

Linearne dif. enačbe 2. reda

(s konstantnimi koeficienti)

$$ay''+by'+cy = g(x)$$

nehomogena enačba:  $g(x) \neq 0$

$$y(x) = y_{\text{hom}}(x) + y_{\text{part}}(x)$$

$g(x)$	$y_{\text{part}}(x)$
$ae^{\kappa x}$	$Ae^{\kappa x}$
$a \sin(\kappa x)$	$A \sin(\kappa x) + B \cos(\kappa x)$
$a \cos(\kappa x)$	
$a \sin(\kappa x) + b \cos(\kappa x)$	
$a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$	$A_n x^n + A_{n-1} x^{n-1} + \dots + A_0$

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(s konstantnimi koeficienti)

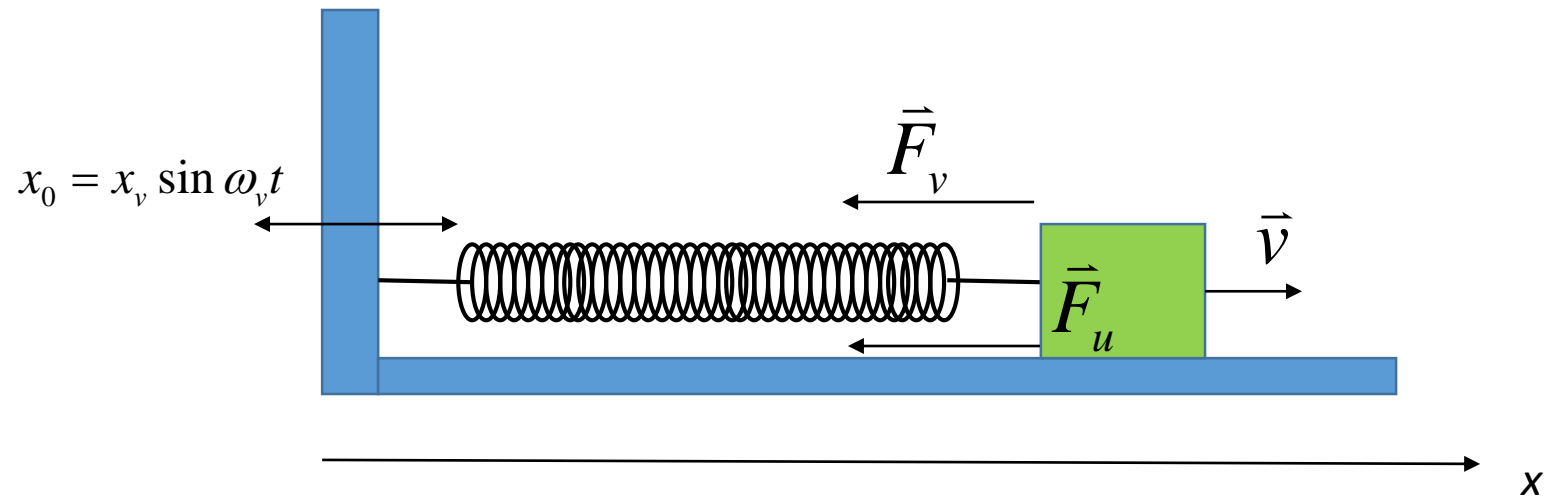
$$ay'' + by' + cy = g(x)$$

nehomogena enačba:  $g(x) \neq 0$

$$y(x) = y_{\text{hom}}(x) + y_{\text{part}}(x)$$

$$y_{\text{hom}}(x) = ae^{\omega_1 x} + be^{\omega_2 x}$$

$$y_{\text{part}}(x) = A \sin(\omega_v t) + B \cos(\omega_v t)$$



$$m\ddot{x} + \beta\dot{x} + kx = x_v \sin \omega_v t$$

$$\omega_{1,2} = -\beta' \pm i\omega' \quad \omega' = \sqrt{\omega_0^2 - \beta'^2}$$

$$x(t=0) = 0 \quad \dot{x}(t=0) = v_0$$

$$x(t) = e^{-\beta't} \left[ \frac{v_0 - A\omega_v + B\beta'}{\omega'} \sin \omega't + B \cos \omega't \right] + A \sin \omega_v t - B \cos \omega_v t$$

