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An iterative method for the analysis of Cherenkov rings in the HERA-B RICH

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Abstract

A new method is presented for the analysis of data recorded with a Ring Imaging Cherenkov (RICH) counter. The method, an iterative sorting of hits on the photon detector, is particularly useful for events where rings overlap considerably. The algorithm was tested on simulated data for the HERA-B experiment. © 1999 Elsevier Science B.V. All rights reserved.

1. Introduction

In the analysis of data recorded by a RICH counter one is confronted with the fact that, in principle, each hit on the photon detector has to be considered when the Cherenkov angle for a given charged track is being evaluated. In cases where the ring radii are rather large and track density is high, a considerable overlap of rings is possible. In such a case, the peak in the Cherenkov angle distribution corresponding to the photons from the considered track might become considerably obscured by the contribution of photons from the neighboring tracks. In the standard approach one estimates the form of background at a given photon detector part, and calculates the likelihood functions for various particle hypotheses [1].

There certainly is room for improvement of this method. In particular, the circumstance that most of the background hits actually belong to other tracks in the event, suggests that some kind of iterative procedure, in which each photon would gradually become predominately associated with one of the tracks, could reduce the background level. One of the possible methods how to proceed in this case, is described in this contribution. Let us note that a similar method, known as expectation–maximization algorithm, is used for image reconstruction in positron emission tomography [2].

2. The iterative method

We shall illustrate the method on a simple example of three overlapping rings, displayed in Fig. 1.

For each of the tracks and for each of the photons we first calculate the corresponding ring radius, and fill a histogram shown in Fig. 2. Here the usual method, a maximum-likelihood analysis of RICH data [1], would calculate the likelihood for each particle hypothesis. In our iterative approach we continue, however, to clear up the histograms. We proceed in the following way: instead of putting

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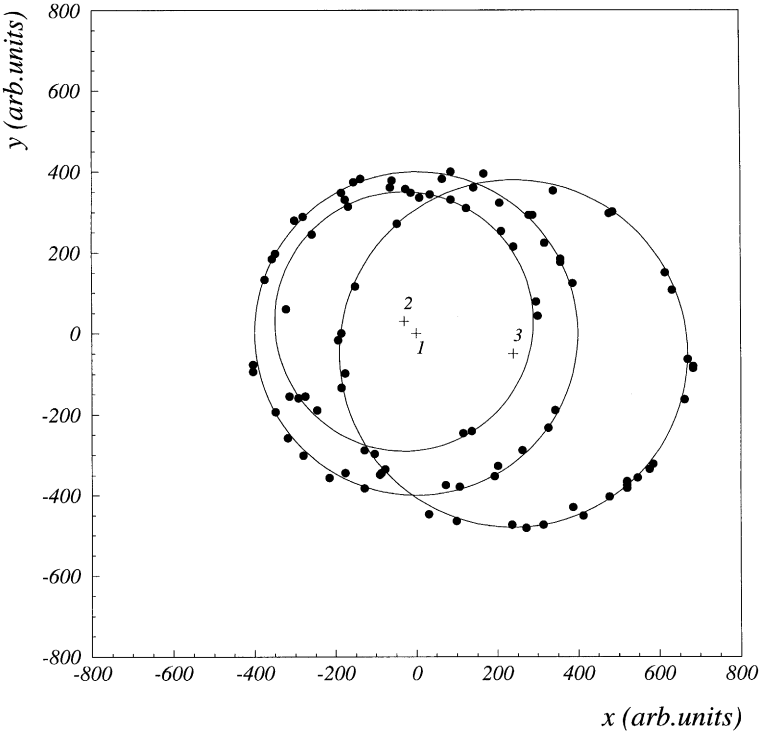

Fig. 1. The test example: three overlapping Cherenkov rings.

