učenja:

predavanja, vaje, domače naloge, konzultacije

#### Learning and teaching methods:

Lectures, exercises, homeworks, consultations

#### Načini ocenjevanja:

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- domače naloge
- izpit iz vaj
- ustni izpit

Delež (v %) /

Weight (in %)

#### Assessment:

Type (examination, oral, coursework, project):

- homeworks
- written exam
- oral exam

10%

50%

40%

Ocene: 1-5 (negativno), 6-10 (pozitivno)  
(po Statutu UL)

Grading: 1-5 (fail), 6-10 (pass) (according  
to the Statute of UL)

**Reference nosilca / Lecturer's references:**

**prof. dr. Roman Drnovšek**

- R. Drnovšek: An irreducible semigroup of idempotents, Stud. Math. 125 (1997), no. 1, 97-99.
- R. Drnovšek: Common invariant subspaces for collections of operators, Integr. Equ. Oper. Theory 39 (2001), no. 3, 253-266.
- R. Drnovšek: Invariant subspaces for operator semigroups with commutators of rank at most one, J. Funct. Anal. 256 (2009), no. 12, 4187-4196.

**prof. dr. Bojan Magajna**

- B. Magajna: On tensor products of operator modules, J. Oper. Theory 54 (2005), no. 2, 317-337.
- B. Magajna: Duality and normal parts of operator modules, J. Funct. Anal. 219 (2005), no. 2, 206-339.
- B. Magajna: On completely bounded bimodule maps over  $W^*$ -algebras, Studia Math. 154 (2003), no. 2, 137-164.

**prof. dr. Peter Šemrl**

- P. Šemrl, Väisälä: Nonsurjective nearisometries of Banach spaces, J. Funct. Anal. 198 (2003), 268-278.
- P. Šemrl: Generalized symmetry transformations on quaternionic indefinite inner product spaces: An extension of quaternionic version of Wigner's theorem, Comm. Math. Phys. 242 (2003), 579-584.
- P. Šemrl: Applying projective geometry to transformations on rank one idempotents, J. Funct. Anal. 210 (2004), 248-257.



## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Uvod v harmonično analizo
<b>Course title:</b>	Introduction to harmonic analysis

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2105

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
45		30			105	6

**Nosilec predmeta / Lecturer:** doc. dr. Oliver Dragičević

<b>Jeziki / Languages:</b>	<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
	<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija

**Prerequisites:**

Enrollment into the program

**Vsebina:**

**Content (Syllabus outline):**

- Fourierove vrste, sumacijske metode, Riesz-Thorinov interpolacijski izrek;
- harmonične funkcije, Poissonovi integrali, Hardyjevi prostori, harmonična konjugiranka, Hilbertova transformacija;
- Schwartzov razred, Fourierova transformacija, distribucije in umirjene distribucije;
- šibki  $L^p$  prostori in Marcinkiewiczev interpolacijski izrek, Paley-Wienerjev izrek ter princip nedoločenosti;
- Hardy-Littlewoodova maksimalna funkcija;
- Calderón-Zygmundovi singularni integralni operatorji;
- linearni parcialni diferencialni operatorji s konstantnimi koeficienti, fundamentalna rešitev, prostori Soboljeva.

- Fourier series, summation methods, Riesz-Thorin interpolation theorem;
- Harmonic functions, Poisson integrals, Hardy spaces, harmonic conjugate, Hilbert transform;
- Schwartz class, Fourier transform, distributions and tempered distributions;
- weak  $L^p$  spaces and the Marcinkiewicz interpolation theorem, the Paley-Wiener theorem and the uncertainty principle;
- Hardy-Littlewood maximal function;
- Calderón-Zygmund singular integral operators;
- linear partial differential operators with constant coefficients, fundamental solution, Sobolev spaces.

#### Temeljni literatura in viri / Readings:

- L. Grafakos: Classical Fourier Analysis, Second Edition, Graduate Texts in Mathematics 249, Springer, 2008.
- E. M. Stein, G. L. Weiss: Introduction to Fourier Analysis on Euclidean Spaces, Princeton University Press, 1971.
- A. Torchinsky: Real-Variable Methods in Harmonic Analysis, Academic Press, 1986.
- Y. Katznelson: An introduction to harmonic analysis, Dover, New York, 1976.
- L. Hörmander: The Analysis of Linear Partial Differential Operators I: Distribution Theory and Fourier Analysis, Berlin Heidelberg New York 1990.

#### Cilji in kompetence:

Spoznavanje temeljnih pojmov in orodij harmonične analize na evklidskih prostorih; umeščanje v kontekst parcialnih diferencialnih enačb.

#### Objectives and competences:

Acquiring knowledge of fundamental notions and tools of euclidean harmonic analysis; placing them into the context of partial differential equations.

#### Predvideni študijski rezultati:

**Znanje in razumevanje:** Obvladovanje osnovnih konceptov harmonične analize na evklidskih prostorih.

**Uporaba:** Parcialne diferencialne enačbe,

#### Intended learning outcomes:

**Knowledge and understanding:** Mastering basic concepts of euclidean harmonic analysis.

**Application:** PDE, mathematical physics, natural sciences, medicine.

matematična fizika, naravoslovje, medicina.

**Refleksija:** Gre za eno temeljnih področij sodobne matematične analize.

**Prenosljive spretnosti – niso vezane le na en predmet:** Prepoznavanje problemov, ki sodijo v področje harmonične analize oziroma formulacija in reševanje nalog s pomočjo metod klasične Fourierove analize.

**Reflection:** The course subject is one of the cornerstones of modern mathematical analysis.

**Transferable skills:** Recognition of problems in the realm of harmonic analysis; formulation and solving problems with methods of classical Fourier analysis.

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations

**Načini ocenjevanja:**

Delež (v %) /  
Weight (in %)

**Assessment:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt): <ul style="list-style-type: none"><li>▪ domače naloge</li><li>▪ ustni zagovor</li></ul>	50%	Type (examination, oral, coursework, project): <ul style="list-style-type: none"><li>▪ homework assignments</li><li>▪ oral exam</li></ul>
Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)	50%	Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

- O. Dragičević, A. Volberg: Linear dimension-free estimates in the embedding theorem for Schrödinger operators, J. London Math. Soc. (2) 85 (2012), 191-222.
- O. Dragičević, A. Volberg: Bilinear embedding for real elliptic differential operators in divergence form with potentials, J. Funct. Anal. 261 no. 10 (2011), 2816-2828.
- O. Dragičević: Weighted estimates for powers of the Ahlfors-Beurling operator, Proc. Amer. Math. Soc. 139 no. 6 (2011), 2113-2120.

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

<b>Predmet:</b>	Izbrana poglavja iz diskretne matematike 1
<b>Course title:</b>	Topics in discrete mathematics 1

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2206

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Sandi Klavžar, doc. dr. Matjaž Konvalinka, prof. dr. Marko Petkovšek, prof. dr. Tomaž Pisanski, prof. dr. Primož Potočnik, prof. dr. Riste Škrekovski

**Jeziki / Languages:**

<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

**Prerequisites:**

Vpis v letnik študija

Enrollment into the program

**Vsebina:** Content (Syllabus outline):

Predavatelj/ica izbere nekatere pomembne teme iz diskretne matematike, kot npr.:

Delno urejene množice.

Ramseyeva teorija.

Matroidi.

Diskretna geometrija.

Načrti in konfiguracije.

Simetrični grafi.

Simetrije kombinatoričnih objektov.

Simetrične funkcije.

Kombinatorno preštevanje.

Diskretna verjetnost.

Metrična teorija grafov.

Teorija dominacije.

Problem hanojskega stolpa.

Pri tem si prizadeva minimizirati prekrivanje z drugimi predmeti tega študijskega programa.

The lecturer selects some important topics in discrete mathematics, such as:

Partially ordered sets.

Ramsey theory.

Matroids.

Discrete geometry.

Designs and configurations.

Symmetric graphs.

Symmetries of combinatorial objects.

Symmetric functions.

Combinatorial enumeration.

Discrete probability.

Metric graph theory.

Domination theory.

The Tower of Hanoi problem.

Special care should be taken to minimize overlap with other courses in this program.

#### Temeljni literatura in viri / Readings:

- Jack H. van Lint, Robin J. Wilson: *A Course in Combinatorics*, Cambridge University Press, Cambridge, 2001.
- R. L. Graham, M. Grötschel and L. Lovász, editors: *Handbook of Combinatorics*, Elsevier Science B.V., Amsterdam; MIT Press, Cambridge, MA, 1995

Predavatelj poleg tega lahko izbere tudi primerne novejšje raziskovalne članke iz znanstvenih revij.

#### Cilji in kompetence:

Študent spozna nekatera pomembna področja diskretne matematike, kot so delno urejene množice, diskretna geometrija, diskretna verjetnost, razčlenitve in simetrične funkcije.

#### Objectives and competences:

Students encounter some of the important areas of discrete mathematics, such as partially ordered sets, discrete geometry, discrete probability, partitions, and symmetric functions.

#### Predvideni študijski rezultati:

**Znanje in razumevanje:** Študentje se seznanijo s tematiko, metodami in glavnimi rezultati različnih področij diskretne matematike.

**Uporaba:** Študent bo znal pridobljeno znanje uporabiti v različnih matematičnih in drugih kontekstih.

#### Intended learning outcomes:

**Knowledge and understanding:** Students get acquainted with the subject matter, the methods, and the main results of various areas of discrete mathematics.

**Application:** Students will be able to use their knowledge in different mathematical and other

**Refleksija:** Študentje spoznajo in razumejo medsebojno prepletanje in oplajanje različnih področij diskretne matematike.

**Prenosljive spretnosti – niso vezane le na en predmet:** Študentje spoznajo nekatere metode, uporabne pri konstrukciji in analizi diskretnih matematičnih modelov .

contexts.

**Reflection:** Students comprehend the interplay and mutual enrichment of various areas of discrete mathematics.

**Transferable skills:** Students learn methods which are useful in construction and analysis of discrete mathematical models.

**Metode poučevanja in učenja:**  
predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**  
Lectures, exercises, homeworks, consultations

		Delež (v %) / Weight (in %)	Assessment:
<b>Načini ocenjevanja:</b> Način (pisni izpit, ustno izpraševanje, naloge, projekt):	▪ izpit iz vaj (2 kolokvija ali pisni izpit)	50%	Type (examination, oral, coursework, project):
	▪ ustni izpit	50%	▪ 2 midterm exams instead of written exam, written exam
Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)			▪ oral exam
			Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

**prof. dr. Sandi Klavžar**

- S. Klavžar: Structure of Fibonacci cubes: a survey, *J. Comb. Optim.* **25** (2013) 505-522.
- S. Klavžar, S. Shpectorov: Convex excess in partial cubes, *J. Graph Theory* **69** (2012) 356-369.
- R. Hammack, W. Imrich, S. Klavžar: *Handbook of Product Graphs: Second Edition*, CRC Press, 2011, 536 str.

**doc. dr. Matjaž Konvalinka**

- M. Konvalinka: Non-commutative extensions of the MacMahon Master Theorem, *Adv.*

*Math.* **216** (2007) 29–61.

- M. Konvalinka: Divisibility of generalized Catalan numbers, *J. Combin. Theory Ser. A* **114** (2007) 1089-1100.
- M. Konvalinka, I. Pak: Triangulations of Cayley and Tutte polytopes. *Adv. Math.* **245** (2013) 1–33.

**prof. dr. Marko Petkovšek**

- M. Petkovšek: Counting Young tableaux when rows are cosets, *Ars Comb.* **37** (1994) 87-95.
- M. Petkovšek, H. S. Wilf, D. Zeilberger: *A=B*, Wellesley (Massachusetts): A K Peters, 1996.
- M. Petkovšek: Letter graphs and well-quasi-order by induced subgraphs, *Discrete Math.* **244** (2002) 375-388.

**prof. dr. Tomaž Pisanski**

- M. Boben, T. Pisanski: Polycyclic configurations, *Eur. J. Comb.* **24** (2003) 431-457.
- T. Pisanski, M. Randić: Use of the Szeged index and the revised Szeged index for measuring network bipartivity. *Discrete Appl. Math.* **158** (2010) 1936-1944.
- T. Pisanski, B. Servatius: *Configurations from a graphical viewpoint*, New York: Birkhäuser, 2013.

**prof. dr. Primož Potočnik**

- P. Potočnik: Tetravalent arc-transitive locally-Klein graphs with long consistent cycles, *European J. Combin.* **36** (2014) 270-281.
- P. Potočnik, P. Spiga, G. Verret: Cubic vertex-transitive graphs on up to 1280 vertices, *J. Symbolic Comp.* **50** (2013) 465-477.
- P. Potočnik: Edge-colourings of cubic graphs admitting a solvable vertex-transitive group of automorphisms, *J. Combin. Theory Ser. B* **91** (2004) 289-300.

**prof. dr. Riste Škrekovski**

- J. Govorčin, M. Knor, R. Škrekovski: Line graph operator and small worlds, *Inform. Process. Lett.* **113** (2013) 196-200.
- Z. Dvorak, B. Lidicky, R. Škrekovski: Randić index and the diameter of a graph, *European J. Comb.* **32** (2011) 434-442.
- T. Kaiser, M. Stehlik, R. Škrekovski: On the 2-resonance of fullerenes, *SIAM J. Discrete Math.* **25** (2011) 1737-1745.

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

**Predmet:** Izbrana poglavja iz diskretne matematike 2  
**Course title:** Topics in discrete mathematics 2

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** še ni dodeljena/not assigned yet

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Sandi Klavžar, prof. dr. Primož Potočnik,izr. prof. dr. Riste Škrekovski

**Jeziki / Languages:**  
**Predavanja / Lectures:** slovenski/Slovene, angleški/English  
**Vaje / Tutorial:** slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija

**Prerequisites:**

Enrollment into the program

**Vsebina:**

**Content (Syllabus outline):**



Predavatelj ob vsakokratnem izvajanju izbere nekaj relevantnih tem iz diskretne matematike, pri čemer je pozoren na prekrivanje z drugimi predmeti iz programa Matematika (prekrivanje naj bo minimalno) in zahtevano predznanje (predznanje naj bo omejeno na obvezne predmete programa Matematika).

The lecturer chooses a few relevant topics in discrete mathematics, while paying attention to a possible overlap with other courses in the program Mathematics (the overlap should be minimal) and prerequisites (those should be bound to obligatory courses of the programme Mathematics).

#### Temeljni literatura in viri / Readings:

- N. L. Biggs, A. T. White: Permutation Groups and Combinatorial Structures, Cambridge University Press, Cambridge, 1979.
- C. Godsil, G. Royle: Algebraic Graph Theory. Springer, New York, 2001.
- Jack H. van Lint, Robin J. Wilson: A Course in Combinatorics, Cambridge University Press, Cambridge, 2001.
- Laszlo Lovasz, Jozsef Pelikan, Katalin Vesztergombi: Discrete Mathematics, Springer, Berlin-Heidelberg-New York, 2003.
- Richard P. Stanley: Enumerative Combinatorics, Vol. 2, Cambridge University Press, New York-Cambridge, 1999.

#### Cilji in kompetence:

Slušatelj spozna predstavljene teme.

#### Objectives and competences:

Students becomes acquainted with the presented topics.

#### Predvideni študijski rezultati:

**Znanje in razumevanje:** Študent bo razumel predstavljene koncepte in rezultate.

**Uporaba:** Študent bo znal pridobljeno znanje uporabiti v različnih matematičnih in drugih kontekstih.

**Refleksija:** Pridobljeno znanje bo študent znal kritično reflektirati.

**Prenosljive spretnosti – niso vezane le na en predmet:** Veščina kritičnega mišljenja, prepoznavanje diskretnih struktur v naravi in

#### Intended learning outcomes:

**Knowledge and understanding:** Student will understand the presented topics and results.

**Application:** Student will know how to use the new knowledge in different mathematical and other contexts.

**Reflection:** Student will be able to critically reflect the topic.

**Transferable skills:** Skill of critical thought, identification of discrete structures in nature and society.

družbi.

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations

**Načini ocenjevanja:**

Način (domače naloge, pisni izpit, ustno izpraševanje, naloge, projekt):

- domače naloge ali projekt
- pisni izpit
- ustni izpit

Ocene: 1-5 (negativno), 6-10 (pozitivno)  
(po Statutu UL)

Delež (v %) /

Weight (in %) /

**Assessment:**

Type (homeworks, examination, oral, coursework, project):

- homeworks or project
- written exam
- oral exam

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

**prof. dr. Primož Potočnik:**

- P. Potočnik, Tetravalent arc-transitive locally-Klein graphs with long consistent cycles, European J. Combin., vol. 36 (2014), 270-281.
- P. Potočnik, P. Spiga, G. Verret, Cubic vertex-transitive graphs on up to 1280 vertices, J. Symbolic Comp. vol. 50 (2013), 465-477.
- P. Potočnik, Edge-colourings of cubic graphs admitting a solvable vertex-transitive group of automorphisms, Journal of Combinatorial Theory Ser. B, vol. 91 (2004), 289-300.

**prof. dr. Sandi Klavžar:**

- S. Klavžar, Structure of Fibonacci cubes: a survey, J. Comb. Optim., vol. 25 (2013), 505-522.
- S. Klavžar, S. Shpectorov, Convex excess in partial cubes, J. Graph Theory, vol. 69 (2012), 356-369.
- R. Hammack, W. Imrich, S. Klavžar, Handbook of Product Graphs: Second Edition, CRC Press, 2011, 536 str.

**Izr. prof. dr. Riste Škrekovski:**

- J. Govorčin, M. Knor, R. Škrekovski, Line graph operator and small worlds, Inform. Process. Lett.

vol. 113 (2013) 196-200.

- Z. Dvorak, B. Lidicky, R. Škrekovski, Randić index and the diameter of a graph, *European J. Comb.* vol. 32 (2011) 434-442.
- T. Kaiser, M. Stehlik, R. Škrekovski, On the 2-resonance of fullerenes, *SIAM J. Disc. Math.* vol. 25 (2011) 1737-1745.

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

**Predmet:** Kardinalna aritmetika  
**Course title:** Cardinal arithmetic

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type**

**Univerzitetna koda predmeta / University course code:**

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
45		30			105	6

**Nosilec predmeta / Lecturer:**

**Jeziki / Languages:**  
**Predavanja / Lectures:**   
**Vaje / Tutorial:**

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

**Prerequisites:**

**Vsebina:**

**Content (Syllabus outline):**

Množice in razredi, aksiomi teorije množic, aksiom izbire, Zornova lema in uporaba, dobra urejenost, transfinitna indukcija, ordinalna števila in računanje z njimi, Schröder-Bernsteinov izrek, kardinalna števila in njihova aritmetika. V odvisnosti od časa še: filtri in ultrafiltri, velika kardinalna števila.

Sets and classes. Axioms of set theory. Axiom of choice, Zorn lemma and its applications, well ordering, transfinite induction, ordinal numbers and their arithmetic, Schröder-Bernstein theorem, cardinal numbers and their arithmetic. If time permits: filters and ultrafilters, large cardinal numbers.

#### Temeljna literatura in viri / Readings:

- W. Just, M. Weese: *Discovering Modern Set Theory I*. AMS, 1991.
- P. R. Halmos: *Naive set theory*, Springer-Verlag, New York, 1974.
- H. Ebbinghaus et al.: *Numbers*, Springer-Verlag, New York, 1990.
- N. Prijatelj: *Matematične strukture I*, DMFA-založništvo, Ljubljana, 1996.

#### Cilji in kompetence:

Poglobliti temeljno znanje o aksiomatski teoriji množic ter se seznaniti z osnovami ordinalne in kardinalne aritmetike.

#### Objectives and competences:

Improvement of knowledge of axiomatic set theory and acquaintance with the basics of ordinal and cardinal arithmetic.

#### Predvideni študijski rezultati:

##### Znanje in razumevanje:

Razumevanje in uporaba aksiomatske teorije množic ter ordinalne in kardinalne aritmetike.

##### Uporaba:

Teorija množic je temeljno matematično področje, ki priskrbi osnovni jezik za druga področja. V tem okviru so Zornova lema in ordinalna ter kardinalna števila nepogrešljiva orodja, uporabna širom matematike, zanimiva pa so tudi za nekatere filozofe.

##### Refleksija:

Teorija množic združuje vse matematične vede v celoto.

##### Prenosljive spretnosti – niso vezane le na en predmet:

Ker za razumevanje predmeta ne bo potrebno kako predhodno specialistično predznanje, bo

#### Intended learning outcomes:

##### Knowledge and understanding:

Understanding and application of axiomatic set theory and ordinal and cardinal arithmetic.

##### Application:

Set theory is a fundamental branch of mathematics that provides the common language of mathematics. The Zorn lemma, ordinal and cardinal numbers are thus basic tools that find applications everywhere in mathematics. They are also interesting for philosophers.

##### Reflection:

Set theory provides a unifying approach to mathematics.

##### Transferable skills:

As no specific technical knowledge is necessary to follow the course, it is generally useful for

zelo primeren tudi za učenje in vadbo matematičnega razmišljanja.

development of mathematical technique and practice of mathematical thinking.

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations

**Načini ocenjevanja:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- izpit iz vaj (2 kolokvija ali pisni izpit)
- ustni izpit

Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)

Delež (v %) /  
Weight (in %)

50%

50%

**Assessment:**

Type (examination, oral, coursework, project):

- 2 midterm exams instead of written exam, written exam
- oral exam

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

**prof. dr. Bojan Magajna**

- B. Magajna: *Infinitezimali*, Obzornik mat. fiz. 30 (1983), no. 2, 33-41.
- B. Magajna: *The minimal operator module of a Banach module*, Proc. Edinburgh Math. Soc. (2) 42 (1999), no. 1, 191-208.
- C. Le Merdy, B. Magajna: *A factorization problem for normal completely bounded mappings*, J. Funct. Anal. 181 (2001), no. 2, 313—345.

**prof. dr. Marko Petkovšek**

- M. Petkovšek: *Ambiguous numbers are dense*, Amer. Math. Monthly 97 (1990), str. 408-411.
- M. Petkovšek, H. S. Wilf, D. Zeilberger: *A = B*, A K Peters, Wellesley MA, 1996.
- M. Petkovšek: *Letter graphs and well-quasi-order by induced subgraphs*, Discrete Math. 244 (2002), str. 375-388.

**izred. prof. dr. Andrej Bauer**

- S. Awodey, A. Bauer: *Propositions as [Types]*, Journal of Logic and Computation. Volume 14, Issue 4, August 2004, pp. 447-471.
- A. Bauer, A. Simpson: *Two constructive embedding-extension theorems with applications to continuity principles and to Banach-Mazur computability*, Mathematical Logic Quarterly, 50(4,5):351-369, 2004.
- A. Bauer: *A relationship between equilogical spaces and Type Two Effectivity*, Math. Logic Quarterly, 2002, vol. 48, suppl. 1, str. 1-15.

UČNI NAČRT PREDMETA / COURSE SYLLABUS						
<b>Predmet:</b>		Kombinatorika				
<b>Course title:</b>		Combinatorics				
<b>Študijski program in stopnja</b> Study programme and level		<b>Študijska smer</b> Study field		<b>Letnik</b> Academic year	<b>Semester</b> Semester	
Magistrski študijski program 2. stopnje Matematika		ni smeri		prvi ali drugi	prvi ali drugi	
Second cycle master study program Mathematics		none		first or second	first or second	
<b>Vrsta predmeta / Course type</b>				izbirni predmet/elective course		
<b>Univerzitetna koda predmeta / University course code:</b>				M2205		
<b>Predavanja</b> Lectures	<b>Seminar</b> Seminar	<b>Vaje</b> Tutorial	<b>Klinične vaje</b> work	<b>Druge oblike študija</b>	<b>Samost. delo</b> Individ. work	<b>ECTS</b>
45		30			105	6
<b>Nosilec predmeta / Lecturer:</b>		prof. dr. Sandi Klavžar, doc. dr. Matjaž Konvalinka, prof. dr. Marko Petkovšek				
<b>Jeziki / Languages:</b>		<b>Predavanja / Lectures:</b> slovenski/Slovene, angleški/English				
		<b>Vaje / Tutorial:</b> slovenski/Slovene, angleški/English				
<b>Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:</b>			<b>Prerequisites:</b>			
Vpis v letnik študija			Enrollment into the program			
<b>Vsebina:</b>			<b>Content (Syllabus outline):</b>			



Dvanajstera pot (binomski koeficienti, Stirlingova števila 1. in 2. vrste, Lahova števila, razčlenitve ...; z rodovnimi funkcijami)

Običajne in eksponentne rodovne funkcije: kombinatorični pomen operacij vsote, produkta, odvoda, kompozicije (eksponentna formula)

Formalne potenčne in Laurentove vrste, Lagrangeeva inverzija

Druge uporabe rodovnih funkcij (računanje povprečij in varianc, asimptotika koeficientov ...)

Pólyeva teorija

Načelo vključitev in izključitev, incidenčna algebra, Möbiusova funkcija, Möbiusova inverzija

Reducirane algebre, Dirichletova rodovna funkcija

Predavatelj izbere še eno izmed naslednjih tem: politopi; incidenčne strukture; simetrične funkcije; diskretna geometrija; upodobitve simetrične grupe

Twelvefold way (binomial coefficients, Stirling numbers of the first and second kind, Lah numbers, partitions etc., using generating functions)

Ordinary and exponential generating functions: combinatorial meaning of sum, product, derivative, composition (exponential formula)

Formal power series, formal Laurent series, Lagrange inversion

Other applications of generating functions (computing the mean and variance, asymptotics of coefficients, etc.)

Pólya theory

Principle of inclusion and exclusion, incidence algebra, Möbius function, Möbius inversion

Reduced algebras, Dirichlet generating function

Instructor chooses an addition topic from the following list: polytopes; incidence structures; symmetric functions; discrete geometry; representations of the symmetric group

### Temeljni literatura in viri / Readings:

- o Richard P. Stanley: *Enumerative Combinatorics*, Vol. 1, Cambridge University Press, New York-Cambridge, 2011.
- o Richard P. Stanley: *Enumerative Combinatorics*, Vol. 2, Cambridge University Press, New York-Cambridge, 1999.
- o Francois Bergeron, Gilbert Labelle, Pierre Leroux: *Combinatorial Species and Tree-like Structures*, Cambridge University Press, Cambridge-New York-Melbourne, 1998.
- o Jack H. van Lint, Robin J. Wilson: *A Course in Combinatorics*, Cambridge University Press, Cambridge, 2001.

### Cilji in kompetence:

Študent spozna glavne tehnike kombinatornega preštevanja.

### Objectives and competences:

The student learns the main techniques of enumerative combinatorics.

### Predvideni študijski rezultati:

**Znanje in razumevanje:** Študentje poznajo in razumejo vlogo rodovnih funkcij in algebrskih struktur pri študiranju kombinatornih problemov.

**Uporaba:** Študentje znajo uporabljati teorijo rodovnih funkcij in algebrskih struktur za reševanje kombinatornih problemov.

**Refleksija:** Študentje spoznajo povezavo med strukturo kombinatornega problema in algebraično naravo pripadajočih rodovnih funkcij oziroma drugih struktur.

**Prenosljive spretnosti – niso vezane le na en predmet:** Uporaba rodovnih funkcij v verjetnosti; poglobljeno razumevanje klasične Möbiusove funkcije; delovanje grup na množici.

### Intended learning outcomes:

**Knowledge and understanding:** Students understand the role of generating functions and algebraic structures in the study of combinatorial problems.

**Application:** Students know how to use generating functions and algebraic structures to solve combinatorial problems.

**Reflection:** The students learn the connection between the structure of the combinatorial problem and the algebraic nature of the corresponding generating functions and other structures

### Transferable skills:

Applications of generating functions in probability; a deeper understanding of the classical Möbius function; action of a group on a set.

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**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**

lectures, exercises, homeworks, consultations

Delež (v %) /

Weight (in %)

**Načini ocenjevanja:**

**Assessment:**

<p>Način (pisni izpit, ustno izpraševanje, naloge, projekt):</p> <ul style="list-style-type: none"> <li>▪ izpit iz vaj (2 kolokvija ali pisni izpit)</li> <li>▪ ustni izpit</li> </ul> <p>Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)</p>	<p>50%</p> <p>50%</p>	<p>Type (examination, oral, coursework, project):</p> <ul style="list-style-type: none"> <li>▪ 2 midterm exams instead of written exam, written exam</li> <li>▪ oral exam</li> </ul> <p>Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)</p>
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**Reference nosilca / Lecturer's references:**

**prof. dr. Sandi Klavžar**

- A. Ilić, S. Klavžar, Y. Rho: The index of a binary word, Theoret. Comput. Sci. 242 (2012) 100-106
- S. Klavžar, S. Shpectorov: Asymptotic number of isometric generalized Fibonacci cubes, European J. Combin. 33 (2012) 220-226
- D. Fronček, J. Jerebic, S. Klavžar, P. Kovář: Strong isometric dimension, biclique coverings, and Sperner's Theorem, Comb. Prob. Comp. 16 (2007) 271-275

**doc. dr. Matjaž Konvalinka**

- M. Konvalinka, I. Pak: Triangulations of Cayley and Tutte polytopes, Adv. Math. 245 (2013) 1-33
- M. Konvalinka: Skew quantum Murnaghan-Nakayama rule, J. Algebraic Combin. 35 (2012) 519-545
- M. Konvalinka: Divisibility of generalized Catalan numbers, J. Combin. Theory Ser. A 114 (2007) 1089-1100

**prof. dr. Marko Petkovšek**

- M. Petkovšek: Counting Young tableaux when rows are cosets, Ars Comb. 37 (1994) 87-95.

- M. Petkovšek, H. S. Wilf, D. Zeilberger:  $A=B$ , Wellesley (Massachusetts): A K Peters, 1996.
- M. Petkovšek: Letter graphs and well-quasi-order by induced subgraphs, *Discrete Math.* 244 (2002) 375-388.

## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Komutativna algebra
<b>Course title:</b>	Commutative algebra

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2203

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
45		30			105	6

**Nosilec predmeta / Lecturer:** doc. dr. David Dolžan, prof. dr. Tomaž Košir, prof. dr. Matjaž Omladič

**Jeziki / Languages:**

<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija

**Prerequisites:**

Enrollment into the program

**Vsebina:**

**Content (Syllabus outline):**

**Osnovni del:**

Komutativni kolobar, spekter kolobarja. Nilradikal in Jacobsonov radikal.

Moduli, podmoduli in homomorfizmi. Operacije na modulih, direktna vsota in produkt. Končno generirani moduli. Eksaktna zaporedja. Tenzorski produkt modulov in njegove eksaktnostne lastnosti. Razširitev in zožitev skalarjev. Algebre in njihovi tenzorski produkti.

Noetherski kolobarji, Hilbertov izrek o bazi. Izrek o noetherski normalizaciji.

Hilbertov izrek o ničlah, topologija Zariskega.

Kolobarji ulomkov, lokalizacija.

Primarni razcep. Prirejani praideali, primarne komponente, izreka o enoličnosti.

**Izbirne vsebine:**

Valuacijski kolobarji.

Filtracija. Artin-Reesova lema.

Napolnitev in Henselova lema.

Uvod v teorijo dimenzije.

Polinomi, Gröbnerjeve baze.

**Basics:**

Commutative ring, spectrum. Nilradical and Jacobson radical.

Modules, submodules and homomorphisms. Module operations, direct sum and product. Finitely generated modules. Exact sequences. Tensor product of modules and its exactness properties. Restriction and extension of scalars. Algebras and their tensor products.

Noetherian rings, Hilbert's Basis theorem. Noetherian normalization theorem.

Hilbert's Nullstellensatz, Zariski topology.

Rings of fractions, localization.

Primary decomposition. Associated prime ideals, primary components, uniqueness theorems.

**Optional themes:**

Valuation rings.

Filtration. Artin-Rees lemma.

Completion and Hensel's lemma.

Introduction to the dimension theory.

Polynomials, Gröbner bases.

**Temeljni literatura in viri / Readings:**

- M. Reid: *Undergraduate Commutative Algebra*, Cambridge Univ. Press, Cambridge, 1995.
- M. F. Atiyah, I. G. MacDonald: *Introduction to Commutative Algebra*, Addison-Wesley, Reading, 1994.
- D. Cox, J. Little, D. O'Shea: *Ideals, Varieties and Algorithms : An Introduction to Computational Algebraic Geometry and Commutative Algebra*, 2nd edition, Springer, New York, 2005.
- N. Lauritzen: *Concrete Abstract Algebra: From Numbers to Gröbner Bases*, Cambridge University Press, Cambridge, 2003.

**Cilji in kompetence:**

Slušatelj spozna osnove teorije komutativne algebre. Dopolni vsebine, ki jih sreča pri algebrskih predmetih na dodiplomskem študiju. Pridobljeno znanje praktično utrdi z domačimi nalogami in samostojnim reševanjem problemov.

**Objectives and competences:**

The student learns the basics of the theory of commutative algebra and upgrades notions and theories that were met during the undergraduate algebraic courses. The knowledge is consolidated by homeworks and individual problem solving exercises.

**Predvideni študijski rezultati:****Intended learning outcomes:**

**Znanje in razumevanje:** Poznavanje osnovnih pojmov in izrekov komutativne algebre in njihovo prepoznavanje v drugih vejah matematike.

**Uporaba:** V algebraični geometriji in algebraični teoriji števil.

**Refleksija:** Razumevanje teorije na podlagi primerov in uporabe.

**Prenosljive spretnosti – niso vezane le na en predmet:** Formulacija problemov v primernem jeziku, reševanje in analiza doseženega na primerih, prepoznavanje algebraičnih struktur v geometriji in teoriji števil.

**Knowledge and understanding:** Learning the basic notions and theorem of commutative algebra and recognizing the concepts in other areas of mathematics.

**Application:** In algebraic geometry and algebraic number theory.

**Reflection:** Understanding the theory on the basis of examples and applications.

**Transferable skills:** Formulations of problems in appropriate language, solving and analysing the results on examples, recognizing algebraic structures in geometry and number theory.

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations

<b>Načini ocenjevanja:</b>	Delež (v %) / Weight (in %)	<b>Assessment:</b>
Način (domače naloge, pisni izpit, ustno izpraševanje, naloge): <ul style="list-style-type: none"> <li>▪ domače naloge</li> <li>▪ pisni izpit</li> <li>▪ ustni izpit</li> </ul> Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)	20% 40% 40%	Type (homeworks, examination, oral, coursework, project): <ul style="list-style-type: none"> <li>▪ homeworks</li> <li>▪ written exam</li> <li>▪ oral exam</li> </ul> Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

- doc. dr. David Dolžan  
Dolžan, David; Oblak, Polona The zero-divisor graphs of rings and semirings. Internat. J.

Algebra Comput. 22 (2012), no. 4, 1250033, 20 pp.

Dolžan, David; Kokol Bukovšek, Damjana; Oblak, Polona Diameters of commuting graphs of matrices over semirings. Semigroup Forum 84 (2012), no. 2, 365–373.

Dolžan, David; Oblak, Polona Commuting graphs of matrices over semirings. Linear Algebra Appl. 435 (2011), no. 7, 1657–1665.

- prof. dr. Tomaž Košir

Grunenfelder, L.; Košir, T.; Omladič, M.; Radjavi, H. Finite groups with submultiplicative spectra. J. Pure Appl. Algebra 216 (2012), no. 5, 1196–1206.

Buckley, Anita; Košir, Tomaž Plane curves as Pfaffians. Ann. Sc. Norm. Super. Pisa Cl. Sci. (5) 10 (2011), no. 2, 363–388.

Košir, Tomaž; Oblak, Polona On pairs of commuting nilpotent matrices. Transform. Groups 14 (2009), no. 1, 175–182.

- prof. dr. Matjaž Omladič

Omladič, Matjaž; Radjavi, Heydar Self-adjoint semigroups with nilpotent commutators. Linear Algebra Appl. 436 (2012), no. 7, 2597–2603.

Grunenfelder, L.; Košir, T.; Omladič, M.; Radjavi, H. Finite groups with submultiplicative spectra. J. Pure Appl. Algebra 216 (2012), no. 5, 1196–1206.

Omladič, Matjaž; Radjavi, Heydar Reducibility of semigroups and nilpotent commutators with idempotents of rank two. Ars Math. Contemp. 3 (2010), no. 1, 99–108.



## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Logika
<b>Course title:</b>	Logic

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2207

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
45		30			105	6

**Nosilec predmeta / Lecturer:** izred. prof. dr. Andrej Bauer  
prof. dr. Marko Petkovšek

**Jeziki / Languages:**

<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

**Prerequisites:**

Vpis v letnik študija

Enrollment into the program

**Vsebina:**

**Content (Syllabus outline):**

Abstraktna sintaksa. Vezane in proste spremenljivke. Substitucija. Naravna dedukcija. Izrek o odstranjevanju rezov. Neprotislovnost naravne dedukcije.

Jezik in teorija prvega reda. Neprotislovnost in polnost teorije. Konzervativna razširitev teorije. Interpretacija teorije. Model teorije prvega reda.

Izrek o zdravju. Gödelov izrek o polnosti. Izrek o kompaktnosti. Posledice.

Peanova aritmetika. Gödelova izreka o nepopolnosti.

Primeri teorij prvega reda in uporaba teorije modelov.

Abstract syntax. Bound and free variables. Substitution. Natural deduction. Cut elimination. Consistency of natural deduction. First-order languages and theories. Consistent and complete theories. Conservative extensions. Interpretation of a language and a model of a theory.

Soundness and Gödel completeness theorem. Compactness theorem and its consequences. Peano arithmetic, Gödel incompleteness theorems.

Examples of first-order theories and applications of model theory.

### Temeljni literatura in viri / Readings:

- N. Prijatelj: *Osnove matematične logike, 2. del: Formalizacija*, DMFA Slovenije, Ljubljana, 1992.
- N. Prijatelj: *Osnove matematične logike, 3. del: Aplikacija*, DMFA Slovenije, Ljubljana, 1994.
- W. Rautenberg: *A Concise Introduction to Mathematical Logic, 3. izdaja*, Springer, 2010.
- E. Mendelson: *Introduction to Mathematical Logic, 4. izdaja*, Chapman and Hall, 1997.
- A.S. Troelstra, H. Schwichtenberg: *Basic Proof Theory, 2. izdaja*, Cambridge University Press, 2000.

### Cilji in kompetence:

Pridobiti znanje iz osnov matematične logike in osnov matematike.

### Objectives and competences:

Basic knowledge of foundations of mathematics and mathematical logic.

### Predvideni študijski rezultati:

#### Znanje in razumevanje:

Razumevanje logičnih osnov matematike in fundamentalnih omejitev aksiomatične metode.

#### Uporaba:

Kot temeljni kamen matematike je logika osnovno sredstvo matematičnega izražanja.

#### Refleksija:

Dejstvo, da obstajajo nerešljivi matematični problemi, zahteva temeljit razmislek o naravi matematike same.

### Intended learning outcomes:

#### Knowledge and understanding:

Understanding of logical foundations of mathematics and the fundamental limitations of the axiomatic method.

#### Application:

Logic, being the foundation of mathematics, provides the means for communication and methodology in mathematics.

#### Reflection:

The fact that there are mathematical problems without solutions invites a thorough

**Prenosljive spretnosti – niso vezane le na en predmet:**  
 Sposobnost formalnega izražanja matematične vsebine. Sposobnost meta-matematične obravnave.

reconsideration of the nature of mathematics.  
**Transferable skills:**  
 Ability to formally express mathematical content. Ability to perform meta-mathematical analysis.

**Metode poučevanja in učenja:**  
 predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**  
 Lectures, exercises, homeworks, consultations

Delež (v %) /  
 Weight (in %)

**Načini ocenjevanja:**

**Assessment:**

<p>Način (pisni izpit, ustno izpraševanje, naloge, projekt):</p> <ul style="list-style-type: none"> <li>▪ izpit iz vaj (2 kolokvija ali pisni izpit)</li> <li>▪ ustni izpit</li> </ul> <p>Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)</p>	<p>50%</p> <p>50%</p>	<p>Type (examination, oral, coursework, project):</p> <ul style="list-style-type: none"> <li>▪ 2 midterm exams instead of written exam, written exam</li> <li>▪ oral exam</li> </ul> <p>Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)</p>
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**Reference nosilca / Lecturer's references:**

- **izred. prof. dr. Andrej Bauer**
- S. Awodey, A. Bauer: *Propositions as [Types]*, Journal of Logic and Computation. Volume 14, Issue 4, August 2004, pp. 447-471.
- A. Bauer, A. Simpson: *Two constructive embedding-extension theorems with applications to continuity principles and to Banach-Mazur computability*, Mathematical Logic Quarterly, 50(4,5):351-369, 2004.
- A. Bauer: *A relationship between equilogical spaces and Type Two Effectivity*, Math. Logic Quarterly, 2002, vol. 48, suppl. 1, str. 1-15.
- **prof. dr. Marko Petkovšek**
- M. Petkovšek: *Ambiguous numbers are dense*, Amer. Math. Monthly 97 (1990), str. 408-411.
- M. Petkovšek, H. S. Wilf, D. Zeilberger: *A = B*, A K Peters, Wellesley MA, 1996.
- M. Petkovšek: *Letter graphs and well-quasi-order by induced subgraphs*, Discrete Math. 244

(2002), str. 375-388.

## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Neasociativna algebra
<b>Course title:</b>	Nonassociative algebra

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2208

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
45		30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Matjaž Omladič, prof. dr. Tomaž Košir

**Jeziki / Languages:**

<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija

**Prerequisites:**

Enrollment into the program

**Vsebina:**

Pomembnejši tipi neasociativnih algeber (alternativne algebre, jordanske algebre).

Definicija Liejeve algebre. Ideali in homomorfizmi. Rešljive in nilpotentne Liejeve algebre.

Liejev in Cartanov izrek. Killingova forma. Povsem razcepne upodobitve. Upodobitve algebre  $sl(2, F)$ . Razcep na korenske podprostore.

Korenski sistemi. Enostavni koreni in Weylova grupa. Klasifikacija (končnorazsežnih)

**Content (Syllabus outline):**

Important types of nonassociative algebras (alternating algebras, Jordan algebras). The definition of Lie algebra. Ideals and homomorphisms. Solvable and nilpotent Lie algebras.

Lie's and Cartan's Theorems. The Killing form. Completely irreducible representations. Representations of  $sl(2, F)$ . Root subspace decomposition.

Root systems. Simple roots and the Weyl group. Classification of (finite-dimensional) simple Lie

enostavnih Liejevih algeber.  
 Univerzalna ovojna algebra. Poicaré-Birkhoff-Wittov izrek.  
 Upodobitve enostavnih Liejevih algeber.

algebras.  
 Universal enveloping algebra. Theorem Poicaré-Birkhoff-Witt.  
 Representation theory of simple Lie algebras.

**Temeljni literatura in viri / Readings:**

- K. A. Zhevlakov, A. M. Slinko, I. P. Shestakov, A. I. Shirshov, Rings that are nearly associative, Academic Press, 1982.
- J. E. Humphreys: Introduction to Lie Algebras and Representation Theory, Springer, New York-Berlin, 1997.
- J. P. Serre: Complex Semisimple Lie Algebras, Springer, Berlin, 2001.
- W. A. de Graaf: Lie Algebras : Theory and Algorithms, North Holland, Amsterdam, 2000.

**Cilji in kompetence:**

Študent spozna osnovne pojme in izreke neasociativne algebre.

**Objectives and competences:**

Student meets the fundamental notions and theorems of the nonassociative algebra.

**Predvideni študijski rezultati:**

**Znanje in razumevanje:** Poznavanje osnovnih pojmov in izrekov neasociativne algebre in njihovo prepoznavanje v drugih vejah matematike.

**Uporaba:** V drugih vejah matematike.

**Refleksija:** Razumevanje teorije na podlagi primerov in uporabe.

**Prenosljive spretnosti – niso vezane le na en predmet:** Formulacija in reševanje problemov z abstraktnimi metodami.

**Intended learning outcomes:**

**Knowledge and understanding:** Understanding of basic concepts and theorems of noncommutative algebra, and their role in some other areas.

**Application:** In other mathematical areas.

**Reflection:** Understanding the theory on the basis of examples and applications.

**Transferable skills:** Formulation and solution of problems using abstract methods.

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations

**Načini ocenjevanja:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):  
 ▪ izpit iz vaj (2 kolokvija ali pisni izpit)

Delež (v %) /

Weight (in %)

50%

**Assessment:**

Type (examination, oral, coursework, project):  
 ▪ 2 midterm exams instead of written

<ul style="list-style-type: none"> <li>▪ ustni izpit</li> </ul> <p>Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)</p>	50%	<p>exam, written exam</p> <ul style="list-style-type: none"> <li>▪ oral exam</li> </ul> <p>Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)</p>
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**Reference nosilca / Lecturer's references:**

**prof. dr. Matjaž Omladič**

- L. Grunenfelder, M. Omladič, H. Radjavi: Jordan analogs of the Burnside and Jacobson density theorems, *Pacific J. Math.*, 1993, let. 161, št. 2, str. 335-346.
- L. Grunenfelder, T. Košir, M. Omladič, H. Radjavi: Maximal Jordan algebras of matrices with bounded number of eigenvalues, *Israel J. Math.*, 2002, vol. 128, str. 53-75.
- L. Grunenfelder, M. Omladič, H. Radjavi: Transitive action of Lie algebras, *J. Pure Appl. Algebra*, 2005, vol. 199, iss. 1-3, str. 87-93.

**prof. dr. Tomaž Košir**

- J. Bernik, R. Drnovšek, D. Kokol Bukovšek, T. Košir, M. Omladič, H. Radjavi. On semitransitive Jordan algebras of matrices. *J. Algebra Appl.*, 2011, Vol. 10, no. 2, str. 319–333.
- L. Grunenfelder, T. Košir, M. Omladič, H. Radjavi: Maximal Jordan algebras of matrices with bounded number of eigenvalues, *Israel J. Math.*, 2002, vol. 128, str. 53-75.
- L. Grunenfelder, R. Guralnick, T. Košir, H. Radjavi: Permutability of Characters on Algebras, *Pacific Journal of Mathematics* 178 (1997), str. 63-70.

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

<b>Predmet:</b>	Nekomutativna algebra
<b>Course title:</b>	Noncommutative algebra

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2200

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
45		30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Matej Brešar, prof. dr. Jakob Cimprič

<b>Jeziki / Languages:</b>	<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
	<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija.

**Prerequisites:**

Enrollment into the program.

**Vsebina:**

**Content (Syllabus outline):**



Nekomutativni obsegi. Frobeniusov izrek. Wedderburnov izrek o končnih obsegih.

Radikal. Polenostavne algebre. Wedderburnov izrek. Maschkejev izrek.

Enostavni in polenostavni moduli. Izrek o gostoti. Jacobsonov radikal.

Tenzorski produkti algeber. Skolem-Noetherin izrek. Izrek o drugem centralizatorju. Brauerjeva grupa.

Noncommutative division rings. Frobenius' theorem. Wedderburn's theorem on finite division rings.

Radical. Semisimple algebras. Wedderburn's theorem. Maschke's theorem.

