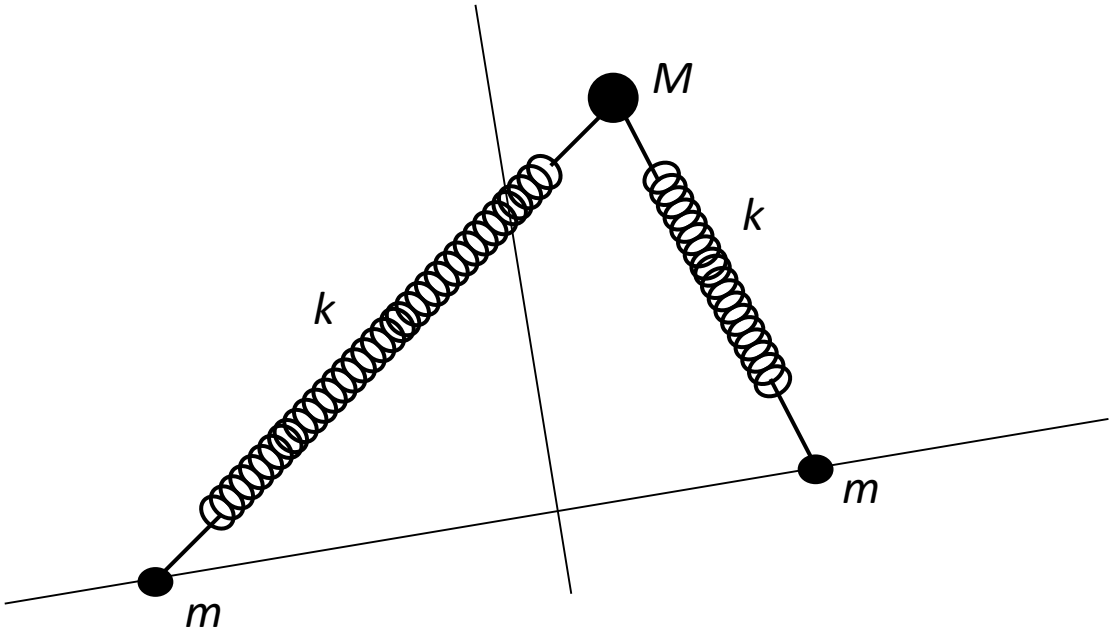
Sklopljeno nihanje – Lagrange-ov formalizem



model nihanja 3-atomske molekule

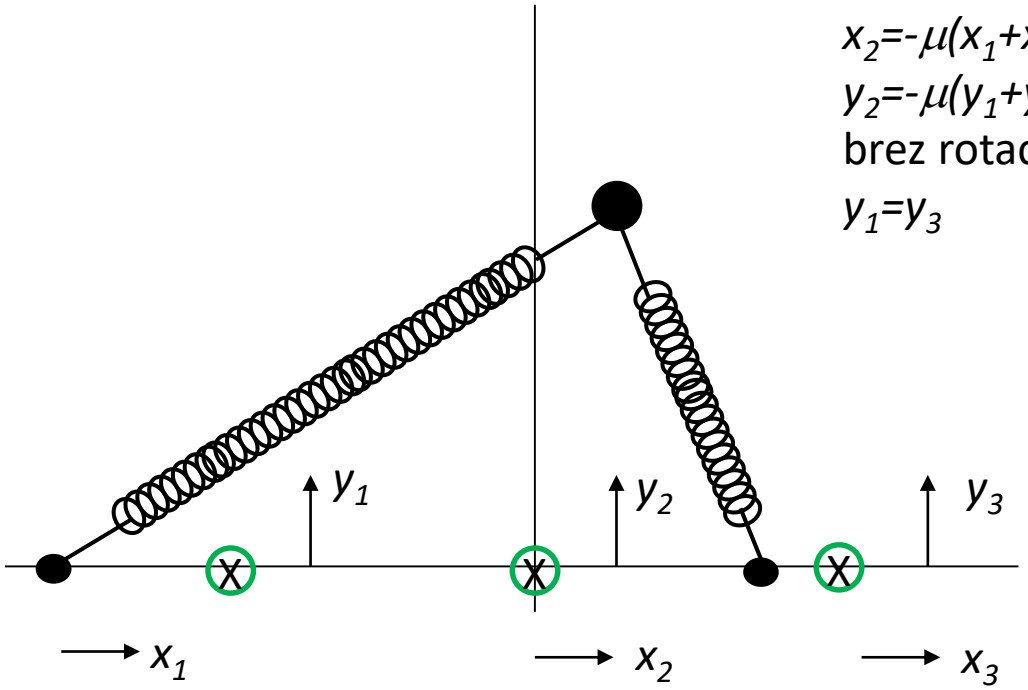
brez translacije:

$x_2=-\mu(x_1+x_3)$

$y_2=-\mu(y_1+y_3)$

brez rotacije:

$y_1=y_3$



X ravnovesna lega

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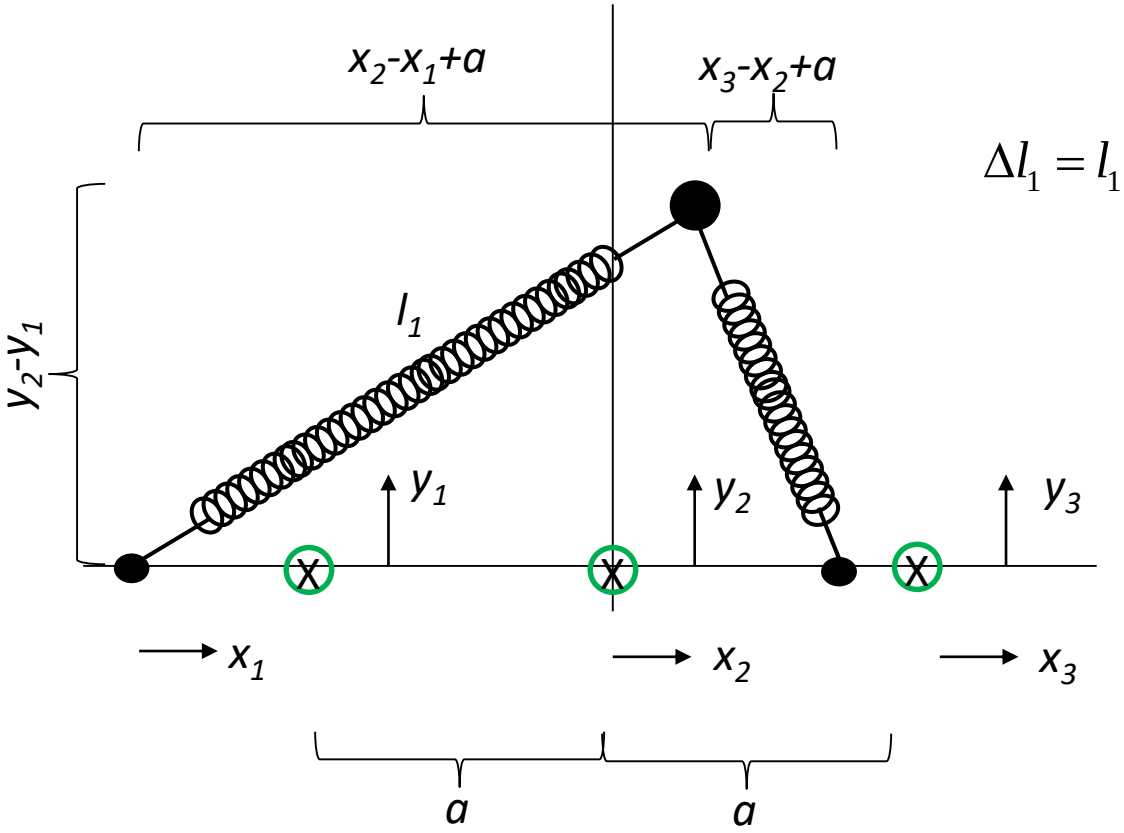