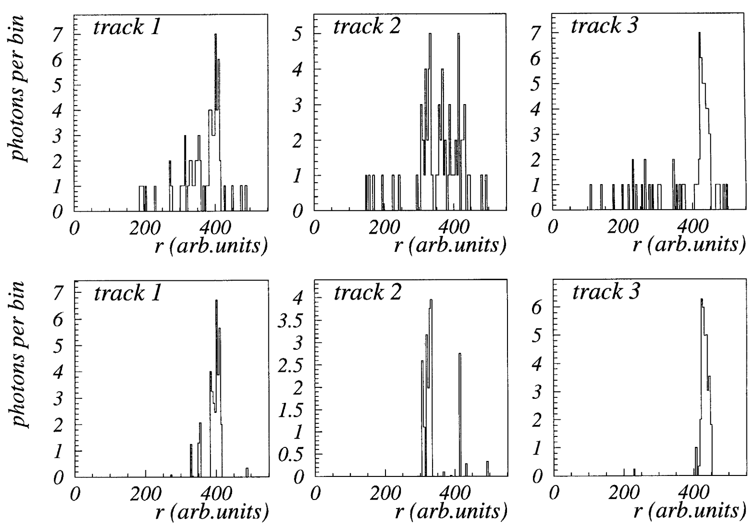

Fig. 2. Photon hit histogram for tracks 1, 2 and 3, from left to right, at the beginning of the iteration procedure (top row) and after 16 iterations (bottom).

a given photon in all the three histograms with weight one, as we did in the first step, we ascribe weights to each photon so that the sum of weights is equal to one for each photon. The weight for a specific photon in the histogram corresponding to the track k is calculated for the next iteration according to the formula

$$w_k = \frac{y_k}{y_1 + y_2 + y_3}$$

where y_1, y_2 and y_3 is the number of entries in the bin, into which the given photon fits, in histograms 1, 2 and 3, respectively. As a result, a photon is given the highest weight in the histogram, where it fits into a peak, and lowest, where it is part of a scarcely populated background.

In the next iterations the procedure is repeated, and the resulting histograms after 16 iterations are shown in the lower row of Fig. 2.

In Fig. 3 the effect of the algorithm on the background reduction is shown for Monte Carlo generated events in HERA-B. In the HERA-B experiment we expect on average 300 charged particles per event, giving rise to 100 Cherenkov rings on the photon detector. As a large number of charged particles originate in the material after the magnet spectrometer or in the RICH radiator or exit the beam pipe after the tracking system, we expect to have on average only 55 measured tracks and momenta per event. Thus a part of the background still remains in the Cherenkov angle histograms for the known tracks.

The speed of convergence, defined here as the maximal difference between weights of adjacent iterations, is shown in Fig. 4. Depending on the number of tracks in the event the difference reaches one percent level after 16 to 32 iterations.

A closer inspection of the resulting histograms shows that the distribution of photons remains Gaussian with the same rms width (Fig. 3). Also, the number of photons in the peak, n_r (sum of weights within $\pm 3\sigma$), remains the same (or is only slightly reduced) if compared with the input. The variation in the number of photons is, however, larger than the Poissonian value (Fig. 5).

This last point needs a further study, since the correct knowledge of the expected distributions is essential for the proper evaluation of the likelihood function. We first note that such a broadening of the distribution over the number of photons is not unexpected, since a part of the information