Simple and semisimple modules. Density theorem. Jacobson radical.

Tensor product of algebras. Skolem-Noether theorem. Double centralizer theorem. Brauer group.

### Temeljni literatura in viri / Readings:

- R. K. Dennis, B. Farb, Noncommutative algebra, Springer, 1993.
- T. Y. Lam, A first course in noncommutative rings, Springer, 2001.
- R. S. Pierce, Associative algebras, Springer, 1982.
- L. Rowen, Graduate algebra: Noncommutative view, AMS, 2008.

### Cilji in kompetence:

Spoznati osnovne pojme in orodja nekomutativne algebre.

### Objectives and competences:

To master basic concepts and tools of noncommutative algebra.

### Predvideni študijski rezultati:

#### Znanje in razumevanje:

Razumevanje osnovnih pojmov in izrekov nekomutativne algebre ter njihove vloge na nekaterih drugih področjih.

#### Uporaba:

V drugih vejah matematike.

#### Refleksija:

Razumevanje teorije na podlagi primerov in uporabe.

### Intended learning outcomes:

#### Knowledge and understanding:

Understanding of basic concepts and theorems of noncommutative algebra, and their role in some other areas.

#### Application:

In other mathematical areas.

#### Reflection:

Understanding the theory on the basis of examples and applications.

**Prenosljive spretnosti – niso vezane le na en predmet:**

Formulacija in reševanje problemov z abstraktnimi metodami.

**Transferable skills:**

Formulation and solution of problems using abstract methods.

**Metode poučevanja in učenja:**

Predavanja, vaje, domače naloge, konzultacije.

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations.

**Načini ocenjevanja:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- domače naloge
- ustni izpit

Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)

Delež (v %) /

Weight (in %) /

50%

50%

**Assessment:**

Type (examination, oral, coursework, project):

- homework assignment
- oral exam

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:****Prof. dr. Matej Brešar**

M. Brešar, M. Chebotar, W. S. Martindale, *Functional identities*, Birkhauser, 2007.

M. Brešar, An elementary approach to Wedderburn's structure theory, *Expositiones Math.* 28 (2010), 79-83.

M. Brešar, An alternative approach to the structure theory of PI-rings, *Expositiones Math.* 29 (2011), 159-164.

**Prof. dr. Jakob Cimprič**

J. Cimprič, Free skew-fields have many  $*$ -orderings, *J. Algebra* 280 (2004), 20-28.

J. Cimprič, Formally real involutions on central simple algebras, *Commun. Algebra*, 165-178.

J. Cimprič, A noncommutative real nullstellensatz corresponds to a noncommutative real ideal: algorithms, *Proc. London Math. Soc.* 106 (2013), 1060-1086.

UČNI NAČRT PREDMETA / COURSE SYLLABUS						
<b>Predmet:</b>	Teorija grafov					
<b>Course title:</b>	Graph theory					
<b>Študijski program in stopnja</b> <b>Study programme and level</b>	<b>Študijska smer</b> <b>Study field</b>	<b>Letnik</b> <b>Academic year</b>	<b>Semester</b> <b>Semester</b>			
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi			
Second cycle master study program Mathematics	none	first or second	first or second			
<b>Vrsta predmeta / Course type</b>		izbirni predmet/elective course				
<b>Univerzitetna koda predmeta / University course code:</b>		M2201				
<b>Predavanja</b> <b>Lectures</b>	<b>Seminar</b> <b>Seminar</b>	<b>Vaje</b> <b>Tutorial</b>	<b>Klinične vaje</b> <b>work</b>	<b>Druge oblike</b> <b>študija</b>	<b>Samost. delo</b> <b>Individ. work</b>	<b>ECTS</b>
45		30			105	6
<b>Nosilec predmeta / Lecturer:</b>		prof. dr. Riste Škrekovski, prof. dr. Sandi Klavžar, prof. dr. Primož Potočnik, doc. dr. Arjana Žitnik				
<b>Jeziki / Languages:</b>		<b>Predavanja / Lectures:</b> slovenski/Slovene, angleški/English				
		<b>Vaje / Tutorial:</b> slovenski/Slovene, angleški/English				
<b>Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:</b>			<b>Prerequisites:</b>			
Vpis v letnik študija			Enrollment into the program			
<b>Vsebina:</b>			<b>Content (Syllabus outline):</b>			

Prerejanja in faktorji (min-max izreki, neodvisne množice in pokritja, Tutterov izrek o 1-faktorju)

Povezanost (struktura 2-povezanih in k-povezanih grafov, dokaz in uporabe Mengerjevih izrekov)

Barvanja grafov (meje, dokaz Brooksovega izreka, struktura k-kromatičnih grafov, Turanov izrek, kromatični polinom, tetivni grafi)

Ravninski grafi (dualni graf, izrek Kuratowskega, konveksne vložitve, barvanja ravninskih grafov, prekrizno število)

Predavatelj izbere še eno izmed naslednjih tem: barvanja povezav in graf povezav, hamiltonski grafi, popolni grafi, ekstremalni problemi, dominacija v grafih, simetrijske lastnosti grafov II.

Matchings and factors (min-max theorem, independent sets and coverings, Tutter's 1-factor theorem)

Graph connectivity (structure of 2-connected and k-connected graphs, Menger theorem and its applications)

Graph Colorings (bounds of the chromatic number, structure of k-chromatic graphs, Turan's theorem, chromatical polynom, chordal graphs)

Planar graphs (dual graph, Kuratowski's theorem, convex embedding, colorings of planar graphs, crossing number)

Instructor chooses an addition topic from the following list: edge colorings and line graphs, Hamiltonian graphs, perfect graphs, extremal graph problems, graph domination, symmetric graph properties II.

### Temeljni literatura in viri / Readings:

- R. Diestel: *Graph Theory*, 3. izdaja, Springer, Berlin, 2005.
- A. Bondy, U.S.R. Murty: *Graph Theory*, 2. izdaja, Springer, Berlin, 2008.
- D. West: *Introduction to Graph Theory*, 2. izdaja, Prentice Hall, Upper Saddle River, 2005.
- R. J. Wilson, M. Watkins: *Uvod v teorijo grafov*, DMFA Slovenije, Ljubljana, 1997.

### Cilji in kompetence:

Študent poglobi in razširi znanje teorije grafov. Spozna uporabnost grafov in omrežij na različnih področjih matematike ter možnosti za njihovo uporabo tudi v drugih vejah znanosti.

### Objectives and competences:

Students will deepen and broaden their knowledge of graph theory. Learn about the usefulness of graphs and networks in different areas of mathematics and their potential use in other branches of science.

### Predvideni študijski rezultati:

**Znanje in razumevanje:** Slušatelj poglobi znanje iz teorije grafov.

**Uporaba:** Grafi omogočajo matematično modeliranje različnih pojavov. Slušatelj se seznanja z vrsto matematičnih rezultatov, ki opisujejo lastnosti grafov in tako omogočajo matematično analizo modelov, opisanih z grafi.

**Refleksija:** Povezovanje teoretičnih spoznanj s praktičnimi uporabami na primer v optimizaciji in pri programiranju. Sposobnost prepoznavanja problemov, ki jih lahko uspešno opišemo z grafi.

**Prenosljive spretnosti – niso vezane le na en predmet:** Sposobnost opisa problemov iz vsakdanjega življenja s pomočjo matematičnih struktur, še posebej z grafi. Sposobnost uporabe matematičnih orodij za reševanje problemov.

### Intended learning outcomes:

**Knowledge and understanding:** The student deepen their knowledge of graph theory.

**Application:** Graphs allow mathematical modeling of variety of phenomena. The student learns various mathematical results that describe the properties of graphs and thus provide a mathematical analysis of the models described by graphs.

**Reflection:** Integration of theoretical knowledge with practical applications such as optimization and programming. Ability to recognize problems that can be successfully described by graphs.

### Transferable skills:

The ability to describe the problems of everyday life with the help of mathematical structures, in particular with graphs. The ability to use mathematical tools to solve problems.

### Metode poučevanja in učenja:

### Learning and teaching methods:

predavanja, vaje, domače naloge, konzultacije	lectures, exercises, homeworks, consultations
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Delež (v %) /  
Weight (in %)

**Načini ocenjevanja:**

**Assessment:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):		Type (examination, oral, coursework, project):
<ul style="list-style-type: none"> <li>▪ izpit iz vaj (pisni izpit)</li> <li>▪ ustni izpit</li> </ul>	50%	<ul style="list-style-type: none"> <li>▪ written exam</li> <li>▪ oral exam</li> </ul>
Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)	50%	Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

<p><b>prof. dr. Primož Potočnik:</b></p> <ul style="list-style-type: none"> <li>• P. Potočnik, Tetraivalent arc-transitive locally-Klein graphs with long consistent cycles, European J. Combin. 36 (2014) 270-281.</li> <li>• P. Potočnik, P. Spiga, G. Verret, Cubic vertex-transitive graphs on up to 1280 vertices, J. Symbolic Comp. 50 (2013) 465-477.</li> <li>• P. Potočnik, Edge-colourings of cubic graphs admitting a solvable vertex-transitive group of automorphisms, J. Comb. Theory Ser. B. 91 (2004) 289-300.</li> </ul> <p><b>prof. dr. Sandi Klavžar:</b></p> <ul style="list-style-type: none"> <li>• S. Klavžar, S. Shpectorov, Convex excess in partial cubes, J. Graph Theory 69 (2012) 356-369.</li> <li>• B. Brešar, S. Klavžar, D.F. Rall, Domination game and an imagination strategy, SIAM J. Discrete Math. 24 (2010) 979-991.</li> <li>• S. Klavžar, On the canonical metric representation, average distance, and partial Hamming graphs, European J. Combin. 27 (2006) 68-73.</li> </ul> <p><b>prof. dr. Riste Škrekovski:</b></p> <ul style="list-style-type: none"> <li>• J. Govorčin, M. Knor, R. Škrekovski, Line graph operator and small worlds, Inform. Process. Lett. 113 (2013) 196-200.</li> <li>• Z. Dvorak, B. Lidicky, R. Škrekovski, Randić index and the diameter of a graph, European J. Comb. 32 (2011) 434-442.</li> <li>• T. Kaiser, M. Stehlik, R. Škrekovski, On the 2-resonance of fullerenes, SIAM J. Discrete Math. 25 (2011) 1737-1745.</li> </ul>
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**doc. dr. Arjana Žitnik:**

- M. Milanič, T. Pisanski, A. Žitnik, Dilation coefficient, plane-width, and resolution coefficient of graphs, *Monatsh. Math.* 170 (2013) 179-193.
- A. Žitnik, B. Horvat, T. Pisanski, All generalized Petersen graphs are unitdistance graphs, *J. Korean Math. Soc.* 49 (2012) 475-491.
- A. Jurišić, P. Terwilliger, A. Žitnik, The Q-polynomial idempotents of a distance-regular graph, *J. Comb. Theory Ser. B* 100 (2010) 683-690.

UČNI NAČRT PREDMETA / COURSE SYLLABUS							
<b>Predmet:</b>		Teorija grup in polgrup					
<b>Course title:</b>		Theory of semigroups and groups					
<b>Študijski program in stopnja</b> <b>Study programme and level</b>		<b>Študijska smer</b> <b>Study field</b>		<b>Letnik</b> <b>Academic year</b>		<b>Semester</b> <b>Semester</b>	
Magistrski študijski program 2. stopnje Matematika		ni smeri		prvi ali drugi		prvi ali drugi	
Second cycle master study program Mathematics		none		first or second		first or second	
<b>Vrsta predmeta / Course type</b>				izbirni predmet/elective course			
<b>Univerzitetna koda predmeta / University course code:</b>				M2202			
<b>Predavanja</b> <b>Lectures</b>	<b>Seminar</b> <b>Seminar</b>	<b>Vaje</b> <b>Tutorial</b>	<b>Klinične vaje</b> <b>work</b>	<b>Druge oblike</b> <b>študija</b>	<b>Samost. delo</b> <b>Individ. work</b>	<b>ECTS</b>	
45		30			105	6	
<b>Nosilec predmeta / Lecturer:</b>		prof. dr. Jakob Cimprič, prof. dr. Tomaž Košir, doc. dr. Primož Moravec, prof. dr. Matjaž Omladič, prof. dr. Primož Potočnik					
<b>Jeziki /</b> <b>Languages:</b>		<b>Predavanja /</b> <b>Lectures:</b>		slovenski/Slovene, angleški/English			
		<b>Vaje / Tutorial:</b>		slovenski/Slovene, angleški/English			
<b>Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:</b>				<b>Prerequisites:</b>			
Vpis v letnik študija				Enrollment into the program			
<b>Vsebina:</b>				<b>Content (Syllabus outline):</b>			



### I. Teorija polgrup

- Osnovni pojmi teorije polgrup; primeri polgrup.
- Greenove relacije.
- Regularne polgrupe; polgrupe z obrati.
- Enostavne polgrupe; povsem enostavne polgrupe.

### II. Teorija grup

- Ponovitev osnovnih pojmov teorije grup.
- Kompozicijska vrsta in Jordan-Hölderjev izrek. Rešljive grupe. Hallov izrek za rešljive grupe. Nilpotentne grupe; p-grupe.
- Razcepne in nerazcepne razširitve grup; semidirektni produkt grup; Schur-Zassenhausov izrek.
- Končne enostavne grupe in problem njihove klasifikacije. Klasične grupe (splošne linearne, simplektične, unitarne in ortogonalne) ter pripadajoče enostavne grupe.
- Osnove teorije upodobitev končnih grup. Teorija karakterjev.

### I. Semigroup theory

- basic notions and examples
- Green relations
- Regular semigroups; inverse semigroups.
- Simple semigroups; completely simple semigroups.

### II. Group theory

- Basic notions
- Composition series, Jordan-Hölder theorem. Solvable groups, Hall's theorem. Nilpotent groups; p-groups.
- Split and non-split extensions of groups; semidirect product; Schur-Zassenhaus theorem.
- Finite simple groups and the classification problem. Classical groups (general linear, symplectic, unitary and orthogonal) and the corresponding simple groups.
- Fundamentals of representation theory of finite groups. Character theory.

**Temeljni literatura in viri / Readings:**

- J. M. Howie: *Fundamentals of semigroup theory*, Oxford University Press, Oxford, 1995.
- P. M. Higgins: *Techniques of semigroup theory*, Oxford University Press, Oxford, 1992.
- J. J. Rotman: *An introduction to the theory of groups*, 4. izd., Springer New York 1995.
- D. J. S. Robinson: *A course in the theory of groups*, 2. izd., Springer New York, 1996.

**Cilji in kompetence:**

Študent spozna osnovne pojme iz teorije polgrup in grup ter njihovo povezanost z drugimi področji matematike.

**Objectives and competences:**

Students get acquainted with basic notions of group theory and semigroup theory. They get familiar with connections between these two theories and other areas of mathematics.

**Predvideni študijski rezultati:****Znanje in razumevanje:**

Poznavanje osnovnih pojmov in izrekov teorije polgrup in grup in njihovo prepoznavanje v drugih vejah matematike.

**Uporaba:**

Teorija polgrup in grup spada med temeljne matematične predmete. Uči nas prepoznavati simetrije v naravi. Uporablja se zlasti v fiziki in kemiji (na primer kristalografija). Znotraj matematike je uporabna v geometriji, asociativni algebri, funkcionalni analizi in teoriji števil.

**Refleksija:**

Razumevanje teorije na podlagi primerov in uporabe.

**Prenosljive spretnosti – niso vezane le na en predmet:**

Formulacija problemov v primernem jeziku, reševanje in analiza doseženega na primerih, prepoznavanje grup v geometriji in analizi.

**Intended learning outcomes:****Knowledge and understanding:**

Basic notions of group theory and semigroup theory, applications in other areas of mathematics.

**Application:**

Group theory and semigroup theory are classical mathematical disciplines. They teach us how to recognize symmetries. They have immense applications in physics and chemistry (crystallography). Within mathematics, they play an important role in geometry, associative algebra, functional analysis, and number theory.

**Reflection:**

Understanding theory based on examples and applications.

**Transferable skills:**

Formulation of problems, solving problems and analysis of results using examples, applying groups in geometry and analysis.

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**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations

**Načini ocenjevanja:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- izpit iz vaj (2 kolokvija ali pisni izpit)
- ustni izpit

Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)

Delež (v %) /  
Weight (in %)

50%

50%

**Assessment:**

Type (examination, oral, coursework, project):

- 2 midterm exams instead of written exam, written exam
- oral exam

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

**prof. dr. Jakob Cimprič:**

- J. Cimprič: *Real spectra of quantum groups*, J. Algebra, 277 (2004), 282-297.
- J. Cimprič: *Preorderings on semigroups and semirings of right quotients*, Semigroup forum, 60 (2000), 396-404.
- J. Cimprič: *On homomorphisms from semigroups onto cyclic groups*, Semigroup forum, 59 (1999), 183-189.

**prof. dr. Tomaž Košir:**

- T. Košir, M. Omladič, H. Radjavi: *Maximal Semigroups Dominated by 0 - 1 Matrices*, Semigroup Forum 54 (1997), 175-189.
- L. Grunenfelder, T. Košir, M. Omladič, H. Radjavi: *On Groups Generated by Elements of Prime Order*, Geometriae Dedicata 75 (1999), 317-332.
- J. Bernik, R. Drnovšek T. Košir, M. Omladič, H. Radjavi: *Irreducible semigroups of matrices with eigenvalue one*, Semigroup Forum 67 (2003), 271-287.

**doc. dr. Primož Moravec:**

- P. Moravec: *Unramified Brauer groups of finite and infinite groups*, Amer. J. Math., 134 (2012), 1679-1704.
- P. Moravec: *On the Schur multipliers of finite  $p$ -groups of given coclass*, Israel J. Math. 185

(2011), 189-205.

- P. Moravec: *Completely simple semigroups with nilpotent structure groups*, Semigroup Forum 77 (2008), 316-324.

**prof. dr. Matjaž Omladič:**

- M. Omladič, M. Radjabalipour, H. Radjavi: *On semigroups of matrices with traces in a subfield*, Linear Algebra Appl. 1994, let. 208/209, str. 419-424.
- M. Omladič: *On 2-groups with submultiplicative spectrum*, J. Pure Appl. Algebra. 2002, vol. 167, iss. 2-3, str. 315-328.
- M. Hladnik, M. Omladič, H. Radjavi: *Trace-preserving homomorphisms of semigroups*, J. Funct. Anal., 2003, vol. 204, no. 2, str. 269-292.

**prof. dr. Primož Potočnik:**

- P. Potočnik: *Edge-colourings of cubic graphs admitting a solvable vertex-transitive group of automorphisms*, Journal of Combinatorial Theory Ser. B, vol. 91 (2004), 289-300.
- A. Malnič, D. Marušič, P. Potočnik: *On cubic graphs admitting an edge-transitive solvable group*, Journal of Algebraic Combinatorics, vol. 20 (2004), 99-113.
- P. Potočnik: *B-groups of order a product of two primes*, Mathematica Slovaca, vol. 51 (2001), 63-67.

## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Teorija števil
<b>Course title:</b>	Number Theory

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** še ni dodeljena/not assigned yet

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
45		30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Tomaž Košir, prof. dr. Boris Lavrič

<b>Jeziki / Languages:</b>	<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
	<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija

**Prerequisites:**

Enrollment into the program

**Vsebina:**

Predavatelj izbere med naslednjimi vsebinami:  
 1. Algebraična števila: diskriminanta, cela algebraična števila, celostna baza, norma in sled. Kvadratični in ciklotomični obsegi. Nerazcepni elementi. Problem enolične faktorizacije. Praelamenti. Evklidski obsegi. Ramanujan-Nagellov izrek. Primarni razcep.  
 2. Mreže v  $R^n$ . Kvocientni torus. Izrek Minkowskega. Razcep celih števil na vsoto

**Content (Syllabus outline):**

The lecturer selects from the following list of contents:  
 1. Algebraic numbers: discriminant, algebraic integers, integral basis, norm and trace. Quadratic and cyclotomic fields. Irreducible elements. The problem of unique factorization. Prime elements. Euclidean fields. The Ramanujan-Nagell theorem. Prime factorization.

kvadratov. Dedekindov izrek. Konstante Minkowskega.

3. Legendrov simbol. Gaussov zakon o kvadratni recipročnosti. Dirichletov izrek o praštevilih v aritmetičnih zaporedjih. Jacobijev simbol.

4. Dirichletov izrek o obrnljivih elementih.

5. Praštevila. Eratostenovo rešeto. Testiranje razcepnosti celih števil. Pseudopraštevila. Fermateva in Mersennova števila. Carmichaelova števila. Razporeditev praštevil. Regularna praštevila. Hevristične metode. Eulerjeva pseudopraštevila. Nelinearne diofantske enačbe. Pitagorejske trojke. Pellova enačba. Kummerjeva teorija za regularna praštevila in Fermatev problem.

6. Lucasova zaporedja. Eulerjev polinom za iracionalna števila. Generiranje praštevil. Transcendentnost znanih števil.

7. Relativna sled in norma. Diskriminanta in diferenta. Primarni razcep v Galoisevih razširitvah. Izrek Kroneckerja in Webra. Teorija razredov.

8.  $p$ -adična števila. Formalne potenčne vrste.

2. Lattices in  $R^n$ . The quotient torus. Minkowski's theorem. Sums of squares. The Dedekind's theorem. Minkowski's constants.

3. The Legendre symbol. Gauss' quadratic reciprocity law. Dirichlet's theorem on primes in arithmetic progression. The Jacobi symbol.

4. Dirichlet's unit theorem. 5. Prime numbers. The sieve of Erathostenes. Testing of factorizability of integers. Pseudoprime numbers. Fermat and Mersenne numbers. Carmichael numbers. The distribution of prime numbers. Regular primes. Heuristic methods. Euler pseudoprimes. Nonlinear diophantine equations. Pythagorean triples. Pell's equation. Kummer's theory of regular primes and Fermat's problem.

6. Lucas sequences. Euler polynomial for irrational numbers. Generating prime numbers. Transcendancy of renown numbers. 7. Relative trace and norm. Discriminant and different. Factoring of prime ideals in Galois extensions. The theorem of Kronecker and Weber. The class-field theory.

8.  $p$ -adic numbers. Formal power series.

### Temeljni literatura in viri / Readings:

- I. Stewart, D. Tall: *Algebraic Number Theory and Fermat's Last Theorem*, AK Peters, Natick, ZDA. 3. izdaja, 2002.
- P. Ribenboim: *Classical Theory of Algebraic Numbers*, Universitext. Springer-Verlag, New York, etc. 2001.
- P. Ribenboim: *The Little Book of Bigger Primes*, Springer-Verlag, New York, etc. 2. izdaja, 2004.
- K. H. Rosen: *Elementary Number Theory and its Applications*, Person, Boston, ZDA. 5. izdaja, 2005.
- P. Ribenboim: *My Numbers, my Friends*, Popular Lectures on Number Theory. Springer-Verlag, New York, etc. 2000.
- A. A. Gioia: *The Theory of Numbers. An Introduction*, Dover Publ. 2001.
- S. Alaca, K. S. Williams: *Introductory Algebraic Number Theory*, Cambridge Univ. Press. 2004.

### Cilji in kompetence:

Študent se seznanj z osnovami teorije števil in njihovo uporabo. Poudarek je na algebraični teoriji števil.

### Objectives and competences:

The student learns the basics of the number theory and its applications. The emphasis is on the algebraic theory of numbers.

**Predvideni študijski rezultati:****Znanje in razumevanje:**

Poznavanje osnovnih pojmov in izrekov teorije števil in njihovo prepoznavanje v drugih vejah matematike.

**Uporaba:**

V drugih vejah matematike, kriptografiji in teoriji kodiranja. Uporaba v računalništvu in informatiki, zlasti pri računalniški varnosti.

**Refleksija:**

Razumevanje teorije na podlagi primerov in uporabe.

**Prenosljive spretnosti – niso vezane le na en predmet:**

Formulacija problemov v primernem jeziku, reševanje in analiza doseženega na primerih, prepoznavanje algebraičnih struktur v teoriji števil.

**Intended learning outcomes:****Knowledge and understanding:**

Knowledge of basic concepts and theorems of the number theory of and their recognition in other areas of mathematics.

**Application:**

In other areas of mathematics, cryptography and coding theory. Application in computer science and informatics, especially in computer safety

**Reflection:**

Understanding the theory on the basis of examples and applications.

**Transferable skills:**

Formulation of problems in appropriate language, solving and analysis of the result on examples, identifying algebraic structures in theory of numbers.

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations

**Načini ocenjevanja:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- izpit iz vaj (2 kolokvija ali pisni izpit)
- ustni izpit

Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)

Delež (v %) /

Weight (in %) /

**Assessment:**

Type (examination, oral, coursework, project):

- 2 midterm exams instead of written exam, written exam
- oral exam

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:****prof. dr. Tomaž Košir**

- L. Grunenfelder, T. Košir, M. Omladič, H. Radjavi: *On Groups Generated by Elements of Prime Order*, Geometriae Dedicata 75 (1999), 317-332.

- T. Košir, B. A. Sethuraman: *Determinantal Varieties Over Truncated Polynomial Rings*, Journal of Pure and Applied Algebra 195 (2005), 75-95.
- L. Grunenfelder, T. Košir: *Geometric Aspects of Multiparameter Spectral Theory*, Transactions Amer. Math. Soc. 350 (1998), 2525-2546.

**prof. dr. Boris Lavrič**

- B. Lavrič: *Urejeni številski obsegi*, Obz. mat. fiz., 1994, let. 41, št. 2, str. 45-50.
- B. Lavrič: *Delno urejeni številski kolobarji*, Obz. mat. fiz., 1994, let. 41, št. 3, str. 83-91.
- B. Lavrič: *Vsote praštevil in vsote njihovih kvadratov*, Obz. mat. fiz., 1996, let. 43, št. 5, str. 161-167.



**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

<b>Predmet:</b>	Uporabna diskretna matematika
<b>Course title:</b>	Applied discrete mathematics

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** še ni dodeljena/not assigned yet

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
<b>30</b>	45				<b>105</b>	<b>6</b>

**Nosilec predmeta / Lecturer:** doc. dr. Alen Orbanić, prof. dr. Primož Potočnik, prof. dr. Riste Škrekovski

**Jeziki / Languages:**

<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija  
Obvladovanje vsaj enega programskega jezika na osnovnem nivoju.

**Prerequisites:**

Enrollment into the program  
Knowledge of a programming language on a basic level.

**Vsebina:**

**Content (Syllabus outline):**

Predstavimo več modelnih problemov, ki jih lahko rešimo s pomočjo modeliranja z metodami iz diskretne matematike.

Osredotočimo se na proces obravnave problema: identifikacija entitet in odnosov med njimi, identifikacija ciljev, postavitve podatkovnega modela, izdelava algoritmov, predvidevanje testiranja in testni podatki, specifikacije, implementacija, ocenjevanje in kakovostna presoja rezultatov.

Glede na izbrane modelne probleme se po potrebi spoznamo z matematičnimi orodji in metodologijo za naslavljanje problemov, npr. orodja in metode iz heuristične optimizacije, vizualizacije in predstavitve podatkov (grafi, diagrami, ...), kvalitativne analize diskretnih dinamičnih sistemov in drugo.

V seminarskem delu predmeta bodo študenti dobili individualne ali skupinske uporabne ter raziskovalne projekte, lahko tudi v sodelovanju s podjetji ali preko vključitve na domače ali mednarodne projekte.

Several model problems are presented and modeled by using methods from discrete mathematics.

We focus on the process of addressing a problem: identification of entities and relationships among them, identification of goals, data model design, algorithm implementation, design of testing procedures and test data, specification, implementation, evaluation and qualitative evaluation of the results.

Depending on the choice of the model problems, students get familiar with various mathematical tools and methodologies for addressing the problems, e.g. heuristic optimization procedures, data visualisation methods (graphs, charts, etc.), qualitative analysis of discrete dynamic systems, etc.

During the course seminar work, students will be assigned individual and team applied and research projects. If possible the students will be involved in projects with companies or in national or international applied research projects.

### Temeljni literatura in viri / Readings:

- E. Zakrajšek: *Matematično modeliranje*, DMFA-založništvo, Ljubljana, 2004.
- R. Aris: *Mathematical modelling techniques*, Dover, 1994.
- M. Jünger, P. Mutzel: *Graph Drawing Software*, Springer-Verlag, Berlin, 2004.
- Z. Michalewicz: *Genetic Algorithms + Data Structures = Evolution Programs*, Springer-Verlag, Berlin, 1999.
- R. A. Holmgren: *A First Course in Discrete Dynamical Systems*, Springer-Verlag, Berlin, 1996.

### Cilji in kompetence:

Študenti se naučijo identificirati problem, ki ga je mogoče obravnavati z matematičnimi tehnikami, problem formulirati v matematično obvladljivi obliki, identificirati orodja, s katerimi se problema lahko lotimo, preiskati kompetentno literaturo, razviti ali prilagoditi ustrezen model za reševanje, poiskati kritične

### Objectives and competences:

Students become capable of identifying problems that can be addressed by various mathematical techniques. They learn how to formulate problems in mathematical form, identify relevant tools to deal with the problem, search through the relevant literature, develop or adapt a relevant model for solving the

dejavnike modela, rešitev problema implementirati v praksi. Pri izdelavi projekta je poudarek tudi na posebnostih skupinskega dela.

problem, find critical aspects of it and implement a solution in practice. Specifics of team work are emphasised during the work on projects.

**Predvideni študijski rezultati:**

**Znanje in razumevanje:** spoznavanje procesa obravnave problema od njegove identifikacije, prek formulacije in obravnave modela do implementacije rešitve.

**Uporaba:** izdelava modelov pri reševanju realnih problemov.

**Refleksija:** presojanje veljavnosti predpostavk teoretičnih modelov, kritično vrednotenje izdelanih rešitev, vrednotenje skupinskega dela.

**Prenosljive spretnosti – niso vezane le na en predmet:** sposobnost prepoznavanja relevantnih dejstev, formuliranja problema, prilagajanja znanih rešitev, predstavitve konceptov.

**Intended learning outcomes:**

**Knowledge and understanding:** Learning of the process of a problem identification and problem addressing, starting by forming of a model, dealing with it and progressing towards a solution implementation.

**Application:** Construction of models for solving of real problems.

**Reflection:** evaluation of validity of assumptions for theoretical models, critical evaluation of constructed solutions, evaluation of team work.

**Transferable skills:** Capabilities of recognizing of relevant facts, problem formulation, adaptation of known solutions, concept presentation.

**Metode poučevanja in učenja:**

predavanja, vaje, skupinsko načrtovanje rešitev, projektno delo, seminarski nastopi, konzultacije

**Learning and teaching methods:**

Lectures, exercises, team solution planning, projects, seminar presentations, consultations

**Načini ocenjevanja:**

Delež (v %) /

Weight (in %) **Assessment:**

<p>Način (domače naloge, pisni izpit, ustno izpraševanje, naloge, projekt):</p> <ul style="list-style-type: none"> <li>▪ projektna naloga (načrt, izvedba, dokumentacija, poročilo, predstavitev, zagovor)</li> </ul> <p>Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)</p>	<p>100%</p>	<p>Type (homeworks, examination, oral, coursework, project):</p> <ul style="list-style-type: none"> <li>▪ project assignment (plan, execution, documentation, report, presentation, defense)</li> </ul> <p>Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)</p>
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**Reference nosilca / Lecturer's references:**

**doc. dr. Alen Orbanić**

- Hubard, A. Orbanić, D. Pellicer, A. I. Weiss. Symmetries of equivelar 4-toroids. *Discrete comput. geom.*, 2012, vol. 48, iss. 4, 1110-1136.
- Orbanić, D. Pellicer, A. I. Weiss. Map operations and k-orbit maps. *J. comb. theory. Ser. A*, 2010, vol. 117, iss. 4, 411-429.
- Hubard, A. Orbanić, A. I. Weiss. Monodromy groups and self-invariance. *Can. j. math.*, 2009, vol. 61, no. 6, str. 1300-1324.

**prof. dr. Primož Potočnik:**

- P. Potočnik, Tetravalent arc-transitive locally-Klein graphs with long consistent cycles, *European J. Combin.*, vol. 36 (2014), 270-281.
- P. Potočnik, P. Spiga, G. Verret, Cubic vertex-transitive graphs on up to 1280 vertices, *J. Symbolic Comp.* vol. 50 (2013), 465-477.
- P. Potočnik, Edge-colourings of cubic graphs admitting a solvable vertex-transitive group of automorphisms, *Journal of Combinatorial Theory Ser. B*, vol. 91 (2004), 289-300.

**prof. dr. Riste Škrekovski**

- J. Govorčin, M. Knor, R. Škrekovski, Line graph operator and small worlds, *Inform. Process. Lett.* 113 (2013) 196-200.
- Z. Dvorak, B. Lidicky, R. Škrekovski, Randić index and the diameter of a graph, *European J. Comb.* 32 (2011) 434-442.
- T. Kaiser, M. Stehlik, On the 2-resonance of fullerenes, *SIAM J. Discrete Math.* 25 (2011) 1737-1745.

## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Urejenostne algebrske strukture
<b>Course title:</b>	Ordered algebraic structures

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2204

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
45		30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Jakob Cimprič, prof. dr. Karin Cvetko-Vah, prof. dr. Boris Lavrič

**Jeziki / Languages:**

<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija.

**Prerequisites:**

Enrollment into the program.

**Vsebina:**

**Content (Syllabus outline):**

Delno urejene množice. Modulske mreže.  
Distributivne mreže in njihove upodobitve.  
Booleove algebre in njihove upodobitve.

Delno urejene grupe in vektorski prostori.  
Konveksne podgrupe. Homomorfizmi.  
Arhimedske in Dedekindovo polne grupe.  
Linearno urejene grupe.

Delno urejeni kolobarji. Ureditve polja  
ulomkov. Formalno realna polja. Realno zaprta  
polja. Arhimedske ureditve. Ureditve in  
valuacije.

Partially ordered sets. Modular lattices.  
Distributive lattices and their representations.  
Boolean algebras and their representations.

Partially ordered groups and vector spaces.  
Convex subgroups. Homomorphisms.  
Archimedean and Dedekind complete groups.  
Linearly ordered groups.

Partially ordered rings. Orderings on the field of  
fractions. Formally real fields. Real closed fields.  
Archimedean orderings. Orderings and  
valuations.

### Temeljni literatura in viri / Readings:

- G. Birkhoff: Lattice Theory, 3rd edition, AMS, Providence, 2006.
- T.S. Blyth: Lattices and Ordered Algebraic Structures, Springer, 2005.
- L. Fuchs: Partially Ordered Algebraic Systems, Pergamon Press, London, 1963.
- A. M. W. Glass: Partially Ordered Groups, World Scientific, River Edge, 1999.
- B. Lavrič: Delno urejene grupe in delno urejeni kolobarji, DMFA-založništvo, Ljubljana, 1993.
- B. Lavrič: Delno urejeni vektorski prostori, DMFA-založništvo, Ljubljana, 1995.

### Cilji in kompetence:

Študent spozna osnovne pojme teorije  
urejenostnih algebrskih struktur.

### Objectives and competences:

The student learns the basics of the theory of  
ordered algebraic structures.

### Predvideni študijski rezultati:

#### Znanje in razumevanje:

Razumevanje osnovnih pojmov in izrekov  
teorije urejenostnih algebrskih struktur ter  
njihove vloge na nekaterih drugih področjih.

#### Uporaba:

V drugih vejah matematike.

#### Refleksija:

Razumevanje teorije na podlagi primerov in  
uporabe.

### Intended learning outcomes:

#### Knowledge and understanding:

Understanding of basic concepts and theorems  
of the theory of ordered algebraic structures,  
and their role in some other areas.

#### Application:

In other mathematical areas.

#### Reflection:

Understanding the theory on the basis of  
examples and applications.

**Prenosljive spretnosti – niso vezane le na en predmet:**

Formulacija in reševanje problemov z abstraktnimi metodami.

**Transferable skills:**

Formulation and solution of problems using abstract methods.

**Metode poučevanja in učenja:**

Predavanja, vaje, domače naloge, konzultacije.

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations.

**Načini ocenjevanja:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- domače naloge
- ustni izpit

Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)

Delež (v %) /

Weight (in %)

50%

50%

**Assessment:**

Type (examination, oral, coursework, project):

- homework assignment
- oral exam

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

**prof. dr. Jaka Cimprič**

- J. Cimprič: Free skew fields have many  $*$ -orderings, *J. Algebra*, 280 (2004), pp. 20-28.
- J. Cimprič, I. Klep: Generalized orderings and rings of fractions., *Algebra Universalis* 55 (2006), pp. 93-109.
- J. Cimprič: A representation theorem for archimedean quadratic modules on  $*$ -rings. *Can. math. bull.* 52 (2009), pp. 39-52

**prof. dr. Boris Lavrič**

- B. Lavrič: Delno urejeni vektorski prostori, *Podiplomski Seminar iz Matematike*, 22. Društvo Matematikov, fizikov in astronomov Slovenije, Ljubljana, 1995. 152 strani. ISBN: 961-212-049-8.
- B. Lavrič: Delno urejene grupe in delno urejeni kolobarji, *Podiplomski Seminar iz Matematike*, 21. Društvo Matematikov, fizikov in astronomov Slovenije, Ljubljana, 1993. 138 strani. ISBN: 961-212-010-2.

- B. Lavrič: Coherent archimedean f-rings, *Comm. Alg.* 28(2), (2000), 1091-1096.

**prof. dr. Karin Cvetko-Vah**

- Cvetko-Vah, Karin: Internal decompositions of skew lattices. *Commun. Algebra*, 2007, vol. 35, no. 1, 243-247
- Cvetko-Vah, Karin: On strongly symmetric skew lattices. *Algebra univers.* (Print. ed.), 2011, vol. 66, no. 1-2, 99-113.
- Bauer, Andrej; Cvetko-Vah, Karin: Stone duality for skew Boolean algebras with intersections. *Houston J. Math.* 39 (2013), no. 1, 73–109.



## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Algebraična topologija 1
<b>Course title:</b>	Algebraic topology 1

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2302

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:** prof.dr.Petar Pavešić, prof.dr.Janez Mrčun, prof.dr.Sašo Strle,  
prof. dr. Dušan Repovš

**Jeziki / Languages:**

<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

**Prerequisites:**

Vpis v letnik študija

Enrollment into the program

**Vsebina:**

**Content (Syllabus outline):**

Homotopija, homotopska ekvivalenca, razširitve in dvigi homotopij, homotopska kategorija.

CW kompleksi, konstrukcija, topološke lastnosti, celularne preslikave.

Fundamentalna grupa, Seifert-van Kampenov izrek, uporaba (osnovni izrek algebre, Brouwerjev in Borsuk-Ulamov izrek, grupa vozla).

Krovni prostori, povezava s fundamentalno grupo, klasifikacija.

Homološke grupe, definicija in osnovne lastnosti, računanje, uporaba (stopnja preslikave, ovojna in spletna števila, indeks vektorskega polja, negibne točke). Očrt konstrukcije homoloških grup, osnove homološke algebre.

Homotopy, homotopy equivalence, extensions and liftings of homotopies, homotopy category.

CW complexes, construction, topological properties, cellular maps.

Fundamental group, Seifert-van Kampen theorem, applications (fundamental theorem of algebra, Brouwer and Borsuk-Ulam theorem, knot groups).

Covering spaces, relation to the fundamental group, classification.

Homology groups, definition and properties, computation, applications (degree of a map, winding and linking numbers, index of a vector field, fixed points). Outline of the construction of homology groups, basic facts of homological algebra.

#### Temeljni literatura in viri / Readings:

- A. Hatcher: *Algebraic Topology*, Cambridge Univ. Press, Cambridge, 2002.

#### Cilji in kompetence:

Študent spozna osnovne pojme algebraične topologije kot so homotopija, celični prostori, fundamentalna grupa in homološke grupe.

#### Objectives and competences:

Student learns basic concepts of algebraic topology: homotopy, cellular spaces, fundamental group, homology groups.

#### Predvideni študijski rezultati:

##### Znanje in razumevanje:

Poznavanje osnovnih pojmov in tehnik za delo s fundamentalno grupo in homološkimi grupami. Razumevanje homotopske invariance in prijemov za obravnavanje geometrijskih

#### Intended learning outcomes:

##### Knowledge and understanding:

Basic concepts and techniques for the computation of the fundamental group and homology groups. Understanding of the concepts of homotopy invariance and of

vprašanj s pomočjo algebre.

**Uporaba:**  
V področjih matematike, ki delajo z geometričnimi objekti (kompleksna in globalna analiza, dinamični sistemi, geometrijska in diferencialna topologija, teorija grafov), v računalništvu (grafika, prepoznavanje vzorcev, **topološka analiza podatkov, robotika**), v teoretični fiziki.

**Refleksija:**  
Razumevanje teorije na podlagi primerov in uporabe.

**Prenosljive spretnosti – niso vezane le na en predmet:**  
Formulacija problemov v primernem jeziku, reševanje in analiza doseženega na primerih, prepoznavanje algebraičnih struktur v geometriji.

approaches to geometric problems by algebraic methods.

**Application:**  
Parts of mathematics with strong geometric content (complex and global analysis, geometric and differential topology, graph theory), computer science (computer graphics, pattern recognition, topological data analysis, robotics), theoretical physics.

**Reflection:**  
Understanding of theoretical concepts through examples and applications.

**Transferable skills:**  
Recognition of algebraic structures in geometry, appropriate formulation of problems.

**Metode poučevanja in učenja:**  
predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**  
Lectures, exercises, homeworks, consultations

<b>Načini ocenjevanja:</b>	Delež (v %) / Weight (in %)	<b>Assessment:</b>
Način (pisni izpit, ustno izpraševanje, naloge, projekt): <ul style="list-style-type: none"> <li>▪ izpit iz vaj</li> <li>▪ ustni izpit</li> </ul> Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)	50% 50%	Type (examination, oral, coursework, project): <ul style="list-style-type: none"> <li>▪ written exam</li> <li>▪ oral exam</li> </ul> Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

## Reference nosilca / Lecturer's references:

### prof. dr. Petar Pavešić

- P. Pavešić: *The Hopf invariant one problem*, (Podiplomski seminar iz matematike, 23). Ljubljana: Društvo matematikov, fizikov in astronomov Slovenije, 1995.
- P. Pavešić: Reducibility of self-homotopy equivalences. *Proc. R. Soc. Edinb., Sect. A, Math.*, 2007, vol. 137, iss 2, str. 389-413.
- P. Pavešić, R.A.Piccinini. *Fibrations and their classification*, (Research and exposition in mathematics, vol. 33). Lemgo: Heldermann, cop. 2013. XIII, 158 str.

### prof. dr. Janez Mrčun

- I. Moerdijk, J. Mrčun: *Introduction to Foliations and Lie Groupoids*, Cambridge Studies in Advanced Mathematics, 91. Cambridge University Press, Cambridge (2003).
- I. Moerdijk, J. Mrčun: *Lie groupoids, sheaves and cohomology*, Poisson Geometry, Deformation Quantisation and Group Representations, 145-272, London Math. Soc. Lecture Note Ser. 323, Cambridge University Press, Cambridge (2005).
- J. Mrčun: Topologija. Izbrana poglavja iz matematike in računalništva 44, DMFA - založništvo, Ljubljana, 2008

### prof. dr. Sašo Strle

- B. Owens, S. Strle: *A characterisation of the  $n\langle 1 \rangle \oplus \langle 3 \rangle$  form and applications to rational homology spheres*. *Math. Res. Lett.*, 2006, vol. 13, iss. 2, str. 259-271.
- B. Owens, S. Strle: *Rational homology spheres and the four-ball genus of knots*. *Adv. Math.* (New York. 1965), 2006, vol. 200, iss. 1, str. 196-216.
- S. Strle: *Bounds on genus and geometric intersections from cylindrical end moduli spaces*. *J. Differ. Geom.*, 2003, vol. 65, no. 3, str. 469-511.

### prof. dr. Dušan Repovš

- U. H. Karimov, D. Repovš: *On the homology of the Harmonic Archipelago*, *Cent. Eur. J. Math.* 10:3 (2012), 863-872.
- U. H. Karimov, D. Repovš: *On noncontractible compacta with trivial homology and homotopy groups*, *Proc. Amer. Math. Soc.* 138:4 (2010), 1525-1531.
- F. Hegenbarth, D. Repovš: *Applications of controlled surgery in dimension 4: Examples*, *J. Math. Soc. Japan* 58:4 (2006), 1151-1162.

## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Algebraična topologija 2
<b>Course title:</b>	Algebraic topology 2

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2303

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Petar Pavešić, prof. dr. Janez Mrčun, prof. dr. Sašo Strle, prof. dr. Dušan Repovš

**Jeziki / Languages:**

<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

**Prerequisites:**

Vpis v letnik študija

Enrollment into the program

**Vsebina:**

**Content (Syllabus outline):**

Homotopija, homotopska ekvivalenca, razširitve in dvigi homotopij, homotopska kategorija.

CW kompleksi, konstrukcija, topološke lastnosti, celularne preslikave.

Homotopske grupe, eksaktna zaporedja para in vlaknenja, homotopski izrez.

Kohomološke grupe, definicija in osnovne lastnosti, računanje, uporaba. Konstrukcija kohomoloških grup. Kohomološki kolobar.

Homotopy, homotopy equivalence, extensions and liftings of homotopies, homotopy category.

CW complexes, construction, topological properties, cellular maps.

Homotopy groups, exact sequences of a pair and of a fibration, homotopy excision.

Cohomology groups, definition and properties, computation, applications. Construction of cohomology groups. Cohomology ring.

### Temeljni literatura in viri / Readings:

- A. Hatcher: *Algebraic Topology*, Cambridge Univ. Press, Cambridge, 2002.

### Cilji in kompetence:

Študent spozna osnovne pojme algebraične topologije kot so homotopija, celični prostori, homotopske grupe in kohomološke grupe.

### Objectives and competences:

Student learns basic concepts of algebraic topology: homotopy, cellular spaces, homotopy groups and cohomology groups.

### Predvideni študijski rezultati:

#### Znanje in razumevanje:

Poznavanje osnovnih pojmov in tehnik za delo s homotopskimi in kohomološkimi grupami. Razumevanje homotopske invariance in prijemov za obravnavanje geometrijskih vprašanj s pomočjo algebre.

#### Uporaba:

V področjih matematike, ki delajo z geometričnimi objekti (kompleksna in globalna analiza, dinamični sistemi, geometrijska in diferencialna topologija, teorija grafov), v računalništvu (grafika, prepoznavanje vzorcev,

### Intended learning outcomes:

#### Knowledge and understanding:

Basic concepts and techniques for the computation of homotopy and cohomology groups. Understanding of the concepts of homotopy invariance and of approaches to geometric problems by algebraic methods.

#### Application:

Parts of mathematics with strong geometric content (complex and global analysis, geometric and differential topology, graph theory), computer science (computer graphics, pattern recognition, topological data analysis, robotics),

topološka analiza podatkov, robotika), v teoretični fiziki.

**Refleksija:**

Razumevanje teorije na podlagi primerov in uporabe.