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$$\beta' = 0,05 \omega_0$$

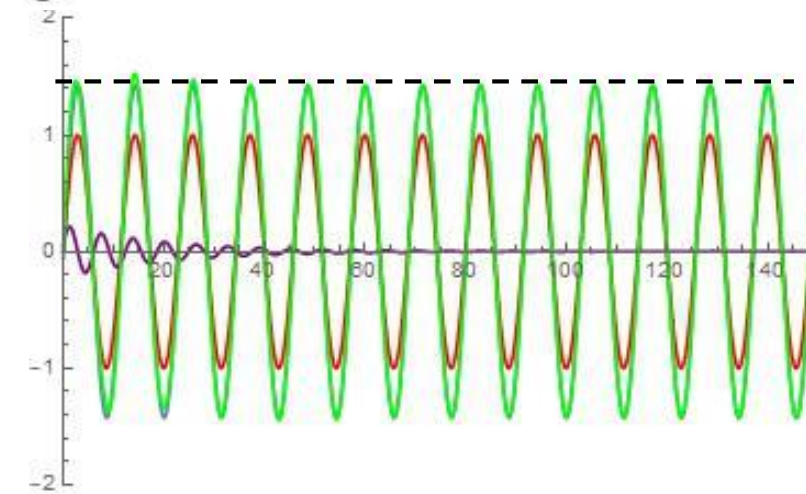
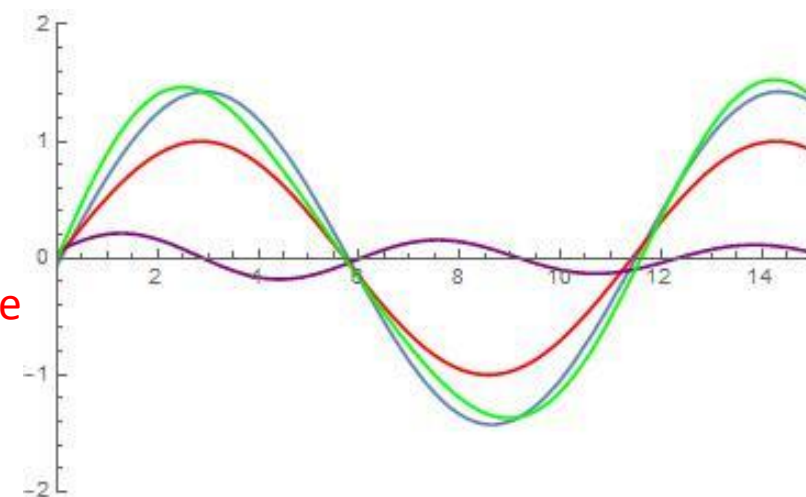
$$\omega_v = 0,55 \omega_0$$