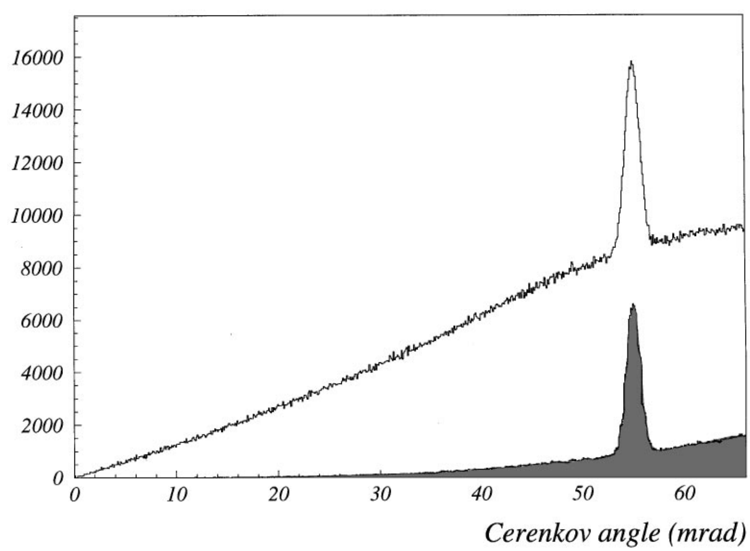

Fig. 3. Cherenkov angle distribution for pions above 15 GeV/c, as expected in HERA-B RICH. Open: before iterations, shaded: after 20 iterations.

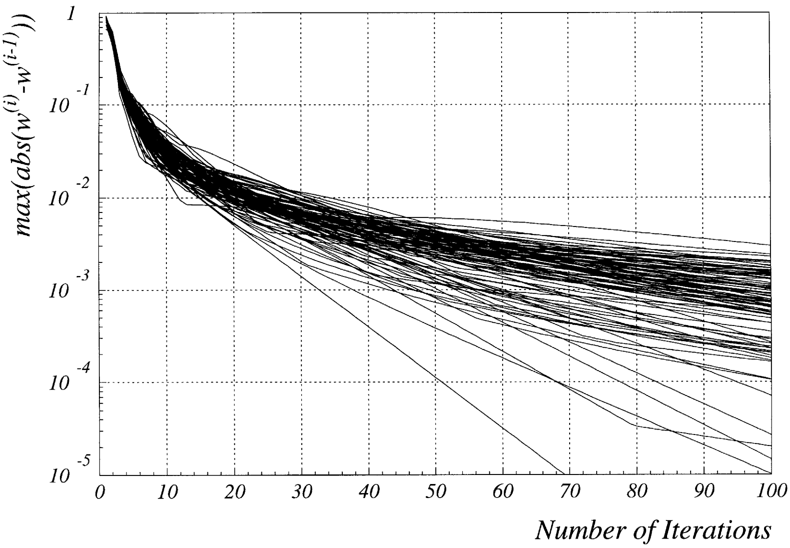

Fig. 4. The convergence of the iterative procedure: maximal difference between weights of adjacent iterations versus the number of iterations for 100 HERA-B Monte Carlo generated events.

