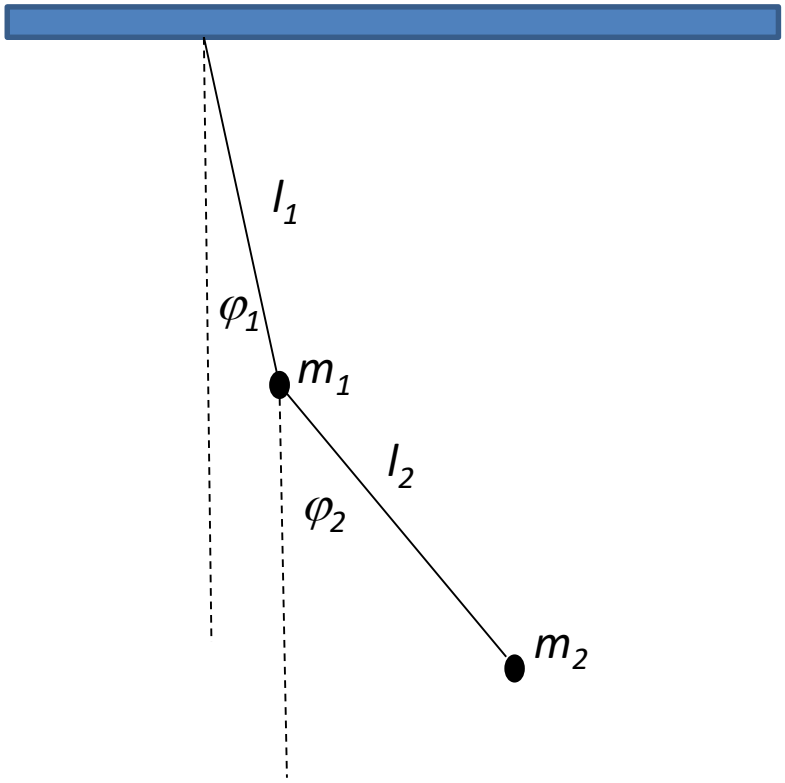
Sklopljeno nihanje – Lagrange-ov formalizem

$$\mathcal{L} = W_{kin} - W_{pot}$$

$$\begin{cases} \dfrac{\partial \mathcal{L}}{\partial \varphi_1} = \dfrac{d}{dt}\left[\dfrac{\partial \mathcal{L}}{\partial \dot{\varphi}_1}\right] \\ \dfrac{\partial \mathcal{L}}{\partial \varphi_2} = \dfrac{d}{dt}\left[\dfrac{\partial \mathcal{L}}{\partial \dot{\varphi}_2}\right] \end{cases}$$

$$\frac{\partial \mathcal{L}(\vec{x}, \dot{\vec{x}})}{\partial x_i} = \frac{d}{dt}\left[\frac{\partial \mathcal{L}(\vec{x}, \dot{\vec{x}})}{\partial \dot{x}_i}\right]$$



$$\ddot{\phi}_1 + \frac{m_2}{m_1 + m_2}\frac{l_2}{l_1}\ddot{\phi}_2 + \frac{g}{l_1}\phi_1 = 0$$

$$\ddot{\phi}_2 + \frac{l_1}{l_2}\ddot{\phi}_1 + \frac{g}{l_2}\phi_2 = 0$$

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$$\ddot{\phi}_1 + \frac{m_2}{m_1+m_2}\frac{l_2}{l_1}\ddot{\phi}_2 + \frac{g}{l_1}\phi_1 = 0$$

$$\ddot{\phi}_2 + \frac{l_1}{l_2}\ddot{\phi}_1 + \frac{g}{l_2}\phi_2 = 0$$

$$\underline{M}\ddot{\vec{\varphi}} + \underline{K}\vec{\varphi} = 0$$

$$\underline{M} = \begin{bmatrix} 1 & \frac{m_2}{m_1+m_2}\frac{l_2}{l_1} \\ \frac{l_1}{l_2} & 1 \end{bmatrix} \quad \underline{K} = \begin{bmatrix} \frac{g}{l_1} & 0 \\ 1 & \frac{g}{l_2} \end{bmatrix}$$

$$\vec{\varphi}_L = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} e^{i\omega t}$$

$$\Rightarrow \left[-\omega^2 \underline{M} + \underline{K}\right]\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = 0$$

$$\det\left[-\omega^2 \underline{M} + \underline{K}\right] = 0$$

$$\omega_1^2 = \left(2-\sqrt{2}\right)\sqrt{\frac{g}{l}} \qquad \omega_2^2 = \left(2+\sqrt{2}\right)\sqrt{\frac{g}{l}}$$

$$\varphi_1 = a_1 e^{i\omega_1 t} + b_1 e^{-i\omega_1 t} + a_2 e^{i\omega_2 t} + b_2 e^{-i\omega_2 t}$$

$$\varphi_2 = \sqrt{2}a_1 e^{i\omega_1 t} + \sqrt{2}b_1 e^{-i\omega_1 t} - \sqrt{2}a_2 e^{i\omega_2 t} - \sqrt{2}b_2 e^{-i\omega_2 t}$$