**Prenosljive spretnosti – niso vezane le na en predmet:**

Formulacija problemov v primernem jeziku, reševanje in analiza doseženega na primerih, prepoznavanje algebraičnih struktur v geometriji.

theoretical physics.

**Reflection:**

Understanding of theoretical concepts through examples and applications.

**Transferable skills:**

Recognition of algebraic structures in geometry, appropriate formulation of problems.

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations

Delež (v %) /

Weight (in %)

**Načini ocenjevanja:**

**Assessment:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- izpit iz vaj
- ustni izpit

50%

50%

Type (examination, oral, coursework, project):

- written exam
- oral exam

Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

**prof. dr. Petar Pavešić**

- P. Pavešić: *The Hopf invariant one problem*, (Podiplomski seminar iz matematike, 23). Ljubljana: Društvo matematikov, fizikov in astronomov Slovenije, 1995.
- P. Pavešić: Reducibility of self-homotopy equivalences. *Proc. R. Soc. Edinb., Sect. A, Math.*, 2007, vol. 137, iss 2, str. 389-413.
- P. Pavešić, R.A.Piccinini. *Fibrations and their classification*, (Research and exposition in

mathematics, vol. 33). Lemgo: Heldermann, cop. 2013. XIII, 158 str.

**prof. dr. Janez Mrčun**

- I. Moerdijk, J. Mrčun: *Introduction to Foliations and Lie Groupoids*, Cambridge Studies in Advanced Mathematics, 91. Cambridge University Press, Cambridge (2003).
- I. Moerdijk, J. Mrčun: *Lie groupoids, sheaves and cohomology*, Poisson Geometry, Deformation Quantisation and Group Representations, 145-272, London Math. Soc. Lecture Note Ser. 323, Cambridge University Press, Cambridge (2005).
- J. Mrčun: *Topologija. Izbrana poglavja iz matematike in računalništva 44, DMFA - založništvo, Ljubljana, 2008.*

**prof. dr. Sašo Strle**

- B. Owens, S. Strle: *A characterisation of the  $n\langle 1 \rangle \oplus \langle 3 \rangle$  form and applications to rational homology spheres*. Math. Res. Lett., 2006, vol. 13, iss. 2, str. 259-271.
  - B. Owens, S. Strle: *Rational homology spheres and the four-ball genus of knots*. Adv. Math. (New York. 1965), 2006, vol. 200, iss. 1, str. 196-216.
- S. Strle: *Bounds on genus and geometric intersections from cylindrical end moduli spaces*. J. Differ. Geom., 2003, vol. 65, no. 3, str. 469-511.

**prof. dr. Dušan Repovš**

- F. Hegenbarth, Yu. V. Muranov, D. Repovš: *Browder-Livesay filtrations and the example of Cappell and Shaneson*, Milan J. Math. 81:1 (2013), 79-97.
- U. H. Karimov, D. Repovš: *Examples of cohomology manifolds which are not homologically locally connected*, Topol. Appl. 155:11 (2008), 1169-1174.
- D. Repovš, M. Skopenkov, F. Spaggiari: *On the Pontryagin-Steenrod-Wu theorem*, Israel J. Math. 145 (2005), 341-348.



## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Analiza na mnogoterostih
<b>Course title:</b>	Analysis on manifolds

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2300

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
45		30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Franc Forstnerič, prof. dr. Janez Mrčun, prof. dr. Pavle Saksida

**Jeziki / Languages:**

<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

**Prerequisites:**

Vpis v letnik študija

Enrollment into the program

**Vsebina:**

**Content (Syllabus outline):**

Definicija gladke mnogoterosti in preslikave. Osnovne konstrukcije in primeri. **Diferencial preslikave**. Tangentni sveženj in tangentna preslikava. Mnogoterosti z robom. Delovanje grupe na mnogoterosti. Krovne in kvocientne mnogoterosti. **Svežnji in vektorski svežnji**. Imerzije in submerzije. **Podmnogoterosti**. Vložitve mnogoterosti v evklidske prostore.

Vektorska polja kot dinamični sistemi. Tok vektorskega polja. Komutator vektorskih polj. Frobeniusov izrek. **Izrek o obstoju cevaste okolice**. Indeks kritične točke vektorskega polja. Poincaré-Hopfov izrek.

**Liejeve grupe**. Eksponentna preslikava. **Invariantna vektorska polja**. **Liejeva algebra**. **Adjungirana reprezentacija**.

Sardov izrek. Thomov izrek o transversalnosti. Presečno število podmnogoterosti. Morsejeve funkcije.

#### **Možne dodatne vsebine:**

Diferencialne forme in integracija. Stokesov izrek. De Rhamova kohomologija. Poincaréjeva dualnost. Eulerjev razred in Thomov razred.

Riemannove mnogoterosti. Volumska forma in integracija. Hodgev \*-operator. Laplaceov operator. Harmonične forme. Hodgejeva dekompozicija.

The notion of a smooth manifold and map. Basic constructions and examples. The differential. The tangent bundle and the tangent map. Manifolds with boundary. Group actions on manifolds. Covering and quotient manifolds. Fiber bundles and vector bundles. Immersions and submersions. Submanifolds. Embedding manifolds to Euclidean spaces.

Vector fields as dynamical systems Flows. Commutator of vector fields. The theorem of Frobenius. The tubular neighborhood theorem. Index of a critical point of a vector field. The Poincaré-Hopf theorem.

Lie groups. The exponential map. Invariant vector fields. The Lie algebra of a Lie group. The adjoint representation.

Sard's theorem. The Thom transversality theorem. The intersection number of submanifolds. Morse functions.

#### **Other possible topics:**

Differential forms and integration. Stokes' theorem. De Rham cohomology. Poincaréjeva dualnost. Eulerjev and Thomov class.

Riemannian manifolds. Volume form and integration. The Hodge \*-operator. Laplace operator. Harmonic forms. Hodge decomposition.

#### **Temeljni literatura in viri / Readings:**

- W. M. Boothby: *An Introduction to Differentiable Manifolds and Riemannian Geometry*, 2nd edition, Academic Press, Orlando, 1986.
- V. Guillemin, A. Pollack: *Differential Topology*, Prentice Hall, Englewood Cliffs, 1974.
- M. W. Hirsch: *Differential Topology*, Springer, New York, 1977.
- M. Spivak: *Calculus on Manifolds*, W. A. Benjamin, New York-Amsterdam, 1965.
- F. W. Warner: *Foundations of Differentiable Manifolds and Lie Groups*, Springer, New York-Berlin, 1983.

**Cilji in kompetence:**

Slušatelj se seznani z osnovami teorije gladkih mnogoterosti in njihovo povezavo s sorodnimi področji matematike kot so analitična in algebraična geometrija, teorija Riemannovih ploskev, teorija Liejevih grup in druga. Pri tem uporabi znanje iz osnovne analize, algebre in topologije.

**Objectives and competences:**

Students learn some of the main basic concepts and methods of the theory of smooth manifolds and its connection to related fields of mathematics such as analytic and algebraic geometry, the theory of Lie groups, the theory of Riemann surfaces, etc. Basic methods of analysis, algebra and topology are applied in the course.

**Predvideni študijski rezultati:**

**Znanje in razumevanje:** Metode matematične analize, algebre in topologije, ki jih je študent spoznal na prvi stopnji študija, se obravnavajo in uporabijo v splošnejšem kontekstu gladkih mnogoterosti.

**Uporaba:** Teorija mnogoterosti je ena najbolj interdisciplinarnih področij sodobne matematike in je osnova vrsti področij kot so analitična, algebraična in diferencialna geometrija, teorija Liejevih grup, teorija Riemannovih ploskev, dinamika, itd. Mnogoterosti so nepogrešljivo orodje v naravoslovju in tehniki.

**Refleksija:** Razumevanje teorije na podlagi primerov. Razvoj sposobnosti uporabe teorije v različnih problemih znanosti in tehnike.

**Prenosljive spretnosti – niso vezane le na en predmet:** Identifikacija, formulacija in reševanje problemov s pomočjo metod teorije gladkih mnogoterosti. Spretnost uporabe domače in tuje literature.

**Intended learning outcomes:**

**Knowledge and understanding:** Methods of mathematical analysis, algebra and topology are applied and further developed in the context of smooth manifolds.

**Application:** The theory of smooth manifolds is one of the most interdisciplinary areas of modern mathematics. It is a basis of a number of areas such as analytic, algebraic and differential geometry, the theory of Lie groups, the theory of Riemann surfaces, dynamics, etc. Manifolds are a major tool in natural and technical sciences.

**Reflection:** Understanding the theory on the basis of examples. Acquiring skills in applying the theory to diverse scientific problems.

**Transferable skills:** The ability to identify, formulate and solve scientific problems using methods of smooth manifolds. Developing skills of using the domestic and foreign literature.

**Metode poučevanja in učenja:****Learning and teaching methods:**

predavanja, vaje, domače naloge, konzultacije	Lectures, exercises, homeworks, consultations
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Načini ocenjevanja:	Delež (v %) / Weight (in %)	Assessment:
<b>Način (domače naloge, pisni izpit, ustno izpraševanje):</b> <ul style="list-style-type: none"> <li>▪ Domače naloge in/ali pisni izpit</li> <li>▪ ustni izpit</li> </ul> <p>Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)</p>	50%  50%	<b>Type (homework, written exam, oral exam):</b> <ul style="list-style-type: none"> <li>▪ Homework and/or written exam</li> <li>▪ oral exam</li> </ul> <p>Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)</p>

**Reference nosilca / Lecturer's references:**

**prof. dr. Franc Forstnerič**

- F. Forstnerič: *Runge approximation on convex sets implies Oka's property*, Annals. of Math., 163 (2006), 689-707.
- F. Forstnerič: *Noncritical holomorphic functions on Stein manifolds*, Acta Math., 191 (2003), 143-189.
- F. Forstnerič: *Manifolds of holomorphic mappings from strongly pseudoconvex domains*. Asian J. Math. 11 (2007) 113-126.

**prof. dr. Janez Mrčun**

- I. Moerdijk, J. Mrčun: *Introduction to Foliations and Lie Groupoids*, Cambridge Studies in Advanced Mathematics, 91, Cambridge Univ. Press, Cambridge, 2003.
- I. Moerdijk, J. Mrčun: *On integrability of infinitesimal actions*, Amer. J. Math., 124 (2002), 567-593.
- J. Mrčun: *Functoriality of the bimodule associated to a Hilsum-Skandalis map*, K-Theory, 18 (1999), 235-253.

**prof. dr. Pavle Saksida**

- P. Saksida: *Integrable anharmonic oscillators on spheres and hyperbolic spaces*, Nonlinearity, 14 (2001), 977-994.
- Pavle Saksida: *Lattices of Neumann oscillators and Maxwell-Bloch equations*, Nonlinearity **19** (2006), pp 747-768
- Pavle Saksida: *On zero-curvature condition and Fourier analysis*. Phys. A: Math. Gen. **44** (2011), pp. 85203-85222

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

**Predmet:** Diferencialna geometrija  
**Course title:** Differential geometry

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type**

**Univerzitetna koda predmeta / University course code:**

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
45		30			105	6

**Nosilec predmeta / Lecturer:**

**Jeziki / Languages:**  
**Predavanja / Lectures:**   
**Vaje / Tutorial:**

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

**Prerequisites:**

**Vsebina:**

**Content (Syllabus outline):**

**Obvezni del:**

Uvod in osnovni pojmi: Vektorska polja in Liejev oklepaj. Temeljni pojmi teorije Liejevih grup in Liejevih algeber. Diferencialne forme. Vektorski svežnji in Riemannove strukture na njih.

Glavni svežnji, pridruženi svežnji, sveženj ogrodij, pojem redukcije svežnja.

Diferencialne forme z vrednostmi v Liejevih algebrah, povezave na glavnem svežnju. Horizontalni dvig poti. Ukrivljenost in holonomija. Različni opisi ukrivljenosti na glavnem svežnju.

Povezave na vektorskih svežnjih, kovariantni odvod. Chernovi razredi.

Temelji Riemannove geometrije: Riemannova metrika, Levi-Civítájeva povezava, Riemannov tenzor ukrivljenosti in njegove lastnosti, Riccijeva in Weylova ukrivljenost, avtoparalelnost, geodetske krivulje. Eksponentna preslikava.

**Izbirni del:**

Podgrupe grupe  $GL(n, \mathbf{C})$  in simetrični prostori. Gaussova ukrivljenost na ploskvah. Poissonove in simplektične mnogoterosti. Pontrjaginovi razredi in Bottov izrek. Konformnost in Weylov tenzor.

**Core topics:**

Introduction: Vector fields and Lie bracket.

Fundamental notions of the Lie theory.

Differential forms. Vector bundles, Riemann structures on vector bundles.

Principal bundles, associated bundles, frame bundles, reductions of bundles.

Differential forms with values in Lie algebras, connections on principal bundles. Horizontal lift of a path. Curvature and holonomy. Various descriptions of the curvature on a principal bundle.

Connections on vector bundles, covariant derivative. Chern classes.

Fundamental notions of Riemann geometry:

Riemannian metric, Levi-Civítá connection, Riemann curvature tensor and its properties, Ricci and Weyl curvatures, autoparallel curves, geodesic curves. Exponential map.

**Additional topics:** Subgroups of the group  $GL(n; \mathbf{C})$  and symmetric spaces. Gaussian curvature on surfaces. Poisson and symplectic manifolds. Pontryagin classes and Bott's theorem. Conformality and Weyl tensor

**Temeljni literatura in viri / Readings:**

- B. A. Dubrovin, A. T. Fomenko, S. P. Novikov: *Modern Geometry - Methods and Applications II : The Geometry and Topology of Manifolds*, Springer, New York, 1985.
- S. Helgason: *Differential Geometry, Lie Groups, and Symmetric Spaces*, AMS, Providence, 2001.
- S. Kobayashi, K. Nomizu: *Foundations of Differential Geometry I, II*, John Wiley & Sons, New York, 1996.
- P. Petersen: *Riemannian Geometry*, Springer, New York, 1997.
- J. Cheeger, D. Ebin, *Comparison Theorems in Riemannian Geometry*, AMS Chelsea Publishing, Providence, 2008

**Cilji in kompetence:****Objectives and competences:**

Študent se spozna s temelji sodobne diferencialne geometrije. Osnovna pojma tega predmeta sta povezava na glavnem ali na vektorskem svežnju in ukrivljenost povezave. Ukrivljenost je predstavljena skozi optiko Frobeniusovega izreka. Vpeljan je pojem holonomije, opisana je zveza med ukrivljenostjo in holonomijo. Te pojme uporabimo pri obravnavi temeljev Riemannove geometrije. Prek Chernovih razredov poudarimo povezavo s topologijo.

Fundamental concepts of modern differential geometry are introduced. The central objects of the course are connections on principal or vector bundles and their curvatures. The curvature is described from the point of view of the Frobenius theorem. The notion of holonomy is introduced and the relationship between holonomy and curvature is described. These notions are then used in the presentation of the fundamentals of the Riemannian geometry. The relationship between differential geometry and topology is illustrated by means of Chern classes.

#### **Predvideni študijski rezultati:**

**Znanje in razumevanje:** Poznavanje in razumevanje osnovnih pojmov in definicij iz diferencialne geometrije.

**Uporaba:** Uporaba teorije pri reševanju problemov.

**Refleksija:** Razumevanje teorije na podlagi uporabe.

**Prenosljive spretnosti – niso vezane le na en predmet:** Spretnosti uporabe domače in tuje literature in drugih virov, identifikacija in reševanje problemov, kritična analiza.

#### **Intended learning outcomes:**

**Knowledge and understanding:** Understanding the fundamental definitions and concepts of differential geometry.

**Application:** Solving problems by applying the relevant theory.

**Reflection:** Understanding the theory through its applications.

**Transferable skills:** Skills in the use of the relevant literature and other sources, formulating problems and solving them, critical analysis.

#### **Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije

#### **Learning and teaching methods:**

Lectures, exercises, homework, consultations

Načini ocenjevanja:	Delež (v %) / Weight (in %)	Assessment:
<ul style="list-style-type: none"> <li>▪ pisni izpit</li> <li>▪ ustni izpit</li> </ul> <p>Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)</p>	<p>50%</p> <p>50%</p>	<ul style="list-style-type: none"> <li>▪ written exam</li> <li>▪ oral exam</li> </ul> <p>Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)</p>

**Reference nosilca / Lecturer's references:**

**prof. dr. Janez Mrčun**

- J. Mrčun: *An extension of the Reeb stability theorem*, Topology Appl. 70 (1996), 25-55.
- I. Moerdijk, J. Mrčun: *On integrability of infinitesimal actions*, Amer. J. Math. 124 (2002) 567-593.
- I. Moerdijk, J. Mrčun: *Introduction to Foliations and Lie Groupoids*, Cambridge Studies in Advanced Mathematics, 91. Cambridge University Press, Cambridge (2003).

**prof. dr. Pavle Saksida**

- P. Saksida: *Nahm's equations and generalizations of the Neumann system*, Proc. London Math. Soc. , 78 (1999), no. 3, 701-720.
- P. Saksida: *Integrable anharmonic oscillators on spheres and hyperbolic spaces*, Nonlinearity 14 (2001), no. 5. 977-994.
- P. Saksida: *Lattices of Neumann oscillators and Maxwell-Bloch equations*, Nonlinearity 19 (2006), no. 3 747-768.

**prof. dr. Sašo Strle**

- A. Stefanovska, S. Strle, P. Krošelj: *On the overestimation of the correlation dimension*, Phys. Lett. A 235 (1997), no. 1, 24-30.
- D. Ruberman, S. Strle: *Mod 2 Seiberg-Witten invariants of homology tori*, Math. Res. Lett. 7 (2000), no. 5-6, 789-799.
- S. Strle: *Bounds on genus and geometric intersections from cylindrical end moduli spaces*, J. Differential Geom. 65 (2003), no. 3, 469-511.



## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Konveksnost
<b>Course title:</b>	Convexity

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2306

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
45		30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Franc Forstnerič, prof. dr. Boris Lavrič

<b>Jeziki / Languages:</b>	<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
	<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

**Prerequisites:**

Vpis v letnik študija

Enrollment into the program

**Vsebina:**

**Content (Syllabus outline):**

Afine in konveksne množice. Topološke lastnosti konveksnih množic. Caratheodorijev in Radonov izrek. Separacijski izreki. Ekstremne točke. Politopi. Stožci in polare. Poliedri. Izrek Weyla in Minkowskega. Sistemi linearnih neenačb. Farkaseva lema in linearno programiranje. Posplošitve Hellyjevega izreka. Metrični prostor konveksnih množic. Blaschkejev izrek. Metrične lastnosti konveksnih množic. Konveksne funkcije. Zveznost, odvedljivost in subgradient. Ekstremi.

Affine and convex sets. Topological properties of convex sets. Theorems of Caratheodory and Radon. Separation theorems. Extreme points. Polytopes. Cones and polars. Polyhedra. The theorem of Weyl and Minkowski. Systems of linear inequations. The Farkas lemma and linear programming. Generalizations of the theorem of Helly. The metric space of convex sets. The Blaschke theorem. Metric properties of convex sets. Convex functions. Continuity, differentiability and the subgradient. Extrema.

### Temeljni literatura in viri / Readings:

- H. G. Eggleston: Convexity, Cambridge Univ. Press, Cambridge, 1958.
- A. Brøndsted: An Introduction to Convex Polytopes, Springer, New York-Berlin, 1983.
- F. A. Valentine: Convex Sets, Robert E. Krieger, Huntington, 1976.
- R. T. Rockafellar: Convex Analysis, Princeton Univ. Press, Princeton, 1996.
- A. W. Roberts, D. E. Varberg: Convex Functions, Academic Press, New York-London, 1973.

### Cilji in kompetence:

Študent spozna osnovne pojme konveksne geometrije in konveksne analize. Seznan se z lastnostmi konveksnih množic in konveksnih funkcij v evklidskih in normiranih prostorih ter z uporabo teorije na raznih področjih matematike. Pri tem povezuje geometrijsko intuicijo z algebro, analizo in kombinatoriko.

### Objectives and competences:

The student learns the basic concepts of convex geometry and convex analysis. The student gets familiar with the properties of convex sets and convex functions in euclidean and normed spaces and applications of the theory in different areas of mathematics. The student combines geometric intuition with algebra, analysis and combinatorics.

### Predvideni študijski rezultati:

#### Znanje in razumevanje:

Poznavanje in razumevanje osnovnih pojmov teorije konveksnih množic in konveksnih funkcij. Sinteza metod iz linearne algebre, analize in geometrije.

#### Uporaba:

Uporaba teorije pri reševanju problemov na raznih področjih matematike in drugih znanosti.

#### Refleksija:

### Intended learning outcomes:

#### Knowledge and understanding:

Knowledge and understanding of basic concepts of the theory of convex sets and convex functions. A synthesis of methods of linear algebra, analysis and geometry.

#### Application:

Solving problems in different areas of mathematics and other sciences using the theory.

#### Reflection:

Razumevanje teorije na podlagi primerov in uporabe.

**Prenosljive spretnosti – niso vezane le na en predmet:**  
 Postavitev problema, njegova matematična formulacija ter reševanje in analiza. Prenos teorije v prakso.

Understanding the theory on the basis of examples and applications.

**Transferable skills:**  
 Posing of a problem, its mathematical formulation, solving and analysis. The transfer of the theory into praxis.

**Metode poučevanja in učenja:**  
 predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**  
 Lectures, exercises, homeworks, consultations

<b>Načini ocenjevanja:</b>	Delež (v %) / Weight (in %)	<b>Assessment:</b>
Način (pisni izpit, ustno izpraševanje, naloge, projekt):		Type (examination, oral, coursework, project):
▪ izpit iz vaj (2 kolokvija ali pisni izpit)	50%	▪ 2 midterm exams instead of written exam, written exam
▪ ustni izpit	50%	▪ oral exam
Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)		Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

**prof. dr. Franc Forstnerič**  
 F. Forstnerič: *Runge approximation on convex sets implies Oka's property*, Annals of Math., 163 (2006), 689-707.  
 F. Forstnerič: *Noncritical holomorphic functions on Stein manifolds*, Acta Math., 191 (2003), 143-189.  
 F. Forstnerič: *Embedding strictly pseudoconvex domains into balls*, Trans. Amer. Math. Soc., 295 (1986), 347-368.

**prof. dr. Boris Lavrič**  
 B. Lavrič: *The isometries of certain maximum norms*, Linear Algebra Appl. 405 (2005), 249-263.  
 B. Lavrič: *The isometries and the G-invariance of certain seminorms*, Linear Algebra Appl. 374 (2003), 31-40.  
 B. Lavrič: *Monotonicity properties of certain classes of norms*, Linear Algebra Appl. 259 (1997), 237-250.

## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Liejeve grupe
<b>Course title:</b>	Lie groups

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2307

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
45		30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Franc Forstnerič, prof. dr. Janez Mrčun, prof. dr. Pavle Saksida

**Jeziki / Languages:**

<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

**Prerequisites:**

Vpis v letnik študija

Enrollment into the program

**Vsebina:**

**Content (Syllabus outline):**

Liejeva grupa in njena Liejeva algebra. Eksponentna preslikava. Adjungirano delovanje.

Liejeva teorija.

Homogeni prostori. Izrek o rezini.

Kompaktne Liejeve grupe. Haarova mera.

Maksimalni torusi.

**Možne dodatne vsebine:**

Weylova grupa. Korenski prostori. Upodobitev kompaktnih Liejevih grup. Rešljive, nilpotentne in polenostavne Liejeve grupe in Liejeve algebre. Harmonična analiza na Liejevi grupi.

Lie group and the associated Lie algebra.

Exponential map. Adjoint action.

Lie theory.

Homogeneous spaces. Slice theorem.

Compact Lie groups. Haar measure. Maximal tori.

**Other possible topics::**

Weyl group. Root spaces. Representations of compact Lie groups. Solvable, nilpotent and semisimple Lie groups and Lie algebras.

Harmonic analysis on a Lie group.

**Temeljni literatura in viri / Readings:**

- J. F. Adams: *Lectures on Lie Groups*, W. A. Benjamin, New York-Amsterdam, 1969.
- F. W. Warner: *Foundations of Differentiable Manifolds and Lie Groups*, Springer, New York-Berlin, 1983.
- J. P. Serre: *Lie Algebras and Lie Groups*, 2nd edition, Springer, Berlin, 2006.
- T. Bröcker, T. T. Dieck: *Representations of Compact Lie Groups*, Springer, New York, 2003.
- J. J. Duistermaat, J. A. C. Kolk: *Lie Groups*, Springer, Berlin, 2000.

**Cilji in kompetence:**

Študent se spozna s pojmom Liejeve grupe in njene Liejeve algebre, ter z Liejevo teorijo. Posebej se seznanja s teorijo upodobitev kompaktnih Liejevih grup in homogenih prostorov. Liejeve grupe so centralni pojem diferencialne geometrije, njihova uporaba pa sega v številna področja matematike in matematične fizike.

**Objectives and competences:**

Student gets familiar with the basic concepts of Lie group with the associated Lie algebra, and with Lie theory. In particular, the student learns the basic theory of representations of compact Lie groups and homogeneous spaces. Lie groups are a central concept of differential geometry and are applied in many areas of mathematics and mathematical physics.

**Predvideni študijski rezultati:**

**Znanje in razumevanje:** Poznavanje in razumevanje osnovnih pojmov in definicij iz teorije Liejevih grup.

**Uporaba:** Uporaba teorije pri reševanju problemov.

**Refleksija:** Razumevanje teorije na podlagi uporabe.

**Intended learning outcomes:**

**Knowledge and understanding:** Knowledge and understanding of basic concepts and definitions of the theory of Lie groups.

**Application:** Solving problems using the theory.

**Reflection:** Understanding of the theory from the applications.

**Transferable skills:** Skills in using the literature and other sources, the ability to identify and

**Prenosljive spretnosti – niso vezane le na en predmet:** Spretnosti uporabe domače in tuje literature in drugih virov, identifikacija in reševanje problemov, kritična analiza.

solve the problem, critical analysis.

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**

lectures, exercises, homeworks, consultations

**Načini ocenjevanja:**

Delež (v %) /  
Weight (in %)

**Assessment:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- 2 kolokvija namesto pisnega izpita, pisni izpit **ali domače naloge**
- ustni izpit

Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)

50%

50%

Type (examination, oral, coursework, project):

- 2 midterm exams instead of written exam, written exam **or homework**
- oral exam

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

**prof. dr. Franc Forstnerič**

- F. Forstnerič, J.-P. Rosay: *Approximation of biholomorphic mappings by automorphisms of  $\mathbb{C}^n$* , Invent. Math. 112 (1993), 323-349.
- F. Forstnerič: *Runge approximation on convex sets implies Oka's property*, Annals. of Math. (2) 163 (2006), 689-707.
- F. Forstnerič: *Actions of  $(\mathbb{R}, +)$  and  $(\mathbb{C}, +)$  on complex manifolds*. Math. Z. 223 (1996), 123-153.

**prof. dr. Janez Mrčun**

- J. Mrčun: *On isomorphisms of algebras of smooth functions*. Proc. Amer. Math. Soc. 133 (2005), 3109-3113.
- I. Moerdijk, J. Mrčun: *Introduction to Foliations and Lie Groupoids*, Cambridge Studies in Advanced Mathematics, 91. Cambridge University Press, Cambridge (2003).
- I. Moerdijk, J. Mrčun: *On the integrability of Lie subalgebroids*. Adv. Math. 204 (2006), 101-115.

**prof. dr. Pavle Saksida**

- P. Saksida: *Maxwell-Bloch equations, C Neumann system and Kaluza-Klein theory*. J. Phys. A 38 (2005), no. 48, 10321-10344.
- P. Saksida: *Neumann system, spherical pendulum and magnetic fields*. J. Phys A 35 (2002), no. 25, 5237-5253.
- P. Saksida: *Lattices of Neumann oscillators and Maxwell-Bloch equations*. Nonlinearity 19 (2006), no. 3, 747-768.

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

**Predmet:** Riemannove ploskve  
**Course title:** Riemann surfaces

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2305

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Miran Černe, prof. dr. Barbara Drinovec Drnovšek,  
prof. dr. Franc Forstnerič, prof. dr. Pavle Saksida

**Jeziki / Languages:**

<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija

**Prerequisites:**

Enrollment into the program

**Vsebina:**

**Content (Syllabus outline):**



Definicija Riemannove ploskve. Osnovni primeri. Holomorfne in meromorfne funkcije in preslikave. Topologija Riemannovih ploskev.

**Krovni prostori in krovne transformacije.**

Analitično nadaljevanje. Algebraične funkcije. Integracija na Riemannovih ploskvah.

**Riemannove ploskve kot kompleksne krivulje.**

Osnovni pojmi teorije snopov.

Konstrukcija meromorfni funkcij z  $L_2$ -metodo.

Weylova lema. Hilbertov prostor kvadratno integrabilnih form. Meromorfne funkcije in diferenciali. Harmonični in analitični diferenciali. Bilinearne relacije. Divizorji in holomorfni vektorski svežnji. Riemannov-Rochov izrek in uporaba.

#### **Možne dodatne vsebine:**

Odrte Riemannove ploskve. Dirichletov problem. Rungejev aproksimacijski izrek. Mittag-Lefflerjev in Weierstrassov izrek.

**Riemann-Koebejev uniformizacijski izrek.**

Riemann-Hilbertov robni problem. **Serrejev izrek o dualnosti.** **Abelov izrek in uporabe.**

Jacobijev inverzni problem. Kompleksni torusi. Eliptične funkcije. Weierstrassova funkcija.

The notion of a Riemann surface. Basic examples. Holomorphic and meromorphic functions and maps. Topology of Riemann surfaces. Covering spaces and deck transformations. Analytic continuation. Algebraic functions. Integration on Riemann surfaces. Riemann surfaces as complex curves. Basics of sheaf theory.

Construction of meromorphic functions by  $L_2$ -method. Weyl lemma. Hilbert space of square integrable forms. Meromorphic functions and differentials. Harmonic and analytic differentials. Bilinear relations. Divisors and holomorphic line bundles. The Riemann-Roch theorem and applications.

**Other possible topics:** Open Riemann surfaces. The Dirichlet problem. The Runge approximation theorem. Theorems of Mittag-Leffler and Weierstrass. Riemann-Koebe uniformization theorem. Riemann-Hilbert boundary value problem. Serre duality. Abel's theorem and applications. Jacobi inverse problem. Complex tori. Elliptic functions. Weierstrass function.

#### **Temeljni literatura in viri / Readings:**

- H. M. Farkas, I. Kra: *Riemann Surfaces*, 2nd edition, Springer, New York, 1992.
- O. Forster: *Lectures on Riemann Surfaces*, Springer, New York, 1999.
- F. Kirwan: *Complex Algebraic Curves*, Cambridge Univ. Press, Cambridge, 1992.
- B. A. Dubrovin, A. T. Fomenko, S. P. Novikov: *Modern Geometry - Methods and Applications III : Introduction to Homology Theory*, Springer, New York, 1990.
- **D. Varolin: *Riemann surfaces by way of complex analytic geometry*. Amer. Math. Soc., Providence, RI, 2011.**

#### **Cilji in kompetence:**

Slušatelj se seznani z osnovami teorije Riemannovih ploskev in njihovo povezavo s sorodnimi področji matematike kot so kompleksna analiza in algebraična geometrija. Pri tem uporabi znanje iz osnovne analize, algebre in topologije.

#### **Objectives and competences:**

Students learn some of the basic concepts and methods of the theory of Riemann surfaces and its connection to related fields of mathematics such as complex analysis and algebraic geometry. Basic methods of analysis, algebra and topology are applied in the course.

**Predvideni študijski rezultati:****Znanje in razumevanje:**

Spoznanje in razumevanje nekaterih bistvenih osnovnih pojmov teorije Riemannovih ploskev.

**Uporaba:** Riemannove ploskve so pojavljajo v vrsti matematični področij (analitična in algebraična geometrija, diferencialna geometrija, simpleksična geometrija), nepogrešljive pa so tudi v mnogih vejah fizike (npr. teorija strun) in širše znanosti. Eliptične krivulje so bistvenega pomena v kriptografiji.

**Refleksija:** Razumevanje teorije na podlagi primerov. Razvoj sposobnosti uporabe teorije v različnih znanstvenih problemih.

**Prenosljive spretnosti – niso vezane le na en predmet:** Identifikacija, formulacija in reševanje problemov s pomočjo metod teorije Riemannovih ploskev. Spretnost uporabe domače in tuje literature. Privajanje na samostojno seminarsko predstavitev gradiva.

**Intended learning outcomes:**

**Knowledge and understanding:** Understanding of fundamental topics in the theory of Riemann surfaces.

**Application:** Riemann surfaces appear naturally in many areas of mathematics (e.g. in analytic and algebraic geometry, differential geometry, symplectic geometry and other areas), as well as in several areas of physics (such as string theory) and in other sciences. Elliptic curves are a fundamental tool in cryptography.

**Reflection:** Understanding the theory on the basis of examples. Acquiring skills in applying the theory to diverse scientific problems.

**Transferable skills:** The ability to identify, formulate and solve scientific problems using methods of Riemann surface theory. Developing skills of using the domestic and foreign literature. Developing skills of independent presentation of the material.

**Metode poučevanja in učenja:**

predavanja, seminarji, vaje, domače naloge, konzultacije

**Learning and teaching methods:**

Lectures, seminar presentations, exercises, homeworks, consultations

**Načini ocenjevanja:**

Način (domače naloge, seminarska naloga, ustno izpraševanje):

- domače naloge, seminarska naloga
- ustni izpit

Delež (v %) /

Weight (in %)

**Assessment:**

Type (homework, seminar paper, oral exam):

- homework and seminar paper
- oral exam

50%

Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)	50%	Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)
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**Reference nosilca / Lecturer's references:**

**prof. dr. Miran Černe**

- M. Černe: *Nonlinear Riemann-Hilbert problem for bordered Riemann surfaces*, Amer. J. Math., 126 (2004), 65-87.
- M. Černe, F. Forstnerič: *Embedding some bordered Riemann surfaces in the affine plane*, Math. Res. Lett., 9 (2002), 683-696.
- M. Černe, M. Flores, *Quasilinear  $\bar{\partial}$ -equation on bordered Riemann surfaces*. Math. Ann. 335 (2006), 379-403.

**prof. dr. Barbara Drinovec Drnovšek**

- B. Drinovec Drnovšek: *Discs in Stein manifolds containing given discrete sets*. Math. Z., 2002, vol. 239, no. 4, str. 683-702.
- B. Drinovec Drnovšek: *Proper discs in Stein manifolds avoiding complete pluripolar sets*. Math. res. lett., 2004, vol. 11, no. 5-6, str. 575-581
- B. Drinovec Drnovšek, F. Forstnerič, *Holomorphic curves in complex spaces*, Duke Math. J., 139 (2007), 203-253.

**prof. dr. Franc Forstnerič**

- F. Forstnerič: *Runge approximation on convex sets implies Oka's property*, Annals. of Math., 163 (2006), 689-707.
- F. Forstnerič, E.F.Wold: *Bordered Riemann surfaces in  $C^2$* . J. Math. Pures Appl. 91 (2009) 100-114.
- F. Forstnerič, E.F. Wold: *Embeddings of infinitely connected planar domains into in  $C^2$* . Analysis & PDE, 6 (2013) 499-514.

**prof. dr. Pavle Saksida**

- P. Saksida: *Maxwell-Bloch equations, C Neumann system and Kaluza-Klein theory*, J. Phys. A, 38 (2005), 10321-10344.
- P. Saksida: *Lattices of Neumann oscillators and Maxwell-Bloch equations*, Nonlinearity 19 (2006), pp 747-768 .
- P. Saksida: *Integrable anharmonic oscillators on spheres and hyperbolic spaces*, Nonlinearity, 14 (2001), 977-994.

## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Uvod v algebraično geometrijo
<b>Course title:</b>	Introduction to algebraic geometry

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2301

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
45		30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Tomaž Košir

<b>Jeziki / Languages:</b>	<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
	<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija

**Prerequisites:**

Enrollment into the program

**Vsebina:**

**Osnovni del:**  
 Afine raznoterosti. Hilbertov izrek o ničlah.  
 Kolobar polinomskih funkcij. Racionalne funkcije.  
 Lokalne lastnosti ravninskih krivulj.  
 Projektivne raznoterosti. Regularne in racionalne funkcije.  
 Projektivne ravninske krivulje. Bezoutov izrek.  
 Izrek Maxa Noetherja.  
 Preslikave med raznoterostmi. Resolucije

**Content (Syllabus outline):**

**Fundamental part:**  
 Affine varieties. Hilbert Nullstellensatz.  
 Ring of polynomial functions. Rational functions.  
 Local properties of plane curves.  
 Projective varieties. Regular and rational functions.  
 Projective plane curves. Bezout's Theorem.  
 Max Noether Theorem.  
 Affine and rational maps. Resolutions of singularities.  
 Hilbert polynomial and Hilbert function.  
 Divisors on varieties.

singularnosti krivulj.

Hilbertov polinom in Hilbertova funkcija.

Delitelji na raznoterostih.

Krivulje. Ravninske kubične krivulje. Linearni sistemi na krivulji. Projektivne vložitve krivulj.

**Izbirne vsebine:**

Riemann-Rochov izrek.

Curves. Plane cubic curves. Linear systems on curves. Projective embeddings of curves.

**Elective topics:**

Riemann-Roch Theorem.

**Temeljni literatura in viri / Readings:**

- B. Hassett. *Introduction to algebraic geometry*. Cambridge Univ. Press, 2007.
- M. C. Beltrametti, E. Carletti, D. Gallarati, G. Monti Bragadin. *Lectures on Curves, Surfaces and Projective Varieties. A Classical View of Algebraic Geometry*, EMS Text-books in Mathematics, 2009.
- I. Shafarevich: *Basic Algebraic Geometry I : Varieties in Projective Space*, 2nd edition, Springer, Berlin, 1994.
- K. Hulek: *Elementary Algebraic Geometry*, AMS, Providence, 2003.
- W. Fulton: *Algebraic Curves*, Addison-Wesley, Redwood City, 1989.
- J. Harris: *Algebraic Geometry : A First Course*, Springer, New York, 1995.

**Cilji in kompetence:**

Študent se spozna z osnovnimi pojmi in izreki algebraične geometrije.

**Objectives and competences:**

Student masters basic concepts and tools of algebraic geometry.

**Predvideni študijski rezultati:**

**Znanje in razumevanje:** Poznavanje pojmov in izrekov algebraične geometrije in njihovo prepoznavanje v drugih vejah matematike.

**Uporaba:** V področjih matematike, ki delajo z geometričnimi objekti, v teoretični fiziki, in drugje.

**Refleksija:** Razumevanje teorije na podlagi primerov in uporabe.

**Prenosljive spretnosti – niso vezane le na en predmet:** Formulacija problemov v primernem jeziku, reševanje in analiza dobljenih rezultatov na primerih, prepoznavanje algebraičnih struktur v geometriji.

**Intended learning outcomes:**

**Knowledge and understanding:** Understanding of basic concepts and theorems of algebraic geometry, and their role in some other areas.

**Application:** In the areas of mathematics that deal with geometric objects, in theoretical physics, and elsewhere.

**Reflection:** Understanding the theory on the basis of examples and applications.

**Transferable skills:** Formulation and solution of problems in an appropriate setup, solution and analysis of the results in examples, recognizing algebraic structure in geometric objects.

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**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations

**Načini ocenjevanja:**

Delež (v %) /

Weight (in %)

**Assessment:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt): <ul style="list-style-type: none"><li>▪ izpit iz vaj (2 kolokvija ali pisni izpit)</li><li>▪ ustni izpit</li></ul> Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)	50%  50%	Type (examination, oral, coursework, project): <ul style="list-style-type: none"><li>▪ 2 midterm exams instead of written exam, written exam</li><li>▪ oral exam</li></ul> Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)
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**Reference nosilca / Lecturer's references:****prof. dr. Tomaž Košir**

- L. Grunenfelder, T. Košir: *Geometric Aspects of Multiparameter Spectral Theory*, Transactions Amer. Math. Soc. 350 (1998), 2525-2546.
- T. Košir, B. A. Sethuraman: *Determinantal Varieties Over Truncated Polynomial Rings*, Journal of Pure and Applied Algebra 195 (2005), 75-95.
- A. Buckley, T. Košir. *Plane curves as Pfaffians*. Ann. Sc. Norm. Super. Pisa Cl. Sci. (5) 10 (2011), no. 2, 363–388.

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

**Predmet:** Iterativne numerične metode v linearni algebri  
**Course title:** Iterative numerical methods in linear algebra

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type**

**Univerzitetna koda predmeta / University course code:**

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:**

**Jeziki / Languages:**  
**Predavanja / Lectures:**   
**Vaje / Tutorial:**

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

**Prerequisites:**

**Vsebina:**

**Content (Syllabus outline):**

Kadar imamo opravka z velikimi razpršenimi matrikami, se moramo numeričnega reševanja linearnega sistema in problema lastnih vrednosti lotiti drugače kot z direktnimi metodami (na primer Gaussova eliminacija oziroma QR metoda), saj nam sicer zmanjka spomina ali pa računanje poteka prepočasi.

Iterativne metode za linearni sistem.

Jacobijeva, Gauss-Seidlova in SOR metoda.

Simetrična SOR metoda s pospešitvijo Čebiševa. Podprostor Krilova. Lanczosev in Arnoldijev algoritem, GMRES, MINRES in sorodne metode. Metoda konjugiranih gradientov. **Bi-konjugirani gradienti.**

Predpogojevanje.

Nelinearni sistemi. Newton-GMRES, Broydova metoda. **GMRES za najmanjše kvadrate.**

Iterativne metode za problem lastnih vrednosti. Rayleigh-Ritzeva metoda, Metode podprostorov Krilova, Jacobi-Davidsonova metoda. Posplošeni problem lastnih vrednosti, polinomski problem lastnih vrednosti.

In case of large sparse matrices we can not apply direct methods (e.g., Gaussian elimination or QR algorithm) to solve a linear system or compute the eigenvalues, as we run out of time or memory.

Iterative methods for linear systems. Jacobi, Gauss-Seidel and SOR method. Symmetric SOR with Chebyshev acceleration. Krylov subspace. Lanczos and Arnoldi algorithm, GMRES, MINRES and similar methods. Conjugate gradients. Bi-conjugate gradients. Preconditioning.

Nonlinear systems. Newton-GMRES, Broyden's method, GMRES for least squares.

Iterative methods for eigenvalue problems. Rayleigh-Ritz method, methods based on Krylov subspaces, Jacobi-Davidson method. Generalized eigenvalue problem, polynomial eigenvalue problem.

### Temeljni literatura in viri / Readings:

- J. W. Demmel: *Uporabna numerična linearna algebra*, DMFA-založništvo, Ljubljana, 2000.
- R. Barrett, M. W. Berry, T. F. Chan, J. Demmel, J. Donato, J. Dongarra, V. Eijkhout, R. Pozo, C. Romine, H. van der Vorst: *Templates for the Solution of Linear Systems : Building Blocks for Iterative Methods*, SIAM, Philadelphia, 1994.
- Z. Bai, J. Demmel, J. Dongarra, A. Ruhe, H. van der Vorst: *Templates for the Solution of Algebraic Eigenvalue Problems : A Practical Guide*, SIAM, Philadelphia, 2000.
- G. H. Golub, C. F. Van Loan: *Matrix Computations*, 3rd edition, Johns Hopkins Univ. Press, Baltimore, 1996.
- C. T. Kelley: *Iterative Methods for Linear and Nonlinear Equations*, SIAM, Philadelphia, 1995.
- H. van der Vorst: *Iterative Krylov methods for large linear systems*, Cambridge University Press, Cambridge, 2003.
- Y. Saad: *Iterative methods for sparse linear systems. Second edition*, SIAM, Philadelphia, 2011.

**Cilji in kompetence:**

**Objectives and competences:**



Slušatelj spozna iterativne numerične metode za reševanje linearnih sistemov in problemov lastnih vrednosti, ki se jih uporablja pri razpršenih matrikah. Dopolni vsebine, ki jih sreča pri Uvodu v numerične metode in Numerični linearni algebr. Pridobljeno znanje praktično utrdi z domačimi nalogami in reševanjem problemov s pomočjo računalnika.

Students learn iterative numerical methods for linear systems and eigenvalue problems where matrices are sparse. New knowledge complements the content of courses Numerical linear algebra and Introduction to numerical methods. The acquired knowledge is consolidated by homework assignments and solving problems using computer programs.

#### **Predvideni študijski rezultati:**

**Znanje in razumevanje:** Razumevanje osnovnih numeričnih algoritmov za razpršene matrike. Obvladanje numeričnega reševanja problemov z velikimi matrikami. Sposobnost izbire najprimernejšega algoritma glede na lastnosti matrike. Znanje programiranja in uporabe Matlaba oziroma drugih sorodnih orodij za reševanje tovrstnih problemov.

**Uporaba:** Ekonomično in natančno numerično reševanje linearnih sistemov oziroma lastnih problemov z razpršenimi matrikami.

**Refleksija:** Razumevanje teorije na podlagi uporabe.

**Prenosljive spretnosti – niso vezane le na en predmet:** Spretnost uporabe računalnika pri reševanju matematičnih problemov. Razumevanje razlik med eksaktnim in numeričnim računanjem. Predmet konstruktivno nadgrajuje zahtevnejša znanja linearne algebre.

#### **Intended learning outcomes:**

**Knowledge and understanding:** Understanding of basic numerical algorithms for sparse matrices. Being able to numerically solve problems with large sparse matrices. The ability to choose an appropriate algorithm based on matrix properties. Knowledge of computer programming package Matlab or other similar software for solving such problems.

**Application:** Economical and accurate numerical computation of linear systems or eigenvalue problems with sparse matrices.

**Reflection:** Understanding of the theory from the applications.

**Transferable skills:** The ability to solve mathematical problems using a computer. Understanding the differences between the exact and the numerical computation. The subject enriches constructively the knowledge of linear algebra.

#### **Metode poučevanja in učenja:**

#### **Learning and teaching methods:**

predavanja, vaje, domače naloge, konzultacije, projekti	Lectures, exercises, homeworks, consultations, projects
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Delež (v %) /

**Načini ocenjevanja:**

Weight (in %) **Assessment:**

<p>Način (domače naloge, pisni izpit, ustno izpraševanje, naloge, projekt):</p> <ul style="list-style-type: none"> <li>▪ domače naloge ali projekt</li> <li>▪ pisni izpit</li> <li>▪ ustni izpit</li> </ul> <p>Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)</p>	<p>20%</p> <p>40%</p> <p>40%</p>	<p>Type (homeworks, examination, oral, coursework, project):</p> <ul style="list-style-type: none"> <li>▪ homeworks or project</li> <li>▪ written exam</li> <li>▪ oral exam</li> </ul> <p>Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)</p>
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**Reference nosilca / Lecturer's references:**

**prof. dr. Gašper Jaklič:**

- JAKLIČ, Gašper, KOZAK, Jernej, VITRIH, Vito, ŽAGAR, Emil. Lagrange geometric interpolation by rational spatial cubic Bézier curves. *Comput. aided geom. des.*, 2012, vol. 29, iss. 3-4, str. 175-188
- JAKLIČ, Gašper, KANDUČ, Tadej, PRAPROTNIK, Selena, ŽAGAR, Emil. Energy minimizing mountain ascent. *J. optim. theory appl.*, 2012, vol. 155, is. 2, str. 680-693
- JAKLIČ, Gašper, MODIC, Jolanda. On properties of cell matrices. *Appl. math. comput.*, 2010, vol. 216, iss. 7, str. 2016-2023.