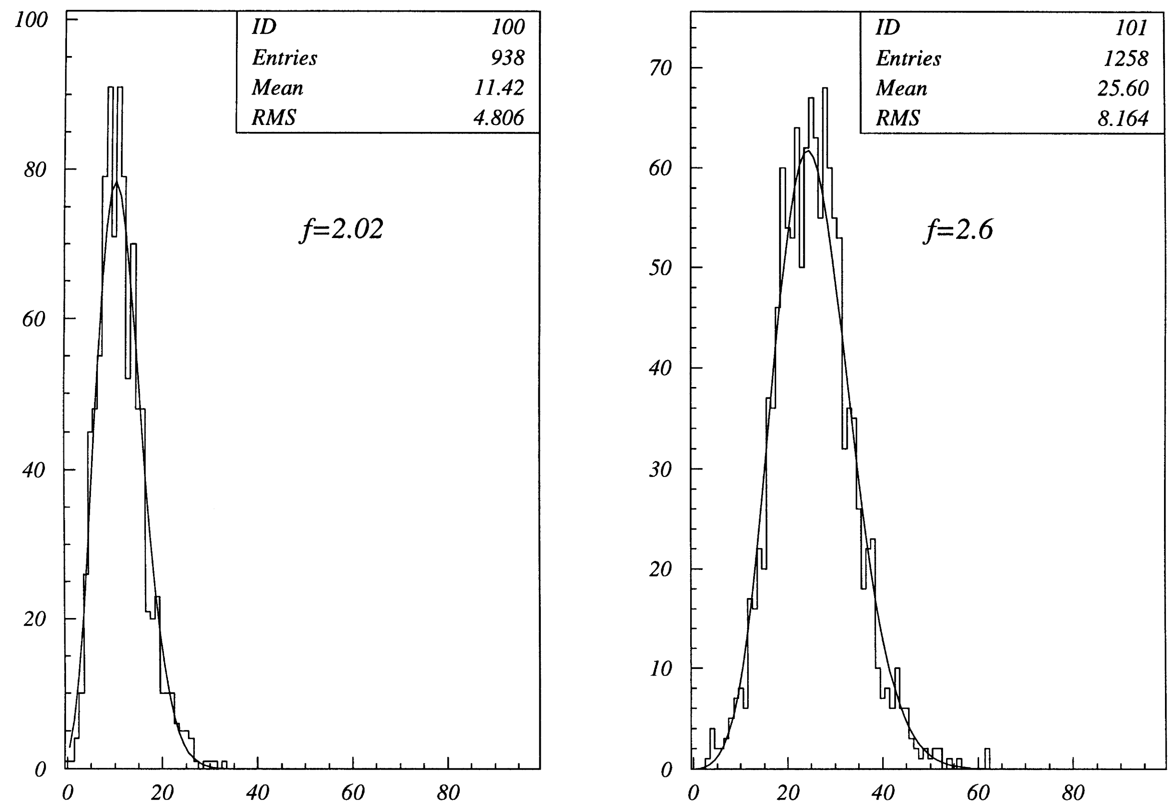

Fig. 5. Number of reconstructed photons in the proper hypothesis window after 20 iterations, for 10 expected photons (left) and 25 expected photons (right). The curves correspond to a Poissonian-like distribution Eq. (1), with the value of factor f (Eq. (2)) indicated for each case. Events analysed were of the type expected in the HERA-B spectrometer.

contained in the initial histograms is used in the separation process. For the specific example displayed in Fig. 5 one expects an rms of five photons if 25 photons are detected on average, but the observed value is 8.2. To model the distribution, we replaced the Poisson distribution $P(n_e, n_r)$ of the number of reconstructed photons n_r with mean n_e, by a Poisson-like function

$$P'(n_e, n_r) = P\left(\frac{n_e}{f}, \frac{n_r}{f}\right) \quad (1)$$

where the factor f is determined from the rms width and mean of the distribution,

$$f = \sigma_{n_r}^2/\bar{n}_r. \quad (2)$$

As can be seen from Fig. 5, the function defined in such a way describes the observed distributions reasonably well. The value of f was calculated for each interval of $\bar{n}_r$, and was later used in a tabular form in the likelihood analysis of data. We note that such a parametrisation of the factor f as a function of only one parameter is a very rough one, in particular since the ring density is a strong function of the coordinate on the photon detector. Studies are being pursued to improve the parametrisation.

3. Likelihood function

The likelihood function is constructed in the following way. We first denote by $F(\theta_i, \theta^{\text{hyp}})$ the probability density function of detected photons in the Cherenkov angle, calculated for a specific track under the assumption of the track mass

