Univerza v Ljubljani THE UNIVERSITY OF TOKYO

Flavour Physics at B-factories and Hadron Colliders

Part 2: CP violation primer

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An arbitrary linear combination of the neutral B-meson flavor eigenstates

$$a\left|B^0\right\rangle + b\left|\overline{B}^0\right\rangle$$

is governed by a time-dependent Schroedinger equation

$$i\frac{d}{dt}\begin{pmatrix} a \\ b \end{pmatrix} = H\begin{pmatrix} a \\ b \end{pmatrix} = (M - \frac{i}{2}\Gamma)\begin{pmatrix} a \\ b \end{pmatrix}$$

M and Γ are 2x2 Hermitian matrices. *CPT* invariance → $H_{11}=H_{22}$

$$M = \begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix}, \Gamma = \begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}$$

diagonalize →

Time evolution in the B system

The light B_L and heavy B_H mass eigenstates with eigenvalues $m_H, \Gamma_H, m_L, \Gamma_L$ are given by

$$\left|B_L\right\rangle = p\left|B^0\right\rangle + q\left|\overline{B}^0\right\rangle$$

$$\left|B_H\right\rangle = p\left|B^0\right\rangle - q\left|\overline{B}^0\right\rangle$$

With the eigenvalue differences

$$\Delta m_B = m_H - m_L, \Delta\Gamma_B = \Gamma_H - \Gamma_L$$

Which are related to the M and Γ matrix elements

$$(\Delta m_B)^2 - \frac{1}{4}(\Delta\Gamma_B)^2 = 4(\left|M_{12}\right|^2 - \frac{1}{4}\left|\Gamma_{12}\right|^2)$$

$$\Delta m_B \Delta\Gamma_B = 4\,\mathrm{Re}(M_{12}\Gamma_{12}^{\ *})$$

The ratio p/q is

$$\frac{q}{p} = -\frac{\Delta m_B - \frac{i}{2}\Delta\Gamma_B}{2(M_{12} - \frac{i}{2}\Gamma_{12})} = -\frac{2(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)}{\Delta m_B - \frac{i}{2}\Delta\Gamma_B}$$

What do we know about Δm_B and $\Delta\Gamma_B$?

Δm_B=(0.502+-0.007) ps^{-1} well measured

→ $\Delta m_B/\Gamma_B = x_d$ =0.771+-0.012

$\Delta\Gamma_B/\Gamma_B$ not measured, expected O(0.01), due to decays common to B and anti-B - O(0.001).

→ **$\Delta\Gamma_B << \Delta m_B$**

Since $\Delta\Gamma_B << \Delta m_B$

$$\Delta m_B = 2|M_{12}|$$

$$\Delta\Gamma_B = 2\,\mathrm{Re}(M_{12}\Gamma_{12}^*)/|M_{12}|$$

and

$$\frac{q}{p} = -\frac{|M_{12}|}{M_{12}}$$

or to next order

$$\frac{q}{p} = -\frac{|M_{12}|}{M_{12}}\left[1 - \frac{1}{2}\mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right)\right]$$

B^0 and $\bar{B}^0$ can be written as an admixture of the states B_H and B_L

$$\left|B^0\right\rangle = \frac{1}{2p}\left(\left|B_L\right\rangle + \left|B_H\right\rangle\right)$$

$$\left|\overline{B}^0\right\rangle = \frac{1}{2q}\left(\left|B_L\right\rangle - \left|B_H\right\rangle\right)$$

Time evolution

Any B state can then be written as an admixture of the states B_H and B_L, and the amplitudes of this admixture evolve in time

$$a_H(t) = a_H(0)e^{-iM_H t}e^{-\Gamma_H t/2}$$

$$a_L(t) = a_L(0)e^{-iM_L t}e^{-\Gamma_L t/2}$$

A B^0 state created at t=0 (denoted by B^0_{phys}) has $a_H(0)= a_L(0)=1/(2p)$; an anti-B at t=0 (anti-B^0_{phys}) has $a_H(0)= a_L(0)=1/(2q)$

At a later time t, the two coefficients are not equal any more because of the difference in phase factors exp(-iMt)

→initial B^0 becomes a linear combination of B and anti-B

→mixing

Time evolution of B's

Time evolution can also be written in the B^0 in $\overline{B}^0$ basis:

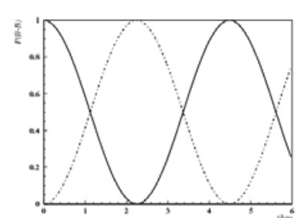
$$\left|B^0_{phys}(t)\right\rangle = g_+(t)\left|B^0\right\rangle + (q/p)g_-(t)\left|\overline{B}^0\right\rangle$$
$$\left|\overline{B}^0_{phys}(t)\right\rangle = (p/q)g_-(t)\left|B^0\right\rangle + g_+(t)\left|\overline{B}^0\right\rangle$$

with
$$g_+(t) = e^{-iMt}e^{-\Gamma t/2}\cos(\Delta m t/2)$$
$$g_-(t) = e^{-iMt}e^{-\Gamma t/2} i\sin(\Delta m t/2)$$

$M = (M_H + M_L)/2$

If B mesons were stable ($\Gamma=0$), the time evolution would look like:

$$g_+(t) = e^{-iMt}\cos(\Delta m t/2)$$
$$g_-(t) = e^{-iMt} i\sin(\Delta m t/2)$$



→Probability that a B turns into its anti-particle **→beat**

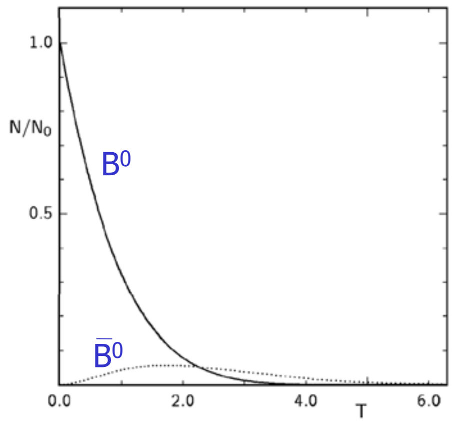
$$\left|\left\langle \overline{B}^0 \middle| B^0_{phys}(t)\right\rangle\right|^2 = |q/p|^2 |g_-(t)|^2 = |q/p|^2 \sin^2(\Delta m t/2)$$