vsiljeno nihanje

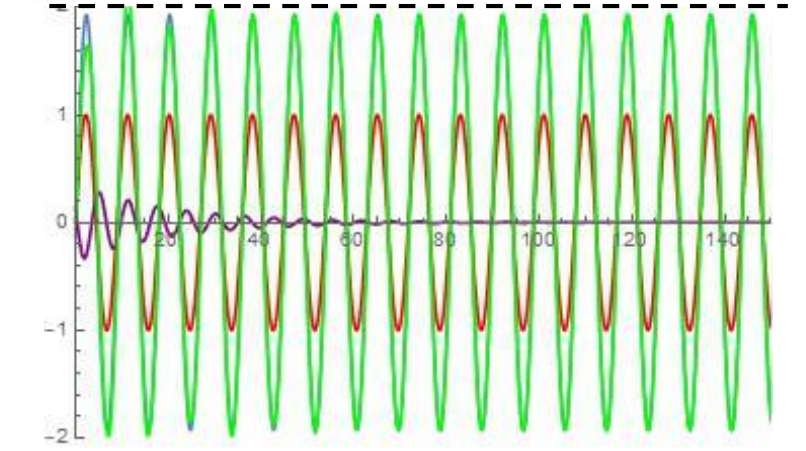
$x_h(t)$

$x_p(t)$

$x_h(t) + x_p(t)$



$$\omega_v = 0,7 \omega_0$$



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$$ay'' + by' + cy = g(x)$$

nehomogena enačba:  $g(x) \neq 0$

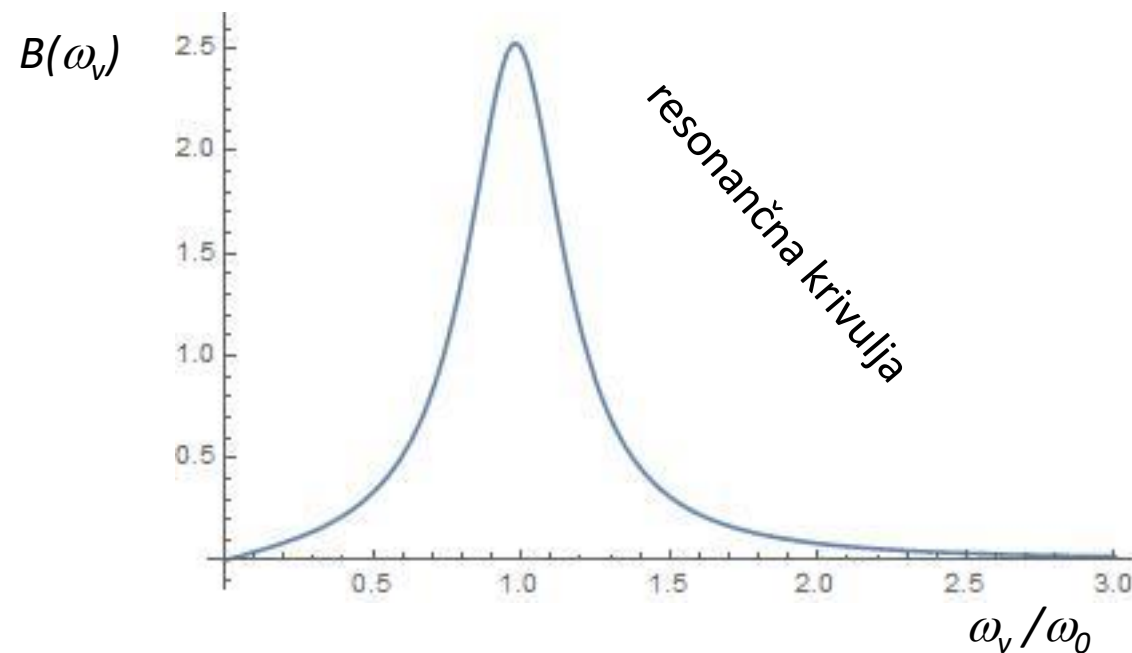
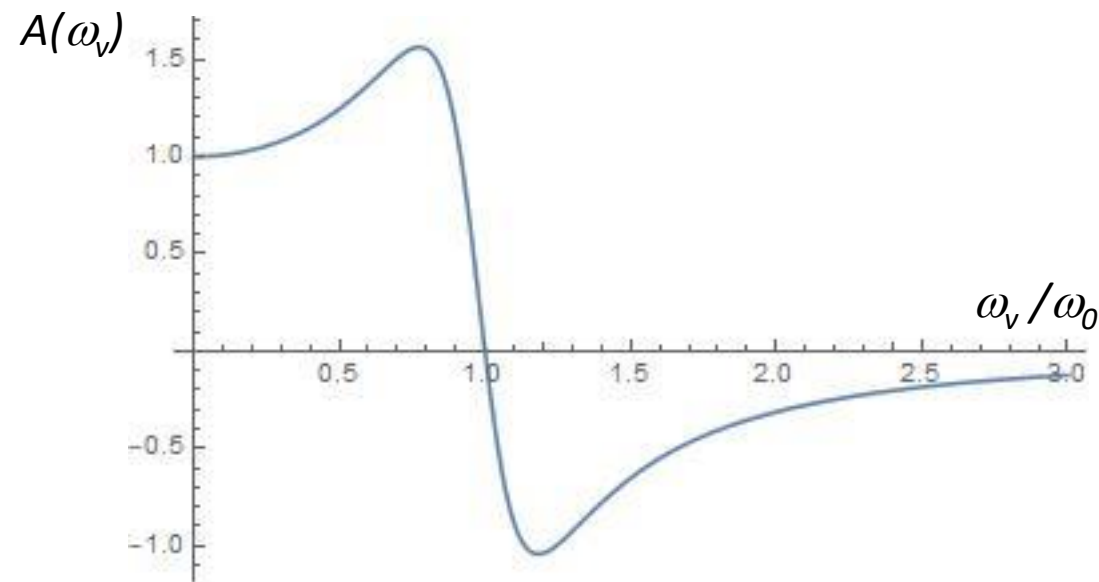
$$m\ddot{x} + \beta\dot{x} + kx = x_v \sin \omega_v t$$

$$\omega_{1,2} = -\beta' \pm i\omega' \quad \omega' = \sqrt{\omega_0^2 - \beta'^2}$$