$$\Delta l_1 = l_1 - a = \sqrt{\left(x_2 - x_1 + a\right)^2 + \left(y_2 - y_1\right)^2} - a$$

$$x_{1,2}, y_{1,2} << 1$$

$$\Delta l_1 \approx x_2 - x_1$$

$$\mu = \frac{m}{M}$$

$$W_k = \frac{m}{2}\left[(1+\mu)(\dot{x}_1^2 + \dot{x}_3^2) + 2\mu\dot{x}_1\dot{x}_3\right]$$

$$W_{pr} = \frac{k}{2}\left[(2\mu^2 + 2\mu + 1)(x_1^2 + x_3^2) + 4\mu(\mu+1)x_1x_3\right]$$

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$$W_k = \frac{m}{2}\left[(1+\mu)(\dot{x}_1^2 + \dot{x}_3^2) + 2\mu\dot{x}_1\dot{x}_3\right]$$

$$W_{pr} = \frac{k}{2}\left[(2\mu^2 + 2\mu + 1)(x_1^2 + x_3^2) + 4\mu(\mu+1)x_1x_3\right]$$

$$\frac{\partial\mathcal{L}(\vec{x},\dot{\vec{x}})}{\partial x_i} = \frac{d}{dt}\left[\frac{\partial\mathcal{L}(\vec{x},\dot{\vec{x}})}{\partial\dot{x}_i}\right]$$

$$\ddot{x}_1 + \frac{k}{m}(x_1 - x_2) = 0$$

$$\ddot{x}_2 + \frac{k}{m}\mu(2x_2 - x_1 - x_3) = 0$$

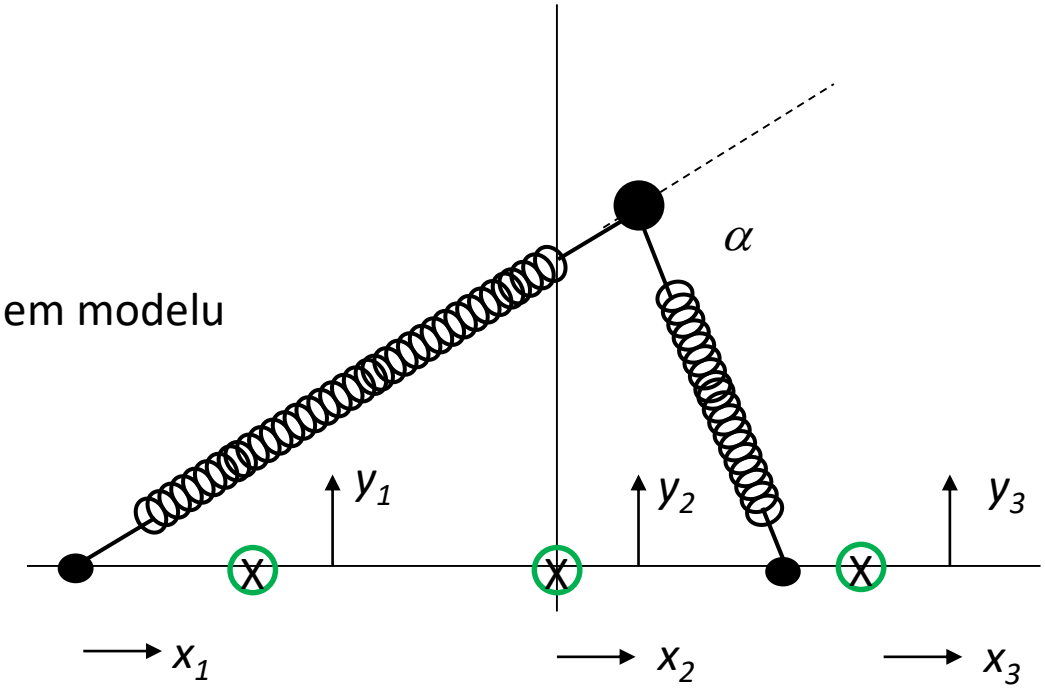
$$\ddot{x}_3 + \frac{k}{m}(x_3 - x_2) = 0$$