→Probability that a B remains a B

$$\left|\left\langle B^0 \middle| B^0_{phys}(t)\right\rangle\right|^2 = |g_+(t)|^2 = \cos^2(\Delta m t/2)$$

→blackboard exercise on the two level system

B mesons of course do decay →



B^0 at t=0

Evolution in time

- Full line: B^0
- dotted: B^0

T: in units of $\tau=1/\Gamma$

Discovery of mixing: ARGUS (1987)

>1000 citations

Phys.Lett. B192 (1987) 245.

Razpadna verjetnost

Decay probability $P(B^0 \to f, t) \propto \left|\left\langle f \left| H \right| B^0_{phys}(t)\right\rangle\right|^2$

Decay amplitudes of B and anti-B to the same final state ***f***

$$A_f = \left\langle f \left| H \right| B^0 \right\rangle$$

$$\overline{A}_f = \left\langle f \left| H \right| \overline{B}^0 \right\rangle$$

Decay amplitude as a function of time:

$$\left\langle f \left| H \right| B^0_{phys}(t)\right\rangle = g_+(t)\left\langle f \left| H \right| B^0 \right\rangle + (q/p) g_-(t) \left\langle f \left| H \right| \overline{B}^0 \right\rangle$$

$$= g_+(t) A_f + (q/p) g_-(t) \overline{A}_f$$

... and similarly for the anti-B

CP violation: three types

Decay amplitudes of B and anti-B to the same final state ***f***

$$A_f = \langle f | H | B^0 \rangle$$

$$\overline{A}_f = \langle f | H | \overline{B}^0 \rangle$$

Define a parameter λ

$$\lambda = \frac{q}{p} \frac{\overline{A}_f}{A_f}$$

Three types of CP violation (CPV):

~~CP~~ in decay: $|\overline{A}/A| \neq 1$
~~CP~~ in mixing: $|q/p| \neq 1$

$$\left.\begin{array}{l} \\ \\ \end{array}\right\} |\lambda| \neq 1$$

~~CP~~ in interference between mixing and decay: even if $|\lambda| = 1$ if only $\mathrm{Im}(\lambda) \neq 0$

CP violation in decay

~~CP~~ in decay: $|\overline{A}/A| \neq 1$

(and of course also $|\lambda| \neq 1$)

$$a_f = \frac{\Gamma(B^+ \to f,t) - \Gamma(B^- \to \overline{f},t)}{\Gamma(B^+ \to f,t) + \Gamma(B^- \to \overline{f},t)} =$$

$$= \frac{1 - |\overline{A}/A|^2}{1 + |\overline{A}/A|^2}$$

Also possible for the neutral B.

CP violation in decay

CPV in decay: $|\overline{A}/A| \neq 1$: how do we get there?

$$A_f = \sum_i A_i e^{i(\delta_i+\varphi_i)}$$

$$\overline{A}_{\overline{f}} = \sum_i A_i e^{i(\delta_i-\varphi_i)}$$

In general, A is a sum of amplitudes with strong phases δ_i and weak phases ϕ_i. The amplitudes for anti-particles have same strong phases and opposite weak phases ->

$$\left|\frac{\overline{A}_{\overline{f}}}{A_f}\right| = \left|\frac{\sum_i A_i e^{i(\delta_i-\varphi_i)}}{\sum_i A_i e^{i(\delta_i+\varphi_i)}}\right|$$

$$|A_f|^2 - |\overline{A}_{\overline{f}}|^2 = \sum_{i,j} A_i A_j \sin(\varphi_i - \varphi_j)\sin(\delta_i - \delta_j)$$

CPV in decay: need at least two interfering amplitudes with different weak and strong phases.

CP violation in mixing

$\not{CP}$ in mixing: $|q/p| \neq 1$ (again $|\lambda| \neq 1$)

In general: probability for a B to turn into an anti-B can differ from the probability for an anti-B to turn into a B.

$$|B^0_{phys}(t)\rangle = g_+(t)|B^0\rangle + (q/p)g_-(t)|\overline{B}^0\rangle$$

$$|\overline{B}^0_{phys}(t)\rangle = (p/q)g_-(t)|B^0\rangle + g_+(t)|\overline{B}^0\rangle$$

Example: semileptonic decays:

$$\langle l^-\nu X|H|B^0_{phys}(t)\rangle = (q/p)g_-(t)A^*$$

$$\langle l^+\nu X|H|\overline{B}^0_{phys}(t)\rangle = (p/q)g_-(t)A$$

CP violation in mixing

$$a_{sl} = \frac{\Gamma(\overline{B}^0_{phys}(t) \to l^+ \nu X) - \Gamma(B^0_{phys}(t) \to l^- \nu X)}{\Gamma(\overline{B}^0_{phys}(t) \to l^+ \nu X) + \Gamma(B^0_{phys}(t) \to l^- \nu X)} =$$

$$= \frac{|p/q|^2 - |q/p|^2}{|p/q|^2 + |q/p|^2} = \frac{1 - |q/p|^4}{1 + |q/p|^4}$$

-> Small, since to first order `|q/p|~1.` Next order:

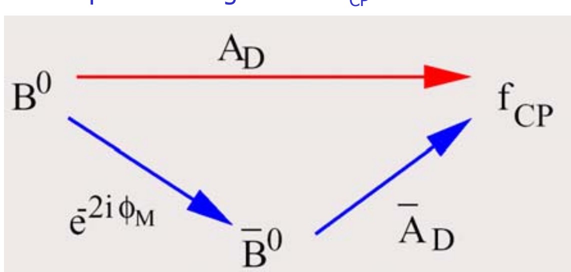
$$\frac{q}{p} = -\frac{|M_{12}|}{M_{12}}\left[1 - \frac{1}{2}\mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right)\right]$$