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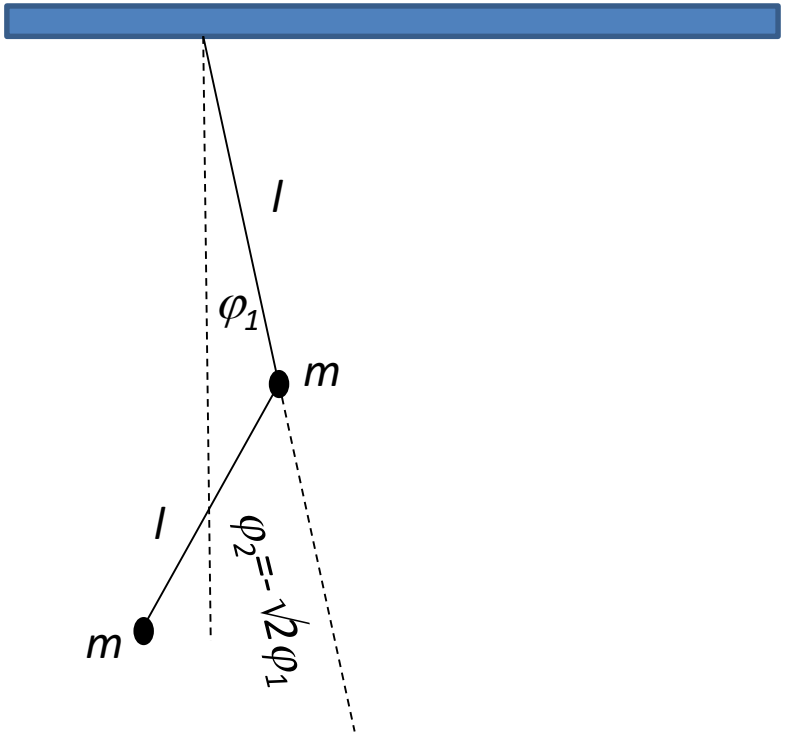
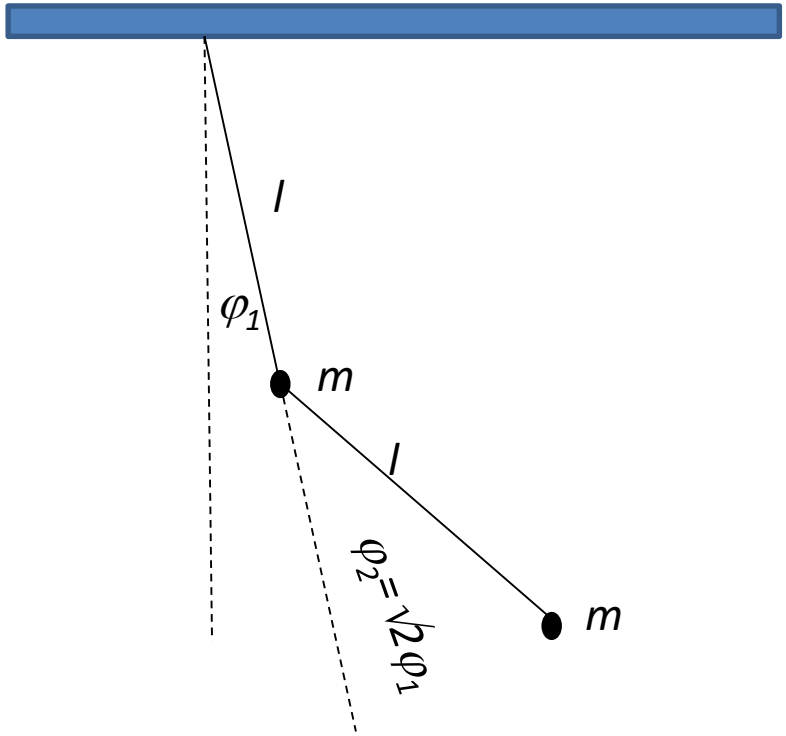


$$\omega_2^2 = (2+\sqrt{2})\frac{g}{l}$$

$$\varphi_1 = a_2 e^{i\omega_2 t} + b_2 e^{-i\omega_2 t}$$

$$\varphi_2 = -\sqrt{2}a_2 e^{i\omega_2 t} - \sqrt{2}b_2 e^{-i\omega_2 t}$$



$$\omega_1^2 = (2-\sqrt{2})\frac{g}{l}$$

$$\varphi_1 = a_1 e^{i\omega_1 t} + b_1 e^{-i\omega_1 t}$$

$$\varphi_2 = \sqrt{2}a_1 e^{i\omega_1 t} + \sqrt{2}b_1 e^{-i\omega_1 t}$$

https://www.myphysicslab.com/pendulum/double-pendulum-en.html