**prof. dr. Bor Plestenjak:**

- M. E. Hochstenbach, A. Muhič, B. Plestenjak: On linearizations of the quadratic two-parameter eigenvalue problems, *Linear Algebra Appl.* 436 (2012) 2725-2743.
- A. Muhič, B. Plestenjak: On the quadratic two-parameter eigenvalue problem and its linearization, *Linear Algebra Appl.* 432 (2010) 2529-2542.
- M. E. Hochstenbach, T. Košir, B. Plestenjak: *A Jacobi-Davidson type method for the two-parameter eigenvalue problem.* *SIAM j. matrix anal. appl.*, 2005, vol. 26, no. 2, str. 477-497.

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

**Predmet:** Numerična aproksimacija in interpolacija  
**Course title:** Numerical approximation and interpolation

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2400

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
45	0	30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Gašper Jaklič, doc. dr. Marjetka Krajnc, prof. dr. Emil Žagar

**Jeziki / Languages:**  
**Predavanja / Lectures:** slovenski/Slovene, angleški/English  
**Vaje / Tutorial:** slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija

**Prerequisites:**

Enrollment into the program

**Vsebina:**

**Content (Syllabus outline):**

**Aproksimacija funkcij:** Izbira prostorov aproksimativnih funkcij. Polinomi. Trigonometrijski polinomi. Odsekoma polinomske funkcije. **Stabilnost baz.** Weierstrassov izrek. Pozitivni operatorji. Optimalni aproksimativni problem. Eksistenca in enoličnost elementa najboljše aproksimacije. Enakomerna konveksnost, stroga normiranost.

**Enakomerna aproksimacija s polinomi:** Enoličnost za diskretni in zvezni primer. Alternacija residuala. Konstrukcija. Prvi in drugi Remesov postopek. Konvergenca. Polinomi Čebiševa. Posplošitve: Čebiševi sistemi funkcij, generalizirani polinomi.

**Metoda najmanjših kvadratov v zveznem in diskretnem primeru:** Ortogonalni polinomi. Tričlenska rekurzivna formula. Gram-Schmidtova ortogonalizacija in numerično stabilnejše izvedbe. Reortogonalizacija. Navezava diskretnega in zveznega primera. Enakomerna konvergenca  $L_2$ -aproksimacij.

**Interpolacija:** Interpolacija s polinomi. Lagrangeva oblika interpolacijskega polinoma in ostanek. **Baricentrična Lagrangeova interpolacija.** Deljene diference. Newtonova oblika interpolacijskega polinoma, posplošena Hornerjeva shema. Divergenca interpolacijskih polinomov.

**Odsekoma polinomske funkcije, zlepki:** Eulerjevi poligoni, interpolacija in aproksimacija v drugi normi. Kubični zlepki. B-zlepki kot baza prostora odsekoma polinomske funkcij. Bézierove krivulje. **Zlepki v dveh dimenzijah.**

**Approximation of functions:** Spaces of approximation functions. Polynomials. Trigonometric polynomials. Piecewise polynomial functions. **Stability of bases.** Weierstrass' Theorem. Positive operators. Optimal approximation. Existence and uniqueness of the best approximation. Uniform convexity and strong normed spaces.

**Uniform approximation by polynomials:** Uniqueness in the discrete and continuous case. Iteration of residuals. Construction. The first and the second Remes algorithm. Convergence. Chebyshev polynomials. Generalizations: Chebyshev systems, generalized polynomials.

**Continuous and discrete least squares:** Orthogonal polynomials. Three-term recurrence. Gram-Schmidt orthogonalization, basic and stable version. Reorthogonalization. Connection between discrete and continuous case. Uniform convergence of  $L_2$ -approximants.

**Interpolation:** Polynomial interpolation. Lagrange form. **Barycentric Lagrange interpolation.** Divided differences. Newton form and generalized Horner scheme. Divergence of interpolating polynomials.

**Piecewise polynomial functions, splines:** Euler polygons, interpolation and approximation in the second norm. Cubic splines. B-spline bases of piecewise polynomial functions. Bézier curves. **Splines in two dimensions.**

#### Temeljna literatura in viri / Readings:

- J. Kozak: *Numerična analiza*, DMFA-založništvo, Ljubljana, 2008.
- R. L. Burden, J. D. Faires: *Numerical Analysis*, 8th edition, Brooks/Cole, Pacific Grove, 2005.
- E. K. Blum: *Numerical Analysis and Computation : Theory and Practice*, Addison-Wesley, Reading, 1998.
- Z. Bohte: *Numerične metode*, DMFA-založništvo, Ljubljana, 1991.
- S. D. Conte, C. de Boor: *Elementary Numerical Analysis : An Algorithmic Approach*, 3rd edition,

McGraw-Hill, Auckland, 1986.

- C. de Boor: *A Practical Guide to Splines*, Springer, New York, 2001.
- E. Isaacson, H. B. Keller: *Analysis of Numerical Methods*, John Wiley & Sons, New York-London-Sydney, 1994.
- D. R. Kincaid, E. W. Cheney: *Numerical Analysis : Mathematics of Scientific Computing*, 3rd edition, Brooks/Cole, Pacific Grove, 2002.

#### Cilji in kompetence:

Slušatelj dopolni poznavanje analitičnih metod aproksimacije in interpolacije z numeričnimi. Ob domačih nalogah pridobljeno znanje praktično utrdi.

#### Objectives and competences:

Student supplements knowledge of analytical methods in approximation and interpolation by numerical aspects. By solving homeworks the obtained theoretical knowledge is consolidated.

#### Predvideni študijski rezultati:

**Znanje in razumevanje:** Razumevanje pojmov interpolacije in aproksimacije. Praktično obvladanje numeričnih postopkov za konstrukcijo interpolacijskih oziroma aproksimacijskih funkcij.

**Uporaba:** Numerična konstrukcija interpolacijskih ali aproksimacijskih funkcij s pomočjo računalnika in ocenjevanje napak na podlagi teorije. Interpolacija in aproksimacija se uporabljata **na mnogih področjih, še posebej** pri računalniško podprtem grafičnem modeliranju.

**Refleksija:** Razumevanje teorije na podlagi uporabe.

**Prenosljive spretnosti – niso vezane le na en predmet:** Spretnost uporabe računalnika pri reševanju matematičnih problemov. Razumevanje razlik med eksaktnim in numeričnim računanjem.

#### Intended learning outcomes:

**Knowledge and understanding:** Understanding of interpolation and approximation. Ability of numerical algorithms for construction of interpolating or approximating functions.

**Application:** Numerical construction of interpolating and approximating functions using a computer and error estimation based on theory. Interpolation and approximation are used in several fields, in particular in computer aided graphical modelling.

**Reflection:** Understanding of theory based through applications.

**Transferable skills:** Skill of using computer for solving numerical problems. Understanding differences between exact and numerical computing.

**Metode poučevanja in učenja:**

Predavanja, vaje, domače naloge, konzultacije.

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations.

Delež (v %) /

**Načini ocenjevanja:**

Weight (in %)

**Assessment:**

<p>Način (domače naloge, pisni izpit, ustno izpraševanje, naloge, projekt):</p> <ul style="list-style-type: none"> <li>▪ domače naloge ali project</li> <li>▪ pisni izpit</li> <li>▪ ustni izpit</li> </ul> <p>Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)</p>	<p>20%</p> <p>40%</p> <p>40%</p>	<p>Type (homeworks, examination, oral, coursework, project):</p> <ul style="list-style-type: none"> <li>▪ homeworks or project</li> <li>▪ written exam</li> <li>▪ oral exam</li> </ul> <p>Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)</p>
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**Reference nosilca / Lecturer's references:****prof. dr. Gašper Jaklič:**

- G. Jaklič in E. Žagar: *Curvature variation minimizing cubic hermite interpolants*. Appl. Math. Comput., 2011, vol. 218, št. 7, str. 3918-3924.
- G. Jaklič in E. Žagar: *Planar cubic  $G^1$  interpolatory splines with small strain energy*. J. Comput. Appl. Math., 2011, vol. 235, str. 2758--2765.
- G. Jaklič: *On the dimension of the bivariate spline space  $S_3^1$* . Int. J. Comput. Math., 2005, vol. 82, št. 11, 1355--1369.

**doc. dr. Marjetka Krajnc:**

- G. Jaklič, J. Kozak, M. Krajnc, V. Vitrih, E. Žagar: *High-order parametric polynomial approximation of conic sections*. Constr. Approx., 2013, vol. 38, št. 1, str. 1--18.
- M. Krajnc: *Interpolation scheme for planar cubic  $G^2$  spline curves*. Acta Appl. Math., 2011, vol. 113, str. 129--143.
- M. Krajnc: *Hermite geometric interpolation by cubic  $G^1$  splines*. Nonlinear Anal.-Theory, 2009, vol. 70, str. 2614-2626.

**prof. dr. Emil Žagar:**

- G. Jaklič, J. Kozak, V. Vitrih in E. Žagar: *Lagrange geometric interpolation by rational spatial cubic Bézier curves*. Comput. Aided Geom. Design, 2012, vol. 29, št. 3-4, str. 175-188.
- J. Kozak in E. Žagar: *On geometric interpolation by polynomial curves*. SIAM J. Numer. Anal., 2004, vol. 42, št. 3, str. 953-967.
- E. Žagar: *On  $G^2$  continuous spline interpolation of curves in  $\mathbb{R}^d$* . BIT, 2002, vol. 42, št. 3, str. 670-688.

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

**Predmet:** Numerična integracija in navadne diferencialne enačbe  
**Course title:** Numerical integration and ordinary differential equations

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type**

**Univerzitetna koda predmeta / University course code:**

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
45	0	30			105	6

**Nosilec predmeta / Lecturer:**

**Jeziki / Languages:**  
**Predavanja / Lectures:**   
**Vaje / Tutorial:**

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

**Prerequisites:**

**Vsebina:**

**Content (Syllabus outline):**

**Numerično odvajanje:** Stabilno računanje odvodov. Diferenčne aproksimacije za odvode.

**Numerična integracija:** Stopnja in konvergenca. Newton–Cotesove formule. Gaussove kvadraturene formule. Sestavljena pravila. Ocene napak. Konvergenca. Euler-McLaurinovo sumacijsko pravilo in Rombergova ekstrapolacija. Singularni integrali. Večkratni integrali. Metode tipa Monte Carlo.

**Reševanje navadnih diferencialnih enačb:** Začetni problem. Enačbe prvega reda. Enačbe višjih redov. Sistemi diferencialnih enačb. Lokalna in globalna napaka. Eksplicitne in implicitne metode.

**Enočlenske metode:** Eulerjeva metoda. Uporaba Taylorjeve vrste. Metode tipa Runge-Kutta. Eksplicitna RK metoda četrtega reda, trapezno pravilo, Mersonova metoda, Runge-Kutta Fehlbergova metoda. Stabilnost, konsistentnost in konvergenca enočlenskih metod. A-stabilnost.

**Veččlenske metode:** Metode, ki temeljijo na numerični integraciji. Adamsove metode. Splošne linearne veččlenske metode. Rodovna polinoma in lokalna napaka. Prediktor–korektor metode. Milnova metoda. Ničelna stabilnost. Ocena reda ničelno stabilne veččlenske metode. Metode, ki temeljijo na diferencialnih aproksimacijah odvoda. Implicitne BDF metode. Ničelna stabilnost, konsistentnost in konvergenca veččlenskih metod.

**Robni problemi:** Linearne enačbe. Prevedba na začetne probleme in strelska metoda. Diferenčna metoda.

**Numerical differentiation:** Stable computing of derivatives. Differential approximations for derivatives.

**Numerical integration:** Degree of a rule and convergence. Newton-Cotes integration rules. Gauss quadratures. Composite rules. Error estimates. Convergence. Euler-Maclaurin formula and Romberg extrapolation. Singular integrals. Multiple integrals. Monte Carlo methods.

**Ordinary differential equations:** Initial value problems. First order ODE equations. Higher order ODE equations. Systems of ODE equations. Local and global error. Explicit and implicit methods.

**One-step methods:** Euler method. Methods based on Taylor's series. Runge-Kutta methods. Explicit RK method of order four, trapezoidal rule, Merson method, Runge-Kutta Fehlberg method. Stability, consistency and convergence of one-step methods. A-stability.

**Multistep methods:** Methods based on numerical integration. Adams methods. Linear multistep methods. Characteristic polynomials and a local error. Predictor-Corrector methods. Milne's method. Zero stability. Order estimation of a zero stable method. Methods based on derivative approximations. Implicit BDF methods. Stability, consistency and convergence of multistep methods.

**Boundary value problems:** Linear equations. Initial value and shooting methods. Finite difference methods.

**Temeljni literatura in viri / Readings:**



• J. Kozak: *Numerična analiza*, DMFA-založništvo, Ljubljana, 2008.

- R. L. Burden, J. D. Faires: *Numerical Analysis*, 8th edition, Brooks/Cole, Pacific Grove, 2005.
- E. K. Blum: *Numerical Analysis and Computation : Theory and Practice*, Addison-Wesley, Reading, 1998.
- Z. Bohte: *Numerične metode*, DMFA-založništvo, Ljubljana, 1991.
- S. D. Conte, C. de Boor: *Elementary Numerical Analysis : An Algorithmic Approach*, 3rd edition, McGraw-Hill, Auckland, 1986.
- E. Isaacson, H. B. Keller: *Analysis of Numerical Methods*, John Wiley & Sons, New York-London-Sydney, 1994.
- D. R. Kincaid, E. W. Cheney: *Numerical Analysis : Mathematics of Scientific Computing*, 3rd edition, Brooks/Cole, Pacific Grove, 2002.

### Cilji in kompetence:

Slušatelj dopolni poznavanje metod za numerično odvajanje, integracijo in numerično reševanje navadnih diferencialnih enačb. Ob domačih nalogah pridobljeno znanje praktično utrdi.

### Objectives and competences:

Student supplements knowledge of numerical differentiation, integration and numerical solving of ODE equations. By solving homeworks the obtained theoretical knowledge is consolidated.

### Predvideni študijski rezultati:

#### Znanje in razumevanje:

Razumevanje delovanja metod za numerično integriranje in reševanje navadnih diferencialnih enačb. Sposobnost numeričnega reševanja navadnih diferencialnih enačb in robnih problemov s pomočjo računalnika. Sposobnost izbire najprimernejšega algoritma glede na lastnosti problema.

**Uporaba:** Numerično računanje integralov in numerično reševanje navadnih diferencialnih enačb s pomočjo računalnika in ocenjevanje napak na podlagi teorije. V praksi (fizika, mehanika, kemija, ekonomija, ...) se pogosto pojavljajo integrali in diferencialne enačbe, ki jih ni možno rešiti drugače kot numerično.

**Refleksija:** Razumevanje teorije na podlagi uporabe.

### Intended learning outcomes:

**Knowledge and understanding:** Understanding methods for numerical integration and ordinary differential equations. Ability of numerical solving of initial and boundary value problems with the help of computers. Capability of choosing the most appropriate algorithm according to some features of the problem.

**Application:** Numerical computing of integrals and numerical solving of ODE equations using a computer and error estimation based on theory. Problems that can not be solved any other way that numerically occurs very often in practise (physics, mechanics, chemistry, economy, ...).

**Reflection:** Understanding of theory through applications.

**Transferable skills:** Skill of using computer for solving numerical problems. Understanding differences between exact and numerical

**Prenosljive spretnosti – niso vezane le na en predmet:** Spretnost uporabe računalnika pri reševanju matematičnih problemov. Razumevanje razlik med eksaktnim in numeričnim računanjem. Predmet konstruktivno nadgrajuje zahtevnejša znanja analize in drugih področij matematike.

computing. Knowledge of analysis and other fields of mathematics is constructively upgraded.

**Metode poučevanja in učenja:**

Predavanja, vaje, domače naloge, konzultacije.

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations.

Delež (v %) /

**Načini ocenjevanja:**

Weight (in %)

**Assessment:**

Način (domače naloge, pisni izpit, ustno izpraševanje, naloge, projekt):

- domače naloge ali projekt
- pisni izpit
- ustni izpit

20%  
40%  
40%

Type (homeworks, examination, oral, coursework, project):

- homeworks or project
- written exam
- oral exam

Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

**prof. dr. Gašper Jaklič:**

- G. Jaklič in E. Žagar: *Curvature variation minimizing cubic hermite interpolants*. Appl. Math. Comput., 2011, vol. 218, št. 7, str. 3918-3924.
- G. Jaklič in E. Žagar: *Planar cubic  $G^1$  interpolatory splines with small strain energy*. J. Comput. Appl. Math., 2011, vol. 235, str. 2758--2765.
- G. Jaklič: *On the dimension of the bivariate spline space  $S_3^1$* . Int. J. Comput. Math., 2005, vol. 82, št. 11, 1355--1369.

**doc. dr. Marjetka Krajnc:**

- G. Jaklič, J. Kozak, M. Krajnc, V. Vitrih, E. Žagar: *High-order parametric polynomial*

*approximation of conic sections*. Constr. Approx., 2013, vol. 38, št. 1, str. 1--18.

- M. Krajnc: *Interpolation scheme for planar cubic  $G^2$  spline curves*. Acta Appl. Math., 2011, vol. 113, str. 129–143.
- M. Krajnc: *Hermite geometric interpolation by cubic  $G^1$  splines*. Nonlinear Anal.-Theory, 2009, vol. 70, str. 2614-2626.

**prof. dr. Emil Žagar:**

- G. Jaklič, J. Kozak, V. Vitrih in E. Žagar: *Lagrange geometric interpolation by rational spatial cubic Bézier curves*. Comput. Aided Geom. Design, 2012, vol. 29, št. 3-4, str. 175-188.
- J. Kozak in E. Žagar: *On geometric interpolation by polynomial curves*. SIAM J. Numer. Anal., 2004, vol. 42, št. 3, str. 953-967.
- E. Žagar: *On  $G^2$  continuous spline interpolation of curves in  $\mathbb{R}^d$* . BIT, 2002, vol. 42, št. 3, str. 670-688.

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

**Predmet:** Numerične metode za linearne sisteme upravljanja  
**Course title:** Numerical methods for linear control systems

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type**

**Univerzitetna koda predmeta / University course code:**

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:**

**Jeziki / Languages:**  
**Predavanja / Lectures:**   
**Vaje / Tutorial:**

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

**Prerequisites:**

**Vsebina:**

**Content (Syllabus outline):**

Linearni sistemi upravljanja. Zvezni in diskretni sistemi. Vzhodno-izhodne diferencialne enačbe, prostor stanj. Stabilnost, vodljivost, spoznavnost. Regulatorji, odprtozankni in zaprtozankni sistemi.

Odziv sistema. Rešitev zveznega sistema. Računanje eksponentne funkcije matrike preko razvoja v Taylorjevo vrsto, Padéjeve aproksimacije in različnih faktorizacij.

Numerično testiranje vodljivosti in spoznavnosti. Oddaljenost od najbližjega nevodljivega sistema. Oddaljenost od najbližjega nestabilnega sistema.

Numerično reševanje in stabilnost Ljapunove in Sylvestrove matrične enačbe. Uporaba Jordanove forme, Bartels–Stewartov algoritem, Hessenberg–Schurova metoda, splošene Schurove metode.

Numerično reševanje in stabilnost Riccatijeve matrične enačbe. Uporaba lastnega razcepa, Schurova metoda, Newtonova metoda, splošene Schurove metode.

Uravnoteženje sistema. Redukcija modela. Stabilizacija s povratno informacijo in razporejanje lastnih vrednosti. Stabilizabilen sistem. Razporejanje polov.

Linear control systems. Continuous-time and discrete-time systems. Input-output differential equations, state space. Stability, controllability, observability. Regulators, open-loop and closed-loop systems.

System response. Solution of a continuous-time system. Numerical computation of matrix exponential using Taylor series, Padé approximation, and matrix factorizations.

Numerical tests for controllability and observability. Distance to the nearest uncontrollable system. Distance to the nearest unstable system.

Numerical methods for and stability of Lyapunov and Sylvester matrix equations. Application of Jordan canonical form, Bartels-Stewart algorithm, Hessenberg-Schur method, generalized Schur methods.

Numerical methods for and stability of Riccati matrix equations. Application of eigendecomposition, Schur method, Newton method, generalized Schur methods.

Internal balancing. Model reduction. State-feedback stabilization and eigenvalue assignment problem. Stabilizable system. Pole assignment.

### Temeljni literatura in viri / Readings:

- K. J. Åström, R. M. Murray: Feedback Systems: An Introduction for Scientists and Engineers, Princeton University Press, Princeton, 2008.
- B. N. Datta: Numerical Methods for Linear Control Systems, Academic Press, San Diego, 2004.
- P. Hr. Petkov, N. D. Christov, M. M. Konstantinov: Computational Methods for Linear Control Systems, Prentice Hall, New York, 1991.

### Cilji in kompetence:

Slušatelj spozna osnove linearnih sistemov upravljanja, poudarek pa je na numeričnih metodah, ki jih potrebujemo za reševanje raznih matričnih problemov, ki se tu pojavijo. Pridobljeno znanje praktično utrdi z domačimi nalogami in reševanjem problemov s pomočjo računalnika.

### Objectives and competences:

Student learns basics of linear control systems with emphasis on numerical methods for various related matrix problems. The acquired knowledge is consolidated by homework assignments and solving problems using computer programs.

**Predvideni študijski rezultati:**

**Znanje in razumevanje:** Razumevanje osnov linearnih sistemov upravljanja. Poznavanje osnovnih numeričnih pristopov za reševanje problemov s tega področja. Znanje programiranja in uporabe Matlaba oziroma drugih sorodnih orodij za reševanje tovrstnih problemov.

**Uporaba:** Numerično reševanje problemov iz linearnih sistemov upravljanja.

**Refleksija:** Razumevanje teorije na podlagi uporabe.

**Prenosljive spretnosti – niso vezane le na en predmet:** Spretnost uporabe računalnika pri reševanju matematičnih problemov.

**Intended learning outcomes:**

**Knowledge and understanding:** Understanding of basics of control linear systems. The knowledge of basic numerical methods for related problems. Knowledge of computer programming package Matlab or other similar software for solving such problems.

**Application:** Numerical computation of problems from linear control theory.

**Reflection:** Understanding of the theory from the applications.

**Transferable skills:** The ability to solve mathematical problems using a computer.

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije, projekti

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations, projects

Delež (v %) /

**Načini ocenjevanja:**

Weight (in %)

**Assessment:**

Način (domače naloge, pisni izpit, ustno izpraševanje, naloge, projekt):

- domače naloge ali projekt
- pisni izpit
- ustni izpit

20%  
40%  
40%

Type (homeworks, examination, oral, coursework, project):

- homeworks or project
- written exam
- oral exam

Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

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**Reference nosilca / Lecturer's references:**

**doc. dr. Bor Plestenjak**

- M. E.Hochstenbach, T. Košir, B. Plestenjak: *A Jacobi-Davidson type method for the two-parameter eigenvalue problem*. SIAM j. matrix anal. appl., 2005, vol. 26, no. 2, str. 477-497.
- M. E.Hochstenbach, B. Plestenjak: *Backward error, condition numbers, and pseudospectra for the multiparameter eigenvalue problem*. Linear Algebra Appl., 2003, vol. 375, str. 63-81.
- B. Plestenjak: *A continuation method for a weakly elliptic two-parameter eigenvalue problem*. IMA j. numer. anal., 2001, vol. 21, no. 1, str. 199-216

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

<b>Predmet:</b>	Numerično reševanje parcialnih diferencialnih enačb
<b>Course title:</b>	Numerical solving of partial differential equations

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2404

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Gašper Jaklič, doc. dr. Marjetka Krajnc, prof. dr. Emil Žagar

**Jeziki / Languages:**

<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija

**Prerequisites:**

Enrollment into the program

**Vsebina:**

**Content (Syllabus outline):**



**Parcialne diferencialne enačbe:** Uvod v PDE in modelni problemi drugega reda.

**Enačbe eliptičnega tipa:** Poissonova enačba. Diferenčna metoda. Diskretni maksimalni princip in ocena globalne napake. Iterativno reševanje diskretiziranih enačb. Jacobijeva, Gauss-Seidelova in SOR metoda. Simetrična SOR metoda s pospešitvijo Čebiševa. ADI metoda. Metode podprostorov Krilova. Večmrežne metode. Variacijske metode. Metoda končnih elementov.

**Enačbe parabolnega tipa:** Prevajanje toplote. Eksplicitne in implicitne numerične sheme. Crank-Nicolsonova metoda. Konsistenca, stabilnost in konvergenca.

**Enačbe hiperboličnega tipa:** Valovna enačba. Karakteristike, karakteristične spremenljivke. Diferenčna metoda. Courantov pogoj. Konvergenca diferenčnih aproksimacij za modelni primer. Metoda karakteristik.

**Partial differential equations:** Introduction to PDE and examples of partial differential equations of the second order.

**Elliptic equations:** Poisson's equation. Finite difference method. Discrete maximum principle and global error estimation. Iterative methods for discretized equations. Jacobi, Gauss-Seidel and SOR iterative methods. Symmetric SOR and Chebyshev acceleration. ADI method. Krilov subspace methods. Multigrid methods. Variational methods. Finite element methods.

**Parabolic equations:** Heat transfer equation. Explicit and implicit numerical schemes. Crank-Nicolson's method. Consistency, stability and convergence.

**Hyperbolic equations:** Wave equation. Characteristics. Characteristic variables. Finite difference method. Courant's condition. Convergence of finite difference approximations for a model equation. Method of characteristics.

### Temeljni literatura in viri / Readings:

- J. Kozak: *Numerična analiza*, DMFA-založništvo, Ljubljana, 2008.
- W. F. Ames: *Numerical Methods for Partial Differential Equations*, 3rd edition, Academic Press, Boston, 1992.
- Z. Bohte: *Numerične metode*, DMFA-založništvo, Ljubljana, 1991.
- S. D. Conte, C. de Boor: *Elementary Numerical Analysis : An Algorithmic Approach*, 3rd edition, McGraw-Hill, Auckland, 1986.
- J. W. Demmel: *Uporabna numerična linearna algebra*, DMFA-založništvo, Ljubljana, 2000.
- E. Isaacson, H. B. Keller: *Analysis of Numerical Methods*, John Wiley & Sons, New York-London-Sydney, 1966.
- D. R. Kincaid, E. W. Cheney: *Numerical Analysis : Mathematics of Scientific Computing*, 3rd edition, Brooks/Cole, Pacific Grove, 2002.
- K. W. Morton, D. F. Mayers: *Numerical Solution of Partial Differential Equations*, 2nd edition, Cambridge Univ. Press, Cambridge, 2005.
- G. D. Smith: *Numerical Solution of Partial Differential Equations : Finite Differences Methods*, 3rd edition, Clarendon Press, Oxford (New York), 2004.

### **Cilji in kompetence:**

Slušatelj spozna metode za numerično reševanje parcialnih enačb. Pridobljeno znanje praktično utrdi z reševanjem domačih nalog.

### **Objectives and competences:**

Student supplements knowledge of numerical differentiation, integration and numerical solving of ODE equations. By solving homeworks the obtained theoretical knowledge is consolidated.

### **Predvideni študijski rezultati:**

#### **Znanje in razumevanje:**

Razumevanje delovanja metod za numerično reševanje parcialnih diferencialnih enačb. Sposobnost numeričnega reševanja parcialnih diferencialnih enačb s pomočjo računalnika. Sposobnost izbire najprimernejšega algoritma glede na lastnosti problema.

**Uporaba:** Numerično reševanje parcialnih diferencialnih enačb s pomočjo računalnika in ocenjevanje napak na podlagi teorije. V praksi (fizika, mehanika, kemija, ekonomija, ...) se pogosto pojavljajo parcialne diferencialne enačbe, ki jih ni mogoče rešiti drugače kot numerično.

**Refleksija:** Razumevanje teorije na podlagi uporabe.

**Prenosljive spretnosti – niso vezane le na en predmet:** Spretnost uporabe računalnika pri reševanju matematičnih problemov. Razumevanje razlik med eksaktnim in numeričnim računanjem. Predmet konstruktivno nadgrajuje zahtevnejša znanja analize in drugih področij matematike.

### **Intended learning outcomes:**

**Knowledge and understanding:** Understanding of numerical methods for solving partial differential equations. Ability of solving partial differential equations with the computer. Capability of choosing the most appropriate algorithm according to some features of the problem.

**Application:** Numerical solution of partial differential equations using a computer and error estimation based on theory. Problems that can not be solved any other way that numerically occurs very often in practise (physics, mechanics, chemistry, economy, ...).

**Reflection:** Understanding of theory through applications.

**Transferable skills:** Skill of using computer for solving numerical problems. Understanding differences between exact and numerical computing. Knowledge of analysis and other fields of mathematics is constructively upgraded.

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**Metode poučevanja in učenja:**

Predavanja, vaje, domače naloge, konzultacije, projekt.

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations, project

<b>Načini ocenjevanja:</b>	Delež (v %) / Weight (in %)	<b>Assessment:</b>
Način (domače naloge, pisni izpit, ustno izpraševanje, naloge, projekt): <ul style="list-style-type: none"> <li>▪ domače naloge ali projekt</li> <li>▪ pisni izpit</li> <li>▪ ustni izpit</li> </ul> Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)	20% 40% 40%	Type (homeworks, examination, oral, coursework, project): <ul style="list-style-type: none"> <li>▪ homeworks or project</li> <li>▪ written exam</li> <li>▪ oral exam</li> </ul> Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

**prof. dr. Gašper Jaklič:**

- G. Jaklič in E. Žagar: *Curvature variation minimizing cubic hermite interpolants*. Appl. Math. Comput., 2011, vol. 218, št. 7, str. 3918-3924.
- G. Jaklič in E. Žagar: *Planar cubic  $G^1$  interpolatory splines with small strain energy*. J. Comput. Appl. Math., 2011, vol. 235, str. 2758--2765.
- G. Jaklič: *On the dimension of the bivariate spline space  $S_3^1$* . Int. J. Comput. Math., 2005, vol. 82, št. 11, 1355--1369.

**doc. dr. Marjetka Krajnc:**

- G. Jaklič, J. Kozak, M. Krajnc, V. Vitrih, E. Žagar: *High-order parametric polynomial approximation of conic sections*. Constr. Approx., 2013, vol. 38, št. 1, str. 1--18.
- M. Krajnc: *Interpolation scheme for planar cubic  $G^2$  spline curves*. Acta Appl. Math., 2011, vol. 113, str. 129--143.
- M. Krajnc: *Hermite geometric interpolation by cubic  $G^1$  splines*. Nonlinear Anal.-Theory, 2009, vol. 70, str. 2614-2626.

**prof. dr. Emil Žagar:**

- G. Jaklič, J. Kozak, V. Vitrih in E. Žagar: *Lagrange geometric interpolation by rational*

*spatial cubic Bézier curves*. *Comput. Aided Geom. Design*, 2012, vol. 29, št. 3-4, str. 175-188.

- J. Kozak in E. Žagar: *On geometric interpolation by polynomial curves*. *SIAM J. Numer. Anal.*, 2004, vol. 42, št. 3, str. 953-967.
- E. Žagar: *On  $G^2$  continuous spline interpolation of curves in  $\mathbb{R}^d$* . *BIT*, 2002, vol. 42, št. 3, str. 670-688.

UČNI NAČRT PREDMETA / COURSE SYLLABUS						
<b>Predmet:</b>	Računalniško podprto (geometrijsko) oblikovanje					
<b>Course title:</b>	Computer aided (geometric) design					
<b>Študijski program in stopnja</b> <b>Study programme and level</b>	<b>Študijska smer</b> <b>Study field</b>		<b>Letnik</b> <b>Academic year</b>	<b>Semester</b> <b>Semester</b>		
Magistrski študijski program 2. stopnje Matematika	ni smeri		prvi ali drugi	prvi ali drugi		
Second cycle master study program Mathematics	none		first or second	first or second		
<b>Vrsta predmeta / Course type</b>						
			izbirni predmet/elective course			
<b>Univerzitetna koda predmeta / University course code:</b>			M2402			
<b>Predavanja</b> <b>Lectures</b>	<b>Seminar</b> <b>Seminar</b>	<b>Vaje</b> <b>Tutorial</b>	<b>Klinične vaje</b> <b>work</b>	<b>Druge oblike študija</b>	<b>Samost. delo</b> <b>Individ. work</b>	<b>ECTS</b>
30	15	30			105	6
<b>Nosilec predmeta / Lecturer:</b>			prof. dr. Gašper Jaklič, prof. dr. Emil Žagar			
<b>Jeziki / Languages:</b>		<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English			
		<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English			
<b>Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:</b>			<b>Prerequisites:</b>			
Vpis v letnik študija			Enrollment into the program			
<b>Vsebina:</b>			<b>Content (Syllabus outline):</b>			

<p><b>Uvod:</b> de Casteljauov algoritem, Bernsteinova oblika Bezierove krivulje, Bezierove krivulje (splošno), zlepki v Bezierovi obliki, racionalne Bezierove krivulje</p> <p><b>Geometrijska zveznost:</b> geometrijska zveznost krivulj in ploskev, geometrijsko zvezni zlepki</p> <p><b>Bezierove ploskve:</b> tenzorski produkti, trikotne krpe, racionalne Bezierove ploskve</p> <p><b>Stožnice:</b> racionalne kvadratne Bezierove krivulje, eksaktna reprezentacija stožnic</p> <p><b>Krivulje B-zlepkov:</b> lastnosti, algoritmi za delo z B-zlepki</p>	<p><b>Introduction:</b> de Casteljau algorithm, Bernstein form of Bezier curve, Bezier curves (general), Bezier splines, rational Bezier curves</p> <p><b>Geometric continuity:</b> geometric continuity of curves and surfaces, geometrically continuous splines</p> <p><b>Bezier surfaces:</b> tensor products, triangular patches, rational Bezier surfaces</p> <p><b>Conics:</b> rational quadratic Bezier curves, exact representation of conics</p> <p><b>B-spline curves:</b> properties, algorithms for manipulating B-spline curves</p>
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#### Temeljni literatura in viri / Readings:

- G. Farin: *Curves and Surfaces for Computer Aided Geometric Design : A Practical Guide*, 4th edition, Academic Press, San Diego, 1997.
- C. de Boor: *A Practical Guide to Splines*, Springer, New York, 2001.
- R. H. Bartels, J. C. Beatty, B. A. Barsky: *An Introduction to Splines for Use in Computer Graphics and Geometric Modeling*: Morgan Kaufmann, Palo Alto, 1996.
- M.-J. Lai, L. L. Schumaker, *Spline functions on triangulations*, Cambridge University Press, 2007

#### Cilji in kompetence:

Študent spozna osnove računalniškega oblikovanja. Uporaba Bezierovih krivulj in ploskev, racionalnih Bezierovih krivulj in geometrijsko zveznih zlepkov.

#### Objectives and competences:

An introduction to computer aided geometric design, use of Bezier curves and surfaces, rational Bezier curves and geometrically smooth splines.

#### Predvideni študijski rezultati:

##### Znanje in razumevanje:

Razumevanje osnovnih pojmov krivulj in ploskev. Osnovno znanje programiranja v Matlabu ali Mathematici. Sposobnost implementacije postopkov na računalniku.

##### Uporaba:

Uporaba postopkov interpolacije in aproksimacije s polinomi in zlepki pri računalniškem oblikovanju.

#### Intended learning outcomes:

##### Knowledge and understanding:

Knowledge of basic facts on curves and surfaces. Basic programming skill in Matlab or Mathematica. Skill to implement algorithms in programming language.

##### Application:

Application of interpolation and approximation with polynomials and splines in CAGD.

**Refleksija:**  
Razumevanje teorije na podlagi uporabe.  
**Prenosljive spretnosti – niso vezane le na en predmet:** Spretnost uporabe teorije v praksi. Sposobnost povezovanja znanj iz numerične matematike, analize in računalništva. Kritično presojanje razlik med teorijo in prakso.

**Reflection:**  
Understanding theory based on application.  
**Transferable skills:**  
Skill of using theory in practical use. Skill of interconnecting knowledge from numerical mathematics, analysis and computer science. Critical judgement of differences between theory and practical applications.

**Metode poučevanja in učenja:**  
predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**  
Lectures, exercises, homeworks, consultations

Delež (v %) /

**Načini ocenjevanja:**

Weight (in %)

**Assessment:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):		Type (examination, oral, coursework, project):
<ul style="list-style-type: none"> <li>▪ projekt</li> <li>▪ ustni izpit</li> </ul>	50%	<ul style="list-style-type: none"> <li>▪ project</li> <li>▪ oral exam</li> </ul>
Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)	50%	Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

- G. Jaklič, J. Kozak, M. Krajnc, V. Vitrih, E. Žagar, High-order parametric polynomial approximation of conic sections, *Constructive Approximation*, Volume 38, Issue 1 (2013), 1–18.
- G. Jaklič, J. Kozak, M. Krajnc, V. Vitrih, E. Žagar, Hermite geometric interpolation by rational Bezier spatial curves, *SIAM Journal on Numerical Analysis*, Vol. 50, No. 5, 2012, pp. 2695–2715.
- G. Jaklič, E. Žagar, Planar cubic G1 interpolatory splines with small strain energy, *Journal of Computational and Applied Mathematics*, 235 (2011), 2758–2765.

## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Aktuarska matematika
<b>Course title:</b>	Actuarial mathematics

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2503

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Mihael Perman, doc. dr. Janez Bernik

<b>Jeziki / Languages:</b>	<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
	<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija

**Prerequisites:**

Enrollment into the program

**Vsebina:**

Modeliranje v zavarovalništvu:

- porazdelitve izgub,
- izračun agregatnih izplačil,
- modeliranje pogostnosti zahtevkov,
- rekurzivne metode za izračun agregatnih škod,
- teorija kredibilnosti,
- verjetnost bankrota,
- modeli za odvisna tveganja,

**Content (Syllabus outline):**

Mathematical models for insurance:

- loss distribution,
- methods to compute aggregate payments,
- modeling of the claim frequencies,
- recursive methods for aggregate loss computation,
- credibility theory,
- probability of default,
- dependent risks modeling,



- modeli za ekstremne dogodke,
- stabilnost.

- extreme events modeling,
- stability.

### Temeljni literatura in viri / Readings:

- H. H. Panjer, G. E. Willmot: *Insurance Risk Models*, Schaumburg, Society of Actuaries, 1992.
- R. Kaas, M. Goovaerts, J. Dhaene, M Denuit: *Modern Actuarial Risk Theory*, Boston, Kluwer, 2001.
- M. Denuit, J. Dhaene, M. Goovaerts, R. Kaas: *Dependent Risks, Measures, Orders and Models*, Wiley, 2005.
- S. A. Klugman, H. H. Panjer, G. E. Willmot: *Loss Models : From Data to Decisions*, Wiley, 1998.
- H. Bühlmann: *Mathematical Methods in Risk Theory*, Springer, 2005.
- P. Embrechts, C. Klüppelberg, T. Mikosch: *Modelling Extremal Events for Insurance and Finance*, Springer, 1997.

### Cilji in kompetence:

Bolj kompleksni zavarovalni produkti zahtevajo bolj poglobljene matematične modele in bolj rafinirane mere tveganja. Tečaj bo prikazal ustaljene načine matematičnega razmišljanja v zavarovalništvu.

Zaradi nepostredne uporabnosti vsebin bodo pri predmetu sodelovali tudi strokovnjaki iz prakse.

### Objectives and competences:

The complexity of the insurance products requires more and more sophisticated mathematical models and more refined measures of risk. The course will cover current mathematical modelling for insurance.

Since the content is of great practical importance we expect that also specialists from financial practice will present their work experience during the course.

### Predvideni študijski rezultati:

**Znanje in razumevanje:** Razumevanje pojma tveganja in merjenja tveganja je bistvenega pomena za vrednotenje in razvoj zavarovalnih produktov. Za oceno tveganja pa je potrebno razumevanje osnovnih stohastičnih modelov, ki jih uporabljajo aktuarji pri svojem delu.

**Uporaba:** Pridobljeno znanje je neposredno uporabno v zavarovalnem sektorju.

**Refleksija:** Medigra med uporabo, statističnim modeliranjem, povratno informacijo iz drugih ved in spodbude iz uporabe za matematično razmišljanje.

**Prenosljive spretnosti – niso vezane le na en**

### Intended learning outcomes:

**Knowledge and understanding:** Understanding of risks and its measuring is a central issue in pricing and development of modern insurance products. Knowledge of the basic stochastic models for insurance is needed to assess the risks involved.

**Application:** The knowledge is directly applicable in insurance sector of the economy.

**Reflection:** Interplay between applications, statistical modelling and feedback information from other fields. Mathematical thinking based on concrete applications.

**Transferable skills:** Skills are transferable to

**predmet:** Spretnosti so prenosljive na druga področja matematičnega modeliranja, še najbolj pa je predmet pomemben zaradi svoje neposredne uporabnosti.

many other fields of mathematical modelling. The value of the course is in concrete applications to insurance.

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations

**Načini ocenjevanja:**

Delež (v %) /

Weight (in %)

**Assessment:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- izpit iz vaj (2 kolokvija ali pisni izpit)
- ustni izpit

Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)

50%

50%

Type (examination, oral, coursework, project):

- 2 midterm exams instead of written exam, written exam
- oral exam

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

**prof. dr. Mihael Perman**

- M. Perman, J. Wellner: *On the distribution of Brownian areas*, Ann. Appl. Probab., 6, no. 4., (1996), 1091-1111.
- M. Huzak, M. Perman, H. Šikić, Z. Vondraček: *Ruin probabilities and decompositions for general perturbed risk processes*, Ann. Appl. Probab., 2004, vol. 14, no. 3, (2004), 1378-1397.
- M. Huzak, M. Perman, H. Šikić, Z. Vondraček: *Ruin probabilities for competing claim processes*, J. Appl. Probab., 41, no. 3, (2004) 679-690.

**doc. dr. Janez Bernik**

- J. Bernik, M. Mastnak, H. Radjavi: *Realizing irreducible semigroups and real algebras of compact operators*, J. Math. Anal. Appl. 348 (2008), 692--707.
- J. Bernik, M. Mastnak, H. Radjavi: *Positivity and matrix semigroups*, Linear Algebra Appl. 434 (2011), 801-812.
- J. Bernik, L.W. Marcoux, H. Radjavi: *Spectral conditions and band reducibility of operators*, J. London Math. Soc. 86 (2012), 214-234.

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

**Predmet:** Bayesova statistika  
**Course title:** Bayesian statistics

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	None	first or second	first or second

**Vrsta predmeta / Course type**

izbirni/elective course

**Univerzitetna koda predmeta / University course code:**

še ni dodeljena/not assigned yet

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:**

red. prof. dr. Matjaž Omladič, doc. dr. Dejan Velušček

**Jeziki /  
Languages:**

**Predavanja /  
Lectures:** slovenski, angleški

**Vaje / Tutorial:** slovenski, angleški

**Pogoji za vključitev v delo oz. za opravljanje  
študijskih obveznosti:**

**Prerequisites:**

Vpis v letnik študija

Enrollment into the program

**Vsebina:**

**Content (Syllabus outline):**

Bayesovi modeli z enim in več parametri. Povezava s standardnimi statističnimi metodami. Hierarhični modeli. Preverjanje modelov in analiza občutljivosti. Bayesovo načrtovanje poskusov.

Bayesov pristop k združevanju rezultatov več raziskav, potenčne apriorne porazdelitve, analiza odvisnosti združene analize od preteklih raziskav.

Uvod v regresijsko analizo. Analiza variance in kovariance, informativne hipoteze in njihovo ovrednotenje. Bayesov faktor, kompleksnost in prilaganje. Aposteriorne verjetnosti hipotez - modelov, vpliv apriorne porazdelitve, učni vzorec.

Povzemanje aposteriorne porazdelitve, ocene parametrov, centralni kredibilnostni interval, pomen konjugiranih porazdelitev. Gibbsov vzorčevalnik, konvergenca ocen, Metropolis Hastingsov algoritem. Aposteriorne simulacije. Drugi specifični modeli Bayesove analize.

Bayesian models with one and more parameters. Connection with standard statistical methods. Hierarchical models. Testing of models and sensitivity analysis. Bayesian design of experiment.

Bayesian approach to evidence synthesis of multiple surveys, power priors, analysis of dependence of synthesis analysis on previous surveys.

Introduction into regression analysis. Analysis of variance and covariance. Hypothesis testing via Bayes factor, complexity and fit. Posterior probabilities of hypotheses – models, and influence of priors on them, training sample.

More on posterior probabilities, estimating parameters, central credibility interval, the importance of conjugated distributions. Gibbs sampler, convergence of estimates, algorithm Metropolis-Hastings. Posterior simulations. Some other specific models of Bayesian analysis.

#### **Temeljni literatura in viri / Readings:**

- A. Gelman, J.B. Carlin, H.S. Stern, D.B. Rubin: Bayesian Data Analysis. Chapman&Hall, 1995.
- H. Hoijtink: Bayesian Data Analysis. In: R.E. Millsap and A. Maydeu-Olivares, The SAGE Handbook of Quantitative Methods in Psychology. London: SAGE, 2009.
- I. Ntzoufras: Bayesian Modeling Using WinBUGS. New York: Wiley, 2009.

#### **Cilji in kompetence:**

Študent spozna temeljne Bayesove metode za obdelavo podatkov.

Spozna se tudi z uporabo teh metod v praksi. Zato je predvideno, da bodo pri predmetu sodelovali tudi strokovnjaki iz prakse.

#### **Objectives and competences:**

Basic knowledge of Bayesian statistics is acquired.

Bayesian methods are of great importance in practice. Therefore, experts with practical knowledge will present their experience in class.

#### **Predvideni študijski rezultati:**

#### **Intended learning outcomes:**

<b>Znanje in razumevanje:</b> Razumevanje osnovnih konceptov Bayesove statistike.	<b>Knowledge and understanding:</b> Understanding of basic concepts of Bayesian statistics.