Expect O(0.01) effect in semileptonic decays

CP violation in the interference between decays with and without mixing

CP violation in the interference between mixing and decay to a state accessible in both B^0 and anti-B^0 decays

For example: a CP eigenstate f_{CP} like $\pi^+ \pi^-$



$$\lambda = \frac{q}{p}\frac{\overline{A}_f}{A_f}$$

`We can get CP violation if Im(λ) ≠ 0, even if |λ| = 1`

CP violation in the interference between decays with and without mixing

Decay rate asymmetry:

$$a_{f_{CP}} = \frac{P(\overline{B}^0 \to f_{CP},t) - P(B^0 \to f_{CP},t)}{P(\overline{B}^0 \to f_{CP},t) + P(B^0 \to f_{CP},t)}$$

Decay rate: $P(B^0 \to f_{CP},t) \propto \left| \langle f_{CP} | H | B^0_{phys}(t) \rangle \right|^2$

Decay amplitudes vs time:

$$\langle f_{CP} | H | B^0_{phys}(t) \rangle = g_+(t) \langle f_{CP} | H | B^0 \rangle + (q/p) g_-(t) \langle f_{CP} | H | \overline{B}^0 \rangle$$

$$= g_+(t) A_{f_{CP}} + (q/p) g_-(t) \overline{A}_{f_{CP}}$$

$$\langle f_{CP} | H | \overline{B}^0_{phys}(t) \rangle = (p/q) g_-(t) \langle f_{CP} | H | B^0 \rangle + g_+(t) \langle f_{CP} | H | \overline{B}^0 \rangle$$

$$= (p/q) g_-(t) A_{f_{CP}} + g_+(t) \overline{A}_{f_{CP}}$$

CP violation in the interference between decays with and without mixing

$$a_{f_{CP}} = \frac{P(\overline{B}^0 \to f_{CP},t) - P(B^0 \to f_{CP},t)}{P(\overline{B}^0 \to f_{CP},t) + P(B^0 \to f_{CP},t)} =$$

$$\lambda = \frac{q}{p} \frac{\overline{A}_f}{A_f}$$

$$= \frac{(1 - |\lambda_{f_{CP}}|^2) \cos(\Delta m t) - 2\,\mathrm{Im}(\lambda_{f_{CP}}) \sin(\Delta m t)}{1 + |\lambda_{f_{CP}}|^2}$$

$$= C \cos(\Delta m t) + S \sin(\Delta m t)$$

Non-zero effect if Im(λ) ≠ 0, even if |λ| = 1

If in addition |λ| = 1 ->

$$a_{f_{CP}} = -\mathrm{Im}(\lambda) \sin(\Delta m t)$$

CP violation in the interference between decays with and without mixing

One more form for λ:

$$\lambda_{f_{CP}} = \frac{q}{p}\frac{\overline{A}_{f_{CP}}}{A_{f_{CP}}} = \eta_{f_{CP}} \frac{q}{p}\frac{\overline{A}_{\overline{f}_{CP}}}{A_{f_{CP}}}$$

η_{fcp}=+-1 CP parity of f_{CP}

-> we get one more (–1) sign when comparing asymmetries in two states with opposite CP parity

$$a_{f_{CP}} = -\mathrm{Im}(\lambda_{f_{CP}})\sin(\Delta m t)$$

B and anti-B from the Υ(4s)

B and anti-B from the Υ(4s) decay are in a l=1 state.

They cannot mix independently (either BB or anti-B anti-B states are forbidden with l=1 due to Bose symmetry).

After one of them decays, the other evolves independently ->

-> only time differences between one and the other decay matter (for mixing).

Assume

•one decays to a CP eigenstate f_{CP} (e.g. $\pi\pi$ or $J/\psi K_S$) at time t_{fCP} and

•the other at t_{ftag} to a flavor-specific state f_{tag} (=state only accessible to a B^0 and not to a anti-B^0 (or vice versa), e.g. $B^0 \rightarrow D^0\pi$, $D^0 \rightarrow K^-\pi^+$)

also known as 'tag' because it tags the flavour of the B meson it comes from

Time evolution for B and anti-B from the Y(4s)

The time evolution for the B anti-B pair from Y(4s) decay

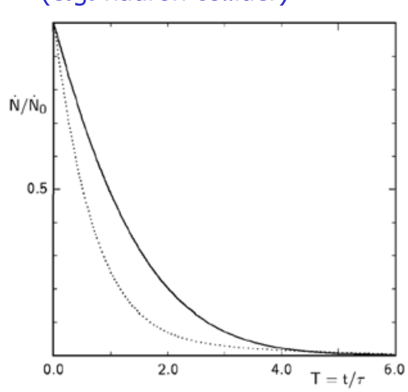
$$R(t_{tag}, t_{f_{CP}}) = e^{-\Gamma(t_{tag}+t_{f_{CP}})} \left|\overline{A_{tag}}\right|^2 \left|A_{f_{CP}}\right|^2$$
$$[1 + \left|\lambda_{f_{CP}}\right|^2 + \cos\left[\Delta m (t_{tag} - t_{f_{CP}})\right](1 - \left|\lambda_{f_{CP}}\right|^2)$$
$$- 2\sin\left(\Delta m (t_{tag} - t_{f_{CP}})\right)\mathrm{Im}(\lambda_{f_{CP}})]$$

with $\lambda_{f_{CP}} = \frac{q}{p}\frac{\overline{A}_{f_{CP}}}{A_{f_{CP}}}$

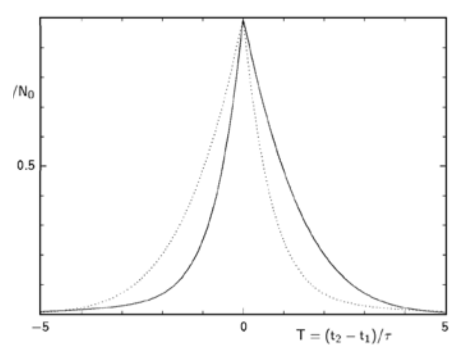
-> in asymmetry measurements at Y(4s) we have to use t_{ftag}-t_{fCP} instead of absolute time t.