lastne frekvence:

$$\omega_2^2 = \frac{k}{m}(1+2\mu), \quad x_{10} = x_{30},\ x_{20} = -2\mu x_{30}$$

$$\omega_1^2 = \frac{k}{m}, \quad x_{10} = -x_{30},\ x_{20} = 0$$

transverzalno nihanje (ga ni v osnovnem modelu z dvema vzmetema):



$W_p \propto \alpha^2$

$$\alpha \propto \frac{2\mu+1}{a} y_1$$

$$W_p = k\gamma^2 \left[(2\mu+1) y_1\right]^2$$

$$\ddot{y}_1 + \gamma^2 \frac{k}{m}(2\mu+1) y_1 = 0 \qquad \omega_0^2 = \gamma^2 \frac{k}{m}(2\mu+1)$$

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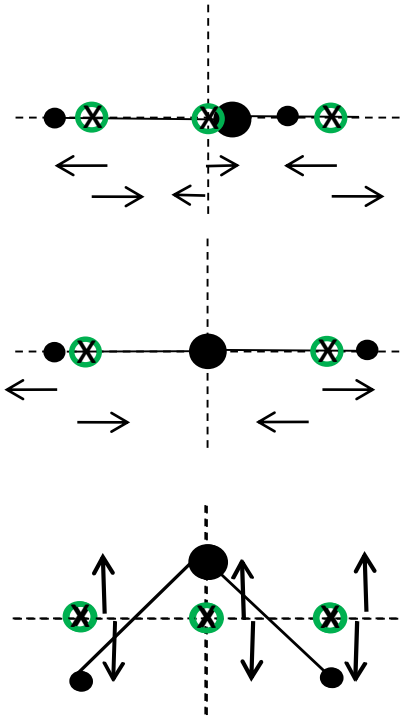
lastna nihanja:

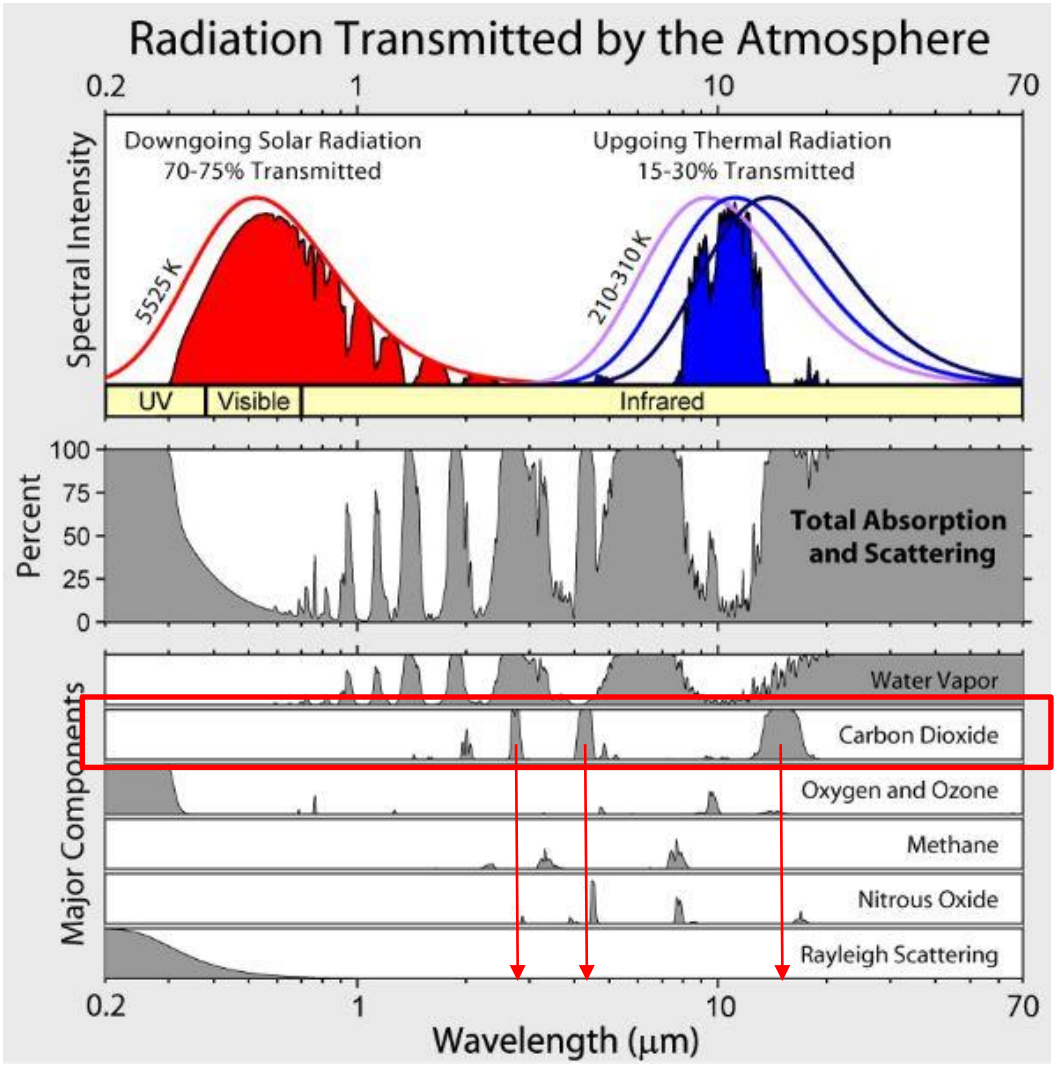


$$\omega_2^2 = \frac{k}{m}(1+2\mu), \quad x_1 = x_3$$

$$\omega_1^2 = \frac{k}{m}, \quad x_1 = -x_3$$

$$\omega_0^2 = \gamma^2 \frac{k}{m}(2\mu+1)$$

Sklopljeno nihanje – Lagrange-ov formalizem



$\omega_+ / \omega_- = \sqrt{(1+2\mu)} = \lambda_2 / \lambda_1$

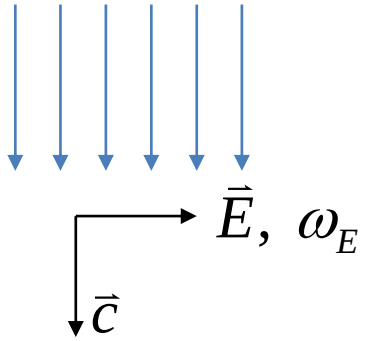
$\sqrt{11/3} = 1{,}91 \quad 1{,}61$

$\omega_0 / \omega_- = \gamma\sqrt{(1+2\mu)} = \lambda_2 / \lambda_3$

$\gamma = (\lambda_2/\lambda_3) / \sqrt{(1+2\mu)} \approx 0{,}16$

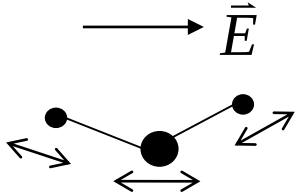
model nihanja CO_2

svetloba – EM valovanje



atmosfera

O_2, CO_2, H_20, N_2,..



zaradi neenakomerne porazdelitve naboja v molekuli $\vec{E}$ povzroči (vsiljeno) nihanje;

največja amplituda ko $\omega_E = \omega_{+,-,0}$

$\lambda_1 \approx 2{,}8\ \mu m$, $\lambda_2 \approx 4{,}5\ \mu m$,

$\lambda_3 \approx 15\ \mu m$