<b>Metode poučevanja in učenja:</b> predavanja, vaje, seminarske naloge, praktične naloge z uporabo statističnih paketov, konzultacije	<b>Learning and teaching methods:</b> Lectures, exercises, seminar type homework, homework that require the use of statistical packages, consultations
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Delež (v %) /	
<b>Načini ocenjevanja:</b>	Weight (in %) <b>Assessment:</b>
Način (pisni izpit, ustno izpraševanje, naloge, projekt): <ul style="list-style-type: none"> <li>▪ Izpit iz vaj</li> <li>▪ izpit iz teorije</li> <li>▪ ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)</li> </ul>	Type (examination, oral, coursework, project): <ul style="list-style-type: none"> <li>▪ 2 midterm exams instead of written exam, written exam</li> <li>▪ oral exam</li> </ul> Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

<p><b>red. prof. dr. Matjaž Omladič:</b></p> <ul style="list-style-type: none"> <li>• M. Omladič: Na prvem koraku do faktorske analize. Obz. mat. fiz., 1986, let. 33, št. 1-2, str. 9-23.</li> <li>• R. E. Hartwig, M. Omladič, P. Šemrl, G. P. H. Styan: On some characterizations of pairwise star orthogonality using rank and dagger additivity and subtractivity. Linear Algebra Appl., 1996, let. 237/238, št. 2, str. 499-507.</li> <li>• M. Omladič, V. Omladič: More on restricted canonical correlations. Linear Algebra Appl.. [Print ed.], 2000, vol. 1/3, no. 321, str. 285-293.</li> </ul> <p><b>doc. dr. Dejan Velušček</b></p> <ul style="list-style-type: none"> <li>• P. Dörsek, J. Teichmann, D. Velušček: Cubature methods for stochastic (partial) differential equations in weighted spaces, accepted for publication in »Stochastic Partial Differential Equations: Analysis and Computations«.</li> <li>• K. Oshima, J. Teichmann, D. Velušček: A new extrapolation method for weak approximation</li> </ul>
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schemes with applications, Ann. Appl. Probab. 22, no. 3 (2012), 1008-1045.

I. Klep, D. Velušček: Central extensions of  $*$ -ordered skew fields , Manuscripta math. 120, no. 4 (2006), 391-402.

## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Časovne vrste
<b>Course title:</b>	Time Series

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	1 or 2	1 or 2

**Vrsta predmeta / Course type** izbirni/elective

**Univerzitetna koda predmeta / University course code:** M2511

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Mihael Perman, prof. dr. Matjaž Omladič

<b>Jeziki / Languages:</b>	<b>Predavanja / Lectures:</b>	slovenski, angleški/slovene, english
	<b>Vaje / Tutorial:</b>	slovenski, angleški/slovene, english

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija

**Prerequisites:**

Enrollment into the program

**Vsebina:**

Uvod: primeri časovnih vrst, trend in sezonska odstopanja, avtokorelacijska funkcija. Krepka in šibka stacionarnost. Hilbertovi prostori in napovedovanje, časovne vrste v R.  
Stacionarni procesi: linearni procesi, ARMA modeli, vzročnost in obrnljivost ARMA procesov. MA procesi neskončnih redov. lastnosti, avtokorelacijska funkcija, napovedovanje stacionarnih procesov.  
ARMA modeli: avtokorelacijska in parcialna

**Content (Syllabus outline):**

Introduction: Examples of time series. Trend and seasonality. Autocorrelation function. Multivariate normal distribution. Strong and weak stationarity. Hilbert spaces and prediction. Introduction to R.  
Stationary sequences: Linear processes. ARMA models. Causality and invertibility of ARMA processes. Infinite order MA processes.  
Partial autocorrelation function. Estimation of autocorrelation function and other parameters.

avtokorelacijska funkcija, ocenjevanje parametrov, diagnostične metode, napovedovanje.

Spektralna analiza: spektralna gostota, Herglotzov izrek, periodogram..

Nestacionarne in nelinearne časovne vrste: ARCH in GARCH modeli, Momenti in stacionarne porzdelitve za GARCH procese. Ekspontentni ARIMA modeli, SARIMA modeli, napovedovanje pri nestacionarnih časovnih vrstah.

Statistika stacionarnih procesov: Asimptotični rezultati, ocenjevanje trendov in sezonskih vplivov. Neparametrične metode.

Večrazsežne časovne vrste: stacionarnost, večrazsežni ARMA in ARIMA modeli, ocenjevanje parametrov, napovedovanje, razcep variance.

Forecasting stationary time series. Modeling and forecasting for ARMA processes. Asymptotic behavior of the sample mean and the autocorrelation function. Parameter estimation for ARMA processes.

Spectral analysis: Spectral density. Spectral density of ARMA processes. Herglotz theorem. Periodogram.

Nonlinear and nonstationary time series models: ARCH and GARCH models. Moments and stationary distributiopn of GARCH process. Exponential GARCH. ARIMA models. SARIMA models. orecasting nonstationary time series.

Statistics for stationary process: Asymptotic results for stationary time series. Estimating trend and seasonality. Nonparametric methods.

Multidimensional time series: stacionarity, multidimensional ARMA and ARIMA models, parameter estimation, forecasting, variance decomposition.

#### Temeljni literatura in viri / Readings:

- P. J. Brockwell, R. A. Davis: *Introduction to Time Series and Forecasting*, 2nd edition, Springer, 2002.
- C. Chatfield: *The Analysis of Time Series: An Introduction*, 6th Edition, Chapman & Hall/CRC, 2003.
- P.J. Brockwell, R.A. Davis: *Time Series: Theory and Methods*, Springer, 1991.
- W.N. Venables, B.D. Ripley: *Modern Applied Statistics with S-Plus*, Springer, 1994.
- W.N. Shumway, D. Stoffer: *Time Series Analysis and Its Applications*, Springer, 2006.

#### Cilji in kompetence:

Časovne vrste so eno od temeljnih področij uporabne statistike z možnimi uporabami tako v tehniki kot tudi v ekonomiji. Osnovni koncepti časovnih vrst so del statistične izobrazbe, poleg tega pa poglobijo in na novo osvetlijo že znane pojme iz statistike.

Zaradi nepostredne uporabnosti vsebin bodo pri predmetu sodelovali tudi strokovnjaki iz prakse.

#### Objectives and competences:

Time series course is one of fundamental courses of applied statistics with several applications to engineering and economics. Basic concepts of the time series analysis are part of necessary background of any statistical education. They deepen and shed new light on basic notions of statistics.

Since the content is of great practical importance we expect that also specialists from financial practice will present their work experience during the course.

#### Predvideni študijski rezultati:

#### Intended learning outcomes:



**Znanje in razumevanje:**

Predmet predstavi pomembno področje statistike, ki je vedno bolj pomembno v modeliranju finančnih in ekonomskih podatkov.

**Uporaba**

Makroekonomski analitiki ter ponudniki električne energije ali goriv uporabljajo časovne vrste za svoje napovedi. Poleg tega področje osvetljuje že prej znane pojme iz statistike.

**Refleksija**

Medigra med uporabo, statističnim modeliranjem, povratno informacijo ekonomije in tehnike in spodbude iz uporabe za matematično razmišljanje.

**Prenosljive spretnosti – niso vezane le na en predmet**

Spretnosti so neposredno uporabne v finančnem in zavarovalnem sektorju, predstavljajo pa tudi pomembno orodje za ekonomiste.

**Knowledge and understanding:**

Understanding of statistical applications to economics, modelling of economics and financial data.

**Application:**

In macroeconomic analysis or on energy markets, time series methods are the fundamental forecasting tool. This analysis deepens and sheds new light on basic notions of statistics.

**Reflection:**

The interplay between application, statistical modelling, economics feedback information, and application stimulation for mathematical reasoning.

**Transferable skills:**

The skills are directly applicable in finance and insurance. They are also an important tool for the economists.

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije, seminarske naloge

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations, seminars

**Načini ocenjevanja:**

Delež (v %) /

Weight (in %)

**Assessment:**

<p>Način (pisni izpit, ustno izpraševanje, naloge, projekt):</p> <ul style="list-style-type: none"> <li>▪ izpit iz vaj (pisni izpit)</li> <li>▪ ustni izpit</li> </ul> <p>Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)</p>	<p>50%</p> <p>50%</p>	<p>Type (examination, oral, coursework, project):</p> <ul style="list-style-type: none"> <li>▪ written exam</li> <li>▪ oral exam</li> </ul> <p>Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)</p>
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**Reference nosilca / Lecturer's references:****prof. dr. Mihael Perman**

- M. Blejec, M. Lovrečič-Saražin, M. Perman, M. Štraus: *Statistika*. Piran: Gea College, Visoka šola za podjetništvo, 2003. X, 150 str., graf. prikazi, tabele.

- M. Perman: *Order statistics for jumps of subordinators*, Stoc. Proc. Appl., 46, 267-281 (1993).
- M. Huzak, M. Perman, H. Šikić, Z. Vondraček: *Ruin probabilities and decompositions for general perturbed risk processes*, Ann. Appl. Probab., 2004, vol. 14, no. 3, (2004), 1378-1397.

**prof. dr. Matjaž Omladič**

- M. Omladič: *Na prvem koraku do faktorske analize*. Obz. mat. fiz., 1986, let. 33, št. 1-2, str. 9-23.
- R. E. Hartwig, M. Omladič, P. Šemrl, G. P. H. Styan: *On some characterizations of pairwise star orthogonality using rank and dagger additivity and subtractivity*. Linear Algebra Appl., 1996, let. 237/238, št. 2, str. 499-507.
- M. Omladič, V. Omladič: *More on restricted canonical correlations*. Linear Algebra Appl.. [Print ed.], 2000, vol. 1/3, no. 321, str. 285-293.

## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Ekonometrija
<b>Course title:</b>	Econometrics

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	None	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2509

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
<b>30</b>	<b>15</b>	<b>30</b>			<b>105</b>	<b>6</b>

**Nosilec predmeta / Lecturer:** prof. dr. Mihael Perman, doc. dr. Dejan Velušček

<b>Jeziki / Languages:</b>	<b>Predavanja / Lectures:</b>	slovenski, angleški/slovene, english
	<b>Vaje / Tutorial:</b>	slovenski, angleški/slovene, english

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

vpis v letnik

**Prerequisites:**

enrollment to the program

**Vsebina:**

Uvod: definicija in mesto ekonometrije v ekonomski znanosti, osnovna metodologija ekonometričnih raziskav.  
Linearna regresija: metoda najmanjših kvadratov, izrek Gauss-Markova, testiranje splošne linearne domneve, diagnostične metode, pomembne opazovane vrednosti, testi za ostanke, testi za linearnost, Cookov test.

**Content (Syllabus outline):**

Introduction: the definition and the place of econometrics in the economics, basic methodology of the econometric research.  
Linear regression: the method of least squares, The Gauss-Markov Theorem, testing of the general linear assumption, diagnostic methods, important empirical values, residue tests, linearity tests, the Cook Test.  
Generalized linear model: heteroskedasticity,

Posplošitve linearnega modela: heteroskedastičnost, avtokorelacija napak, stohastične neodvisne spremenljivke, nelinearni regresijski modeli, modeli z nepravilnimi spremenljivkami. **Kointegracija.**

**Logit in probit modeli za dihotope in politome podatke.**

Panelni podatki: ozadje modelov in definicije, ocenjevanje parametrov, preizkušanje domnev, večstopenjski panelni podatki, **modeli diskretne izbire.**

Simultani sistemi več regresijskih enačb: zapisi simultane sistema regresijskih enačb, identifikacija enačb sistema, izbrane metode ocenjevanja simultane sistema enačb.

**Vektorska avtoregresija, preverjanje pravilnosti modela. Kointegrirana vektorska avtokorelacija.**

autokorelacija napak, stohastično neodvisne spremenljivke, nelinearni regresijski modeli, modeli z instrumentalnimi spremenljivkami.

**Cointegracija, logit in probit modeli za dihotope in politome podatke.**

Panelni podatki: modeliranje in definicije, ocenjevanje parametrov, preizkušanje domnev, večstopenjski panelni podatki, **diskretne izbire.**

Simultani sistemi regresijskih enačb: različne oblike sistemov, identifikacijska enačba sistema, različne metode ocenjevanja simultane sistema enačb.

**Vector autoregression, model verification. Cointegrated vector autocorrelation.**

#### Temeljni literatura in viri / Readings:

- W. H. Greene: *Econometric analysis*, 3rd edition, Prentice Hall, 1997.
- **M. Verbeek: A Guide to Modern Econometrics, Wiley, 2004.**
- J. Woolridge: *Introductory Econometrics: A modern Approach*, 2nd Edition, South-Western College Pub, 2002.
- N. Gujarati: *Basic Econometrics*. 4th ed. Boston: McGraw Hill, 2003. Part 1 (str. 15-333) in Part 2 (str. 335-560).
- R. Ramanathan: *Introductory Econometrics with Applications*. 5th ed.
- J. Johnston: *Econometric Methods*, 3rd Edition, McGraw-Hill, New York, 1984.
- R. S. Pindyck in D. S. Rubinfeld: *Econometric Models and Economic Forecast*, 4th Edition,, McGraw-Hill, New York 1998.
- S. Weisberg: *Applied Linear Regression*, Wiley & Sons, 1985.
- B. H. Baltagi: *Econometrics*, Springer, 1998.

#### Cilji in kompetence:

Uporaba statistike v ekonomskih vedah nujno vodi do ekonometrije. S tem nastane nov in globlji pogled na statistiko samo na eni strani, po drugi strani pa predmet da občutek za soigro ekonomskega in statističnega razmišljanja. Predmet je tudi nujen korak do uporabe statistike za ekonomsko analizo.

**Zaradi neposredne uporabnosti vsebin bodo**

#### Objectives and competences:

Statistical applications in economics naturally lead to econometrics. This gives new, deeper perspective to the statistics itself on one side, and to the interplay between statistics and economics on the other side. The course is a necessary prerequisite for anybody who will use statistics for the analysis of the processes in the economics.

pri predmetu sodelovali tudi strokovnjaki iz prakse.

Since the content is of great practical importance we expect that also specialists from financial practice will present their work experience during the course.

#### **Predvideni študijski rezultati:**

##### **Znanje in razumevanje**

Predmet omogoča neposreden vpogled v uporabo statistike v ekonomiji, nakaže načine razmišljanja in osvetli medigro med ekonomskim in statističnim razmišljanjem.

##### **Uporaba**

Statistika je jezik bolj kvantitativno usmerjene ekonomije. Ta predmet bo omogočal neposredno uporabo statistike po eni strani, po drugi pa se bodo diplomanti lahko brez težav vpisali tudi na doktorski študij ekonomije.

##### **Refleksija**

Medigra med uporabo, statističnim modeliranjem, povratno informacijo ekonomije in spodbude iz uporabe za matematično razmišljanje.

##### **Prenosljive spretnosti – niso vezane le na en predmet**

Spretnosti so prenosljive na druga področja matematičnega modeliranja, še najbolj pa je predmet pomemben zaradi svoje neposredne uporabnosti in brušenja zmožnosti matematičnega modeliranja.

#### **Intended learning outcomes:**

##### **Knowledge and understanding:**

Understanding of statistical applications to economics, interplay between statistical reasoning and economics.

##### **Application:**

Statistics is the language of the quantitative economics. On one side, application is immediate, on the other side the knowledge will satisfy to persue doctoral studies in economics.

##### **Reflection:**

The interplay between application, statistical modelling, economics feedback information, and application stimulation for mathematical reasoning.

##### **Transferable skills:**

The skills obtained are transferable to other areas of mathematical modelling, but the gist of the course is its immediate applicability.

#### **Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije

#### **Learning and teaching methods:**

Lectures, exercises, homeworks, consultations

Delež (v %) /

**Načini ocenjevanja:**

Weight (in %) **Assessment:**

<p>Način (pisni izpit, ustno izpraševanje, naloge, projekt):</p> <ul style="list-style-type: none"> <li>▪ domače naloge,</li> <li>▪ izpit iz vaj</li> <li>▪ ustni izpit.</li> </ul> <p>ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)</p>	<p>50%</p> <p>50%</p>	<p>Type (examination, oral, coursework, project):</p> <ul style="list-style-type: none"> <li>▪ homework</li> <li>▪ written exam</li> <li>▪ oral exam</li> </ul> <p>Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)</p>
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**Reference nosilca / Lecturer's references:**

**prof. dr. Mihael Perman**

- M. Blejec, M. Lovrečič-Saražin, M. Perman, M. Štraus: *Statistika*. Piran: Gea College, Visoka šola za podjetništvo, 2003. X, 150 str., graf. prikazi, tabele.
- M. Perman: *Order statistics for jumps of subordinators*, Stoc. Proc. Appl., 46, 267--281 (1993).
- M. Huzak, M. Perman, H. Šikić, Z. Vondraček: *Ruin probabilities and decompositions for general perturbed risk processes*, Ann. Appl. Probab., 2004, vol. 14, no. 3, (2004).

**doc. dr. Dejan Velušček**

- P. Dörsek, J. Teichmann, D. Velušček: Cubature methods for stochastic (partial) differential equations in weighted spaces, accepted for publication in »Stochastic Partial Differential Equations: Analysis and Computations«.
- K. Oshima, J. Teichmann, D. Velušček: A new extrapolation method for weak approximation schemes with applications, Ann. Appl. Probab. 22, no. 3 (2012), 1008-1045.
- I. Klep, D. Velušček: Central extensions of  $\ast$ -ordered skew fields, Manuscripta math. 120, no. 4 (2006), 391-402.

## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Finančna matematika 2
<b>Course title:</b>	Financial mathematics 2

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	1 ali 2	1
Second cycle master study program Mathematics	none	1 or 2	1

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2508

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
45		30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Mihael Perman, doc. dr. Janez Bernik, doc. dr. Dejan Velušček

**Jeziki / Languages:**

<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

<b>Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:</b>	<b>Prerequisites:</b>
Vpis v letnik študija	Enrollment into the program

**Vsebina:** **Content (Syllabus outline):**

**Stohastična integracija:**

Pregled sredstev iz analize, teorije mere in verjetnosti, Brownovo gibanje, martingali v zveznem času, stohastični integral, Itôva formula, stohastične diferencialne enačbe.

**Vrednotenje izvedenih inštrumentov:**

Black-Scholesov model, izvedeni inštrumenti, arbitraža in varovanje v splošnem, kompletost modelov, zamenjava mere in izrek Girsanova, paritetne enakosti.

**Modeli obrestnih mer:**

Obveznice in obresti, nekaj klasičnih martingalskih modelov, vrednotenje opcij na obrestne mere.

Po potrebi predavatelj v tečaj vključi tudi druge aktualne teme iz novejše znanstvene periodike.

**Stochastic integration:**

Recapitulation of prerequisites from analysis, measure theory and probability, Brownian motion, continuous time martingales, stochastic integral, Itô formula, stochastic differential equations.

**Pricing of financial derivatives:**

Black-Merton-Scholes model, derivatives, arbitrage and hedging in general, model completeness, change of measure and Girsanov theorem, parity equations.

**Interest rate models:**

Bonds and interest, some classical martingale models, pricing of interest rate options. The lecturer can also include other current topics from recent scientific periodicals in the course.

**Temeljni literatura in viri / Readings:**

- T. Björk: Arbitrage Theory in Continuous Time, 2nd edition, Oxford Univ. Press, Oxford, 2004.
- S. E. Shreve: Stochastic Calculus for Finance II: Continuous-Time Models, Springer, New York, 2004.
- D. Lamberton, B. Lapeyre: Introduction to Stochastic Calculus Applied to Finance, Chapman & Hall/CRC, Boca Raton, 2000.
- J. C. Hull: Options, Futures, and Other Derivative Securities, 6th edition, Pearson/Prentice Hall, Upper Saddle River NJ, 2006.
- B. Øksendal: Stochastic Differential Equations: An Introduction with Applications, 6th edition, Springer, Berlin, 2006.

**Cilji in kompetence:**

Modernejši modeli trga slonijo na stohastičnem računu. Predmet bi najprej predstavil stohastično integracijo do mere, ki je nujno potrebna za razumevanje modelov v finančni matematiki v zveznem času. Stohastične diferencialne enačbe potem omogočajo po eni strani sredstvo za modeliranje trgov, obrestnih mer in portfeljev, po drugi strani pa omogočajo njihovo učinkovito obravnavo, ki vodi do

**Objectives and competences:**

Modern market models are based on stochastic calculus. The course starts with a short introduction of stochastic integration which is needed for understanding the continuous time models in financial mathematics. Stochastic differential equations present on one hand the means for modeling the financial markets, interest rates and portfolios and on the other hand the tool for their efficient study, which leads to optimal



problemov optimalnega ustavljanja in stohastične kontrole.

stopping problems and to stochastic control theory.

**Predvideni študijski rezultati:**

**Znanje in razumevanje:**

Razumevanje matematičnih modelov, ki se uporabljajo v matematičnih financah in sredstev za njihovo obravnavo.

**Uporaba:**

Pridobljeno znanje je po eni strani neposredno prenosljivo, po drugi strani pa je izhodišče za kombiniranje matematičnega znanja z ekonomskimi vsebinami.

**Refleksija:**

Področje, in s tem posledično predmet, združuje številna znanja iz matematike od linearne algebre, do parcialnih diferencialnih enačb.

**Prenosljive spretnosti – niso vezane le na en predmet:**

Pridobljeno znanje je neposredno uporabno v finančnih ustanovah kot so banke in zavarovalnice. Vsebina predmeta tudi pomaga izostriti sposobnost matematičnega modeliranja.

**Intended learning outcomes:**

**Knowledge and understanding:**

Understanding of mathematical models, which are used in mathematical finance, and the means for their treatment.

**Application:**

The acquired knowledge is both: directly transferable and it also serves as a base for combining mathematical knowledge with economical content.

**Reflection:**

The area itself, and hence also the course, combines various mathematical disciplines: from linear algebra to partial differential equations.

**Transferable skills:**

The acquired knowledge is directly applicable in financial institutions, e.g. banks, insurance companies, ... The content of the course contributes to the sharpening of the ability of mathematical modeling.

**Metode poučevanja in učenja:**

predavanja, vaje, samostojna seminarska naloga

**Learning and teaching methods:**

Lectures, exercises, one's own seminar assignment

Delež (v %) /

Weight (in %) **Assessment:**

**Načini ocenjevanja:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- samostojna seminarska naloga
- izpit iz vaj (2 kolokvija ali pisni izpit)

50%

Type (examination, oral, coursework, project):

- one's own seminar assignment
- 2 midterm exams instead of written

<ul style="list-style-type: none"> <li>▪ ustni izpit</li> </ul> <p>Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)</p>	<p>50%</p>	<p>exam, written exam</p> <ul style="list-style-type: none"> <li>▪ oral exam</li> </ul> <p>Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)</p>
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**Reference nosilca / Lecturer's references:**

**prof. dr. Mihael Perman:**

- M. Perman, J. Wellner: On the distribution of Brownian areas, *Ann. Appl. Probab.*, 6, no. 4., (1996), 1091-1111.
- M. Huzak, M. Perman, H. Šikić, Z. Vondraček: Ruin probabilities and decompositions for general perturbed risk processes, *Ann. Appl. Probab.*, 2004, vol. 14, no. 3, (2004), 1378-1397.
- M. Huzak, M. Perman, H. Šikić, Z. Vondraček: Ruin probabilities for competing claim processes, *J. Appl. Probab.*, 41, no. 3, (2004) 679-690.

**doc. dr. Dejan Velušček**

- P. Dörsek, J. Teichmann, D. Velušček: Cubature methods for stochastic (partial) differential equations in weighted spaces, accepted for publication in »Stochastic Partial Differential Equations: Analysis and Computations«.
- K. Oshima, J. Teichmann, D. Velušček: A new extrapolation method for weak approximation schemes with applications, *Ann. Appl. Probab.* 22, no. 3 (2012), 1008-1045.
- I. Klep, D. Velušček: Central extensions of  $*$ -ordered skew fields, *Manuscripta math.* 120, no. 4 (2006), 391-402.

**doc. dr. Janez Bernik**

- J. Bernik, M. Mastnak, H. Radjavi: *Realizing irreducible semigroups and real algebras of compact operators*, *J. Math. Anal. Appl.* 348 (2008), 692--707.
- J. Bernik, M. Mastnak, H. Radjavi: *Positivity and matrix semigroups*, *Linear Algebra Appl.* 434 (2011), 801-812.
- J. Bernik, L.W. Marcoux, H. Radjavi: *Spectral conditions and band reducibility of operators*, *J. London Math. Soc.* 86 (2012), 214-234.

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

**Predmet:** Finančna matematika 3  
**Course title:** Financial Mathematics 3

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	None	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** še ni dodeljena/not assigned yet

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Matjaž Omladič, doc. dr. Dejan Velušček

**Jeziki / Languages:**  
**Predavanja / Lectures:** slovenski/Slovene, angleški/English  
**Vaje / Tutorial:** slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija

**Prerequisites:**

Enrollment into the program

**Vsebina:**

Osnove: obrestne mere, krivulje donosov, struktura obveznic, LIBOR obrestne mere. Nekaj elementarnih modelov, kratkoročni modeli, pojem arbitraže v teh modelih, Vasičkov model, model Cox-Ingersoll-Ros, model Hull-White.

Modeli terminskih obrestnih mer v modelih z diskretnim in z zveznim časom. Klasični modeli, teorija Heatha. Jarrova in Mortona (HJM), modeli terminskih obrestnih mer, ki jih ženejo slučajna polja.

**Content (Syllabus outline):**

Basic notions: interest rates, yield curves, bond structures, LIBOR rates.

Some elementary models, short rate models, no-arbitrage in short rate models, Vasicek, Cox-Ingersoll-Ross, Hull-White models.

Forward interest rate models in discrete and continuous time settings. Classical cases, Heath-Jarrow-Morton (HJM) framework and forward rate models driven by random fields.

No arbitrage criteria and drift conditions, change of numeraire, martingale methods.

Kriterij neobstoja arbitraže in pogoji usmeritve, zamenjava numerarja, martingalske metode.

Posebne teme: LIBOR modeli, obveznice in možnost propada, problemi vrednotenja izvedenih instrumentov na obrestne mere.

Statistična vprašanja v modelih obrestnih mer: metode za kalibracijo modelov, ocenjevanje parametrov.

Some special topics: LIBOR models, defaultable bonds, pricing problems of certain interest rate derivatives.

Statistical questions in interest rate models, calibration methods, parameter estimation.

#### **Temeljni literatura in viri / Readings:**

- T. Bjork., Arbitrage Theory in Continuous Time, Oxford University Press, Oxford, New York, 1998.
- D. Brigo, F. Mercurio. Interest Rate Models - Theory and Practice: With Smile, Inflation and Credit, Springer, Berlin, Heidelberg, New York, 2006.
- R. A. Jarrow. Modeling Fixed Income Securities and Interest Rate Options, The McGraw-Hill Companies, Inc., New York, 1996.
- M. Musiela, M. Rutkowski. Martingale Methods in Financial Modeling, Springer-Verlag, Berlin, Heidelberg, 1997.
- A. Pelsser. Ecient Methods for Valuing Interest Rate Derivatives, Springer-Verlag, London, 2000.

#### **Cilji in kompetence:**

Predmet pokriva poglavja iz matematičnih financ, ki so pomembna za modeliranje krivulj obrestnih mer.

Zaradi nepostredne uporabnosti vsebin bodo pri predmetu sodelovali tudi strokovnjaki iz prakse.

#### **Objectives and competences:**

The course covers the chapter of mathematical finance that deal with modelling of the interest rate curves.

Since the content is of great practical importance we expect that also specialists from financial practice will present their work experience during the course.

#### **Predvideni študijski rezultati:**

#### **Intended learning outcomes:**

**Znanje in razumevanje:**

Razumevanje matematičnih modelov, ki se uporabljajo v matematičnih financah, in sredstev za njihovo obravnavo.

**Uporaba:**

Pridobljeno znanje je po eni strani neposredno prenosljivo, po drugi strani pa je izhodišče za kombiniranje matematičnega znanja s finančnimi vsebinami.

**Refleksija:**

Področje, in s tem posledično predmet, združuje številne znanja iz matematike, predvsem tistih povezanih s teorijo verjetnosti in matematično statistiko.

**Prenosljive spretnosti – niso vezane le na en predmet:**

Pridobljeno znanje je neposredno uporabno v finančnih ustanovah kot so banke in investicijske družbe.

**Knowledge and understanding:**

Understanding of mathematical models used in finance. Mathematical tools necessary in modelling.

**Application:**

The knowledge is directly usable in practice, it is also the source for combining of mathematical theories with finance.

**Reflection:**

The subject connects many mathematical topics, specially those of probability theory and statistics, with application.

**Transferable skills:**

The knowledge is directly applicable in everyday practice in financial institutions such as banks and investment companies.

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije, seminarske naloge

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations, seminars

**Načini ocenjevanja:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- samostojna seminarska naloga
- ustni izpit

Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)

Delež (v %) /

Weight (in %)

**Assessment:**

Type (examination, oral, coursework, project):

- seminar work
- oral exam

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

**prof. dr. Matjaž Omladič**

- M. Omladič, V. Omladič: *Hierarchical dynamics for power and control in society. J. Math. Sociol.*,

1993/94, let. 18, št. 14, str. 293-313.

- R. E. Hartwig, M. Omladič, P. Šemrl, G. P. H. Styan: *On some characterizations of pairwise star orthogonality using rank and dagger additivity and subtractivity*. *Linear Algebra Appl.*, 1996, let. 237/238, št. 2, str. 499-507.
- M. Omladič, V. Omladič: *More on restricted canonical correlations*. *Linear Algebra Appl.* [Print ed.], 2000, vol. 1/3, no. 321, str. 285-293.

**doc. dr. Dejan Velušček**

- P. Dörsek, J. Teichmann, D. Velušček: Cubature methods for stochastic (partial) differential equations in weighted spaces, accepted for publication in »Stochastic Partial Differential Equations: Analysis and Computations«.
- K. Oshima, J. Teichmann, D. Velušček: A new extrapolation method for weak approximation schemes with applications, *Ann. Appl. Probab.* 22, no. 3 (2012), 1008-1045.
- I. Klep, D. Velušček: Central extensions of  $*$ -ordered skew fields, *Manuscripta math.* 120, no. 4 (2006), 391-402.

## UČNI NAČRT PREDMETA / COURSE SYLLABUS

**Predmet:** Izbrana poglavja iz finančne matematike 1  
**Course title:** Topics in financial mathematics 1

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M3043

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Matjaž Omladič, prof. dr. Mihael Perman

**Jeziki / Languages:**  
**Predavanja / Lectures:** slovenski/Slovene, angleški/English  
**Vaje / Tutorial:** slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija

**Prerequisites:**

Enrollment into the program

**Vsebina:**

**Predavatelj izbira med naslednjimi pa tudi drugimi aktualnimi področji finančne matematike:**

**Modeli za kreditno tveganje:** osnovne definicije, osnovni modeli, vrednotenje izvedenih vrednostnih papirjev vezanih na kreditno tveganje.

**Upravljanje s tveganjem:** mere tveganja, koherenca, dinamične mere tveganja, modeli s

**Content (Syllabus outline):**

**Lecturer can choose among the following and some other current topics in financial mathematics:**

**Credit risk models:** basic definitions, basic models, pricing of credit derivatives.

**Risk management:** risk measures, coherence, dynamic risk measures, copula models, extreme

kopulami, teorija ekstremnih vrednosti, optimalne strategije, modeli za obvladovanje tveganja.

value theory, optimal strategies, risk management models.

### Temeljni literatura in viri / Readings:

- M. Ammann: Credit Risk Valuation : Methods, Models and Applications, 2nd edition, Springer, Berlin, 2001.
- J. Grandell: Aspects of Risk Theory, Springer, New York, 1992.
- I. Karatzas, S. E. Shreve: Methods of Mathematical Finance, Springer, New York, 2001.
- T. Björk: Arbitrage Theory in Continuous Time, 2nd edition, Oxford Univ. Press, Oxford, 2004.
- P. Wilmott: Derivatives : The Theory and Practice of Financial Engineering, Wiley, New York, 2000.
- A. J. McNeil, R. Frey, P. Embrechts, Paul: Quantitative risk management: Concepts, techniques and tools, Princeton Series in Finance, Princeton University Press, Princeton, NJ, 2005.
- P. Embrechts, C. Klüppelberg, T. Mikosch: Modelling extremal events for insurance and finance, Springer-Verlag, Berlin, 1997.

### Cilji in kompetence:

Predmet pokriva poglavja iz matematičnih financ, pri katerih se prepleta ekonomsko razmišljanje z zapletenimi matematičnimi orodji. Nekatera poglavja so nadgradnja prejšnjih z dodatnimi interpretacijami, nekatera pa so pomemben del razmišljanja o tveganju.

Zaradi nepostredne uporabnosti vsebin bodo pri predmetu sodelovali tudi strokovnjaki iz prakse.

### Objectives and competences:

The course covers topics in mathematical finance in which economic reasoning is combined with advanced mathematical tools. Some of them are based on previous courses and give additional interpretation, some contribute to understanding of the risks.

Since the content is of great practical importance we expect that also specialists from financial practice will present their work experience during the course.

### Predvideni študijski rezultati:

#### Znanje in razumevanje:

Razumevanje matematičnih modelov, ki se uporabljajo v matematičnih finančnih sredstvih za njihovo obravnavo.

#### Uporaba:

Pridobljeno znanje je po eni strani neposredno prenosljivo, po drugi strani pa je izhodišče za kombiniranje matematičnega znanja z ekonomskimi vsebinami.

### Intended learning outcomes:

#### Knowledge and understanding:

Understanding of mathematical models used in mathematical finance and the mathematical tools used in solutions.

#### Application:

The knowledge and skills acquired are directly transferable and can also serve for combining mathematical reasoning with economic topics.

#### Reflection:



**Refleksija:**

Področje, in s tem posledično predmet, združuje številne znanja iz matematike od linearna algebre do parcialnih diferencialnih enačb.

**Prenosljive spretnosti – niso vezane le na en predmet:**

Pridobljeno znanje je neposredno uporabno v finančnih ustanovah kot so banke in zavarovalnice. Vsebina predmeta tudi pomaga izostritvi sposobnosti matematičnega modeliranja.

The subject of the course, hence the course itself, combines numerous mathematical skills starting from linear algebra to partial differential equations.

**Transferable skills:**

The knowledge and skills acquired are immediately applicable in financial institutions such as banks and insurance companies. The content also serves to deepen the ability to use mathematical models.

**Metode poučevanja in učenja:**

predavanja, vaje, konzultacije, seminarske naloge

**Learning and teaching methods:**

Lectures, exercises, consultations, seminars

**Načini ocenjevanja:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- samostojna seminarska naloga
- opravljena seminarska naloga za pristop k teoretičnemu delu izpita

Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)

Delež (v %) /

Weight (in %) /

50%

50%

**Assessment:**

Type (examination, oral, coursework, project):

- individual seminar
- completed seminar work is required for the exam on the course content

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:****prof. dr. Mihael Perman:**

- M. Perman, J. Wellner: *On the distribution of Brownian areas*, Ann. Appl. Probab., 6, no. 4., (1996), 1091-1111.
- M. Huzak, M. Perman, H. Šikić, Z. Vondraček: *Ruin probabilities and decompositions for general perturbed risk processes*, Ann. Appl. Probab., 2004, vol. 14, no. 3, (2004), 1378-1397.
- M. Huzak, M. Perman, H. Šikić, Z. Vondraček: *Ruin probabilities for competing claim processes*, J. Appl. Probab., 41, no. 3, (2004) 679-690.

**prof. dr. Matjaž Omladič**

- M. Omladič, V. Omladič: *A linear algebra approach to non-transitive expected utility. Soc. choice welf.*, 2001, vol. 18, str. 251-267.
  - M. Omladič, V. Omladič: *Matematika in denar*, (Knjižnica Sigma, 58). Ljubljana: Društvo matematikov, fizikov in astronomov Slovenije, 1995. 142 str.
- M. Omladič, V. Omladič: *Optimal solutions to the problem of restricted canonical correlations*. V: The International Conference on Measurement and Multivariate Analysis, May 11-14, 2000, Alberta, Canada : proceedings. Volume Two. Alberta: ICMMA, 2000, str. 238-240.

## UČNI NAČRT PREDMETA / COURSE SYLLABUS

**Predmet:** Izbrana poglavja iz finančne matematike 2  
**Course title:** Topics in financial mathematics 2

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type:** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** še ni dodeljena/not assigned yet

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:** prof. dr Matjaž Omladič, prof. dr. Mihael Perman

**Jeziki / Languages:**  
**Predavanja / Lectures:** slovenski/Slovene, angleški/English  
**Vaje / Tutorial:** slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija

**Prerequisites:**

Enrollment into the program

**Vsebina:**

**Predavatelj izbira med naslednjimi in drugimi aktualnimi temami:**

**Modeli pri upravljanju portfeljev:** model povprečje-varianca. Markowitzeva teorija. Razpršenost donosov in njeno merjenje. Optimalne strategije. Teorija vrednotenja brez arbitraže. CAPM model. Enofaktorski in večfaktorski modeli. Modeli Bayesovega tipa. Black- Littermanov algoritem. Enoobdobni in večobdobni modeli. Linearni faktorski modeli. Vrednotenje naložb v zveznem času.

**Content (Syllabus outline):**

**Lecturer can choose among the following and some other current topics:**

**Portfolio management:** mean-variance model. Markowitz theory. Volatility of returns and its measurement. Arbitrage pricing. CAPM model. One and multifactor models. Bayesian models. Black-Litterman algorithm. One period and multiperiod models. Pricing in continuous time. Mathematical models for high frequency trading.  
**Consumption and investment:** definitions, optimization problems, general equilibrium,

Matematični modeli za algoritmično in visokofrekvenčno trgovanje.

**Potrošnja in naložbe:** definicije, optimizacijski problemi, ravnovesje, problemi s stranskimi pogoji, nepolni trgi.

**Stohastična optimizacija:** stohastična teorija upravljanja, Malliavinov račun. Viskoznostne rešitve.

side conditions, incomplete markets.

**Stochastic optimization:** stochastic control theory, Malliavin calculus. Viscosity solutions.

### Temeljni literatura in viri / Readings:

- I. Aldridge: High frequency trading: A practical guide to algorithmic strategies and trading systems. Wiley, 2013.
- M. Capinski, T. Zastawniak, Mathematics for Finance, An Introduction to Financial Engineering, London, Springer, 2. izdaja, 2011.
- D. G. Luenberger, Investment science, New York, Oxford University Press, 2. izdaja, 2013.
- E. J. Elton, M. J. Gruber, S. J. Brown, W. N. Goetzmann, Modern Portfolio Theory and Investment Analysis, New York, Wiley, 8. izdaja, 2009.
- G. Da Prato, Introduction to stochastic analysis and Malliavin calculus, Pisa : Edizioni della Normale, 2. izdaja, 2008.
- D. Nualart, The Malliavin calculus and related topics, Berlin, Heidelberg, New York: Springer, 2006.

### Cilji in kompetence:

Predmet pokriva poglavja iz matematičnih financ, pri katerih se prepleta ekonomsko razmišljanje z zapletenimi matematičnimi orodji. Nekatera poglavja so nadgradnja prejšnjih z dodatnimi interpretacijami, nekatera pa so pomemben del razmišljanja o tveganju.

Zaradi nepostredne uporabnosti vsebin bodo pri predmetu sodelovali tudi strokovnjaki iz prakse.

### Objectives and competences:

The course covers topics in mathematical finance in which economic reasoning is combined with advanced mathematical tools. Some of them are based on previous courses and give additional interpretation, some contribute to understanding of the risks.

Since the content is of great practical importance we expect that also specialists from financial practice will present their work experience during the course.

### Predvideni študijski rezultati:

#### Znanje in razumevanje:

Razumevanje matematičnih modelov, ki se uporabljajo v matematičnih financah in sredstev za njihovo obravnavo.

#### Uporaba:

### Intended learning outcomes:

#### Knowledge and understanding:

Understanding of mathematical models used in mathematical finance and the mathematical tools used in solutions.

#### Application:

Pridobljeno znanje je po eni strani neposredno prenosljivo, po drugi strani pa je izhodišče za kombiniranje matematičnega znanja z ekonomskimi vsebinami.

**Refleksija:**

Področje, in s tem posledično predmet, združuje številne znanja iz matematike od linearna algebre do parcialnih diferencialnih enačb.

**Prenosljive spretnosti – niso vezane le na en predmet:**

Pridobljeno znanje je neposredno uporabno v finančnih ustanovah kot so banke in zavarovalnice. Vsebina predmeta tudi pomaga izostriti sposobnosti matematičnega modeliranja.

The knowledge and skills acquired are directly transferable and can also serve for combining mathematical reasoning with economic topics.

**Reflection:**

The subject of the course, hence the course itself, combines numerous mathematical skills starting from linear algebra to partial differential equations.

**Transferable skills:**

The knowledge and skills acquired are immediately applicable in financial institutions such as banks and insurance companies. The content also serves to deepen the ability to use mathematical models.

**Metode poučevanja in učenja:**

predavanja, vaje, konzultacije, seminarske naloge

**Learning and teaching methods:**

Lectures, exercises, consultations, seminars

Delež (v %) /

Weight (in %)

**Načini ocenjevanja:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- samostojna seminarska naloga
- opravljena seminarska naloga za pristop k teoretičnemu delu izpita

Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)

50%

50%

**Assessment:**

Type (examination, oral, coursework, project):

- individual seminar
- completed seminar work is required for the exam on the course content

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

**prof. dr. Mihael Perman:**

- M. Perman, J. Wellner: *On the distribution of Brownian areas*, Ann. Appl. Probab., 6, no. 4., (1996), 1091-1111.
- M. Huzak, M. Perman, H. Šikić, Z. Vondraček: *Ruin probabilities and decompositions for general perturbed risk processes*, Ann. Appl. Probab., 2004, vol. 14, no. 3, (2004), 1378-1397.
- M. Huzak, M. Perman, H. Šikić, Z. Vondraček: *Ruin probabilities for competing claim processes*, J. Appl. Probab., 41, no. 3, (2004) 679-690.

**prof. dr. Matjaž Omladič**

- M. Omladič, V. Omladič: *A linear algebra approach to non-transitive expected utility. Soc. choice welf.*, 2001, vol. 18, str. 251-267.
- M. Omladič, V. Omladič: *Matematika in denar*, (Knjižnica Sigma, 58). Ljubljana: Društvo matematikov, fizikov in astronomov Slovenije, 1995. 142 str.
- M. Omladič, V. Omladič: *Optimal solutions to the problem of restricted canonical correlations*. V: The International Conference on Measurement and Multivariate Analysis, May 11-14, 2000, Alberta, Canada : proceedings. Volume Two. Alberta: ICMMA, 2000, str. 238-240.

UČNI NAČRT PREDMETA / COURSE SYLLABUS						
<b>Predmet:</b>	Izbrana poglavja iz teorije iger					
<b>Course title:</b>	Topics in game theory					
<b>Študijski program in stopnja</b>	<b>Študijska smer</b>		<b>Letnik</b>		<b>Semester</b>	
<b>Study programme and level</b>	<b>Study field</b>		<b>Academic year</b>		<b>Semester</b>	
Magistrski študijski program 2. stopnje Matematika	ni smeri		prvi ali drugi		prvi ali drugi	
Second cycle master study program Mathematics	None		first or second		first or second	
<b>Vrsta predmeta / Course type</b>			izbirni predmet/elective course			
<b>Univerzitetna koda predmeta / University course code:</b>			M2504			
<b>Predavanja</b>	<b>Seminar</b>	<b>Vaje</b>	<b>Klinične vaje</b>	<b>Druge oblike</b>	<b>Samost. delo</b>	<b>ECTS</b>
<b>Lectures</b>	<b>Seminar</b>	<b>Tutorial</b>	<b>work</b>	<b>študija</b>	<b>Individ. work</b>	
30	15	30			105	6
<b>Nosilec predmeta / Lecturer:</b>			doc. dr. Matjaž Konvalinka, prof. dr. Matjaž Omladič			
<b>Jeziki / Languages:</b>		<b>Predavanja / Lectures:</b>	slovenski, angleški/Slovene, English			
		<b>Vaje / Tutorial:</b>	slovenski, angleški/Slovene, English			
<b>Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:</b>			<b>Prerequisites:</b>			
Vpis v letnik študija			Enrollment into the program			

**Vsebina:**

**Content (Syllabus outline):**

Predavatelj izbere nekatere pomembne teme s področja teorije iger, kot so na primer:  
Bimatrične igre. Število ravnovesij, njihovo učinkovito odkrivanje, stabilnost.  
Kombinatorne igre. Igre na grafih.  
Igre s ponavljanji.  
Pogajanja, dražbe.  
Uporabe teorije iger v družboslovju.  
Teorija odločanja. Teorija socialne izbire.  
Evolucijska teorija iger.  
Eksperimentalna teorija iger.  
Diferencialne igre.

The lecturer choose some important topics in game theory, for example:  
Bimatrix games. Number of equilibria, efficient methods for finding equilibria, stability.  
Combinatorial games. Games on graphs.  
Repeated games.  
Bargaining, auctions.  
Applications of game theory in social sciences.  
Decision theory. Social choice theory.  
Evolutionary game theory.  
Experimental game theory.  
Differential games.

### **Temeljni literatura in viri / Readings:**

A. Fraenkel: Combinatorial Games, Electron. J. Combinatorics, DS2, zadnja dopolnitev, 2006.  
D. Fudenberg, J. Tirole: Game Theory, MIT Press, Cambridge MA, 1991.  
P. Morris: Introduction to Game Theory, Springer, New York, 1994.  
M. J. Osborne: An Introduction to Game Theory, Oxford University Press, Oxford, 2004.  
M. J. Osborne, A. Rubinstein: A Course in Game Theory, 10. natis, MIT Press, Cambridge MA, 2004.

### **Cilji in kompetence:**

Študent podrobneje spozna eno ali več pomembnejših področij teorije iger. Pri tem spozna nekatere najnovejše rezultate z obravnavanega področja.

### **Objectives and competences:**

The student gains a deeper knowledge of some areas of game theory, including recent results.

### **Predvideni študijski rezultati:**

Znanje in razumevanje:  
Slušatelj natančneje spozna izbrano področje teorije iger. Seznan se z najnovejšimi rezultati tega področja in z njegovimi uporabami v praksi.  
Uporaba: Modeliranje vsaj potencialno konfliktnih situacij in njihovo razreševanje s pomočjo formalnih metod.

### **Intended learning outcomes:**

Knowledge and understanding:  
The student gains a deeper understanding of the chosen area of game theory. He or she learns the newest results in the field and their applications.  
Application:  
Modelling in situations with a potential for conflict, finding the solution using formal



Refleksija: Uporabe in pomanjkljivosti opisovanja in raziskovanja pojavov iz vsakdanjega življenja s pomočjo formalnih modelov.

Prenosljive spretnosti – niso vezane le na en predmet: Sposobnost natančnega matematičnega opisa in zavedanje njegovih pomanjkljivosti. Sposobnost samostojnega študija sodobne strokovne in izbrane znanstvene literature.

methods.

Reflection:

Applications and shortcomings of descriptions and study of everyday life with the help of formal models.

Transferable skills:

Ability to set up a rigorous mathematical framework and understand its shortcomings. Ability to study modern scientific papers and monographs independently.