Decay rate to f_{CP}

Incoherent production (e.g. hadron collider)

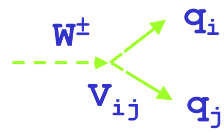


coherent production at Y(4s)



CP violation in SM

CP violation: consequence of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix



$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

CP violation in SM

$$\mathcal{L} = V_{ij} \overline{U}_i \gamma^{\mu} (1-\gamma_5) D_j W_{\mu}^{+} + V_{ij}^{*} \overline{D}_i \gamma^{\mu} (1-\gamma_5) U_j W_{\mu}$$

$$\Updownarrow CP$$

$$\mathcal{L}_{CP} = V_{ij} \overline{D}_i \gamma^{\mu} (1-\gamma_5) U_j W_{\mu} + V_{ij}^{*} \overline{U}_i \gamma^{\mu} (1-\gamma_5) D_j W_{\mu}$$

If $V_{ij}=V_{ij}^{*}$ ► $\mathcal{L}=\mathcal{L}_{CP}$ ► CP is conserved

CKM matrix

3x3 ortogonal matrix: 3 parameters - angles

3x3 unitary matrix: 18 parameters, 9 conditions = 9 free parameters, 3 angles and 6 phases

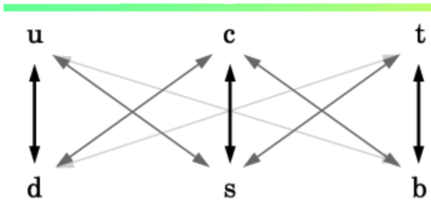
6 quarks: 5 relative phases can be transformed away (by redefinig the quark fields)

1 phase left -> the matrix is in general complex

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{13} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

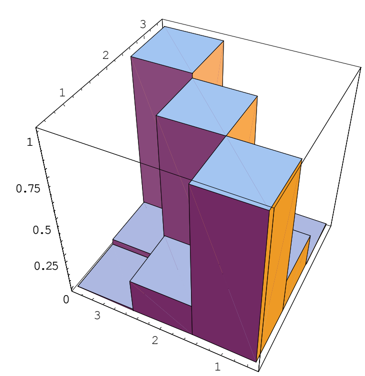
$s_{12}=\sin\theta_{12}$, $c_{12}=\cos\theta_{12}$ etc.

CKM matrix



Transitions between members of the same family more probable (=thicker lines) than others

-> CKM: almost a diagonal matrix, but not completely ->



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CKM matrix

Almost a diagonal matrix, but not completely ->

Wolfenstein parametrisation: expand in the parameter λ (=$\sin\theta_c$=0.22)

A, ρ and η: all of order one

$$V = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

CKM matrix

define $$s_{12} \equiv \lambda, s_{23} \equiv A\lambda^2, s_{13}e^{-i\delta} \equiv A\lambda^3(\rho - i\eta)$$

Then to O(λ^6)

$$\begin{aligned} V_{us} &= \lambda, V_{cb} = A\lambda^2, \\ V_{ub} &= A\lambda^3(\bar{\rho} - i\bar{\eta}), \\ V_{td} &= A\lambda^3(1-\bar{\rho} - i\bar{\eta}), \\ \mathrm{Im}\, V_{cd} &= -A\lambda^5\eta, \\ \mathrm{Im}\, V_{ts} &= -A\lambda^4\eta, \\ \bar{\rho} &= \rho(1-\frac{\lambda^2}{2}), \bar{\eta} = \eta(1-\frac{\lambda^2}{2}) \end{aligned}$$

Unitary relations

Rows and columns of the V matrix are orthogonal

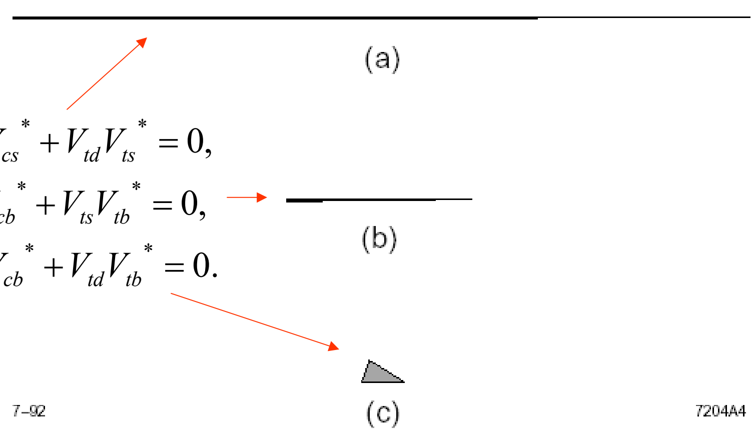
Three examples: 1st+2nd, 2nd+3rd, 1st+3rd columns

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0,$$
$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0,$$
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0.$$

Geometrical representation: triangles in the complex plane.