$$x(t=0) = 0 \quad \dot{x}(t=0) = v_0$$

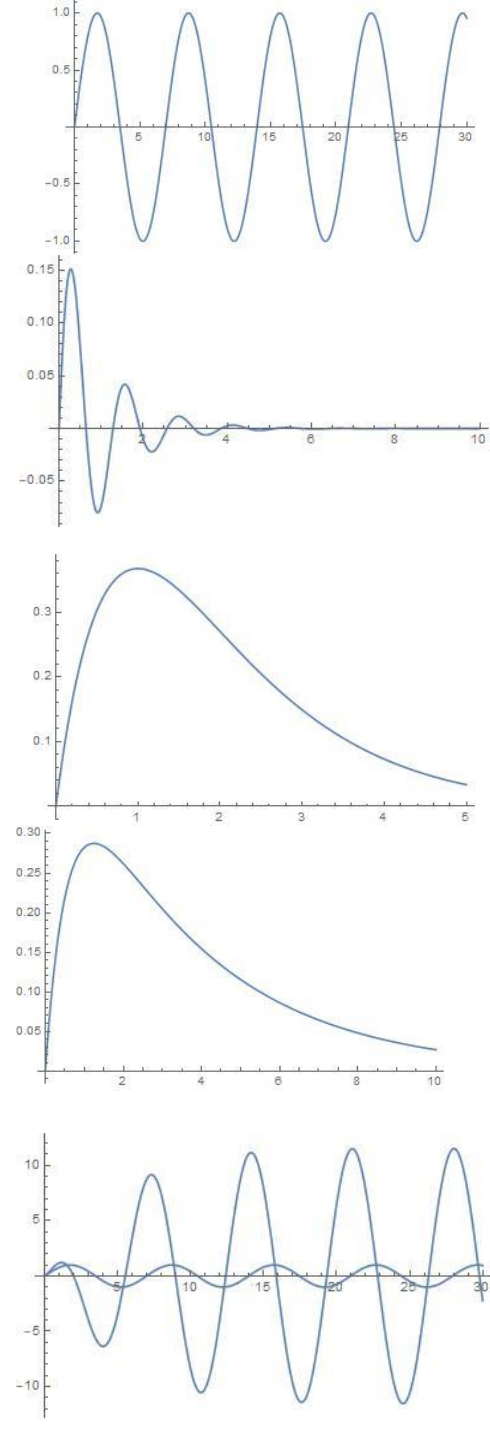
$$x(t) = e^{-\beta't} \left[ \frac{v_0 - A\omega_v + B\beta'}{\omega'} \sin \omega't + B \cos \omega't \right] +$$

$$+ A \sin \omega_v t - B \cos \omega_v t$$



# Nihanje

način	enačba	lastne vredn.	oznake	rešitev
sinusno	$\ddot{x} + \frac{k}{m} x = 0$	$\omega_{1,2} = \pm i \omega_0$	$\omega_0 = \sqrt{k/m}$	$x(t) = \frac{v_0}{\omega_0} \sin \omega_0 t$
podkritično dušeno	$\ddot{x} + \frac{\beta}{m} \dot{x} + \frac{k}{m} x = 0$	$\omega_{1,2} = -\beta' \pm i \omega'$	$\omega' = \sqrt{\omega_0^2 - \beta'^2},$ $\beta' = \beta / 2m$	$x(t) = \frac{v_0}{\omega'} e^{-\beta' t} \sin \omega' t$
kritično dušeno	$\ddot{x} + \frac{\beta}{m} \dot{x} + \frac{k}{m} x = 0$	$\omega_{1,2} = -\beta'$		$x(t) = v_0 t e^{-\beta' t}$
nadkritično dušeno	$\ddot{x} + \frac{\beta}{m} \dot{x} + \frac{k}{m} x = 0$	$\omega_{1,2} = -\beta' \pm i \omega''$	$\omega'' = \sqrt{\beta'^2 - \omega_0^2}$	$x(t) = \frac{v_0}{\beta'} e^{-\beta' t} \sinh \omega'' t$
vsiljeno	$\ddot{x} + \frac{\beta}{m} \dot{x} + \frac{k}{m} x = x_v \sin \omega_v t$	$\omega_{1,2} = -\beta' \pm i \omega'$		$x(t) = e^{-\beta' t} [(c / \omega') \sin \omega' t - B \cos \omega' t] + A \sin \omega_v t - B \cos \omega_v t$



začetni pogoji:  $x(t=0) = 0, \dot{x}(t=0) = v_0$

## Nihanje

$$x(t) = e^{-\beta t} [(c / \omega') \sin \omega' t - x_b \cos \omega' t] + x_a \sin \omega_v t + x_b \cos \omega_v t$$