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije, seminarske naloge

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations, seminars

Delež (v %) /

**Načini ocenjevanja:**

Weight (in %)

**Assessment:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- samostojna seminarska naloga
- pisni ali ustni izpit

50%  
50%

Type (examination, oral, coursework, project):

- seminar work
- written or oral exam

Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

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**doc. dr. Matjaž Konvalinka:**

- M. Konvalinka, I. Pak: Geometry and complexity of O'Hara's algorithm, *Adv. Appl. Math.* 42 (2) (2009), 157-175.
- M. Konvalinka, I. Pak: Triangulations of Cayley and Tutte polytopes, *Adv. Math.*, Vol. 245 (2013), 1-33
- D. Dolžan, M. Konvalinka, P. Oblak: Diameters of components of commuting graphs, *Electron. J. Linear Al.*, Vol. 26 (2013), 433-445

**prof. dr. Matjaž Omladič:**

- M. Omladič, V. Omladič: A linear algebra approach to non-transitive expected utility. *Soc. choice welf.*, 2001, vol. 18, str. 251-267.
- M. Omladič, V. Omladič: *Matematika in denar*, (Knjižnica Sigma, 58). Ljubljana: Društvo matematikov, fizikov in astronomov Slovenije, 1995. 142 str
- H. Radjavi, M. Omladič: Self-adjoint semigroups with nilpotent commutators. *Linear Algebra Appl.* 436 (2012), no. 7, 2597–2603.

## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Modeliranje s slučajnimi procesi
<b>Course title:</b>	Modelling with stochastic processes

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2505

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Mihael Perman, prof. dr. Matjaž Omladič, doc. dr. Janez Bernik

**Jeziki / Languages:**

<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija

**Prerequisites:**

Enrollment into the program

**Vsebina:**

**Aktuarski del:**  
Lundbergov proces, verjetnosti bankrota, martingalske metode, verjetnosti bankrota v končnem času, posplošitve Lundbergovega modela.  
Modeliranje z markovskimi verigami, enačbe Kolmogorova, Thielejeve diferencialne enačbe, izračun matematičnih rezervacij, zvarovalni produkti z izplačili odvisnimi od matematičnih

**Content (Syllabus outline):**

**Actuarial part:**  
Lundberg process, the probability of ruin, martingale methods, the probability of ruin in finite time, generalized Lundberg model. Markov chain models, Kolmogorov equations, Thiele differential equation, mathematical reserves calculation, reserves dependent payoffs, stochastic interest rates via Markov chains.

rezervacij, vpeljava slučajnih obrestnih mer z markovskimi verigami.

#### **Finančni del:**

Optimalna kontrola: formulacija problema, Hamilton-Jacobi-Bellmanove enačbe, linearni regulator, primeri uporabe.

Optimalno ustavljanje: formulacija problema, primeri, ameriške opcije.

Osnovni izrek vrednotenja opcij: formulacija, dokaz, enačbe za varovanje, povezava s parcialnimi diferencialnimi enačbami, primeri nekompletnih trgov.

Nekompletni trgi: Lévyjevi modeli, super-varovanje, vrednotenje, optimizacija.

#### **Financial part:**

Optimal control: formulation of the problem, Hamilton-Jacobi-Bellman equations, linear regulator, applications.

Optimal stopping: formulation of the problem, examples, American options.

Fundamental theorem of asset pricing: formulation, proof, hedging equations, connections to partial differential equations, examples of incomplete markets.

Incomplete markets: Lévy models, superhedging, pricing, optimization.

#### **Temeljni literatura in viri / Readings:**

- M. Denuit, J. Dhaene, M. Goovaerts, R. Kaas: *Dependent Risks, Measures, Orders and Models*, Wiley, New York, 2005.
- J. Grandell: *Aspects of Risk Theory*, Springer, New York, 1991.
- M. Koller: *Stochastische Modelle in der Lebensversicherung*, Springer, Berlin, 2000.
- H. Bühlmann: *Mathematical Methods in Risk Theory*, Springer, New York, 2005.
- T. Björk: *Arbitrage Theory in Continuous Time*, Oxford University Press, Oxford, 1998.
- B. Øksendal: *Stochastic Differential Equations, An Introduction with Applications*, Springer, New York, 2003.
- D. Wong: *Generalised Optima Stopping Problems and Financial Markets*, Longman, 1996.
- M.H.A. Davis: *Stochastic Modelling and Control*, Chapman & Hall, 1995.
- Karatzas, S. E. Shreeve: *Methods of Mathematical Finance*, Springer, New York, 1998.
- W. Schoutens: *Lévy Processes in Finance: Pricing Financial Derivatives*, Wiley, New York, 2003.

#### **Cilji in kompetence:**

Slučajni procesi so osnova za številne modele, ki se uporabljajo v finančnem in aktuarskem svetu. Tečaj se navezuje na teoretična tečaja iz slučajnih procesov in po eni strani odpira pot do uporabe, po drugi pa na drugačen način osvetli teoretične osnove.

Zaradi nepostredne uporabnosti vsebin bodo pri predmetu sodelovali tudi strokovnjaki iz prakse.

#### **Objectives and competences:**

Stochastic processes form a basis for numerous models in finance and insurance. The course links theoretical parts learned in other courses on stochastic processes by showing their applications on one side and elucidates the theoretical background on the other.

Since the content is of great practical importance we expect that also specialists from financial practice will present their work experience during the course.

**Predvideni študijski rezultati:**

**Znanje in razumevanje:** Razumevanje modeliranja s slučajnimi procesi v financah in aktuarstvu in razumevanja matematičnih orodij in predpostavk.

**Uporaba:** Uporaba je neposredna, saj so obravnavani modeli izhodišče za vrednotenje mnogih finančnih in zavarovalnih produktov.

**Refleksija:** Uporaba slučajnih procesov utrdi znanje iz verjetnosti in slučajnih procesov po eni strani, po drugi pa odpira pot do praktične uporabe teorije slučajnih procesov.

**Prenosljive spretnosti – niso vezane le na en predmet:** Spretnosti so prenosljive na druga področja matematičnega modeliranja, še najbolj pa je predmet pomemben zaradi svoje neposredne uporabnosti.

**Intended learning outcomes:**

**Knowledge and understanding:** Understanding of stochastic modelling in finance and insurance and understanding of mathematical framework.

**Application:** Application is immediate as the models under consideration form a basis for Pricing many financial and insurance products.

**Reflection:** The application of stochastic processes deepens the knowledge of probability calculus and stochastic processes and paves the way for their application.

**Transferable skills:** The skills obtained are transferable to other areas of mathematical modelling, but the gist of the course is its immediate applicability.

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije, seminarsek naloge

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations, seminars

Delež (v %) /

Weight (in %)

**Načini ocenjevanja:****Assessment:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- samostojna seminarska naloga
- ustni izpit

50%

50%

Type (examination, oral, coursework, project):

- seminar work
- oral exam

Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

**prof. dr. Mihael Perman**

- M. Perman, J. Pitman, M. Yor: *Size-Biased Sampling of Poisson Processes and Excursions*, Prob.

Theory. Rel. Fields, 92, 21-32 (1992).

- M. Perman, J. Wellner: *On the distribution of Brownian areas*, Ann. Appl. Probab., 6, no. 4., (1996), 1191-1111. .
- M. Perman: An excursion approach to Ray-Knight theorems for perturbed Brownian motion, Stoch. Proc. Appl., 63,(1998), 67-74.

**prof. dr. Matjaž Omladič**

- M. Omladič, V. Omladič: *Hierarchical dynamics for power and control in society*. J. Math. Sociol., 1993/94, let. 18, št. 14, str. 293-313.
- R. E. Hartwig, M. Omladič, P. Šemrl, G. P. H. Styan: *On some characterizations of pairwise star orthogonality using rank and dagger additivity and subtractivity*. Linear Algebra Appl., 1996, let. 237/238, št. 2, str. 499-507.
- M. Omladič, V. Omladič: *More on restricted canonical correlations*. Linear Algebra Appl.. [Print ed.], 2000, vol. 1/3, no. 321, str. 285-293.

**doc. dr. Janez Bernik**

- J. Bernik, M. Mastnak, H. Radjavi: *Realizing irreducible semigroups and real algebras of compact operators*, J. Math. Anal. Appl. 348 (2008), 692--707.
- J. Bernik, M. Mastnak, H. Radjavi: *Positivity and matrix semigroups*, Linear Algebra Appl. 434 (2011), 801-812.
- J. Bernik, L.W. Marcoux, H. Radjavi: *Spectral conditions and band reducibility of operators*, J. London Math. Soc. 86 (2012), 214-234.

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

**Predmet:** Numerične metode v finančni matematiki  
**Course title:** Numerical methods for financial mathematics

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type:** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** še ni dodeljena/not assigned yet

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Matjaž Omladič, doc. dr. Dejan Velušček

**Jeziki / Languages:**  
**Predavanja / Lectures:** slovenski/Slovene, angleški/English  
**Vaje / Tutorial:** slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija

**Prerequisites:**

Enrollment into the program

**Vsebina:**

Algoritmi za vrednotenje opcij v diskretnem času. Monte Carlo metode za evropske opcije. Simulacije klasičnih porazdelitev. Metoda inverzne transformacije. Izračun matematičnega upanja.  
 Tehnike za zmanjšanje variance. Drevesne metode za evropske in ameriške opcije. Red konvergence v binomskih metodah.  
 Ocenjevanje občutljivosti. Numerični algoritmi za zaščito portfeljev. Drevesne metode in metode Monte Carlo za eksotične opcije

**Content (Syllabus outline):**

Algorithms for option pricing in discrete models. Monte Carlo Methods for European options.  
 Simulation methods of classical law. Inverse transform method. Computation of expectation.  
 Variance reduction techniques. Tree methods for European and American options. Convergence orders of binomial methods. Estimating sensitivities. Numerical algorithms for portfolio insurance. Tree methods and Monte Carlo methods for Exotic options (barrier options,

(opcije z mejo, azijske opcije, povratne opcije, mavrične opcije).

Monte Carlo metode za ameriške opcije.

Metode končnih diferenc za Black-Scholesovo parcialno diferencialno enačbo.

asian options, lookback options, rainbow options).

American Monte Carlo methods.

Finite difference methods for the Black-Scholes partial differential equation.

### Temeljna literatura in viri / Readings:

- J. Hull. Options, Futures, and Other Derivatives. Prentice Hall, 2011.
- N. H. Bingham, R. Kiesel. Risk-Neutral Valuation: Pricing and Hedging of Financial Derivatives. Springer Finance, 2004.
- P. Glasserman. Monte Carlo Methods in Financial Engineering. Springer, 2003.

### Cilji in kompetence:

Predmet pokriva poglavja iz matematičnih financ, ki so pomembna za numerične izračune pri vrednotenju izvedenih finančnih instrumentov vseh vrst.

Zaradi nepostredne uporabnosti vsebin bodo pri predmetu sodelovali tudi strokovnjaki iz prakse.

### Objectives and competences:

The course covers the chapter of mathematical finance that deal with numerical methods for pricing of derived financial instruments of all kinds.

Since the content is of great practical importance we expect that also specialists from financial practice will present their work experience during the course.

### Predvideni študijski rezultati:

#### Znanje in razumevanje:

Razumevanje matematičnih modelov, ki se uporabljajo v matematičnih financah, in sredstev za njihovo obravnavo.

#### Uporaba:

Pridobljeno znanje je po eni strani neposredno prenosljivo, po drugi strani pa je izhodišče za kombiniranje matematičnega znanja s finančnimi vsebinami.

#### Refleksija:

Področje, in s tem posledično predmet, združuje številne znanja iz matematike, predvsem tistih povezanih s numeričnimi metodami in teorijo verjetnosti.

**Prenosljive spretnosti – niso vezane le na en predmet:**

### Intended learning outcomes:

#### Knowledge and understanding:

Understanding of mathematical models used in finance. Mathematical tools necessary in modelling.

#### Application:

The knowledge is directly usable in practice, it is also the source for combining of mathematical theories with finance.

#### Reflection:

The subject connects many mathematical topics, specially those of numerical methods and probability theory, with application.

#### Transferable skills:

The knowledge is directly applicable in everyday practice in financial institutions.



Pridobljeno znanje je neposredno uporabno v finančnih ustanovah .

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije, seminarske naloge

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations, seminars

**Načini ocenjevanja:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- samostojna seminarska naloga
- ustni izpit

Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)

Delež (v %) /

Weight (in %) /

**Assessment:**

Type (examination, oral, coursework, project):

- seminar work
- oral exam

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

**prof. dr. Matjaž Omladič**

- M. Omladič, V. Omladič: *Hierarchical dynamics for power and control in society. J. Math. Sociol.*, 1993/94, let. 18, št. 14, str. 293-313.
- R. E. Hartwig, M. Omladič, P. Šemrl, G. P. H. Styan: *On some characterizations of pairwise star orthogonality using rank and dagger additivity and subtractivity. Linear Algebra Appl.*, 1996, let. 237/238, št. 2, str. 499-507.
- M. Omladič, V. Omladič: *More on restricted canonical correlations. Linear Algebra Appl.* [Print ed.], 2000, vol. 1/3, no. 321, str. 285-293.

**doc. dr. Dejan Velušček**

- P. Dörsek, J. Teichmann, D. Velušček: Cubature methods for stochastic (partial) differential equations in weighted spaces, accepted for publication in »Stochastic Partial Differential Equations: Analysis and Computations«.
- K. Oshima, J. Teichmann, D. Velušček: A new extrapolation method for weak approximation schemes with applications, *Ann. Appl. Probab.* 22, no. 3 (2012), 1008-1045.
- I. Klep, D. Velušček: Central extensions of \*-ordered skew fields, *Manuscripta math.* 120, no. 4 (2006), 391-402.

## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Optimizacija v financah
<b>Course title:</b>	Optimization in finance

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2502

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Bojan Mohar, prof. dr. Matjaž Omladič, doc. dr. Dejan Velušček

**Jeziki / Languages:**

<b>Predavanja / Lectures:</b>	slovenski, angleški/Slovene, English
<b>Vaje / Tutorial:</b>	slovenski, angleški/Slovene, English

<b>Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:</b>	<b>Prerequisites:</b>
Vpis v letnik študija	Enrollment into the program

**Vsebina:** \_\_\_\_\_ **Content (Syllabus outline):** \_\_\_\_\_

### **Linearno Programiranje:**

Teorija in algoritmi, metoda simpleksov, metode notranjih točk, programski paketi za praktično reševanje. Linearni modeli v financah: osnovni izrek o vrednotenju, vrednotenje izvedenih finančnih instrumentov v odsotnosti arbitraže, uporaba linearnega programiranja pri klasifikaciji podatkov ipd

### **Kvadratično programiranje:**

Pogoj optimalnosti, dualnost, metode notranjih točk, programska orodja za praktično reševanje. Finančni modeli: različni načini izbire in upravljanja portfelja, maksimiziranje Sharpeovega razmerja, mean-variance optimizacija idr.

### **Optimizacija na stožcih:**

Pregled teorije in praktičnih algoritmov. Finančni modeli: arbitraža z minimalnim tveganjem, aproksimacija kovariantnih matrik idr.

### **Stohastično programiranje:**

Uporaba stohastičnih modelov, modeliranje ob upoštevanju negotovosti, metode za reševanje. Primeri finančnih modelov: izbor in upravljanje s portfelji, optimizacija z izogibanjem tveganja ipd.

### **Dinamično programiranje:**

Pregled teorije in osnovnih metod za reševanje, dinamično programiranje v diskretnem in zveznem času, zvezni prostor stanj, optimalno upravljanje. Primeri finančnih modelov: dinamična analiza portfelja, problem optimalnega ustavljanja idr.

Po potrebi predavatelj v tečaj vključi tudi druge aktualne teme iz novejše znanstvene periodike.

Zaradi nepostredne uporabnosti vsebin bodo pri predmetu sodelovali tudi strokovnjaki iz prakse.

### **Linear programming:**

Theory and algorithms, simplex method, interior point methods, software packages for practical problem solving. Linear models in finance: the basic theorem of asset pricing, the pricing of financial derivatives in the arbitrage-free setting, use of linear programming for data classification, etc.

### **Quadratic programming:**

Condition for optimality, duality, interior point methods, software packages for practical problem solving. Financial models: various methods for creating and managing a portfolio, maximization of the Sharpe's ratio, mean-variance optimization, etc.

### **Cone programming:**

Overview of the theory and of the practical algorithms.

Financial models: minimal risk arbitrage, covariant matrix approximation, etc.

### **Stochastic programming:**

Use of stochastic models, modeling with uncertainty, methods for solving various stochastic programming problems. Examples in finance: portfolio building and management, risk averse optimization, etc.

### **Dynamic programming:**

Overview of the theory and of the basic methods for problem solving, dynamic programming in discrete and continuous time, continuous state space, optimal control. Examples in financial models: dynamic portfolio analysis, optimal stopping problem, etc.

The lecturer can also include other current topics from recent scientific periodicals in the course.

Since the content is of great practical importance we expect that also specialists from financial practice will present their work experience during the course.

### Temeljni literatura in viri / Readings:

- D. P. Bertsekas, Dynamic programming and optimal control, Athena Scientific, 2005.
- V. Chvátal: Linear Programming, Freeman, New York, 1983.
- G. Cornuejols, R. Tütüncü: Optimization Methods in Finance, Cambridge Univ. Press, Cambridge, 2007.
- A. Shapiro, D. Dentscheva, A. Ruszczyński: Lectures on Stochastic Programming: Modeling and Theory, MPS/SIAM Series on Optimization 9, SIAM, 2009.
- S. Zenios: Financial Optimization, Cambridge Univ. Press, Cambridge, 1996.

### Cilji in kompetence:

Študent spozna nekatere osnovne vrste optimizacijskih problemov, še posebej tiste, s katerimi lahko modeliramo probleme s področja financ. Seznan se z osnovnimi matematičnimi prijemi za njihovo reševanje, hkrati pa za praktično reševanje uporablja tudi primerne programske pakete.

### Objectives and competences:

Students acquire knowledge on the basic types of optimization problems, the stress being on the problems suitable for modeling problems coming from the field of finance. The students get acquainted with the basic mathematical approaches for solving the above optimization problems and use suitable software packages for solving practical problems.

### Predvideni študijski rezultati:

#### Znanje in razumevanje:

Sposobnost dobro opisati različne probleme s področja financ z matematičnim modelom. Poznavanje osnovnih prijemov in računalniških orodij za učinkovito reševanje dobljenih optimizacijskih problemov.

#### Uporaba:

Reševanje zahtevnejših praktičnih optimizacijskih problemov s področja financ.

#### Refleksija:

Pomen predstavitve praktičnih problemov v formalizirani obliki, ki omogoča njihovo učinkovito in pravilno reševanje.

#### Prenosljive spretnosti – niso vezane le na en predmet:

Modeliranje nalog iz vsakdanjega življenja v obliki matematičnih optimizacijskih nalog, zmožnost razločevanja med računsko obvladljivimi in neobvladljivimi problemi, sposobnost samostojnega modeliranja in reševanja z računalnikom.

### Intended learning outcomes:

#### Knowledge and understanding:

The ability to describe various problems from the field of finance with a mathematical model. Knowledge on the basic approaches and software tools for efficient solving of the acquired optimization problems.

#### Application:

Solving more demanding practical optimization problems in finance.

#### Reflection:

The importance of presenting practical problems in formalized form which enables their efficient and correct solving.

#### Transferable skills:

Modeling the real-life problems in the form of a mathematical optimization problem, the ability to distinguish between computationally tractable and intractable problems, the ability to model and solve the problem on one's own using the computer.

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**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije, seminarske naloge

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations, seminars

Delež (v %) /

**Načini ocenjevanja:**

Weight (in %)

**Assessment:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- izpit iz vaj (pisni izpit)
- ustni izpit

Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)

50%

50%

Type (examination, oral, coursework, project):

- written exam
- oral exam

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

**prof. dr. Bojan Mohar**

- B. Mohar: A linear time algorithm for embedding graphs in an arbitrary surface, SIAM J. Discrete Math. 12 (1999), 6–26.
- B. Mohar: Circle packings of maps in polynomial time, European J. Combin. 18 (1997), 785–805.
- B. Mohar: Projective planarity in linear time, J. Algorithms 15 (1993), 482–502.

**prof. dr. Matjaž Omladič**

- M. Omladič, V. Omladič: A linear algebra approach to non-transitive expected utility, Soc. choice welf. 18 (2001), 251–267.
- M. Omladič, V. Omladič: Matematika in denar, (Knjižnica Sigma, 58). Ljubljana: Društvo matematikov, fizikov in astronomov Slovenije, 1995. 142 str.
- M. Omladič, V. Omladič: Optimal solutions to the problem of restricted canonical correlations. V: The International Conference on Measurement and Multivariate Analysis, May 11-14, 2000, Alberta, Canada : proceedings. Volume Two. Alberta: ICMMA, 2000, str. 238–240.

**doc. dr. Dejan Velušček**

- P. Dörsek, J. Teichmann, D. Velušček: Cubature methods for stochastic (partial) differential equations in weighted spaces, accepted for publication in »Stochastic Partial Differential Equations: Analysis and Computations«.
- K. Oshima, J. Teichmann, D. Velušček: A new extrapolation method for weak approximation schemes with applications, Ann. Appl. Probab. 22, no. 3 (2012), 1008-1045.
- I. Klep, D. Velušček: Central extensions of  $\ast$ -ordered skew fields, Manuscripta math. 120, no. 4 (2006), 391-402.

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

**Predmet:** Rieszovi prostori v matematični ekonomiji  
**Course title:** Riesz spaces in mathematical economics

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type**

**Univerzitetna koda predmeta / University course code:**

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:**

**Jeziki / Languages:**  
**Predavanja / Lectures:**   
**Vaje / Tutorial:**

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

**Prerequisites:**

**Vsebina:**

**Content (Syllabus outline):**

Arrow-Debreujev model za izmenjalne ekonomije s končno mnogo dobrinami in porabniki.

Kakutanijev izrek o negibni točki.

Walrasovo ravnovesje v neoklasični izmenjalni ekonomiji.

Izreka o blagostanju.

Rieszovi prostori. Linearni funkcionali in linearni operatorji.

Rieszovi prostori dobrin in cen.

Model izmenjalne ekonomije z neskočnorazsežnim prostorom dobrin in števno mnogo porabniki.

The Arrow-Debreu model for exchange economies with a finite number of commodities and consumers.

Kakutani fixed-point theorem.

A Walras equilibrium in a neoclassical exchange economy.

Welfare theorems.

Riesz spaces. Linear functionals and linear operators.

Riesz spaces of commodities and prices.

Model for exchange economy with infinite-dimensional space of commodities and countably many consumers.

#### Temeljni literatura in viri / Readings:

- C. D. Aliprantis, D. J. Brown, O. Burkinshaw: *Existence and optimality of competitive equilibria*, Springer-Verlag, Berlin, 1990.
- C. D. Aliprantis: *Problems in equilibrium theory*, Springer-Verlag, Berlin, 1996.
- C. D. Aliprantis, O. Burkinshaw: *Locally solid Riesz spaces with applications to economics*, Mathematical Surveys and Monographs 105, American Mathematical Society, Providence, RI, 2003.

#### Cilji in kompetence:

Študent spozna uporabo teorije Rieszovih prostorov v matematični ekonomiji. Pri tem se seznanja z nekaterimi modeli za izmenjalne ekonomije.

#### Objectives and competences:

Students learn about the application of the theory of Riesz spaces in mathematical economics. They get acquainted with some models of exchange economies.

#### Predvideni študijski rezultati:

##### Znanje in razumevanje:

Poznavanje in razumevanje osnovnih pojmov teorije Rieszovih prostorov. Sposobnost njene uporabe v matematični ekonomiji.

##### Uporaba:

Uporaba teorije Rieszovih prostorov na modelih za izmenjalne ekonomije.

#### Intended learning outcomes:

##### Knowledge and understanding:

Knowledge and understanding of the basic concepts of the theory Riesz spaces. The ability of its use in mathematical economics.

##### Application:

Using the theory of Riesz spaces on models of exchange economies.



**Refleksija:**

Razumevanje teorije na podlagi primerov in uporabe.

**Prenosljive spretnosti – niso vezane le na en predmet:**

Identifikacija in reševanje problemov.

Formulacija nematematičnih problemov v matematičnem jeziku.

Spretnost uporabe domače in tuje literature.

**Reflection:**

Understanding of the theory and the ability to apply it to concrete examples.

**Transferable skills:**

Identifying and solving problems.

Formulation of nonmathematical problems in mathematical language.

Ability to use domestic and foreign literature.

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije, seminarske naloge

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations, seminars

**Načini ocenjevanja:**

- domače naloge
- izpit

Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)

Delež (v %) /

Weight (in %) /

**Assessment:**

- homeworks
- exam

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

**prof. dr. Roman Drnovšek**

- R. Drnovšek: *Triangularizing semigroups of positive operators on an atomic normed Riesz space*, Proc. Edinb. Math. Soc. 43 (2000), 43-55.
- R. Drnovšek: *On positive unipotent operators on Banach lattices*, Proc. Amer. Math. Soc. 135 (2007), no. 12, 3833-3836.
- R. Drnovšek: *An infinite-dimensional generalization of Zenger's lemma*, J. Math. Anal. Appl. 388 (2012), no. 2, 1233-1238.

**prof. dr. Matjaž Omladič**

- M. Omladič, V. Omladič: *Hierarchical dynamics for power and control in society*. J. Math. Sociol., 1993/94, let. 18, št. 14, str. 293-313.
- M. Omladič, V. Omladič: *Positive root vectors*. Proc. R. Soc. Edinb., Sect. A, Math., 1995, let. 125, št. 4, str. 701-717.
- M. Omladič, V. Omladič: *A linear algebra approach to non-transitive expected utility*, Soc. Choice Welf. 18 (2001), no. 2, 251-267.

## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Slučajni procesi 2
<b>Course title:</b>	Stochastic Processes 2

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2501

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:** izred. prof. dr. Mihael Perman, prof. dr. Matjaž Omladič, doc. dr. Janez Bernik

**Jeziki / Languages:**

<b>Predavanja / Lectures:</b>	slovenski/Slovene
<b>Vaje / Tutorial:</b>	slovenski/Slovene

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

**Prerequisites:**

Vpis v letnik študija

Enrollment into the program

**Vsebina:**

**Content (Syllabus outline):**

**Brownovo gibanje:**

Osnovne lastnosti, obstoj, lastnosti trajektorij, naravna filtracija, čas prvega dotika, markovske lastnosti, krepka lastnost Markova, princip zrcaljenja, pridruženi procesi (proces tekočega supremuma, Brownov most itd.), kvadratična variacija.

**Martingali v zveznem času:**

Filtracije, časi ustavljanja, martingali, izreki o ostavljanju, enakomerna integrabilnost, maksimalne neenakosti, konvergenca martingalov.

**Stohastični integral:**

Stohastični integral glede na Brownovo gibanje, Itôva izometrija, zvezni polmartingali, zvezni lokalni martingali, kvadratična variacija in kovariacija, stohastični integral glede na zvezne polmartingale, Itôva formula, izrek Girsanova, izrek o reprezentaciji martingalov.

**Brownian motion:**

Basic properties, existence, path properties, natural filtration, first hitting time, Markov properties, strong Markov property, reflection principle, associated processes (running supremum process, Brownian bridge etc.), quadratic variation.

**Continuous time martingales:**

Filtrations, stopping times, stopping theorems, uniform integrability, maximal inequalities, convergence of martingales.

**Stochastic integral:**

Stochastic integral wrt Brownian motion, Itô isometry, continuous semimartingales, local martingales, quadratic variation and covariation, stochastic integral wrt continuous semimartingales, Itô's formula, Girsanov Theorem, representation of martingales.

**Temeljni literatura in viri / Readings:**

- S. Resnick: *Adventures in Stochastic Processes*, Birkhäuser Boston, 2002.
- I. Karatzas, S. E. Shreve: *Brownian Motion and Stochastic Calculus*, 2nd Edition, Springer, 2005.
- M. Yor, D. Revuz: *Continuous Martingales and Stochastic Calculus*, 2nd Edition, Springer, 2004
- J. M. Steele: *Stochastic Calculus and Financial Applications*, Springer, New York, 2001.

**Cilji in kompetence:****Objectives and competences:**

Predmet predstavlja uvod v teorijo slučajnih procesov v zveznem času z zveznimi trajektorijami. Rigorozno obravnava Brownovo gibanje kot osnovni primer in gradnik, vpelje martingale v zveznem času, Itôv stohastični račun in Itovo formulo.

This course is an introduction to the theory of stochastic processes in continuous time with continuous sample paths. It rigorously treats Brownian motion as a basic example and building block, introduces martingales in continuous time, stochastic calculus and Ito's formula.

**Predvideni študijski rezultati:**

**Znanje in razumevanje:**

Matematična orodja za strogo obravnavo in uporabo slučajnih procesov.

**Uporaba:**

Osnova za modeliranje v mnogih vejah matematike in njene uporabe.

**Refleksija:**

Vsebina predmeta pomaga za nazaj poglobiti razumevanje konceptov verjetnosti, koncepta odvisnosti in časa.

**Prenosljive spretnosti – niso vezane le na en predmet:**

Spretnosti so prenosljive na druga področja matematičnega modeliranja, še najbolj pa je predmet pomemben zaradi svoje neposredne uporabnosti pri finančnem modeliranju.

**Intended learning outcomes:**

**Knowledge and understanding:**

Mathematical tools for rigorous treatment and applications of stochastic processes.

**Application:**

Basic tools for modelling in many branches of Mathematics and its applications.

**Reflection:**

The contents of the course help in retrospect to deepen the understanding of the concepts of probability, dependence and time.

**Transferable skills:**

The skills acquired are transferable to other areas of mathematical modelling, in particular it is immediately applicable to financial models.

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations

**Načini ocenjevanja:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- pisni izpit

Ocene: 1-5 (negativno), 6-10

Delež (v %) /

Weight (in %)

**Assessment:**

Type (examination, oral, coursework, project):

- written exam

Grading: 1-5 (fail), 6-10 (pass)

(pozitivno) (po Statutu UL)

(according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

**izred. prof. dr. Mihael Perman**

- M. Perman, J. Pitman, M. Yor,: *Size-Biased Sampling of Poisson Processes and Excursions*, Prob. Theory. Rel. Fields, 92, 21-32 (1992).
- M. Perman, J. Wellner: *On the distribution of Brownian areas*, Ann. Appl. Probab., 6, no. 4., (1996), 1091-1111. .
- M. Perman: *An excursion approach to Ray-Knight theorems for perturbed Brownian motion*, Stoch. Proc. Appl., 63,(1998), 67-74.

**prof. dr. Matjaž Omladič**

- M. Omladič, V. Omladič: *Hierarchical dynamics for power and control in society*. J. Math. Sociol., 1993/94, vol. 18, 293-313.
- R. E. Hartwig, M. Omladič, P. Šemrl, G. P. H. Styan: *On some characterizations of pairwise star orthogonality using rank and dagger additivity and subtractivity*. Linear Algebra Appl. 237/238, (1986), 499-507.
- M. Omladič, V. Omladič: *More on restricted canonical correlations*. Linear Algebra Appl. 321, (2000), 285-293.

**doc. dr. Janez Bernik**

- J. Bernik, M. Mastnak, H. Radjavi: *Realizing irreducible semigroups and real algebras of compact operators*, J. Math. Anal. Appl. 348 (2008), 692--707.
- J. Bernik, M. Mastnak, H. Radjavi: *Positivity and matrix semigroups*, Linear Algebra Appl. 434 (2011), 801-812.
- J. Bernik, L.W. Marcoux, H. Radjavi: *Spectral conditions and band reducibility of operators*, J. London Math. Soc. 86 (2012), 214-234.

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

**Predmet:** Slučajni procesi 3  
**Course title:** Stochastic Processes 3

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** še ni dodeljena/not assigned yet

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Mihael Perman, prof. dr. Matjaž Omladič, doc. dr. Oliver Dragičević

**Jeziki / Languages:**  
**Predavanja / Lectures:** slovenski/Slovene  
**Vaje / Tutorial:** slovenski/Slovene

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija

**Prerequisites:**

Enrollment into the program

**Vsebina:**

Lévyjevi procesi, Lévy-Hinčinova formula, skočne mere, konstrukcija Lévyjevih procesov; Potencialna teorija, reševanje parcialnih diferencialnih enačb s pomočjo stohastičnih procesov; Osnove stohastičnih diferencialnih enačb, Ornstein-Uhlenbeckov proces.

**Content (Syllabus outline):**

Lévy processes, Lévy-Khintchine formula, jump measures, construction of Lévy processes; Potential theory, solving PDE by means of stochastic processes; Basic concepts of stochastic differential equations, the Ornstein-Uhlenbeck process.

**Temeljni literatura in viri / Readings:**

- N.V. Krylov: Introduction to the Theory of Random Processes, Graduate Studies in Mathematics, vol. 43, American Mathematical Society, 2002.
- D.W. Stroock: Probability Theory: an analytic view, Cambridge University Press, 2003.
- R. Bass: Probabilistic Techniques in Analysis, Springer-Verlag, 1995.
- R. Durrett: Stochastic Calculus: A Practical Introduction, CRC Press, 1996.

**Cilji in kompetence:**

V okviru predmeta opravimo uvod v teorijo Lévyjevih procesov, prikaz probabilističnega pristopa k potencialni teoriji ter parcialnim diferencialnim enačbam, na koncu pa spoznamo še osnove stohastičnih diferencialnih enačb.

**Objectives and competences:**

Within the course we present an introduction to the theory of Lévy processes, we learn about the probabilistic approach to the potential theory and partial differential equations, and finally we meet the basics of stochastic differential equations.

**Predvideni študijski rezultati:****Znanje in razumevanje:**

Poglobitev študija in rigorozna obravnava nekaterih posebnih lastnosti slučajnih procesov, verjetnostni pristop k problemom iz parcialnih diferencialnih enačb.

**Uporaba:**

Osnova za modeliranje v mnogih vejah matematike in njene uporabe.

**Refleksija:**

Spoznavanje globljih povezav med različnimi vejami matematike, podrobna obravnava skokov.

**Prenosljive spretnosti – niso vezane le na en predmet:**

Spretnosti so prenosljive na druga področja matematičnega modeliranja, med drugim finančnem modeliranju.

**Intended learning outcomes:****Knowledge and understanding:**

Deepening of study and rigorous treatment of certain particular features of stochastic processes, probabilistic approach to problems from PDE.

**Application:**

Basic tools for modelling in many branches of mathematics and its applications.

**Reflection:**

Learning about deeper connections between various areas of mathematics, meticulous treatment of jumps.

**Transferable skills:**

The skills acquired are transferable to other areas of mathematical modelling, among the rest to financial models.

**Metode poučevanja in učenja:****Learning and teaching methods:**



predavanja, vaje, domače naloge, seminarske naloge	Lectures, exercises, homeworks, seminars
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Delež (v %) /

**Načini ocenjevanja:**

Weight (in %)

**Assessment:**

<p>Način:</p> <ul style="list-style-type: none"> <li>▪ domače in seminarske naloge</li> <li>▪ ustni izpit</li> </ul> <p>Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)</p>	<p>50%</p> <p>50%</p>	<p>Type:</p> <ul style="list-style-type: none"> <li>▪ homework and seminar assignments</li> <li>▪ oral exam</li> </ul> <p>Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)</p>
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**Reference nosilca / Lecturer's references:**

**prof. dr. Mihael Perman**

- M. Perman, J. Pitman, M. Yor: *Size-Biased Sampling of Poisson Processes and Excursions*, Prob. Theory. Rel. Fields, 92, 21-32 (1992).
- M. Perman, J. Wellner: *On the distribution of Brownian areas*, Ann. Appl. Probab., 6, no. 4., (1996), 1191-1111. .
- M. Perman: *An excursion approach to Ray-Knight theorems for perturbed Brownian motion*, Stoch. Proc. Appl., 63,(1998), 67-74.

**prof. dr. Matjaž Omladič**

- M. Omladič, V. Omladič: *Hierarchical dynamics for power and control in society*. J. Math. Sociol., 1993/94, let. 18, št. 14, str. 293-313.
- R. E. Hartwig, M. Omladič, P. Šemrl, G. P. H. Styan: *On some characterizations of pairwise star orthogonality using rank and dagger additivity and subtractivity*. Linear Algebra Appl., 1996, let. 237/238, št. 2, str. 499-507.
- M. Omladič, V. Omladič: *More on restricted canonical correlations*. Linear Algebra Appl.. [Print ed.], 2000, vol. 1/3, no. 321, str. 285-293.

**doc. dr. Oliver Dragičević**

- A. Carbonaro, O. Dragičević: *Bellman function and dimension-free estimates in a theorem of Bakry*, J. Funct. Anal. 265 no. 7 (2013), 1085--1104.
- O. Dragičević, S. Petermichl, A. Volberg: *A rotation method which gives linear  $L_p$  estimates for powers of the Ahlfors-Beurling operator*, J. Math. Pures Appl. 86 (2006), 492--509.
- O. Dragičević, A. Volberg: *Bellman function, Littlewood-Paley estimates and asymptotics for the Ahlfors-Beurling operator in  $L_p(C)$* , Indiana Univ. Math. J. 54 (2005), no. 4, 971--996.



## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Statistika 2
<b>Course title:</b>	Statistics 2

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	1 or 2	1 or 2

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2507

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
45		30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Mihael Perman, prof. dr. Matjaž Omladič

<b>Jeziki / Languages:</b>	<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
	<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

<b>Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:</b>	<b>Prerequisites:</b>
Vpis v letnik študija	Enrollment into the program

<b>Vsebina:</b>	<b>Content (Syllabus outline):</b>
<p><b>Linearne metode pri obdelavi podatkov:</b> Linearna regresija, multipli in parcialni korelacijski koeficient, cenilke po metodi najmanjših kvadratov, izrek Gauss-Markova, kanonična redukcija linearnega modela, preizkušanje domnev, diagnostične metode, napovedovanje, posplošitve linearne regresije.</p> <p><b>Analiza variance:</b> Klasifikacija po enem faktorju, klasifikacija po dveh faktorjih, preizkusi značilnosti.</p>	<p><b>Linear methods for data analysis:</b> Linear regression, multiple and partial correlation coefficients), canonical correlation analysis, least square estimators, Gauss-Markov theorem, canonical reduction of the linear model, hypothesis testing, prediction, generalizations of linear regression.</p> <p><b>Analysis of variance:</b> One factor classification, two-factor classification, test of significance.</p>

**Ocenjevanje parametrov:** zadostnost, kompletnost, nepristranskost, cenilke z enakomerno najmanjšo disperzijo, Rao-Cramérjeva meja, metoda največjega verjetja, metoda minimax, asimptotične lastnosti cenilk.

**Preizkušanje domnev:** Osnove (neslučajne in slučajne domneve, napake pri preizkušanju, moč preizkusa). Enakomerno najmočnejši preizkusi, Neyman-Pearsonova lema, preizkušanje v splošnih parametričnih modelih, preizkušanje na podlagi razmerja verjetij, Wilksov izrek, preizkušanje v neparametričnih modelih.

**Območja zaupanja:** Konstrukcija, pivotne količine, lastnosti, asimptotična območja zaupanja. Konstrukcija intervalov zaupanja s bootstrap metodo.

**Multivariatne metode:** Metoda glavnih komponent, faktorska analiza, diskriminantna analiza, razvrščanje.

**Osnove Bayesove statistike** Bayesova formula, podatki, verjetje, apriorne in aposteriorne porazdelitve, konjugirani pari porazdelitev, ocenjevanje parametrov v Bazseovi statistiki, preizkušanje domnev v Bayesovem okviru.

**Parameter estimation:** consistency, completeness, unbiased estimators, efficient estimators, best linear estimator, Rao-Cramer boundary, maximum likelihood method, minimax method, asymptotical properties of estimators.

**Testing of hypotheses:** Fundamentals (probablistic and nonprobablistic hypotheses, types of errors, best tests). Neyman-Pearson lemma, uniformly most powerful tests, test in general parametric models, Wilks theorem, non-parametric tests.

**Confidence intervals:** Constructions, pivots, properties of confidence regions, asymptotic properties, the bootstrap.

**Multivariate analysis:** Principal component analysis, factor analysis, discriminant analysis, classification methods.

**Basic Bayesian statistics:** Bayes formula, data, likelihood, apriori and aposteriory distributions, conjugate distributions pairs, Bayesian parameter estimation, Bayesian hypothesis testing.

#### Temeljni literatura in viri / Readings:

- A. Gelman, J.B. Carlin, H.S. Stern, D.B. Rubin: Bayesian Data Analysis. 2nd edition, Chapman & Hall, 1995.
- J. Rice: *Mathematical Statistics and Data Analysis*, Second edition, Duxbury Press, 1995.
- G.G. Roussas: *A course in mathematical statistics*, 2nd edition, Academic Press, 1997.
- D. R. Cox, D. V. Hinkley: *Theoretical Statistics*, Chapman & Hall/ CRC, 2000.
- S. Weisberg, *Applied Linear Regression*: 3rd edition, Wiley, 2005.
- K. V. Mardia, J. T. Kent, J. M. Bibby: *Multivariate Analysis*, Academic Press, 1979.

**Cilji in kompetence:**

**Objectives and competences:**

Pri predmetu bi postavili teoretične osnove statističnega modeliranja in obdelali osnovne sklope statističnega razmišljanja. Nekaj globlje matematično znanje je potrebno za dobro utemeljeno uporabo statistike. **Spoznali bomo tudi osnove Bayesove statistike.**

Theoretical basis for the statistical modeling will be presented. Deeper mathematical methods are needed for well grounded statistical applications. **Fundamentals of Bayesian analysis will be presented.**

**Predvideni študijski rezultati:**

**Znanje in razumevanje:**

Razumevanje pojma statističnega modela in matematičnega ozadja modeliranja, ocenjevanja in testiranja statističnih modelov.

**Uporaba:**

Statistika je eno najbolj uporabnih področij matematike. Študent bo na podlagi samostojnih projektov usposobljen za uporabo statistike na vseh področjih.

**Refleksija:**

Medigra med uporabo, statističnim modeliranjem, povratno informacijo iz drugih ved in spodbude iz uporabe za matematično razmišljanje.

**Prenosljive spretnosti – niso vezane le na en predmet:**

Spretnosti so prenosljive na druga področja matematičnega modeliranja, še najbolj pa je predmet pomemben zaradi svoje neposredne uporabnosti.

**Intended learning outcomes:**

**Knowledge and understanding:**

Understanding of statistical applications to economics, interplay between statistical reasoning and economics.

**Application:**

Statistics is the language of the quantitative economics. On one side, application is immediate, on the other side the knowledge will satisfy to persue doctoral studies in economics.

**Reflection:**

The interplay between application, statistical modelling, economics feedback information, and application stimulation for mathematical reasoning.

**Transferable skills:**

The skills obtained are transferable to other areas of mathematical modelling, but the gist of the course is its immediate applicability.

**Metode poučevanja in učenja:**

predavanja, vaje, 2 samostojna projekta

**Learning and teaching methods:**

lectures, tutorials, 2 individual projects

**Načini ocenjevanja:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- 2 kolokvija namesto izpita iz vaj
- izpit iz vaj
- ustni izpit

Delež (v %) /

Weight (in %)

**Assessment:**

Type (examination, oral, coursework, project):

- written exam or 2 midterm type exams
- oral exam that can be partially

50%  
50%

<ul style="list-style-type: none"> <li>▪ ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)</li> </ul>		<p>replaced by theoretical tests</p> <ul style="list-style-type: none"> <li>▪ grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)</li> </ul>
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**Reference nosilca / Lecturer's references:**

**prof. dr. Mihael Perman**

- M. Blejec, M. Lovrečič-Saražin, M. Perman, M. Štraus: *Statistika*. Piran: Gea College, Visoka šola za podjetništvo, 2003. X, 150 str., graf. prikazi, tabele.
- M. Perman: *Order statistics for jumps of subordinators*, Stoc. Proc. Appl., 46, 267-281 (1993).
- M. Huzak, M. Perman, H. Šikić, Z. Vondraček: *Ruin probabilities and decompositions for general perturbed risk processes*, Ann. Appl. Probab., 2004, vol. 14, no. 3, (2004).

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**prof. dr. Matjaž Omladič**

- M. Omladič: *Na prvem koraku do faktorske analize*. Obz. mat. fiz., 1986, let. 33, št. 1-2, str. 9-23.
- R. E. Hartwig, M. Omladič, P. Šemrl, G. P. H. Styan: *On some characterizations of pairwise star orthogonality using rank and dagger additivity and subtractivity*. Linear Algebra Appl., 1996, let. 237/238, št. 2, str. 499-507.
- M. Omladič, V. Omladič: *More on restricted canonical correlations*. Linear Algebra Appl.. [Print ed.], 2000, vol. 1/3, no. 321, str. 285-293.

## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Verjetnost 2
<b>Course title:</b>	Probability 2

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi
Second cycle master study program Mathematics	none	first or second	first

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2500

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
45		30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Matjaž Omladič, prof. dr. Mihael Perman

<b>Jeziki / Languages:</b>	<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
	<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:** **Prerequisites:**

Vpis v letnik študija

Enrollment into the program

**Vsebina:** **Content (Syllabus outline):**

**Markovske verige v diskretnem času:** Slučajni procesi in markovska lastnost. Teorija markovskih verig. Povezava s teorijo grafov in linearno algebro. Osnovna struktura verig. Časi prvih prehodov in vrnitev. Povrnjiva in minljiva stanja. Poljubno mnogo obiskov stanja. Ergodično obnašanje verige. Limitni izreki. Posebnosti v končnem.

**Markovske verige v zveznem času:** Poissonov tok in Poissonov proces. Rojstni procesi: reševanje enačb Kolmogorova. Zvezna markovska lastnost. Naprejšnje in nazajšnje enačbe Kolmogorova in njihove rešitve. Stacionarna porazdelitev. Obratna pot do markovskih verig. Stabilnost in eksplozije. Diferencialne enačbe in generator polgrupe.

**Uporaba markovskih verig:** Čakalni sistemi (rojstno smrtni čakalni sistem, čakalni sistem M/M/1, osnovni pojmi teorije strežnih sistemov, nekateri pomembni primeri čakalnih sistemov). Metoda Monte Carlo markovskih verig (Bayesova statistika in Monte Carlo simulacije, algoritma Gibbsov vzorčevalnik in Metropolis-Hastings, konvergenca algoritmov, aplikacije v finančni matematiki).

**Discrete time markov chains:** Random processes and Markov property. Markov chain theory. Connections to graph theory and linear algebra. Basic structure of a chain. Times of first passage and first return. Recurrent and transient states. Infinitely many visits of a state. Ergodic behaviour of a chain. Limit theorems. Specific results for the case of finite number of states.

**Continuous time markov chains:** Poisson flow and Poisson process. Birth processes: solving Kolmogorov equations. Continuous time Markov property. Forward and backward Kolmogorov equations and their solutions. Stationary distribution. Reverse approach. Stability and explosions. Differential equations and generator of a one-parameter semigroup.

**Applications of markov chains:** Waiting queue systems (birth&death system, M/M/1, introduction into the general theory, some important cases of waiting queue systems). Monte Carlo markov chains (Bayesian statistics and Monte Carlo simulations, Gibbs sampler and Metropolis-Hastings algorithm, convergence of MCMC algorithms, applications in Financial Mathematics).