Unitary triangles

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0,$$
$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0,$$
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0.$$



All triangles have the same area J/2 (about 4x10^{-5})

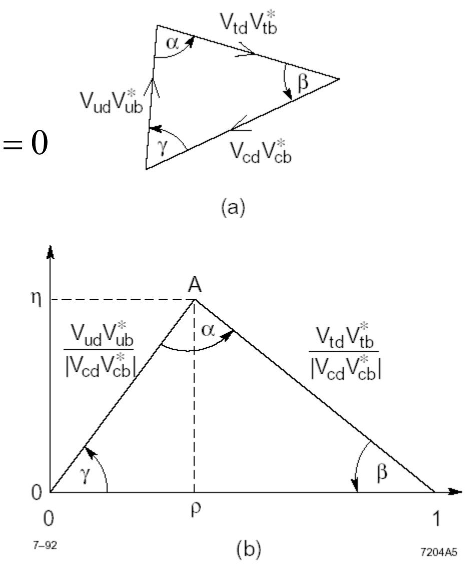
$$J = c_{12}c_{23}c_{13}^2 s_{12}s_{23}s_{13}\sin\delta$$

Jarlskog invariant

Unitarity triangle

THE unitarity triangle:

$$V_{ud}V_{ub}^{\ *}+V_{cd}V_{cb}^{\ *}+V_{td}V_{tb}^{\ *}=0$$



Two notations:

$\phi_1=\beta$

$\phi_2=\alpha$

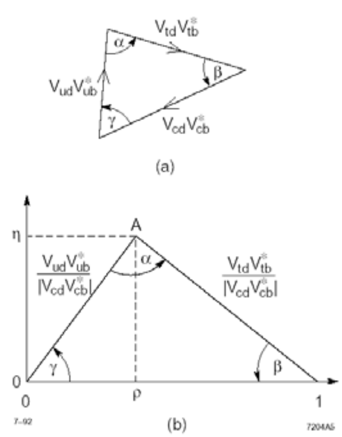
$\phi_3=\gamma$

Angles of the unitarity triangle

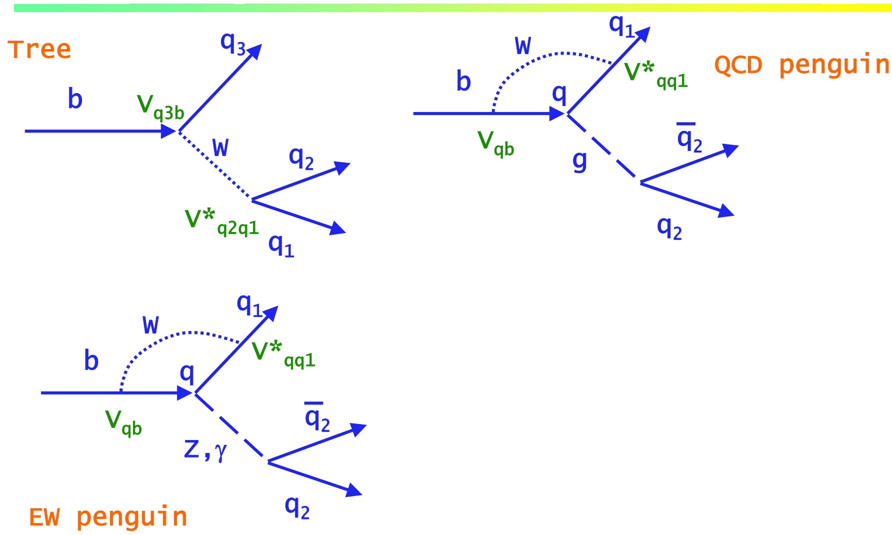
$$\alpha \equiv \phi_2 \equiv \arg\left(\frac{V_{td}V_{tb}^{\ *}}{V_{ud}V_{ub}^{\ *}}\right)$$

$$\beta \equiv \phi_1 \equiv \arg\left(\frac{V_{cd}V_{cb}^{\ *}}{V_{td}V_{tb}^{\ *}}\right)$$

$$\gamma \equiv \phi_3 \equiv \arg\left(\frac{V_{ud}V_{ub}^{\ *}}{V_{cd}V_{cb}^{\ *}}\right) \equiv \pi-\alpha-\beta$$

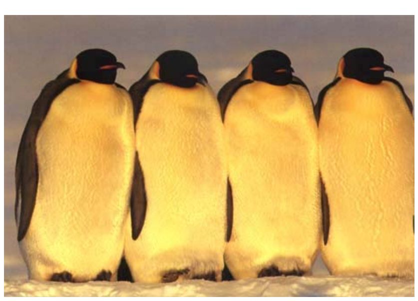


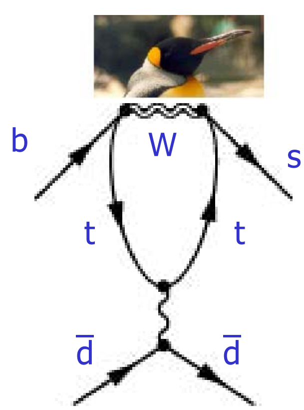
b decays



Why penguin?

Example: b→s transition





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Decay amplitude structure

Quark diagrams: classified in tree (T), penguin and electroweak penguin contributions (P).

Describe the weak-phase structure of B-decay amplitude for the trasition $b \rightarrow q\bar{q}q'$: sum of three terms with definite CKM coefficients:

$$A(q\bar{q}q') = V_{tb}V_{tq'}{}^{*}P^{t}_{q'} + V_{cb}V_{cq'}{}^{*}(T_{c\bar{c}q'}\delta_{qc} + P^{c}_{q'}) + V_{ub}V_{uq'}{}^{*}(T_{u\bar{u}q'}\delta_{qu} + P^{u}_{q'})$$

Decay amplitude structure: *qqs* and *qqd* decays

Use the unitarity condition to simplify the expressions for individual amplitudes:

$$A(c\bar{c}s) = V_{cb}V_{cs}{}^{*}(T_{c\bar{c}s} + P^{c}_{s} - P^{t}_{s}) + V_{ub}V_{us}{}^{*}(P^{u}_{s} - P^{t}_{s}),$$
$$A(u\bar{u}s) = V_{cb}V_{cs}{}^{*}(P^{c}_{s} - P^{t}_{s}) + V_{ub}V_{us}{}^{*}(T_{u\bar{u}s} + P^{u}_{s} - P^{t}_{s}),$$
$$A(s\bar{s}s) = V_{cb}V_{cs}{}^{*}(P^{c}_{s} - P^{t}_{s}) + V_{ub}V_{us}{}^{*}(P^{u}_{s} - P^{t}_{s}).$$

Nice feature: penguin amplitudes only come as differences.

$$A(c\bar{c}d) = V_{tb}V_{td}{}^{*}(P^{t}_{d} - P^{u}_{d}) + V_{cb}V_{cd}{}^{*}(T_{c\bar{c}d} + P^{c}_{d} - P^{u}_{d}),$$
$$A(u\bar{u}d) = V_{tb}V_{td}{}^{*}(P^{t}_{d} - P^{c}_{d}) + V_{ub}V_{ud}{}^{*}(T_{u\bar{u}d} + P^{u}_{d} - P^{t}_{d}),$$
$$A(s\bar{s}d) = V_{tb}V_{td}{}^{*}(P^{t}_{d} - P^{u}_{d}) + V_{cb}V_{cd}{}^{*}(P^{c}_{d} - P^{u}_{d}).$$

Decay asymmetry predictions - overview

Five classes of B decays.

Classes 1 and 2 are expected to have relatively small direct CP violations -> particularly interesting for extracting CKM parameters from interference of decays with and without mixing.

In the remaining three classes, direct CP violations could be significant, decay asymmetries cannot be cleanly interpreted in terms of CKM phases.

1. Decays dominated by a single term: b->ccs and b-> sss. SM cleanly predicts zero (or very small) direct CP violations because the second term is Cabibbo suppressed. Any observation of large direct CP-violating effects in these cases would be a clue to beyond Standard Model physics. The modes $B^+ \to J/\psi K^+$ and $B^+ \to \phi K^+$ are examples of this class. The corresponding neutral modes have cleanly predicted relationships between CKM parameters and the measured asymmetry from interference between decays with and without mixing.

Decay asymmetry predictions - overview

2. Decays with a small second term: b->ccd and b->uud. The expectation that penguin-only contributions are suppressed compared to tree contributions suggests that these modes will have small direct CP violation effects, and an approximate prediction for the relationship between measured asymmetries in neutral decays and CKM phases can be made.

3. Decays with a suppressed tree contribution: b->uus. The tree amplitude is suppressed by small mixing angles, $V_{ub}V_{us}$. The no-tree term may be comparable or even dominate and give large interference effects. An example is $B \to \rho K$.

Decay asymmetry predictions - overview

4. Decays with no tree contribution: b->ssd. Here the interference comes from penguin contributions with different charge 2/3 quarks in the loop. An example is B->KK.

5. Radiative decays: b->sγ . The mechanism here is the same as in case 4 except that the leading contributions come from electromagnetic penguins. An example is B->K$^*\gamma$.

Decay asymmetry predictions – overview
b->qqs

$B \to q\bar{q}s$ Decay Modes

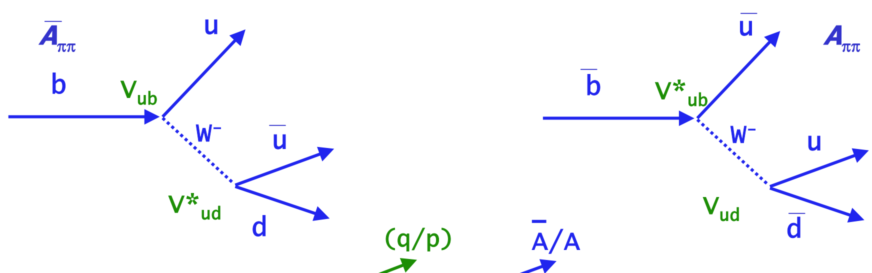
Quark Process	Leading Term	Secondary Term	Sample B_d Modes	B_d Angle	Sample B_s Modes	B_s Angle
$b \to c\bar{c}s$	$V_{cb}V_{cs}^* = A\lambda^2$ tree + penguin $(c-t)$	$V_{ub}V_{us}^* = A\lambda^4(\rho - i\eta)$ penguin only $(u-t)$	$J/\psi\ K_S$	β	$\psi\eta'$ $D_s\overline{D}_s$	β_S
$b \to s\bar{s}s$	$V_{cb}V_{cs}^* = A\lambda^2$ penguin only $(c-t)$	$V_{ub}V_{us}^* = A\lambda^4(\rho - i\eta)$ penguin only $(u-t)$	ϕK_S	β	$\phi\eta'$	0
$b \to u\bar{u}s$ $b \to d\bar{d}s$	$V_{cb}V_{cs}^* = A\lambda^2$ penguin only $(c-t)$	$V_{ub}V_{us}^* = A\lambda^4(\rho - i\eta)$ tree + penguin $(u-t)$	$\pi^0\ K_S$ ρK_S	competing terms	$\phi\pi^0$ K_SK_S	competing terms
$b \to c\bar{u}s$ $b \to u\bar{c}s$	$V_{cb}V_{us}^* = A\lambda^3$ $V_{ub}V_{cs}^* = A\lambda^3(\rho - i\eta)$	0	$D^0K \searrow$ common $\overline{D}^0K \nearrow$ modes	γ	$D^0\phi \searrow$ common $\overline{D}^0\phi \nearrow$ modes	γ