#### Temeljni literatura in viri / Readings:

- G. Grimmett, D. Stirzaker: *Probability and Random Processes*, 3rd edition, Oxford Univ. Press, Oxford, 2001.
- D. Williams: *Probability with Martingales*, Cambridge Univ. Press, Cambridge, 1995.
- L. C. G. Rogers, D. Williams: *Diffusions, Markov Processes, and Martingales I : Foundations*, 2nd edition, Cambridge Univ. Press, Cambridge, 2000.
- J. R. Norris: *Markov Chains*, Cambridge Univ. Press, Cambridge, 1999.
- S. I. Resnick: *Adventures in Stochastic Processes*, Birkhäuser, Boston, 1992.

#### Cilji in kompetence:

Pri predmetu obravnavamo vrsto posebnih verjetnostnih vsebin, pri katerih ni potrebno globoko teoretično predznanje, so pa pomembne za uporabo. Poudarek je predvsem na ergodični teoriji, tako v diskretnem kot zveznem času. Uporabe vključujejo teorijo čakalnih sistemov ter MCMC metode.

#### Objectives and competences:

The course provides a certain number of probability themes that do not need deep theoretical knowledge. However they are important in view of applications. The emphasis is on ergodic theory, both in discrete and continuous time. Applications include waiting queue systems and MCMC methods.



**Predvideni študijski rezultati:****Znanje in razumevanje:**

Spoznavanje nekaterih najpomembnejših aplikacij verjetnosti.

**Intended learning outcomes:****Knowledge and understanding:**

The knowledge of some of the most important applications of probability is acquired.

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations

**Načini ocenjevanja:**

Delež (v %) /

Weight (in %)

**Assessment:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- izpit iz vaj, ki ga je možno nadomestiti z 2 kolokvijema
- izpit iz teorije, ki ga je možno delno nadomestiti s teoretičnimi testi
- ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)

50%

50%

Type (examination, oral, coursework, project):

- written exam or 2 midterm type exams
- oral exam that can be partially replaced by theoretical tests
- grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:****prof. dr. Matjaž Omladič**

- M. Omladič: *Na prvem koraku do faktorske analize*. Obz. mat. fiz., 1986, let. 33, št. 1-2, str. 9-23.
- M. Omladič, V. Omladič: *Optimal solutions to the problem of restricted canonical correlations*. V: The International Conference on Measurement and Multivariate Analysis, May 11-14, 2000, Alberta, Canada : proceedings. Volume Two. Alberta: ICMMA, 2000, str. 238-240.
- M. Omladič, V. Omladič: *More on restricted canonical correlations*. Linear Algebra Appl., 2000, vol. 321, no. 1-3, str. 285-293.

**prof. dr. Mihael Perman**

- M. Perman, J. Pitman, M. Yor: *Size-Biased Sampling of Poisson Processes and Excursions*, Prob. Theory. Rel. Fields, 92, 21-32 (1992).
- M. Perman, J. Wellner: *On the distribution of Brownian areas*, Ann. Appl. Probab., 6, no. 4.,

(1996), 1191-1111.

- M. Perman: *An excursion approach to Ray-Knight theorems for perturbed Brownian motion*, Stoch. Proc. Appl., 63,(1998), 67-74.

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

**Predmet:** Matematični seminar  
**Course title:** Mathematical seminar

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	drugi	prvi in drugi
Second cycle master study program Mathematics	none	second	first and second

**Vrsta predmeta / Course type** obvezni predmet/compulsory course

**Univerzitetna koda predmeta / University course code:** še ni določena/not assigned yet

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30					60	3

**Nosilec predmeta / Lecturer:** doc. dr. Matjaž Konvalinka, doc. dr. Primož Moravec

**Jeziki / Languages:**  
**Predavanja / Lectures:** slovenski/Slovene, angleški/English  
**Vaje / Tutorial:** slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija

**Prerequisites:**

Enrollment into the program

**Vsebina:**

**Content (Syllabus outline):**

Predmet je sestavljen iz treh delov.

Na srečanjih z matematiki študenti poslušajo predavanja diplomiranih matematikov, ki delajo v gospodarstvu. Predavatelji na kratko predstavijo svojo študijsko in poklicno pot in se po predavanju s študenti pogovorijo. Na srečanja so vabljeni matematiki s čim bolj različnimi profili, da študenti dobijo čim boljši vpogled v to, kakšne možne kariere so jim na voljo.

Srečanja z vodjo seminarja so namenjena predstavitev magistrskih del in praks. S predstavitev magistrskega dela se študent bolj poglobi v svojo izbrano temo in izboljšuje svoje sposobnosti podajanja matematične snovi zahtevnejšemu občinstvu; poslušalci podrobneje spoznajo novo matematično področje. Predstavitve prakse pa spodbujajo študente, da se tudi sami odločijo za praktično usposabljanje in s tem izboljšajo svoje zaposlitvene možnosti.

Organizirane so še delavnice (s pomočjo Kariernih centrov Univerze v Ljubljani), na katerih se študenti naučijo pisanja življenjepisa in se pripravijo na iskanje zaposlitve in razgovor za delovno mesto.

The course consists of three parts.

At meetings with mathematicians the students attend lectures of mathematicians who have chosen a career in industry. The lecturers present their careers, and converse with the students after their lecture. Mathematicians with a wide spectrum of careers are invited, in order for the students to get a better understanding of their career options.

At meetings with the seminar organizer, the students present their Master's theses and the results of their internships. By presenting his or her Master's thesis, the student gains a deeper understanding of the subject and improves his or her presentation skills; the listeners learn more about the chosen area of mathematics. Presentations of internships encourage other students to improve their career potential by finding a work-study as well.

Additional workshops are organized (with the help of the Career Centers of the University of Ljubljana) to help students write a CV and to be better prepared for a job hunt and the first job interview.

#### **Temeljni literatura in viri / Readings:**

Članki v raziskovalnih revijah in znanstvene monografije, ki jih študentje potrebujejo pri pisanju svojega magistrskega dela.

#### **Cilji in kompetence:**

#### **Objectives and competences:**

Študent spozna delo matematika, izpopolni sposobnost predstavitve svojega dela, pripravi se na stik z delodajalcem.

The student learns more about work done by mathematicians, improves his or her presentation skills, becomes better prepared for the first contact with potential employers.

**Predvideni študijski rezultati:**

**Znanje in razumevanje:** Poznavanje osnovnih načel pisanja življenjepisa in prijave na razpisano delovno mesto, sposobnost predstavitve svojega dela

**Uporaba:** Pridobljene informacije in spretnosti bodo uporabne pri iskanju zaposlitve in stiku z delodajalci

**Refleksija:** Razumevanje možnosti zaposlitve na osnovi predstavljenih primerov.

**Prenosljive spretnosti – niso vezane le na en predmet:** Na osnovi predstavitev primerov zaposlitev matematikov študent dobi jasnejšo sliko o svoji bodoči poklicni karieri.

**Intended learning outcomes:**

**Knowledge and understanding:** Preparation of CV and job applications, oral presentation of one's work

**Application:** Information and skills obtained are useful for finding employment and contact with employers.

**Reflection:** Understanding career options based on presentations of workers with a degree in Mathematics

**Transferable skills:** A better understanding of their career options.

**Metode poučevanja in učenja:**

predavanja matematikov, zaposlenih v praksi, predavanja strokovnjakov Kariernih centrov UL, študentske predstavitve tem za magistrsko delo, študentske predstavitve opravljenega dela pri praksi

**Learning and teaching methods:**

Lectures of mathematicians who work in the industry, lectures prepared by Career centers of UL, student presentations of Master's theses, student presentations of internships

Delež (v %) /

**Načini ocenjevanja:**

Weight (in %)

**Assessment:**

Opravljen/neopravljen

Pass/fail

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**Reference nosilca / Lecturer's references:**

- M. Konvalinka, I. Pak: Triangulations of Cayley and Tutte polytopes, *Adv. Math.*, Vol. 245 (2013), 1-33
- M. Konvalinka: Skew quantum Murnaghan-Nakayama rule, *J. Algebraic Combin.*, Vol. 35 (4) (2012), 519-545
- M. Konvalinka: Divisibility of generalized Catalan numbers, *J. Combin. Theory Ser. A*, Vol. 114 (6) (2007), 1089-1100
- P. Moravec: Unramified Brauer groups of finite and infinite groups, *Amer. J. Math.* 134 (2012), 1679-1704.
- P. Moravec: On the Schur multipliers of finite p-groups of given coclass, *Israel J. Math.* 185 (2011), 189-205.
- P. Moravec: On pro-p groups with potent filtrations, *J. Algebra* 322 (2009), no. 1, 254-258.

## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Astronomija
<b>Course title:</b>	Astronomy

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi in drugi
Second cycle master study program Mathematics	none	first or second	first and second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2721

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
60		30			120	7

**Nosilec predmeta / Lecturer:** prof. dr. Tomaž Zwitter

<b>Jeziki / Languages:</b>	<b>Predavanja / Lectures:</b>	Slovensko/Slovene
	<b>Vaje / Tutorial:</b>	Slovensko/Slovene

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik.

**Prerequisites:**

Enrollment in academic year.

**Vsebina:** Content (Syllabus outline):

**Zgodovinski uvod:** osnove koledarja, mrki, oblika, velikost in razdalje Zemlje, Lune in Sonca, razdalje v Osončju, spreminjanje hitrosti vrtenja Zemlje, prestopna sekunda.

**Orientacija po nebu:** koncept nebesne krogle, izreki za krogelni trikotnik, izračun višine in azimuta, trenutka vzhoda/zahoda, kulminacije, časa nad obzorjem, uvedba zvezdnega časa, popravki: lom, aberacija, precesija, paralaksa, lastno gibanje.

**Navidezno gibanje Sonca:** koordinate, pravi in srednji Sončev čas, obsevanost, nastop letnih časov.

**Astronomski teleskopi:** leča, zrcalo, sestavljanje 2 elementov, tipi teleskopov, zbiralna moč, merilo slike, svetlost slike, globinska ostrina, povečava, postavitve teleskopov.

**Digitalni detektorji:** njihova velikost, prezentacije slik, fotometrični filtri.

**Astronomske magnitude:** navidezna in absolutna magnituda, osnovna obdelava fotometričnih opazovanj.

**Osnove astronomske spektroskopije:** spektrograf, merljive količine.

**Sonce kot tipična zvezda:** masa Zemlje in Sonca, njuna povprečna gostota, izsev, efektivna temperatura, površinski težnostni in rotacijski pospešek.

**Struktura Soncu podobnih zvezd:** hidrostatično ravnovesje, dinamični čas, središčni tlak in temperatura, utemeljitev privzetka idealnega plina, politropni model, virialni teorem, termični čas, prozornost snovi, ocena proste poti fotonov, sevalni in konvekcijski prenos energije.

**Starost zvezd:** primer Zemlje in Sonca, jedrske

**Historical introduction:** basis of calendar, eclipses, size and distances of the Earth, Moon and Sun, distances in the Solar system, rotational period of the Earth, leap second.

**Positional astronomy:** concept of celestial sphere, spherical trigonometry, calculation of altitude and azimuth, culmination, time above horizon, sidereal time, effects of atmospheric refraction, aberration, precession, parallax and proper motion.

**Solar apparent motion:** coordinates, mean and true Solar time, illumination, occurrence of seasons.

**Astronomical telescopes:** lens, mirror, combining the two elements, types of telescopes, light collecting power, image scale, image brightness, depth of field, magnification, telescope mounts.

**Digital detectors:** their size, image presentation, photometric filters.

**Astronomical magnitudes:** apparent and absolute magnitude, basics of reduction of photometric observations.

**Basics of astronomical spectroscopy:** spectrograph, measurable quantities.

**The Sun as a typical star:** mass of the Earth and Sun, their average density. Solar luminosity, effective temperature, surface gravity and rotational acceleration.

**Structure of Solar-like stars:** hydrostatic equilibrium, dynamical time-scale, central pressure and temperature, justification of calculation with ideal gas, polytropic model, virial theorem, thermal time-scale, optical opacity, free path of photons, energy transport with radiation and convection.

**Ages of stars:** the case of the Earth and the Sun,



reakcije, njihova stabilnost in nuklearni čas, odvisnost izseva od mase za Soncu podobne zvezde, Eddingtonov izsev.

**Razvoj zvezd:** nastanek in Jeansova masa, faza orjakinj, končne faze razvoja, odvisnost razvoja od mase.

**Opazovanje razvoja:** Hertzsprung-Russellov diagram, zvezdne kopice, merjenje razdalj, spektri kemijskih elementov v zvezdnih atmosferah v odvisnosti od temperature, kemične sestave, radialne hitrosti in težnostnega pospeška, prekrivalne spektroskopske dvojne zvezde, opazovanje končnih stopenj razvoja zvezd.

**Medzvezdni prostor:** absorpcija v plinu in prahu, vrste meglic, opazljive lastnosti.

nuclear fusion, its stability and timescale, dependence of luminosity on mass for Solar-like stars, Eddington luminosity.

**Evolution of stars:** formation and Jeans mass, giant phase, final stages of evolution, dependence of evolution on mass.

**Observation of stellar evolution:** Hertzsprung-Russell diagram, star clusters, distance measurement, spectra of chemical elements in stellar atmospheres, their dependence on temperature, chemical composition, radial velocity and gravity, eclipsing spectroscopic binaries, observations of final stages of stellar evolution.

**Interstellar medium:** absorption in gas and dust, types of nebulae, observable properties.

#### Temeljni literatura in viri / Readings:

1. H. Karttunen et al.: *Fundamental Astronomy*, Fifth Edition, Springer, 2007.
2. R.M. Green: *Spherical astronomy*, Cambridge University Press, 1993.
3. F. H. Shu, *The Physical Universe*. University Science Books, 1982.
4. A. Čadež: *Fizika zvezd*, DMFA, 1984.
5. Gordon Walker: *Astronomical observations: an optical perspective*. Cambridge University Press, 1987.
6. T. Zwitter: *Pot skozi vesolje*, Modrijan, 2002.
7. *Presekova zvezdna karta*, DMFA, 2000; *Spikina vrtljiva zvezdna karta*.
8. *Naše nebo, astronomske efemeride*, DMFA, 2012-.

**Cilji in kompetence:**

**Objectives and competences:**

Seznanitev z osnovami astronomskih opazovanj, s fenomenologijo objektov in pojavov v vesolju ter fizikalno sliko razumevanja vesolja in raznolikostjo pri tem uporabljenih pristopov, ki se vklapljajo v širšo sliko našega vedenja o svetu v katerem živimo.

Mastering of basics of astronomical observations, phenomenology of objects and processes in the Universe and understanding of the Universe using laws of physics. This connects to a wider picture of the world we live in.

**Predvideni študijski rezultati:**

**Znanje in razumevanje:** Razumevanje osnov fizikalne slike vesolja in kako smo do nje prišli, zavedanje omejitev našega znanja.

**Uporaba:** Abeceda znanja astronomije in astrofizike, primeri univerzalnosti fizikalnega pristopa in inventivnosti pri meritvah v težavnih pogojih.

**Refleksija:** Vesolje je polno zelo raznovrstnih okolij, ki pa jih lahko razumemo s fizikalno matematičnimi pristopi, ki smo jih razvili.

**Prenosljive spretnosti - niso vezane le na en predmet:** Kritično vrednotenje informacij, primeri obdelave digitalnih podatkov, pridobivanje spretnosti uporabe matematično-fizikalnega znanja pri reševanju odprtih problemov.

**Intended learning outcomes:**

**Knowledge and understanding:** Understanding of the basics of the physical picture of the Universe, its foundations, and acknowledging its present limitations.

**Application:** Basic knowledge of astronomy and astrophysics, experience of the universality of physics, experience of inventive measurements in difficult conditions.

**Reflection:** The Universe contains most diverse environments, but it can be understood with physics and mathematics as we know it.

**Transferable skills:** Critical evaluation of information, examples of manipulation of digital data, experience in the use of apparatus of mathematics and physics to solve open problems.

**Metode poučevanja in učenja:**

Predavanja, računske in praktične vaje, izdelava projektnih nalog.

**Learning and teaching methods:**

Lectures, computational and practical exercises, astro-lab reports.

**Načini ocenjevanja:**

Delež (v %) /

Weight (in %) **Assessment:**

<ul style="list-style-type: none"> <li>• pisni izpit oziroma kolokvij, izdelava projektne naloge,</li> <li>• ustni izpit.</li> </ul> <p>Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)</p>	<p><b>50</b></p> <p><b>50</b></p>	<ul style="list-style-type: none"> <li>▲ written exam, presentation of astro-lab report,</li> <li>▲ oral exam.</li> </ul> <p>Grades: 1-5 (fail), 6-10 (pass), according to University rules.</p>
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#### Reference nosilca / Lecturer's references:

1. Žerjal, M., Zwitter, T., Matijević, G., et al.: Chromospherically Active stars in the Radial Velocity Experiment survey (RAVE). I. The Catalog. *Astrophysical Journal*, Volume 776, Issue 2, article id. 127, 12 pp. (2013).
2. Kos, J., Zwitter, T.: Properties of Diffuse Interstellar Bands at Different Physical Conditions of the Interstellar Medium. *Astrophysical Journal*, Volume 774, Issue 1, article id. 72, 16 pp. (2013).
3. Matijević, G., Zwitter, T., Bienayme, O., et al.: Exploring the Morphology of RAVE Stellar Spectra. *The Astrophysical Journal Supplement*, Volume 200, Issue 2, article id. 14, 14 pp. (2012).
4. Zwitter, T., Matijević, G., Breddels, M., et al.: Distance determination for RAVE stars using stellar models . II. Most likely values assuming a standard stellar evolution scenario. *Astronomy and Astrophysics*, Volume 522, id.A54, 15 pp. (2010).
5. Zwitter, T., Siebert, A., Munari, U., et al.: The Radial Velocity Experiment (rave): Second Data Release, *The Astronomical Journal*, Volume 136, Issue 1, pp. 421-451 (2008).
6. Prša, A., Zwitter, T.: A Computational Guide to Physics of Eclipsing Binaries. I. Demonstrations and Perspectives. *The Astrophysical Journal*, Volume 628, Issue 1, pp. 426-438 (2005).
7. Zwitter, T., Castelli, F., Munari U.: An extensive library of synthetic spectra covering the far red, RAVE and GAIA wavelength ranges. *Astronomy and Astrophysics*, v.417, p.1055-1062 (2004)
8. Munari, U., Zwitter, T.: Equivalent width of Na I and K I lines and reddening. *Astronomy and Astrophysics*, v.318, p.269-274 (1997).
9. D'Odorico, S., Oosterloo, T., Zwitter, T., Calvani, M.: Evidence that the compact object in SS433 is a neutron star and not a black hole. *Nature*, vol. 353, Sept. 26, 1991, p. 329-331 (1991).

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

**Predmet:** Delovna praksa 1  
**Course title:** Work experience 1

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	drugi	drugi
Second cycle master study program Mathematics	none	drugi	drugi

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** še ni določena/not assigned yet

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
0	15	0			165	6

**Nosilec predmeta / Lecturer:** doc. dr. Matjaž Konvalinka, doc. dr. Primož Moravec

**Jeziki / Languages:**  
**Predavanja / Lectures:** slovenski/Slovene, angleški/English  
**Vaje / Tutorial:** slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija.

**Prerequisites:**

Enrollment into the program

**Vsebina:**

**Content (Syllabus outline):**

V dogovoru s strokovnimi sodelavci v podjetjih bomo na Oddelku za matematiko pripravili seznam možnih podjetij in ustanov, na katerih lahko študenti opravljajo praktično usposabljanje. Usposabljanje bo koordinirano in pripravljeno v sodelovanju med učiteljem na fakulteti in zaposlenimi v podjetjih.

Department of Mathematics will prepare a list of possible providers of working experience (based on previous agreement). Working experience will be planned and coordinated by the lecturer and the responsible person from the company.

#### Temeljna literatura in viri / Readings:

- Navodila za delo/work instructions.
- Priročniki/manuals.
- Notranji akti organizacije, ki nudi praktično usposabljanje/ Organization's internal acts.

#### Cilji in kompetence:

Študenti se ob praktičnem usposabljanju povežejo pridobljeno znanje s prakso. Pridobijo praktične izkušnje v delovnem okolju. Spoznajo se s problematiko sodobnega informacijskega ali tehnološkega podjetja ali druge ustanove.

#### Objectives and competences:

Students combine working experience and professional knowledge. They acquire practical experiences in the frame of working environment. Students acquire knowledge about modern information or technological company or some other institution.

#### Predvideni študijski rezultati:

**Znanje in razumevanje:** Poznavanje in razumevanje zapletenih odnosov praktičnega sodelovanja matematika v delovnem okolju.

**Uporaba:** Uporaba praktičnih izkušenj pri oblikovanju poklicne poti.

**Refleksija:** Razumevanje praktičnega dela v konkretnem delovnem okolju in uporaba pridobljenega znanja pri praktičnih problemih.

**Prenosljive spretnosti – niso vezane le na en predmet:** Spretnost uporabe matematičnega znanja v delovnem okolju.

#### Intended learning outcomes:

**Knowledge and understanding:** Knowledge and understanding of complicated relationships between a mathematician and working environment.

**Application:** Application of practical experiences into working carrier.

**Reflection:** Understanding of practical work in a particular working environment and application of the academic knowledge for solving practical problems.

**Transferable skills:** Ability of transferring mathematical knowledge into a working environment.

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**Metode poučevanja in učenja:**

praktično usposabljanje

**Learning and teaching methods:**

working experience

<b>Načini ocenjevanja:</b>	Delež (v %) / Weight (in %)	<b>Assessment:</b>
Praktično delo, zaključno poročilo o praktičnem usposabljanju Ocene: opravi/ni opravi (po Statutu UL)	100%	Practice, final report Grading: passed/not passed (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**
**doc. dr. Matjaž Konvalinka:**

- M. Konvalinka: Skew quantum Murnaghan-Nakayama rule. *J. algebr. comb.*, 35 (2012), 519-545.
- M. Konvalinka, I. Pak: Geometry and complexity of O'Hara's algorithm. *Adv. appl. math.*, 42 (2009), 157-175.
- M. Konvalinka: On quantum immanants and the cycle basis of the quantum permutation space. *Ann. comb.* 16 (2012), 289-304.

**doc. dr. Primož Moravec:**

- P. Moravec: Groups with all centralizers subnormal of defect at most two. *J. algebra*, 2013, vol. 374, str. 132-140.
- P. Moravec: Unramified Brauer groups of finite and infinite groups. *Am. j. math.*, 2012, vol. 134, no. 6, str. 1679-1704
- P. Moravec: Groups of order  $p^2$  and their unramified Brauer groups. *J. algebra*, 2012, vol. 372, str. 420-427

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

**Predmet:** Delovna praksa 2  
**Course title:** Work experience 2

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	drugi	drugi
Second cycle master study program Mathematics	none	drugi	drugi

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** še ni določena/not assigned yet

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
0	15	0			165	6

**Nosilec predmeta / Lecturer:** doc. dr. Matjaž Konvalinka, doc. dr. Primož Moravec

**Jeziki / Languages:**  
**Predavanja / Lectures:** slovenski/Slovene, angleški/English  
**Vaje / Tutorial:** slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija.  
Vpis predmeta Delovna praksa 1.

**Prerequisites:**

Enrollment into the program.  
Enrollment into the course Work experience 1.

**Vsebina:**

**Content (Syllabus outline):**

V dogovoru s strokovnimi sodelavci v podjetjih bomo na Oddelku za matematiko pripravili seznam možnih podjetij in ustanov, na katerih lahko študenti opravljajo praktično usposabljanje. Usposabljanje bo koordinirano in pripravljeno v sodelovanju med učiteljem na fakulteti in zaposlenimi v podjetjih.

Department of Mathematics will prepare a list of possible providers of working experience (based on previous agreement). Working experience will be planned and coordinated by the lecturer and the responsible person from the company.

#### Temeljna literatura in viri / Readings:

- Navodila za delo/work instructions.
- Priročniki/manuals.
- Notranji akti organizacije, ki nudi praktično usposabljanje/ Organization's internal acts.

#### Cilji in kompetence:

Študenti se ob praktičnem usposabljanju povežejo pridobljeno znanje s prakso. Pridobijo praktične izkušnje v delovnem okolju. Spoznajo se s problematiko sodobnega informacijskega ali tehnološkega podjetja ali druge ustanove.

#### Objectives and competences:

Students combine working experience and professional knowledge. They acquire practical experiences in the frame of working environment. Students acquire knowledge about modern information or technological company or some other institution.

#### Predvideni študijski rezultati:

**Znanje in razumevanje:** Poznavanje in razumevanje zapletenih odnosov praktičnega sodelovanja matematika v delovnem okolju.

**Uporaba:** Uporaba praktičnih izkušenj pri oblikovanju poklicne poti.

**Refleksija:** Razumevanje praktičnega dela v konkretnem delovnem okolju in uporaba pridobljenega znanja pri praktičnih problemih.

**Prenosljive spretnosti – niso vezane le na en predmet:** Spretnost uporabe matematičnega znanja v delovnem okolju.

#### Intended learning outcomes:

**Knowledge and understanding:** Knowledge and understanding of complicated relationships between a mathematician and working environment.

**Application:** Application of practical experiences into working carrier.

**Reflection:** Understanding of practical work in a particular working environment and application of the academic knowledge for solving practical problems.

**Transferable skills:** Ability of transferring mathematical knowledge into a working environment.



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**Metode poučevanja in učenja:**

praktično usposabljanje

**Learning and teaching methods:**

working experience

<b>Načini ocenjevanja:</b>	Delež (v %) / Weight (in %)	<b>Assessment:</b>
Praktično delo, zaključno poročilo o praktičnem usposabljanju Ocene: opravi/ni opravi (po Statutu UL)	100%	Practice, final report Grading: passed/not passed (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**
**doc. dr. Matjaž Konvalinka:**

- M. Konvalinka: Skew quantum Murnaghan-Nakayama rule. *J. algebr. comb.*, 35 (2012), 519-545.
- M. Konvalinka, I. Pak: Geometry and complexity of O'Hara's algorithm. *Adv. appl. math.*, 42 (2009), 157-175.
- M. Konvalinka: On quantum immanants and the cycle basis of the quantum permutation space. *Ann. comb.* 16 (2012), 289-304.

**doc. dr. Primož Moravec:**

- P. Moravec: Groups with all centralizers subnormal of defect at most two. *J. algebra*, 2013, vol. 374, str. 132-140.
- P. Moravec: Unramified Brauer groups of finite and infinite groups. *Am. j. math.*, 2012, vol. 134, no. 6, str. 1679-1704
- P. Moravec: Groups of order  $p^2$  and their unramified Brauer groups. *J. algebra*, 2012, vol. 372, str. 420-427

## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Matematični modeli v biologiji
<b>Course title:</b>	Mathematical Models in Biology

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type**

izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:**

M2700

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:**

prof. dr. Jasna Prezelj

**Jeziki /  
Languages:**

<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

- Vpis v letnik študija
- Potrebno predznanje linearne algebre, splošne analize, diferencialnih enačb in verjetnosti.
- **Zaželjen** izpit iz Dinamičnih sistemov.

**Prerequisites:**

- Enrollment into the program
- Acquaintance with linear algebra, general analysis, differential equations and probability theory is necessary
- A positive exam in Dynamical systems is **desirable**.

**Vsebina:**

*Osnovni principi matematičnega modeliranja, motivacijski zglede iz biologije.*

*Diskretni modeli populacijske dinamike.*

**Stabilnost v linearnih in nelinearnih sistemih**, Lesliejev matrični model, modeli za eno populacijo, diskretni modeli za več populacij (modeli zajedavstva in sodelovanja, tekmovalni modeli, epidemiološki modeli).

*Verjetnostni modeli v biologiji.* Uporaba verjetnosti v ekologiji (Mendelova dednost, izumiranje linij), Osnovni genetski modeli (Hardy-Weinbergov in Fisher-Haldane-Wrightov zakon), evolucijski modeli.

*Zvezni modeli v biologiji.* Uporaba dinamičnih sistemov v populacijski dinamiki, **stabilnost v linearnih in nelinearnih sistemih** (teorija Ljapunova), različni modeli rasti, **osnove Poincare-Bendixsonove teorije**, modeli tipa plen-plenilec (Lotka-Volterra), modeli simbioze in tekmovanja ter posplošitve, konkretni ekološki in epidemiološki modeli (biološka pestrost, sistemi SIR ipd.), molekularna kinetika (Menten-Michaelis) in osnovni nevrološki modeli (Hodgkin-Huxley, Fitzhugh-Nagumo).

**Content (Syllabus outline):**

*Fundamental principles of mathematical modeling, biological motivation.*

*Discrete models of population dynamics.*

Stability in linear and nonlinear systems, Leslie matricial model, models for a single population, discrete models for interacting populations (models of parasitism and mutualism, competition, and epidemiological models).

*Stochastic models in biology.* Application of probability theory in ecology (Mendelian heritage, lineage extinction), Fundamental genetic models (Hardy-Weinberg and Fisher-Haldane-Wright law), evolutionary models.

*Continuous models in biology.* Application of dynamical systems in population dynamics, stability in linear and nonlinear systems (Lyapunov theory), various growth models, fundamentals of Poincare-Bendixson theory, predator-prey models (Lotka-Volterra), models of symbiosis, of competition, and their generalizations, concrete ecological and epidemiological models (biodiversity, systems of type SIR), molecular kinetics (Menten-Michaelis) and basic neurological models (Hodgkin-Huxley, Fitzhugh-Nagumo).

**Temeljni literatura in viri / Readings:**

- L.J.S. Allen, *An Introduction to Mathematical Biology*, Prentice Hall, New York 2007.
- J.D. Murray: *Mathematical Biology*, Springer, 1993.
- L. Edelstein-Keshet: *Mathematical Models in Biology*, McGraw-Hill, 2005.
- N.F. Britton: *Essential Mathematical Biology*, Springer 2003.
- J. Hofbauer, K. Sigmund: *Evolutionary Game Dynamics*, Cambridge University Press, 1998.
- A.W.F. Edwards: *Foundation of Mathematical Genetics*, Cambridge University Press, 2000.

**Cilji in kompetence:**

Glavni cilj je uporaba doslej osvojenega matematičnega znanja v opisovanju bioloških procesov. Študent bo po tečaju pripravljen na interdisciplinarno delo in sodelovanje z raziskovalci v drugih vedah.

**Objectives and competences:**

The main goal is the application of already obtained mathematical knowledge to the description of biological processes. The student will be prepared to the interdisciplinary work and to the collaboration with experts from other disciplines.

**Predvideni študijski rezultati:****Znanje in razumevanje:**

Razumevanje principov matematičnega modeliranja v naravoslovju. Poznavanje osnovnih bioloških modelov.

**Uporaba:**

Formulacija in reševanje preprostih problemov v biologiji (modeliranje, napovedovanje pojavov).

**Refleksija:**

Preko številnih zgledov študent spoznava uporabnost matematike v naravoslovju.

**Prenosljive spretnosti – niso vezane le na en predmet:**

Zmožnost opisa bioloških (in drugih) procesov v matematičnem jeziku, splošna razgledanost po uporabni matematiki. Razvijanje spretnosti uporabe domače in tuje literature ter različnih računalniških programov.

**Intended learning outcomes:****Knowledge and understanding:**

To achieve understanding of principles of mathematical modeling in science. To be acquainted with basic biological models.

**Application:**

Formulating and solving simple problems in biology (modeling, forecasting of phenomena).

**Reflection:**

Through various examples one begins to appreciate the applicability of mathematics in science.

**Transferable skills:**

One can learn how to describe biological (and other) processes using mathematical language and achieve a general feeling for mathematical applications. The goal is also to develop the skills for using existent literature and various computer programs.

**Metode poučevanja in učenja:**

Predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations

**Načini ocenjevanja:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- predstavitev domače naloge
- izpit iz vaj (2 kolokvija ali pisni izpit)
- ustni izpit

Delež (v %) /

Weight (in %) /

**Assessment:**

Type (examination, oral, coursework, project):

- Presentation of home exercises
- 2 midterm exams instead of written exam, written exam
- oral exam

<p>Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)</p> <ul style="list-style-type: none"> <li>▪ Ocenjevanje delovnega znanja na kolokvijih, pri domačih nalogah, na pisnem delu izpita, ocenjevanje teoretičnega razumevanja na ustnem izpitu. Predvidena sta 2 kolokvija (namesto pisnega izpita), individualna domača naloga, pisni izpit, ustni izpit.</li> </ul>		<p>Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)</p> <p>Estimating working knowledge on two midterm tests, home exercises and possible on the written test as well as estimating theoretical knowledge on the final oral exam.</p>
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**Reference nosilca / Lecturer's references:**

- J. Prezelj, Interpolation of embeddings of Stein manifolds over discrete sets, *Math. Ann.* 326 (2003) 275-296
- J. Prezelj: Weakly holomorphic embeddings of Stein spaces with isolated singularities, *Pacific Journal of Mathematics* 220 (1): (2005) 141-152
- F. Forstnerič, B. Ivarsson, F. Kutschebauch, J. Prezelj, An interpolation theorem for proper holomorphic embeddings, *Math. Ann.* 338 (2007), 545-554
- J. Prezelj: A relative Oka-Grauert principle for holomorphic submersions over 1-convex spaces, *Trans. Amer. Math. Soc.* 362 (2010), 4213-4228

## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Moderna fizika
<b>Course title:</b>	Modern Physics

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2722

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
45		30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Svjetlana Fajfer, prof .dr. Peter Križan

**Jeziki / Languages:**

<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

**Prerequisites:**

Vpis v letnik študija

Enrollment into the program

**Vsebina:**

**Content (Syllabus outline):**

#### Elektromagnetno polje:

- Električna in magnetna polja;
- Integralska in diferencialna oblika Maxwellovih enačb;

- Elektromagnetno valovanje;

#### Posebna teorija relativnosti:

- Transformacija prostor-časa
- Transformacije električnega in magnetnega polja, Maxwellove enačbe v kovariantni obliki

#### Kvantna fizika:

- Valovne lastnosti delcev;
- Schroedingerjeva enačba in probabilistična interpretacija;
- Postulati kvantne fizike, Heisenbergove relacije;
- Harmonični oscilator;
- Vodikov atom;
- Standardni model osnovnih delcev: leptoni in kvarki, osnove umeritvenih teorij elektromagnetne, šibke in močne interakcije.
- Modeli vesolja

#### Electromagnetic field:

- Electric and magnetic fields;
  - Integral and differential form of Maxwell's equations;
  - Electromagnetic waves;
- Special theory of relativity:
- Transformation of space-time
  - Transformations of electric and magnetic fields, covariant form of Maxwell's equations
- Quantum physics:
- Wave properties of particles;
  - Schrödinger equation and the probabilistic interpretation;
  - Postulates of quantum physics, Heisenberg relations;
  - Harmonic oscillator;
  - Hydrogen atom;
  - The Standard Model of elementary particles: quarks and leptons, the basics of gauge theories of electromagnetic, weak and strong interactions.
  - Models of the Universe

#### Temeljna literatura in viri / Readings:

- J. Strnad: Fizika 3 in Fizika 4
- J. Vanderlinde, Classical electromagnetic theory, Wiley, New York, 1993
- F. Schwabl, Quantum Mechanics, Springer, Berlin, 1991

#### Cilji in kompetence:

Študent spozna osnovne zakonitosti na področju klasične elektrodinamike, posebne teorije relativnosti, kvantne fizike, fizike osnovnih delcev, ter modelov vesolja. Predmetno specifične kompetence: poznavanje in razumevanje konstrukcije teorije elektromagnetnega polja, posebne teorije relativnosti kvantne fizike in interakcij med osnovnimi delci. Sposobnost za reševanje

#### Objectives and competences:

Students learn about the basic laws of classical electrodynamics, special relativity, quantum physics, elementary particle physics, as well as models of the universe. Subject-specific competencies: knowledge and understanding of the of electromagnetic field theory, special theory of relativity and quantum physics interactions between elementary particles and their bound states; the ability to

konkretnih problemov s teh področij. Sposobnost povezovanja teoretičnih napovedi in meritev. Kritično ovrednotenje in uporaba novih spoznanj na področju moderne fizike.

solve practical problems in these areas; the ability to link the theoretical predictions and measurements; critical evaluation and application of new knowledge in the field of modern physics.

**Predvideni študijski rezultati:**

**Znanje in razumevanje:** Pridobitev osnovnega znanja teorij moderne fizike. Sposobnost povezovanja teoretičnih napovedi in meritev.

**Uporaba:** Razumevanje fizikalnih zakonitosti moderne fizike in vloge matematičnih pristopov.

**Refleksija:** Kritično ovrednotenje teoretičnih napovedi z rezultati meritev.

**Prenosljive spretnosti – niso vezane le na en predmet:** Sposobnost razumevanja pojavov ter razlaganja in vrednotenja rezultatov meritev.

**Intended learning outcomes:**

**Knowledge and understanding:** Acquire basic knowledge on theories of modern physics. The ability to link the theoretical predictions and measurements.

**Application:** Understanding of the laws of modern physics and applications of mathematical approaches.

**Reflection:** A critical assessment of the theoretical predictions with the results of the measurements.

**Transferable skills:** Ability to understand the phenomena and the interpretation and evaluation of the results of measurements.

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije, seminarji

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations, seminars

Delež (v %) /

Weight (in %)

**Načini ocenjevanja:**

**Assessment:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):		Type (examination, oral, coursework, project):
▪ izpit iz vaj (2 kolokvija ali pisni izpit)	50%	▪ 2 midterm exams instead of written exam, written exam
▪ ustni izpit	50%	▪ oral exam



Ocene: 1-5 (negativno), 6-10 (pozitivno)  
(po Statutu UL)

Grading: 1-5 (fail), 6-10 (pass) (according  
to the Statute of UL)

**Reference nosilca / Lecturer's references:**

prof. dr. Peter Križan

- I. Adachi et al. [Belle Collaboration], Measurement of  $B^- \rightarrow \tau^- \nu$  with a Hadronic Tagging Method Using the Full Data Sample of Belle, Phys. Rev. Lett. 110 (2013) 131801
- I. Adachi et al., Precise measurement of the CP violation parameter  $\sin 2\phi_1$  in  $B^0 \rightarrow c \bar{c} K^0$  decays, Phys. Rev. Lett. 108 (2012) 171802
- P. Križan, Overview of particle identification techniques, Nucl. Instrum. Meth. A706 (2013) 48.

prof. dr. Svjetlana Fajfer

- S. Fajfer, J. F. Kamenik and I. Nišandžić, On the  $B \rightarrow D^* \tau \nu$  Sensitivity to New Physics, Phys. rev. D Part. fields gravit. cosm. 85 (2012) 094025
- I. Doršner, S. Fajfer, J. F. Kamenik and N. Kosnik, Light colored scalars from grand unification and the forward-backward asymmetry in  $t \bar{t}$  production. Phys. rev. D Part. fields gravit. cosm. 81 (2010) 055009.
- S. Fajfer, J. F. Kamenik, I. Nišandžić and J. Zupan, Implications of Lepton Flavor Universality Violations in B Decays, Phys. Rev. Lett. 109 (2012) 161801.

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

**Predmet:** Teoretična fizika  
**Course title:** Theoretical physics

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type**

**Univerzitetna koda predmeta / University course code:**

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
60		30			120	7

**Nosilec predmeta / Lecturer:**

**Jeziki / Languages:**  
**Predavanja / Lectures:**   
**Vaje / Tutorial:**

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik.

**Prerequisites:**

Enrollment status.

**Vsebina:**

**Content (Syllabus outline):**

**Kratek pregled fizike:** Zgodovinski pregled vseh fizikalnih področjih od fizike osnovnih delcev do kozmologije.

**Osnovne konstante v fiziki:** Fizikalne količine. Simetrije fizikalnih zakonov. Področja veljavnosti klasične nerelativistične in relativistične mehanike ter kvantne relativistične in nerelativistične mehanike.

**Osnove klasične mehanike:** Princip najmanjše akcije, Lagrangeove enačbe gibanja in ohranitveni zakoni. Siplanje delcev na centralno simetričnih potencialih. Siplalni preseki in doseg potencialov. Majhne oscilacije in harmonski oscilatorji. Gibanje togega telesa.

**Posebna teorija relativnosti:** Princip relativnosti in metrika Minkowskega. Lorentzove transformacije. Ohranitveni zakoni v relativistični mehaniki. Skalarji, vektorji, tenzorji. Relativistične enačbe gibanja v eni dimenziji.

**Elektrodinamika:** Delec v elektromagnetnem polju, vektor četverec elektromagnetnega polja. Prosto elektromagnetno polje, tenzor elektromagnetnega polja in Lagrangeova gostota. Maxwellove enačbe za prosto polje in za polje z izviri. Primeri.

**Splošna teorija relativnosti:** Gravitacijska sila. Newtonova mehanika in homogeni model vesolja. Ekvivalenčni princip. Kvalitativni pregled Einsteinovih enačb gibanja. Primeri.

**Short overview of physics:** Historic overview of all physics fields, from particle physics to cosmology.

**Basics constants in physics:** Physics quantities. Symmetries of laws of physics. Validity of classical non-relativistic and relativistic mechanics. Validity of relativistic and non-relativistic quantum mechanics.

**Basics classical mechanics:** Principle of least action, Lagrange equations of motion and conservation laws. Scattering of particles on spherically symmetric potentials. Scattering cross section. Small oscillations of harmonic oscillators. Rigid body.

**Special theory of relativity:** Principle of relativity and Minkowski metric. Lorentz transformations. Conservation laws in relativistic mechanics. Scalars, vectors, tensors. Relativistic equations in one dimension. Examples.

**Electrodynamics:** Particles in electromagnetic field, four-vector of electromagnetic field. Free field, tensor of field, Lagrange density. Maxwell's equations for fields with sources. Examples.

**General theory of relativity:** Gravitational force. Newton mechanics and homogeneous model of Universe. Equivalence principle. Qualitative overview of Einstein equations of motion. Examples.

#### **Temeljni literatura in viri / Readings:**

- M. Mizushima: Theoretical physics, Wiley, New York, 1972.
- A. S. Kompaneec: Teoretičeskaja fizika, Moskva, 1961.
- L.D.Landau, E.M.Lifshitz: Mechanics and electrodynamics, Butterworth Heineman, 1996
- The Feynman lectures on physics, Addison - Wesley, Massachusetts, 1966.

#### **Cilji in kompetence:**

#### **Objectives and competences:**

**Cilji:** Razumevanje osnovnih fizikalnih teorij. Uporaba matematičnega opisa za razumevanje fizikalnih pojavov ter samostojnega reševanja preprostih fizikalnih problemov. Analiza problema, iskanja enačb gibanja in robnih pogojev za dani problem, razpoznavanja simetrij, reševanje enačb ter interpretacije rešitev.

**Kompetence:**

- Teoretično razumevanje.
- Sposobnost modeliranja in reševanja fizikalnih problemov.
- Globlje poznavanje teorije kvantne mehanike.
- Sposobnost iskanja po strokovni literaturi.

**Objectives:** Understanding of basic physical theories. Application of mathematical description to understand physical phenomena. The analysis of problems, search for equations of motion, boundary conditions, symmetries, solving and critical interpretation of solutions.

**Acquired competence:**

- Theoretical understanding.
- Modeling and solving the models of physical systems.
- In depth knowledge of the quantum mechanics.
- Acquired capacity to do independent literature search.

**Predvideni študijski rezultati:**

**Znanje in razumevanje:** Sposobnost analize preprostih fizikalnih problemov, jih opisati z matematičnimi modeli in interpretirati rezultate.

**Uporaba:** Vsaj nekaj pri tem predmetu pridobljenega znanja bo kot učitelj lahko prenesel na dijake. Širše poznavanje problemov pa mu bo pomagalo, da bo dijakom predstavil realne probleme.

**Refleksija:** Uporaba že osvojenih matematičnih znanj v fiziki pomaga študentu poglobiti razumevanje osnovnih matematičnih metod.

**Prenosljive spretnosti – niso vezane le na en predmet:** Sposobnost opaziti problem, ga analizirati, poiskati način reševanja problema in razpoznati, ali je rešitev, ki jo je našel, smiselna.

**Intended learning outcomes:**

**Knowledge and understanding:** Analysis of physical problems, description with mathematical models and interpretation of results.

**Application:** A broader understanding of problems will help to motivate students to connect mathematical formalism to practical problems.

**Reflection:** Application of mathematical methods in physics will help to get a deeper understanding of mathematical background.

**Transferable skills:** Ability to spot a problem, to analyse it, to find a method to solve it and finally, critically to discuss the solution.

**Metode poučevanja in učenja:**

Predavanja, individualne konzultacije, računske vaje, domače naloge.

**Learning and teaching methods:**

Lectures, numerical exercises, homeworks and consultations.

Delež (v %) /

**Načini ocenjevanja:**

Weight (in %)

**Assessment:**

<ul style="list-style-type: none"> <li>• 2 pisna kolokvija iz vaj, ustni izpit.</li> </ul>	<b>50%</b>	<ul style="list-style-type: none"> <li>• 2 tests on numerical exercises or a written examination, oral examination.</li> </ul>
<ul style="list-style-type: none"> <li>• Ocene 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL).</li> </ul>	<b>50%</b>	<ul style="list-style-type: none"> <li>• Grading: 1-5 (negative), 6-10 (positive).</li> </ul>

**Reference nosilca / Lecturer's references:**

1. A non-adiabatically driven electron in a quantum wire with spin-orbit interaction, T. Čadež, J.H. Jefferson, and A. Ramšak, *New J. Phys.* **15**, 013029 (2013).
2. Geometric analysis of entangled qubit pairs, A. Ramšak, *New J. Phys.* **13**, 103037 (2011).
3. Geometrical view of quantum entanglement, A. Ramšak, *Europhys. Lett.* **96**, 40004 (2011).
4. Spin qubits in double quantum dots - entanglement versus the Kondo effect, A. Ramšak, J. Mravlje, R. Žitko, and J. Bonča, *Phys. Rev. B* **74**, 241305(R) (2006).
5. Entanglement of two delocalized electrons, A. Ramšak, I. Sega, and J.H. Jefferson, *Phys. Rev. A* **74**, 010304(R) (2006).

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

**Predmet:** Izbrana poglavja iz optimizacije  
**Course title:** Topics in optimization

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2601

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Vladimir Batagelj, prof. dr. Sergio Cabello, prof. dr. Bojan Mohar, prof. dr. Emil Žagar

**Jeziki / Languages:**  
**Predavanja / Lectures:** slovenski/Slovene, angleški/English  
**Vaje / Tutorial:** slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija

**Prerequisites:**

Enrollment into the program

**Vsebina:**

**Content (Syllabus outline):**

Predavatelj izbere nekatere pomembne teme s področja optimizacije, kot so na primer:

- Matematične osnove metod notranjih točk.
- Zahtevnejši problemi kombinatorične optimizacije.
- Celoštevilsko programiranje.
- Iterativne metode v optimizaciji.
- Hevristike, **evolucijsko in** genetsko programiranje.
- **Praktična uporaba** optimizacijskih metod v financah, ekonomiji, **logistiki**, telekomunikacijah ipd.
- Stohastično programiranje, itd.