Decay asymmetry predictions – overview
b->qqd

$b \to q\bar{q}d$ Decay Modes

Quark Process	Leading Term	Secondary Term	Sample B_d Modes	B_d Angle * (leading terms only)	Sample B_s Modes	B_s Angle * (leading term)
$b \to c\bar{c}d$	$V_{cb}V_{cd}^* = -A\lambda^3$ tree + penguin $(c-u)$	$V_{tb}V_{td}^* = A\lambda^3(1-\rho+i\eta)$ penguin only $(t-u)$	D^+D^-	$^*\beta$	ψK_S	β_S
$b \to s\bar{s}d$	$V_{tb}V_{td}^* = A\lambda^3(1-\rho+i\eta)$ penguin only $(t-u)$	$V_{cb}V_{cd}^* = A\lambda^3$ penguin only $(c-u)$	$\phi\pi$ $K_S\overline{K}_S$	competing terms	ϕK_S	competing terms
$b \to u\bar{u}d$ $b \to d\bar{d}d$	$V_{ub}V_{ud}^* = A\lambda^3(\rho-i\eta)$ tree + penguin (uc)	$V_{tb}V_{td}^* = A\lambda^3(1-\rho+i\eta)$ penguin only $(t-c)$	$\pi\pi$; $\pi\rho$ πa_1	$^*\alpha$	$\pi^0 K_S$ $\rho^0 K_S$	competing terms
$b \to c\bar{u}d$ $b \to u\bar{c}d$	$V_{cb}V_{ud}^* = A\lambda^2$ $V_{ub}V_{cd}^* = -A\lambda^4(\rho-i\eta)$	0	$D^0\pi^0 \searrow$ common $\overline{D}^0\pi^0 \nearrow$ modes	γ	$D^0K_S \searrow$ common $\overline{D}^0K_S \nearrow$ modes	γ

Decay asymmetry predictions – example $\pi^+ \pi^-$



$$\lambda_{\pi\pi} = \eta_{\pi\pi} \boxed{\left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right)} \boxed{\left(\frac{V_{ud}^* V_{ub}}{V_{ud} V_{ub}^*}\right)}$$

$$\mathrm{Im}(\lambda_{\pi\pi}) = \sin 2\phi_2$$

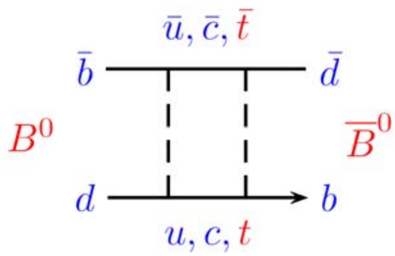
$$\alpha \equiv \phi_2 \equiv \arg\left(\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*}\right)$$

N.B.: for simplicity we have neglected possible penguin amplitudes (which is wrong as we shall see later, and will do it properly).

A reminder:

$$\frac{q}{p} = -\frac{|M_{12}|}{M_{12}}$$

$$\Delta m_B = 2|M_{12}|$$



$\Delta m \propto$

$$|V_{tb}^* V_{td}|^2 m_t^2 \propto \lambda^6 m_t^2$$

$$|V_{cb}^* V_{cd}|^2 m_c^2 \propto \lambda^6 m_c^2$$

Decay asymmetry predictions – example J/ψK$_S$

$\boxed{b \to c\bar{c}s:}$ tree + penguin contribution ~ $V_{cb}V_{cs}^* = A\lambda^2$
penguin only contribution ~ $V_{ub}V_{us}^* = A\lambda^4(\rho - i\eta)$

Take into account that we measure the $\pi^+\pi^-$ component of K_S – also need the $(q/p)_K$ for the K system

$$\lambda_{\psi Ks} = \eta_{\psi Ks} \overbrace{\boxed{\left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right)}}^{(q/p)_B} \overbrace{\boxed{\left(\frac{V_{cs}^* V_{cb}}{V_{cs} V_{cb}^*}\right)}}^{\bar{A}/A} \overbrace{\boxed{\left(\frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*}\right)}}^{(q/p)_K} =$$

$$= \eta_{\psi Ks}\left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right)\left(\frac{V_{cb}}{V_{cb}^*}\frac{V_{cd}^*}{V_{cd}}\right) \qquad \beta \equiv \phi_1 \equiv \arg\left(\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

$$\mathrm{Im}(\lambda_{\psi Ks}) = \sin 2\phi_1$$

b-> c anti-c s
CP=+1 and CP=-1 eigenstates

$$a_{f_{CP}} = -\mathrm{Im}(\lambda_{f_{CP}})\sin(\Delta m t)$$

Asymmetry sign depends on the CP parity of the final state f_{CP}, η_{fcp}=+-1

$$\lambda_{f_{CP}} = \eta_{f_{CP}} \frac{q}{p} \frac{\bar{A}_{\bar{f}_{CP}}}{A_{f_{CP}}}$$

$J/\psi\ K_S\ (\pi^+\pi^-)$: CP=-1

- J/ψ: P=-1, C=-1 (vector particle $J^{PC}=1^{--}$): CP=+1
- K_S (->$\pi^+\pi^-$): CP=+1, orbital ang. momentum of pions=0 -> P $(\pi^+\pi^-)=(\pi^-\pi^+)$, C$(\pi^-\pi^+)$ =$(\pi^+\pi^-)$
- orbital ang. momentum between J/ψ and K_S l=1, P=$(-1)^1$=-1

$J/\psi\ K_L(3\pi)$: CP=+1

Opposite parity to $J/\psi\ K_S\ (\pi^+\pi^-)$, because $K_L(3\pi)$ has CP=-1

The kaon case

The two K states have very different lifetimes

$$\tau_L = (5.17 \pm 0.04)\times 10^{-8} s$$
$$\tau_S = (0.8927 \pm 0.009)\times 10^{-10} s$$

The eigenstates are in this case defined by lifetimes

$$|K_S\rangle = p|K^0\rangle + q|\overline{K}^0\rangle$$
$$|K_L\rangle = p|K^0\rangle - q|\overline{K}^0\rangle$$

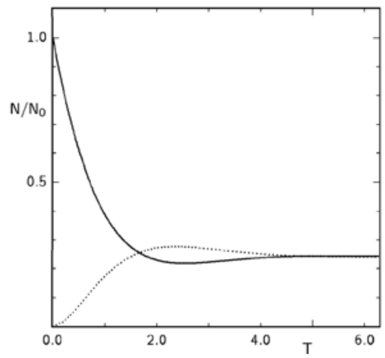
With the mass difference

$$\Delta m_K = m_L - m_S = (3.491 \pm 0.009)\times 10^{-15} GeV$$

The kaon case

In this case

$$\Delta\Gamma_K \approx -2\Delta m_K$$



K^0 at t=0, evolution in time

Full line: K^0, dotted: $\overline{K}^0$

T: in units of τ_s

After a few τ_s: left ony K_L, roughly equal mixture of K^0 and $\overline{K}^0$

The kaon case

Define ϕ_{12} with

$$\frac{M_{12}}{\Gamma_{12}} = -\frac{|M_{12}|}{|\Gamma_{12}|} e^{i\phi_{12}}$$

It turns out that for the K system ϕ_{12}<<1

From

(see above)

$$(\Delta m_B)^2 - \frac{1}{4}(\Delta\Gamma_B)^2 = 4(|M_{12}|^2 - \frac{1}{4}|\Gamma_{12}|^2)$$

$$\Delta m_B \Delta\Gamma_B = 4\,\mathrm{Re}(M_{12}\Gamma_{12}{}^*)$$

To the leading order

$$\Delta\Gamma_K = -2|\Gamma_{12}|$$

$$\Delta m_K = 2|M_{12}|$$

Define $$\Gamma_{12} = |\Gamma_{12}| e^{-2i\xi_K}$$

Use same expression for q/p as for the B case:

$$\frac{q}{p} = -\frac{\Delta m_B - \frac{i}{2}\Delta\Gamma_B}{2(M_{12} - \frac{i}{2}\Gamma_{12})} = -\frac{2(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)}{\Delta m_B - \frac{i}{2}\Delta\Gamma_B}$$

The kaon case

$$\left(\frac{q}{p}\right)_K = e^{2i\xi_K}\left[1 - i\phi_{12}\frac{1 + i\frac{\Delta\Gamma_K}{2\Delta m_K}}{1 + \left(\frac{\Delta\Gamma_K}{2\Delta m_K}\right)^2}\right]$$

The ratio p/q is almost a pure phase (similar as in the B case)

-> CPV in mixing small in both cases (but for different reasons: small lifetime diff in B, small phase in K system)

CPV in interference between mixing and decay:
λ=1 to O(0.001) -> small

To next order ->

$$\frac{q}{p}\frac{\overline{A}_{\pi\pi}}{A_{\pi\pi}} = 1 - i\phi_{12}\frac{1 + i\frac{\Delta\Gamma_K}{2\Delta m_K}}{1 + \left(\frac{\Delta\Gamma_K}{2\Delta m_K}\right)^2}$$

-> can be used to extract ϕ_{12}

But: it is not easy to transform from ϕ_{12} to electroweak parameters because of long distance (strong interaction) contribution M_{12}.

Backup slides

Direct and indirect CP violation

Indirect: CP violating phases appear in $\Delta B=2$ (mixing) amplitudes

Direct: CP violating phases appear in $\Delta B=1$ (decay) amplitudes

CPV in decay = direct

CPV in mixing = indirect

CPV in interference of decays with and without mixing = indirect

However: if we have two final states with different $\mathrm{Im}(\lambda)$, we do not have the freedom in choosing the phase, there must also be direct CP (see Y. Nir in Heavy flavour physics).

Backup slides

Parity of B^0

P: space inversion $P|B^0\rangle = -|B^0\rangle$

Why is the parity of B^0 (pseudoscalar meson) -1?

B^0 is composed of two quarks with spin ½, with total spin J=0.
The two quark spins are combined to ½ ⊕ ½ = 0, the relative angular momentum is l=0 (ground bound state of $\bar{b}$ in d).
Parity of the spatial part of the wave function is $(-1)^l=+1$.
Quark and antiquark have opposite parities
=> additional factor -1

Transformation of bispinor

$x' = \Lambda x$ transformation

$$i\gamma^\mu \frac{\partial \psi(x)}{\partial x^\mu} - m\psi(x) = 0 \quad \text{Dirac equation}$$

compare

$$i\gamma^\mu \frac{\partial \psi'(x')}{\partial x'^\mu} - m\psi'(x') = 0$$

substitute

$$\Downarrow$$

$$\psi'(x') = S\psi(x) \quad \text{transformation for a bispinor}$$

$$S^{-1}\gamma^\mu S = \Lambda^\mu_\nu \gamma^\nu$$

conclude

$$\Lambda^\mu_\nu = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \quad \text{space inversion}$$

spinor for particles (E≥0)

$$S^{-1}\gamma^0 S = \gamma^0 \qquad S^{-1}\gamma^k S = -\gamma^k \quad \Rightarrow \quad \begin{aligned} \psi'_{1,2} &= S\psi_{1,2} = \psi_{1,2} \\ \psi'_{3,4} &= S\psi_{3,4} = -\psi_{3,4} \end{aligned}$$

spinor za for anti-particles (E≤0)

Low-energy effective Hamiltonians

Low-energy effective Hamiltonians: constructed using the operator product expansion (OPE):

$$\langle f|\mathcal{H}_{\rm eff}|i\rangle \propto \sum_k \langle f|Q_k(\mu)|i\rangle\, C_k(\mu)$$

μ is an appropriate renormalization scale O(m_b). The OPE allows one to separate the "long-distance" contributions to that decay amplitude from the "short-distance" parts.
"long-distance" contributions not calculable -> nonperturbative hadronic matrix elements
"short-distance" described by perturbatively calculable Wilson coefficient functions $C_k(\mu)$.

For B decays:

$$\mathcal{H}_{\rm eff}(\Delta B=-1)=\frac{G_{\rm F}}{\sqrt{2}}\left[\sum_{j=u,c} V_{jq}^{*}V_{jb}\left\{\sum_{k=1}^{2} Q_k^{jq}\, C_k(\mu)+\sum_{k=3}^{10} Q_k^{q}\, C_k(\mu)\right\}\right]$$