The lecturer selects some important topics in optimization, such as:

- Mathematical foundations of interior-point methods.
- Advanced problems of combinatorial optimization.
- Integer programming.
- Iterative methods in optimization.
- Heuristics, evolutionary and genetic programming.
- Applications of optimization methods in finance, economy, logistics, telecommunications, etc.
- Stochastic programming, etc.

### Temeljni literatura in viri / Readings:

- S. Boyd, L. Vandenberghe: *Convex Optimization*, Cambridge University Press, Cambridge, 2004.
- J. Renegar: *A Mathematical View of Interior-Point Methods in Convex Optimization*, Society for Industrial and Applied Mathematics, Philadelphia, 2001.
- B. H. Korte, J. Vygen: *Combinatorial Optimization: Theory and Algorithms*, 3. izdaja, Springer, Berlin, 2006.
- L. A Wolsey: *Integer Programming*, Wiley, New York, 1998.
- C. T. Kelley: *Iterative Method for Optimization*, Society for Industrial and Applied Mathematics, Philadelphia, 1999.
- Z. Michalewicz, D. B. Fogel: *How to Solve It: Modern Heuristics*, 2. izdaja, Springer, Berlin, 2004.

### Cilji in kompetence:

Študent podrobneje spozna eno ali več pomembnejših področij optimizacije.

### Objectives and competences:

Students become acquainted with one or several of the more important areas of optimization.

### Predvideni študijski rezultati:

**Znanje in razumevanje:** Slušatelj se natančneje seznanja z izbranim področjem optimizacije. Spozna teoretične osnove ter praktične prijeme pri reševanju optimizacijskih

### Intended learning outcomes:

**Knowledge and understanding:** Students gain deeper knowledge of selected optimization areas. They become familiar with both the theoretical foundations and the techniques for

nalog z izbranega področja.

**Uporaba:** Reševanje optimizacijskih problemov iz vsakdanjega življenja.

**Refleksija:** Pomen **ustreznega modeliranja optimizacijskih problemov, kar omogoča njihovo učinkovito** reševanje.

**Prenosljive spretnosti – niso vezane le na en predmet:** Modeliranje nalog iz vsakdanjega življenja v obliki matematičnih optimizacijskih nalog, zmožnost razločevanja med računsko obvladljivimi in neobvladljivimi problemi, sposobnost samostojnega snovanja modelov in njihove analize s pomočjo računalnika.

solving optimization problems in these areas.

**Application:** Solving optimization problems which arise in practice.

**Reflection:** The importance of adequate modelling of optimization problems which facilitates their efficient solving.

**Transferable skills:** Capabilities to model practical problems as mathematically formulated optimization problems, to distinguish between computationally feasible and infeasible problems, to construct models and to analyze them by means of appropriate software tools.

**Metode poučevanja in učenja:**

predavanja, **seminar**, vaje, domače naloge, konzultacije **in samostojno delo študentov**

**Learning and teaching methods:**

Lectures, **seminar**, exercises, homework, consultations, and **independent work by the students**

Delež (v %) /

Weight (in %)

**Načini ocenjevanja:**

**Assessment:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- izpit iz vaj (2 kolokvija ali pisni izpit)
- ustni izpit

Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)

50%

50%

Type (examination, oral, coursework, project):

- 2 midterm exams instead of written exam, written exam
- oral exam

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

**prof. dr. Vladimir Batagelj**



- W. de Nooy, A. Mrvar, V. Batagelj: *Exploratory social network analysis with Pajek* (Structural analysis in the social sciences 27), Cambridge Univ. Press, New York, 2005.
- P. Doreian, V. Batagelj, A. Ferligoj: *Generalized Blockmodeling* (Structural analysis in the social sciences 25), Cambridge Univ. Press, Cambridge, 2005.
- V. Batagelj, S. Korenjak-Černe, S. Klavžar: *Dynamic programming and convex clustering*, *Algorithmica* 11 (1994), 93–103.

**prof. dr. Sergio Cabello**

- S. Cabello, J. M. Díaz-Báñez, P. Pérez-Lantero: *Covering a bichromatic point set with two disjoint monochromatic disks*, *Computational Geometry: Theory and Applications* **46** (2013) 203–212.
- S. Cabello, P. Giannopoulos, C. Knauer, D. Marx, G. Rote: *Geometric clustering: fixed-parameter tractability and lower bounds with respect to the dimension*, *ACM Transactions on Algorithms* 7 (2011), članek 43.
- S. Cabello, G. Rote: *Obnoxious centers in graphs*, *SIAM Journal on Discrete Mathematics* 24 (2010) 1713–1730.

**prof. dr. Bojan Mohar**

- B. Mohar: *A linear time algorithm for embedding graphs in an arbitrary surface*, *SIAM J. Discrete Math.* 12 (1999), 6–26.
- B. Mohar: *Circle packings of maps in polynomial time*, *European J. Combin.* 18 (1997), 785–805.
- B. Mohar: *Projective planarity in linear time*, *J. Algorithms* 15 (1993), 482–502.

**prof. dr. Emil Žagar**

- G. Jaklič, M. L. Sampoli, A. Sestini, E. Žagar: *C1 rational interpolation of spherical motions with rational rotation-minimizing directed frames*, *Comput. aided geom. design* **30** (2013) 159–173.
- G. Jaklič, T. Kanduč, S. Praprotnik, E. Žagar: *Energy minimizing mountain ascent*. *J. optim. theory appl.* **155** (2012) 680–693.
- G. Jaklič, E. Žagar: *Curvature variation minimizing cubic Hermite interpolants*. *Appl. math. comput.* **218** (2011) 3918–3924.

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

<b>Predmet:</b>	Izbrana poglavja iz računalniške matematike
<b>Course title:</b>	Topics in Computational Mathematics

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2606

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
30	15	30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Andrej Bauer, prof. dr. Sergio Cabello, prof. dr. Bojan Mohar, prof. dr. Marko Petkovšek

**Jeziki / Languages:**

<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija

**Prerequisites:**

Enrollment into the program

**Vsebina:**

**Content (Syllabus outline):**

Predavatelj izbere nekatere pomembne teme s področja računalniške matematike, kot so na primer:

- Računska geometrija in geometrijska optimizacija.
- Računska topologija.
- Algoritmi na grafih.
- Vizualizacija grafov in podatkov.
- Računalniška grafika.
- Računalniški vid.
- Matroidi.
- Algoritmčna teorija iger.
- Aproksimacijski algoritmi.
- Vzporedni algoritmi.
- Algoritmi za tokove podatkov.
- Simbolno računanje.
- Bioinformatika.

The lecturer selects some important topics in computational mathematics, such as:

- Computational geometry and geometric optimization.
- Computational topology.
- Graph algorithms.
- Graph and data visualization.
- Computer graphics.
- Computer vision.
- Matroids.
- Algorithmic game theory.
- Approximation algorithms.
- Parallel algorithms.
- Algorithms for data streams.
- Symbolic computation.
- Bioinformatics.

#### Temeljni literatura in viri / Readings:

- M. de Berg, O. Cheong, M. van Kreveld, M. Overmars: *Computational Geometry: Algorithms and Applications*, 3. izdaja, Springer-Verlag, 2008.
- S. Har-Peled: *Geometric approximation algorithms*, AMS, 2011.
- H. Edelsbrunner, J.L. Harer: *Computational Topology. An Introduction*, AMS, 2010.
- G. Di Battista, P. Eades, R. Tamassia, I.G. Tollis: *Graph Drawing: Algorithms for the Visualization of Graphs*, Prentice Hall, 1998.
- C. H. Lampert: *Kernel Methods in Computer Vision*, Foundations and Trends in Computer Graphics and Vision 4 (2009) 193-285.
- B. Mohar: *Teorija matroidov*, DMFAS, Ljubljana, 1996.
- N. Nisan, T. Roughgarden, E. Tardos (ur.): *Algorithmic Game Theory*, Cambridge University Press, 2007.
- D.P. Williamson, D.B. Shmoys: *The Design of Approximation Algorithms*, Cambridge University Press, 2011.
- J. Jaja. *Introduction to parallel algorithms*. Addison-Wesley, 1992.
- S. Muthukrishnan: *Data Streams: Algorithms and Applications*, Foundations & Trends in Theoretical Computer Science, 2005.
- J. von zur Gathen, J. Gerhard: *Modern Computer Algebra*, 3rd ed., Cambridge University Press, 2013.
- M. Kauers, P. Paule: *The concrete tetrahedron. Symbolic sums, recurrence equations, generating functions, asymptotic estimates*, Springer, 2011.

- N. C. Jones, P. A. Pevzner: An Introduction to Bioinformatics Algorithms, MIT Press, Cambridge MA, 2004.
- Znanstveni članki.

#### Cilji in kompetence:

Študent spozna osnove nekaterih pomembnih področij računalniške matematike.

#### Objectives and competences:

The students get acquainted with some important and actual areas of computational mathematics.

#### Predvideni študijski rezultati:

**Znanje in razumevanje:** Slušatelj se natančneje seznanja z izbranim področjem računalniške matematike. Spozna teoretične osnove ter praktične prijeme z izbranega področja.

**Uporaba** Reševanje računalniških problemov iz različnih področij.

**Refleksija:** Študentje spoznajo računalniške probleme in modeliranje. Povezanost med teorijo in prakso.

**Prenosljive spretnosti – niso vezane le na en predmet:** Uporaba algoritmičnega mišljenja pri reševanju nepopolno definiranih problemov

#### Intended learning outcomes:

**Knowledge and understanding:** Students gain deeper knowledge of selected areas in computational mathematics. They become familiar with both the theoretical foundations and the techniques for solving problems in these areas.

**Application:** Solving computational problems from different areas.

**Reflection:** The students see computational problems and modelling. Connection between theory and praxis.

**Transferable skills:** Use of algorithmic thinking for solving imperfectly defined problems.

#### Metode poučevanja in učenja:

#### Learning and teaching methods:

predavanja, seminar, vaje, domače naloge, konzultacije in samostojno delo študentov	Lectures, seminar, exercises, homework, consultations and independent work by the students
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Delež (v %) /

Weight (in %) **Assessment:**

**Načini ocenjevanja:**

Način:	Delež (v %) / Weight (in %)	Type:
izpit iz vaj (2 kolokvija ali pisni izpit) or homework	50%	exam of exercises (2 midterm exams or written exam) or homework
ustni izpit	50%	oral exam.
Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)		Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

**prof. dr. Andrej Bauer**

- Bauer, C. A. Stone: *RZ: a tool for bringing constructive and computable mathematics closer to programming practice*. Journal of Logic and Computation, 2009, vol. 19, no. 1, str. 17-43.
- Bauer, E. Clarke, X. Zhao: *Analytica — An Experiment in Combining Theorem Proving and Symbolic Computation*. Journal of Automated Reasoning, Vol. 21, no. 3 (1998) 295-325.
- Bauer, M. Petkovšek: *Multibasic and mixed hypergeometric Gosper-type algorithms*. Journal of Symbolic Computation, Vol. 28 (1999) 711-736.

**prof. dr. Sergio Cabello**

- S. Cabello, M. van Kreveld: *Approximation algorithms for aligning points*, Algorithmica 37 (2003) 211-232.
- S. Cabello: *Approximation algorithms for spreading points*, Journal of Algorithms 62 (2007) 49-73.
- S. Cabello, M. van Kreveld, H. Haverkort, B. Speckmann: *Algorithmic aspects of proportional symbol maps*, Algorithmica 58 (2010) 543-565.

**prof. dr. Bojan Mohar**

- B. Mohar: *A linear time algorithm for embedding graphs in an arbitrary surface*, SIAM J. Discrete Math. 12 (1999), 6–26.
- B. Mohar: *Circle packings of maps in polynomial time*, European J. Combin. 18 (1997), 785–805.
- B. Mohar: *Projective planarity in linear time*, J. Algorithms 15 (1993), 482–502.

**prof. dr. Marko Petkovšek**

- M. Petkovšek: *Counting Young tableaux when rows are cosets*. *Ars comb.*, 1994, let. 37, str. 87-95.
- M. Petkovšek, H. S. Wilf, D. Zeilberger: *A=B*. Wellesley (Massachusetts): A. K. Peters, cop. 1996. VII, 212 str. ISBN 1-56881-063-6.
- M. Petkovšek: *Letter graphs and well-quasi-order by induced subgraphs*. *Discrete math.*, 2002, vol. 244, no. 1-3, str. 375-388.

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

<b>Predmet:</b>	Matematika z računalnikom
<b>Course title:</b>	Mathematics with computers

<b>Študijski program in stopnja</b> Study programme and level	<b>Študijska smer</b> Study field	<b>Letnik</b> Academic year	<b>Semester</b> Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2600

<b>Predavanja</b> Lectures	<b>Seminar</b> Seminar	<b>Vaje</b> Tutorial	<b>Klinične vaje</b> work	<b>Druge oblike</b> študija	<b>Samost. delo</b> Individ. work	<b>ECTS</b>
<b>15</b>	<b>30</b>	<b>30</b>			<b>105</b>	<b>6</b>

**Nosilec predmeta / Lecturer:** prof. dr. Andrej Bauer, prof. dr. Marko Petkovšek

<b>Jeziki /</b> <b>Languages:</b>	<b>Predavanja /</b> <b>Lectures:</b>	slovenski/Slovene, angleški/English
	<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija

**Prerequisites:**

Enrollment into the program

**Vsebina:**

**Content (Syllabus outline):**

Študent spozna programsko opremo za reševanje matematičnih problemov. Poudarek je predvsem na praktični uporabi in spoznavanju programske opreme. Predstavljena so naslednja področja (v oklepajih je predlagana programska oprema):

- analiza (Mathematica, Sage)
- diskretna matematika (Mathematica, Sage, Vega, Pajek)
- algebra (Mathematica, Sage, Magma, GAP)
- topologija in geometrija (Mathematica, Sage, GeoGebra, programska oprema za računanje topoloških invariant)
- statistika in finančna matematika (R)
- logika (Isabelle, Coq, HOL, Agda)

Students learn how to use software for solving mathematical problems. The course focuses on the practical aspects and proficient use of software. The following areas of computerized mathematics are covered (suggested software is listed in parentheses):

- analysis (Mathematica, Sage)
- discrete mathematics (Mathematica, Sage, Vega, Pajek)
- algebra (Mathematica, Sage, Magma, GAP)
- topology and geometry (Mathematica, Sage, GeoGebra, various specialized programs for topology invariants)
- statistics and financial mathematics (R)
- logic (Isabelle, Coq, HOL, Agda)

#### Temeljni literatura in viri / Readings:

- Uporabniški priročniki za programsko opremo.
- User manuals and other documentation for the software at hand.

#### Cilji in kompetence:

Spoznavanje in uporaba programske opreme za reševanje matematičnih problemov.

#### Objectives and competences:

Introduction to and application of specialized software for doing mathematics.

#### Predvideni študijski rezultati:

##### Znanje in razumevanje:

Praktično znanje iz uporabe zahtevnih programskih paketov za reševanje matematičnih problemov.

##### Uporaba:

Uporaba računalnikov v matematiki.

##### Refleksija:

Moderna računalniška tehnologija je postala

#### Intended learning outcomes:

##### Knowledge and understanding:

Practical knowledge and use of advanced programs for solving mathematical problems.

##### Application:

Application of computers in mathematics.

##### Reflection:

Modern computer technology has become an indispensable tool in mathematics.



nepogrešljivo orodje za matematika.  
**Prenosljive spretnosti – niso vezane le na en predmet:**

Predmet ima izrazito aplikativno naravnost in študentom omogoči uporabo sodobnih orodij pri ostalih študijskih predmetih.

**Transferable skills:**

The emphasis on practical use and applications enables the student to use computers in all other courses.

**Metode poučevanja in učenja:**

predavanja, vaje, projekti, domače naloge, konzultacije

**Learning and teaching methods:**

Lectures, exercises, project course, homeworks, consultations

**Načini ocenjevanja:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):

- projektno delo
- predstavitev in zagovor projekta

Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)

Delež (v %) /

Weight (in %)

**Assessment:**

Type (examination, oral, coursework, project):

- course project
- project presentation and defense

Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

**prof. dr. Andrej Bauer**

- A. Bauer, C. A. Stone: *RZ: a tool for bringing constructive and computable mathematics closer to programming practice*. Journal of Logic and Computation, 2009, vol. 19, no. 1, str. 17-43.
- A. Bauer, E. Clarke, X. Zhao: *Analytica — An Experiment in Combining Theorem Proving and Symbolic Computation*. Journal of Automated Reasoning, Vol. 21, no. 3 (1998) 295-325.
- A. Bauer, M. Petkovšek: *Multibasic and mixed hypergeometric Gosper-type algorithms*. Journal of Symbolic Computation, Vol. 28 (1999) 711-736.

**prof. dr. Marko Petkovšek**

- M. Petkovšek: *Symbolic computation with sequences*. Program. comput. softw., 2006, vol. 32, no. 2, str. 65-70
- M. Petkovšek, H. S. Wilf, D. Zeilberger: *A = B*, A K Peters, Wellesley MA, 1996. xii + 212 str. (ISBN 1-56881-063-6)
- A. Bauer, M. Petkovšek: *Multibasic and mixed hypergeometric Gosper-type algorithms*. Journal of Symbolic Computation, Vol. 28 (1999) 711-736.

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

<b>Predmet:</b>	Optimizacija 2
<b>Course title:</b>	Optimization 2

<b>Študijski program in stopnja</b> Study programme and level	<b>Študijska smer</b> Study field	<b>Letnik</b> Academic year	<b>Semester</b> Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2604

<b>Predavanja</b> Lectures	<b>Seminar</b> Seminar	<b>Vaje</b> Tutorial	<b>Klinične vaje</b> work	<b>Druge oblike</b> študija	<b>Samost. delo</b> Individ. work	<b>ECTS</b>
30	15	30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Vladimir Batagelj, prof. dr. Sergio Cabello, prof. dr. Bojan Mohar, prof. dr. Emil Žagar

**Jeziki / Languages:**

<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

**Prerequisites:**

Vpis v letnik študija

Enrollment into the program

**Vsebina:** \_\_\_\_\_ **Content (Syllabus outline):** \_\_\_\_\_

Konveksne množice in funkcije, konveksno programiranje. Lagrangeova prirejenost, dualna naloga, šibka in krepka dualnost. Slaterjev pogoj, Karush-Kuhn-Tuckerjev izrek.

Optimizacijski problemi z linearnimi omejitvami, kvadratično in semidefinitno programiranje s posplošitvami. Numerični postopki, kazenske metode. Celoštevilsko programiranje.

Kratek pregled računalniških orodij za reševanje optimizacijskih problemov.

Convex sets and functions, convex programming. Lagrange duality, dual problem, weak and strong duality. Slater's condition, the Karush-Kuhn-Tucker theorem.

Linearly constrained optimization problems, quadratic and semidefinite programming with generalizations. Numerical procedures, penalty functions. Integer programming.

A short overview of software tools for solving optimization problems.

### Temeljni literatura in viri / Readings:

- S. Boyd, L. Vandenberghe: *Convex Optimization*, Cambridge Univ. Press, Cambridge, 2004.
- B. H. Korte, J. Vygen: *Combinatorial Optimization: Theory and Algorithms*, 3. izdaja, Springer, Berlin, 2006.

### Cilji in kompetence:

Študent spozna osnovne vrste **problemov matematičnega programiranja** s poudarkom na konveksnih problemih. Seznan je z osnovnimi matematičnimi prijemi za njihovo reševanje, hkrati pa za praktično reševanje uporablja tudi ustrezne računalniške pakete.

### Objectives and competences:

Students encounter the fundamental types of problems in mathematical programming, with emphasis on the convex ones. They get to know the basic mathematical tools for tackling these problems, using appropriate software packages for solving them in practice.

### Predvideni študijski rezultati:

**Znanje in razumevanje:** Študent je sposoben z matematičnim modelom dobro opisati različne **pomembne uporabne** probleme. Pozna osnovne prijeme in računalniška orodja za učinkovito reševanje dobljenih optimizacijskih problemov.

**Uporaba:** Reševanje optimizacijskih problemov iz **prakse**.

**Refleksija:** Pomen predstavitve praktičnih problemov v formalizirani obliki, ki omogoča njihovo učinkovito in pravilno reševanje.

### Intended learning outcomes:

**Knowledge and understanding:** Students are able to model various important applied problems accurately. They are familiar with the basic techniques and software tools that can be used to solve the resulting optimization problems efficiently.

**Application:** Solving optimization problems which appear in practice.

**Reflection:** The importance of representing practical problems in a formal way which helps to solve them efficiently and adequately.

**Prenosljive spretnosti – niso vezane le na en predmet:** Modeliranje nalog iz vsakdanjega življenja v obliki matematičnih optimizacijskih nalog, zmožnost razločevanja med računsko obvladljivimi in neobvladljivimi problemi, sposobnost samostojnega snovanja modelov in njihove analize s pomočjo računalnika.

**Transferable skills:** Ability to model practical problems as mathematically formulated optimization problems, to distinguish between computationally feasible and infeasible problems, to construct models on one's own and to analyze them by means of appropriate software tools.

**Metode poučevanja in učenja:**

predavanja, seminar, vaje, domače naloge, konzultacije in samostojno delo študentov

**Learning and teaching methods:**

Lectures, seminar, exercises, homework, consultations, and independent work by the students

**Delež (v %) /**

**Načini ocenjevanja:**

**Weight (in %) /**

**Assessment:**

<p>Način (pisni izpit, ustno izpraševanje, naloge, projekt):</p> <ul style="list-style-type: none"> <li>▪ izpit iz vaj (2 kolokvija ali pisni izpit)</li> <li>▪ ustni izpit</li> </ul> <p>Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)</p>	<p>50%</p> <p>50%</p>	<p>Type (examination, oral, coursework, project):</p> <ul style="list-style-type: none"> <li>▪ 2 midterm exams instead of written exam, written exam</li> <li>▪ oral exam</li> </ul> <p>Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)</p>
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**Reference nosilca / Lecturer's references:**

**prof. dr. Vladimir Batagelj**

- W. de Nooy, A. Mrvar, V. Batagelj: *Exploratory social network analysis with Pajek* (Structural analysis in the social sciences 27), Cambridge Univ. Press, New York, 2005.
- P. Doreian, V. Batagelj, A. Ferligoj: *Generalized Blockmodeling* (Structural analysis in the social sciences 25), Cambridge Univ. Press, Cambridge, 2005.
- V. Batagelj, S. Korenjak-Černe, S. Klavžar: *Dynamic programming and convex clustering*, *Algorithmica* 11 (1994), 93–103.

**prof. dr. Sergio Cabello**

- S. Cabello, J. M. Díaz-Báñez, P. Pérez-Lantero: *Covering a bichromatic point set with two disjoint monochromatic disks*, *Computational Geometry: Theory and Applications* **46**

(2013) 203–212.

- S. Cabello, P. Giannopoulos, C. Knauer, D. Marx, G. Rote: *Geometric clustering: fixed-parameter tractability and lower bounds with respect to the dimension*, ACM Transactions on Algorithms 7 (2011), članek 43.
- S. Cabello, G. Rote: *Obnoxious centers in graphs*, SIAM Journal on Discrete Mathematics 24 (2010) 1713–1730.

**prof. dr. Bojan Mohar**

- B. Mohar: *A linear time algorithm for embedding graphs in an arbitrary surface*, SIAM J. Discrete Math. 12 (1999), 6–26.
- B. Mohar: *Circle packings of maps in polynomial time*, European J. Combin. 18 (1997), 785–805.
- B. Mohar: *Projective planarity in linear time*, J. Algorithms 15 (1993), 482–502.

**prof. dr. Emil Žagar**

- G. Jaklič, M. L. Sampoli, A. Sestini, E. Žagar: *C1 rational interpolation of spherical motions with rational rotation-minimizing directed frames*, *Comput. aided geom. design* **30** (2013) 159-173.
- G. Jaklič, T. Kanduč, S. Praprotnik, E. Žagar: *Energy minimizing mountain ascent*. *J. optim. theory appl.* **155** (2012) 680-693.
- G. Jaklič, E. Žagar: *Curvature variation minimizing cubic Hermite interpolants*. *Appl. math. comput.* **218** (2011) 3918-3924.

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

<b>Predmet:</b>	Podatkovne strukture in algoritmi 3
<b>Course title:</b>	Data structures and algorithms 3

<b>Študijski program in stopnja</b> Study programme and level	<b>Študijska smer</b> Study field	<b>Letnik</b> Academic year	<b>Semester</b> Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type**

**Univerzitetna koda predmeta / University course code:**

<b>Predavanja</b> Lectures	<b>Seminar</b> Seminar	<b>Vaje</b> Tutorial	<b>Klinične vaje</b> work	<b>Druge oblike</b> študija	<b>Samost. delo</b> Individ. work	<b>ECTS</b>
30	15	30			105	6

**Nosilec predmeta / Lecturer:**

**Jeziki / Languages:**

<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija

**Prerequisites:**

Enrollment into the program

**Vsebina:**

**Content (Syllabus outline):**

Predavatelj izbere teme iz naslednjega seznama:

- Uravnotežena iskalna drevesa.
- Zgoščene tabele.
- Binomske in Fibonaccijeve kopice.
- Vodenje disjunktnih množic.
- Algoritmi na nizih (Rabina in Karpa; Knutha, Morrisa in Pratta; Boyerja in Moora).
- Računanje konveksne ovojnice.
- Voronojev diagram in Delauneyeva triangulacija.
- Iskanje maksimalnega pretoka s pretokom.
- Iskanje največjega (uteženega) prirejanja v splošnem grafu.
- Algoritem alpha-beta.
- Algoritmi za ravninske grafe.
- Algoritmi za zunanji pomnilnik.
- Vztrajne podatkovne strukture.
- Podatkovne strukture za cela števila.
- Enostavni vzporedni algoritmi.
- Dinamična drevesa.

The lecturer chooses topics from the following list:

- Balanced search trees.
- Hash tables.
- Binomial and Fibonacci heaps.
- Union-find for disjoint sets.
- Algorithms for strings (Rabin and Karp; Knuth, Morris and Pratt; Boyer and Moore).
- Computation of convex hulls.
- Voronoi diagram and Delaunay triangulation.
- Finding maximum flows with preflows.
- Finding largest (weighted) matchings in general graphs.
- Alpha-beta algorithm.
- Algorithms for planar graphs.
- Algorithms for external memory.
- Persistent data structures.
- Data structures for integers.
- Simple parallel algorithms.
- Dynamic trees.

#### Temeljni literatura in viri / Readings:

- T. H. Cormen, C. E. Leiserson, R. L. Rivest, C. Stein: *Introduction to Algorithms*, 2. izdaja, MIT Press, 2001.
- D. C. Kozen: *The Design and Analysis of Algorithms*, Springer, 1991.
- D. E. Knuth: *Selected Papers on Analysis of Algorithms*, Cambridge University Press, 2000.
- S. Even, G. Even: *Graph Algorithms*, 2. izdaja, Cambridge University Press, 2011.
- Znanstveni članki.

#### Cilji in kompetence:

Študent nadgradi poznavanje podatkovnih struktur in z njimi povezanih algoritmov, ki se uporabljajo pri načrtovanju učinkovitih algoritmov. Ob tem poglobi znanje o matematični analizi pravilnosti ter časovne in prostorske zahtevnosti algoritmov.

#### Objectives and competences:

The students improve their knowledge of data structures and related algorithmic techniques used in the design of efficient algorithms. They also develop the knowledge of mathematical analysis for the correctness and the time/space complexity of algorithms.

**Predvideni študijski rezultati:**

**Znanje in razumevanje:** Poznavanje zahtevnejših podatkovnih struktur in algoritmov, praktičnih in teoretičnih problemov, pri katerih se jih lahko smiselno uporabi, ter poznavanje osnov teorije računske zahtevnosti.

**Uporaba:** Snovanje učinkovitih računalniških programov in napovedovanje njihovega obnašanja v praksi s pomočjo matematičnih metod.

**Refleksija:** Povezanost med teoretičnimi napovedmi o obnašanju računalniških programov in dejanskim obnašanjem.

**Prenosljive spretnosti – niso vezane le na en predmet:** Pomen matematične analize računskih postopkov in njena praktična uporabnost. Ločevanje med računsko zahtevnimi in manj zahtevnimi problemi.

**Intended learning outcomes:**

**Knowledge and understanding:** Learning more about complex data structures and algorithms, practical and theoretical problems where this knowledge can be applied, and the basics of computational complexity.

**Application:** The design of efficient computer programs and prediction of their behavior in practice by using mathematical methods.

**Reflection:** The correlation between theoretical predictions about the behavior of computer programs and their actual behavior.

**Transferable skills:** The importance of mathematical analysis of computational processes and its practical application. Classification into difficult and less complex problems.

**Metode poučevanja in učenja:**

predavanja, seminar, vaje, domače naloge, konzultacije in samostojno delo študentov

**Learning and teaching methods:**

Lectures, seminar, exercises, homework, consultations and independent work by the students

**Načini ocenjevanja:**

Delež (v %) /

Weight (in %)

**Assessment:**

Način:

izpit iz vaj (2 kolokvija ali pisni izpit) ali domače naloge,

50%

Type:

exam of exercises (2 midterm exams or written exam) or homework

ustni izpit

50%

oral exam.



Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)		Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)
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**Reference nosilca / Lecturer's references:**

**prof. dr. Sergio Cabello**

- S. Cabello, É. Colin de Verdière, F. Lazarus: *Algorithms for the edge-width of an embedded graph*, Computational Geometry: Theory and Applications 45 (2012): 215–224.
- S. Cabello: *Finding shortest contractible and shortest separating cycles in embedded graphs*, ACM Transactions on Algorithms 6 (2010) članek #24.
- S. Cabello: *Many distances in planar graphs*, Algorithmica 62 (2012) 361–381.

**prof. dr. Bojan Mohar**

- B. Mohar: *A linear time algorithm for embedding graphs in an arbitrary surface*, SIAM J. Discrete Math. 12 (1999), 6–26.
- B. Mohar: *Circle packings of maps in polynomial time*, European J. Combin. 18 (1997), 785–805.
- B. Mohar: *Projective planarity in linear time*, J. Algorithms 15 (1993), 482–502.

**UČNI NAČRT PREDMETA / COURSE SYLLABUS**

<b>Predmet:</b>	Računska zahtevnost
<b>Course title:</b>	Computational complexity

<b>Študijski program in stopnja</b> Study programme and level	<b>Študijska smer</b> Study field	<b>Letnik</b> Academic year	<b>Semester</b> Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type**

**Univerzitetna koda predmeta / University course code:**

<b>Predavanja</b> Lectures	<b>Seminar</b> Seminar	<b>Vaje</b> Tutorial	<b>Klinične vaje</b> work	<b>Druge oblike</b> študija	<b>Samost. delo</b> Individ. work	<b>ECTS</b>
30	15	30			105	6

**Nosilec predmeta / Lecturer:**

**Jeziki / Languages:**

<b>Predavanja / Lectures:</b>	slovenski/Slovene, angleški/English
<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

Vpis v letnik študija

**Prerequisites:**

Enrollment into the program

**Vsebina:**

**Content (Syllabus outline):**

Modeli računanja. Časovna in prostorska zahtevnost. Determinizem in nedeterminizem. Redukcije in polnost.

Fenomen NP-polnosti. Nekaj izbranih NP-polnih problemov. Tehnike dokazovanja NP-polnosti. Struktura razreda NP.

Verjetnostni algoritmi. Vrste verjetnostnih algoritmov. Verjetnostni razredi zahtevnosti. Generatorji psevdonaključnosti.

Aproksimativni algoritmi. Kakovost aproksimacije. Težavnost aproksimacije. Aproksimacijske sheme. Nekaj izbranih aproksimacijskih algoritmov.

Dodatno vsebino lahko predavatelj izbere med naslednjimi temami: Booleova vezja, interaktivni dokazi, kvantno računalništvo, izreki PCP, komunikacijska zahtevnost, parametrična zahtevnost.

Models of computation. Time and space complexity. Determinism and nondeterminism. Reductions and completeness.

NP-completeness. Some selected NP-complete problems. Techniques to prove NP-completeness. Structure of the class NP.

Probabilistic algorithms. Types of probabilistic algorithms. Related computational classes. Pseudorandom generators.

Approximation algorithms. Quality of approximation. Hardness of approximation. Approximation schemes. Selected approximation algorithms.

Additional content may be selected among the following topics: Boolean circuits, interactive proofs, quantum computing, PCP theorems, communication complexity, parameterized complexity.

#### Temeljni literatura in viri / Readings:

- S. Arora, B. Barak: *Computational Complexity: A Modern Approach*, Cambridge University Press, 2009.
- M. R. Garey, D. S. Johnson: *Computers and intractability. A guide to the theory of NP-completeness*, W. H. Freeman and Co., 2003.
- R. Motwani, P. Raghavan: *Randomized Algorithms*, Cambridge University Press, Cambridge, 1995.
- V. V. Vazirani: *Approximation algorithms*, Springer-Verlag, 2001.

#### Cilji in kompetence:

Študent se seznani z osnovnimi modeli računanja, teorijo NP-polnosti, verjetnostnimi algoritmi in z reševanjem težkih problemov z aproksimativnimi algoritmi.

#### Objectives and competences:

Students become acquainted with the basic models of computation, the theory of NP-completeness, probabilistic algorithms, and with solving hard problems approximately.

#### Predvideni študijski rezultati:

#### Intended learning outcomes:

**Znanje in razumevanje:** Študentje poznajo:

- povezave med modeli računanja
- teorijo NP-polnosti
- pojem verjetnostnega algoritma
- pojem aproksimativnega algoritma

**Uporaba:** Študentje znajo:

- analizirati časovno zahtevnost algoritmov
- dokazovati NP-polnost
- načrtovati verjetnostne algoritme
- načrtovati aproksimativne algoritme

**Refleksija:** Študentje spoznajo:

- hierarhijo problemov glede na njihovo časovno zahtevnost
- inherentno težke probleme
- relaksacijske pristope k reševanju težkih problemov

**Prenosljive spretnosti – niso vezane le na en predmet:** Analiza težavnosti problemov s pomočjo redukcij med njimi.

**Knowledge and understanding:** The students understand:

- connections between models of computation;
- theory of NP-completeness;
- the concept of probabilistic algorithm;
- the concept of approximation algorithm.

**Application:** The students are able to:

- analyze time complexity of algorithms;
- prove NP-completeness;
- design probabilistic algorithms;
- design approximation algorithms.

**Reflection:** The students meet:

- problem hierarchies by time complexity;
- inherently hard problems;
- relaxations to solve hard problems.

**Transferable skills:** Analysis of the hardness of problems using reductions between them.

**Metode poučevanja in učenja:**

predavanja, seminar, vaje, domače naloge, konzultacije in samostojno delo študentov

**Learning and teaching methods:**

Lectures, seminar, exercises, homework, consultations and independent work by the students

**Načini ocenjevanja:**

Način:  
izpit iz vaj (2 kolokvija ali pisni izpit) or homework

Delež (v %) /

Weight (in %)

**Assessment:**

Type:  
exam of exercises (2 midterm exams or written exam) or homework

50%

50%

ustni izpit		oral exam.
Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)		Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:**

**prof. dr. Sergio Cabello**

- S. Cabello, J. Cardinal, S. Langerman: *The clique problem in ray intersection graphs*, Discrete & Computational Geometry 50 (2013) 771–783.
- S. Cabello: *Hardness of approximation for crossing number*, Discrete & Computational Geometry 49 (2013) 348–358.
- S. Cabello, P. Lukšič: *The complexity of obtaining a distance-balanced graph*, The Electronic Journal of Combinatorics 18 (2011) članek #49.

**prof. dr. Marko Petkovšek**

- M. Petkovšek, T. Pisanski: *Izbrana poglavja iz računalništva. Del 1, Izračunljivost in rešljivost. Jeziki. NP-polnost. Naloge.*, (Matematični rokopisi, 1.a.). Ljubljana: Društvo matematikov, fizikov in astronomov SRS, 1986. 120 str.
- M. Petkovšek, H. S. Wilf, D. Zeilberger: *A=B*. Wellesley (Massachusetts): A. K. Peters, cop. 1996. VII, 212 str. ISBN 1-56881-063-6.
- M. Petkovšek: *Letter graphs and well-quasi-order by induced subgraphs*. *Discrete math.*, 2002, vol. 244, no. 1-3, str. 375-388.

**prof. dr. Tomaž Pisanski**

- M. Petkovšek, T. Pisanski: *Izbrana poglavja iz računalništva. Del 1, Izračunljivost in rešljivost. Jeziki. NP-polnost. Naloge.*, (Matematični rokopisi, 1.a.). Ljubljana: Društvo matematikov, fizikov in astronomov SRS, 1986. 120 str.
- D. Marušič, T. Pisanski: *Weakly flag-transitive configurations and half-arc-transitive graphs*. *Eur. j. comb.*, 1999, let. 20, št. 6, str. 559-570.
- T. Pisanski, M. Randić: *Bridges between geometry and graph theory*. V: GORINI, Catherine A. (ur.). *Geometry at work : a collection of papers showing applications of geometry*, (MAA notes, no. 53). [Washington, DC]: Mathematical Association of America, cop. 2000, str. 174-194.

## UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	Teorija izračunljivosti
<b>Course title:</b>	Computability theory

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Magistrski študijski program 2. stopnje Matematika	ni smeri	prvi ali drugi	prvi ali drugi
Second cycle master study program Mathematics	none	first or second	first or second

**Vrsta predmeta / Course type** izbirni predmet/elective course

**Univerzitetna koda predmeta / University course code:** M2602

Predavanja Lectures	Seminar Seminar	Vaje Tutorial	Klinične vaje work	Druge oblike študija	Samost. delo Individ. work	ECTS
45		30			105	6

**Nosilec predmeta / Lecturer:** prof. dr. Andrej Bauer , prof. dr. Marko Petkovšek

**Jeziki / Languages:** Predavanja / Lectures: slovenski/Slovene, angleški/English

Vaje / Tutorial: slovenski/Slovene, angleški/English

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

**Prerequisites:**

Vpis v letnik študija

Enrollment into the program

**Vsebina:**

**Content (Syllabus outline):**

Turingovi stroji in izračunljive funkcije.

Univerzalni stroj. Neodločljivi problemi in neizračunljive funkcije.

Osnovni izreki in pojmi: Izrek s-m-n, izrek u-t-m, izrek o rekurziji, izračunljive in izračunljivo preštevne množice, njihove lastnosti, neseparabilne množice, Riceov izrek, Rice-Shapiro izrek.

Računanje z oraklji, Turingove redukcije in stopnje.

Dodatna vsebina: izračunljivi funkcionali, zveznost funkcionalov, izrek KLS, izračunljiva realna števila, osnovni rezultati izračunljive realne analize.

Turing machines and computable functions.

Universal machine. Undecidable problems and non-computable functions.

Basic theorems and notions: s-m-n and u-t-m theorems, recursion theorem, computable and computably enumerable sets and their properties, non-separable sets, Rice's theorem, Rice-Shapiro theorem.

Oracle computations, Turing reducibility and degrees.

If time permits: computable functionals, continuity of functionals, KLS theorem, computable real numbers, basic results in computable analysis.

### Temeljni literatura in viri / Readings:

- J. E. Hopcroft, J. D. Ullman: *Uvod v teorijo avtomatov, jezikov in izračunov*, FER, Ljubljana, 1990.
- P. Odifreddi: *Classical Recursion Theory*, North-Holland, 1989.

### Cilji in kompetence:

Znanje osnovnih pojmov in rezultatov v teoriji izračunljivosti.

### Objectives and competences:

Knowledge of basic notions and results in computability theory.

### Predvideni študijski rezultati:

#### Znanje in razumevanje:

Razumevanje povezav med računskimi pojmi, kot so Turingovi stroji, in osnovnimi matematičnimi pojmi, kot so množice števil.

#### Uporaba:

Snov predstavlja teoretično matematično podlago za računalništvo v splošnem smislu.

#### Refleksija:

Vpliv pojma izračunljivosti na osnove matematike.

### Intended learning outcomes:

#### Knowledge and understanding:

Understanding of the connections between computability notions, such as Turing machines, and basic mathematical notions, such as sets of numbers.

#### Application:

The subject matter provides a general theoretical foundation for computer science.

#### Reflection:

The influence of the notion of computability on

**Prenosljive spretnosti – niso vezane le na en predmet:**

Analitično in abstraktno razmišljanje o teoretičnih mejah računalništva.

foundations of mathematics.

**Transferable skills:**

Analytic and abstract thinking about the theoretical frontiers of computer science.

**Metode poučevanja in učenja:**

predavanja, vaje, domače naloge, konzultacije

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations

<b>Načini ocenjevanja:</b>	Delež (v %) / Weight (in %)	<b>Assessment:</b>
Način (pisni izpit, ustno izpraševanje, naloge, projekt): <ul style="list-style-type: none"> <li>▪ izpit iz vaj (2 kolokvija ali pisni izpit)</li> <li>▪ ustni izpit</li> </ul> Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)	50%  50%	Type (examination, oral, coursework, project): <ul style="list-style-type: none"> <li>▪ 2 midterm exams instead of written exam, written exam</li> <li>▪ oral exam</li> </ul> Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)

**Reference nosilca / Lecturer's references:****doc. dr. Andrej Bauer**

- S. Awodey, A. Bauer: *Propositions as [Types]*. Journal of Logic and Computation. Volume 14, Issue 4, August 2004, pp. 447-471.
- A. Bauer: *First Steps in Synthetic Computability*. Proceedings of Mathematical Foundations of Programming Semantics XXI, Birmingham 2005. Published in Electronic Notes in Theoretical Computer Science.
- A. Bauer: *A relationship between equilogical spaces and Type Two Effectivity*. Math. Logic Quarterly, 2002, vol. 48, suppl. 1, str. 1-15.

**prof. dr. Marko Petkovšek**

- M. Petkovšek: *Ambiguous numbers are dense*. Amer. Math. Monthly 97 (1990), str. 408-411.
- M. Petkovšek, H. S. Wilf, D. Zeilberger: *A = B*, A K Peters, Wellesley MA, 1996. xii + 212 str. (ISBN 1-56881-063-6).
- M. Petkovšek: *Letter graphs and well-quasi-order by induced subgraphs*. Discrete Math. 244 (2002), str. 375-388.



UČNI NAČRT PREDMETA / COURSE SYLLABUS						
<b>Predmet:</b>	Magistrsko delo in magistrski izpit					
<b>Course title:</b>	Masters thesis and exam					
<b>Študijski program in stopnja</b> <b>Study programme and level</b>	<b>Študijska smer</b> <b>Study field</b>		<b>Letnik</b> <b>Academic year</b>	<b>Semester</b> <b>Semester</b>		
Magistrski študijski program 2. stopnje Matematika	ni smeri		drugi	prvi ali drugi		
Second cycle master study program Mathematics	none		second	first or second		
<b>Vrsta predmeta / Course type</b>			Obvezen predmet /compulsory course			
<b>Univerzitetna koda predmeta / University course code:</b>						
<b>Predavanja</b> <b>Lectures</b>	<b>Seminar</b> <b>Seminar</b>	<b>Vaje</b> <b>Tutorial</b>	<b>Klinične vaje</b> <b>work</b>	<b>Druge oblike</b> <b>študija</b>	<b>Samost. delo</b> <b>Individ. work</b>	<b>ECTS</b>
					750	25
<b>Nosilec predmeta / Lecturer:</b>		izred. prof. dr. Primož Moravec				
<b>Jeziki /</b> <b>Languages:</b>	<b>Predavanja /</b> <b>Lectures:</b>	slovenski/Slovene, angleški/English				
	<b>Vaje / Tutorial:</b>	slovenski/Slovene, angleški/English				
<b>Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:</b>			<b>Prerequisites:</b>			
Vpis v letnik študija			Enrollment into the program			
<b>Vsebina:</b>			<b>Content (Syllabus outline):</b>			

<p>V magistrskem delu študent podrobno predstavi izbrano temo.</p> <p>Študent dobi na zaključnem izpitu tri vprašanja: po eno iz matematične analize in algebre ter eno iz izbranega področja študija (geometrija, topologija, verjetnostni račun, numerične metode, diskretna in računalniška matematika). Vprašanja so zajeta iz vnaprej pripravljenega seznama izpitnih vprašanj, ki obsegajo zgolj osnovno matematično znanje.</p>	<p>In the Master's thesis the student presents the chosen topic in detail.</p> <p>The final exam consists of three questions: one about mathematical analysis, one about algebra, and one about a chosen mathematical discipline (geometry, topology, probability theory, numerical methods, discrete and computational mathematics). The questions are taken from a given list of questions and cover only basic notions of mathematics.</p>

**Temeljni literatura in viri / Readings:**

Za magistrsko delo so viri izbrane članke in monografije.

Za magistrski izpit se študent uči po svojih zapiskih ali po literaturi za temeljne predmete.

References for the masters thesis are selected papers and monographies.

The material for masters exam consists of student's notes and main textbooks given by the selected courses.

**Cilji in kompetence:**

Študent se nauči novega področja in obnovi znanje osnovnih področij matematike.

**Objectives and competences:**

The student learns a new area and refreshes his or her knowledge of basic areas of mathematics.

**Predvideni študijski rezultati:****Znanje in razumevanje:**

Poznavanje osnov matematike, ki se jo predela na drugi stopnji študija.

**Uporaba:**

V matematiki in praksi.

**Refleksija:**

Poznavanje teorije, ki temelji na primerih in uporabah.

**Prenosljive spretnosti – niso vezane le na en predmet:**

Formulacija problema, reševanje problema in analiza rezultatov na primerih.

**Intended learning outcomes:****Knowledge and understanding:**

Basic notions of mathematics at masters level.

**Application:**

In mathematics and practice.

**Reflection:**

Understanding theory based on examples and applications.

**Transferable skills:**

Formulation of problems, solving problems and analysis of results using examples.

**Metode poučevanja in učenja:****Learning and teaching methods:**

Konzulacije, samostojno delo

Consultations, individual work

Delež (v %) /  
Weight (in %)

**Načini ocenjevanja:**

**Assessment:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt): <ul style="list-style-type: none"><li>▪ Ustni izpit</li><li>▪ Zagovor magistrskega dela</li></ul> Ocene: 1-5 (negativno), 6-10 (pozitivno) (po Statutu UL)	50%  50%	Type (examination, oral, coursework, project): <ul style="list-style-type: none"><li>▪ Oral exam</li><li>▪ Thesis defence</li></ul> Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)
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**Reference nosilca / Lecturer's references:**

**izred. prof. dr. Primož Moravec:**

- P. Moravec, Unramified Brauer groups of finite and infinite groups. *Amer. J. Math.* **134** (2012), no. 6, 1679-1704.
- P. Moravec, On the Schur multipliers of finite  $p$ -groups of given coclass. *Israel J. Math.* **185** (2011), 189-205.
- P. Moravec, Powerful actions and nonabelian tensor products of powerful  $p$ -groups. *J. Group Theory* **13** (2010), no. 3, 417-427.