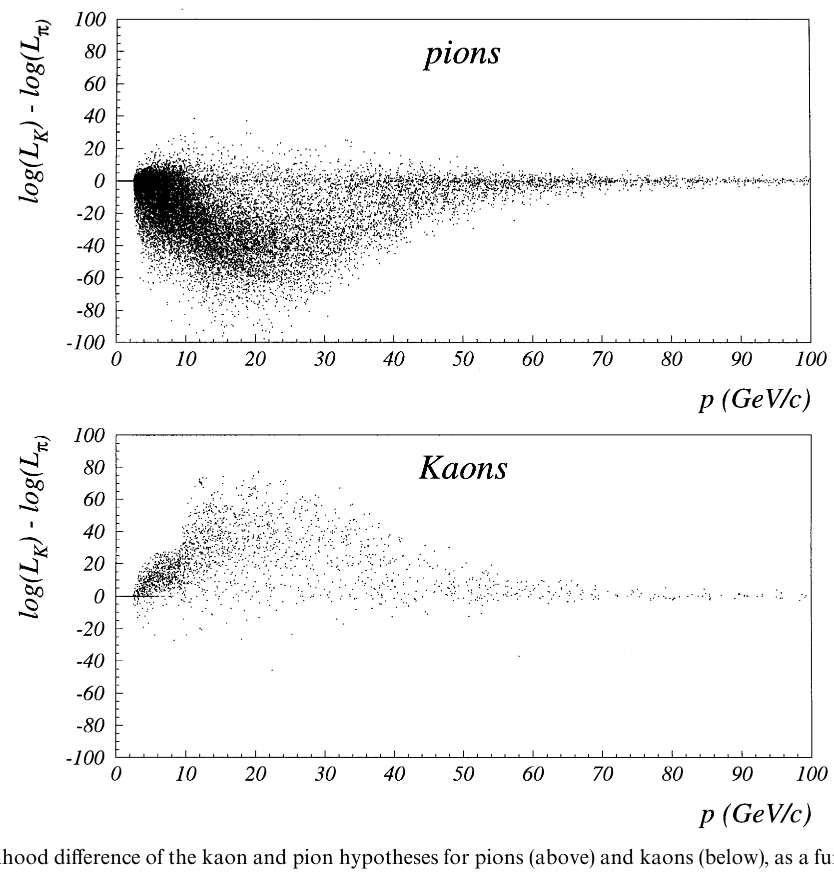

Fig. 6. The log-likelihood difference of the kaon and pion hypotheses for pions (above) and kaons (below), as a function of momentum. Events analysed were of the type expected in the HERA-B spectrometer.

hypothesis hyp,

$$F(\theta_i, \theta^{\mathrm{hyp}}) = pS(\theta_i, \theta^{\mathrm{hyp}}) + (1-p)B(\theta_i, \theta^{\mathrm{hyp}}) \quad (3)$$

where S and B are the probability density functions corresponding to the signal and background, respectively, and p is the ratio of the expected numbers of signal photons ($n_{\mathrm{e}}^{\mathrm{s}}$) to all photons,

$$p = \frac{n_{\mathrm{e}}^{\mathrm{s}}}{n_{\mathrm{e}}^{\mathrm{s}} + n_{\mathrm{e}}^{\mathrm{b}}}$$

here $n_{\mathrm{e}}^{\mathrm{b}}$ being the expected number of background photons.

For the signal S a normalized Gaussian distribution is assumed with mean θ^{hyp} and rms estimated from known detector parameters. The shape of the background distribution B and the number of expected background photons $n_{\mathrm{e}}^{\mathrm{b}}$ could be obtained from the histograms of already identified particles (electrons and muons) after the last iteration.

As discussed in the previous section, when we construct the extended likelihood function, we have to account for the somewhat larger variations in the number of reconstructed photons, as observed in Fig. 5, and documented in Eq. (1). Also, we have to consider each photon i in a given histogram with the corresponding weight w_i. Thus log-likelihood function reads:

$$\log L^{\mathrm{hyp}} = \sum w_i \log F(\theta_i, \theta^{\mathrm{hyp}}) + f \log P'(n_{\mathrm{e}}, n_{\mathrm{r}}) \quad (4)$$

where the index i runs over all photons in the π/K/p hypotheses windows, $n_{\mathrm{e}} = n_{\mathrm{e}}^{\mathrm{s}} + n_{\mathrm{e}}^{\mathrm{b}}$ and $n_{\mathrm{r}} = \sum w_i$.

4. K/π separation in the HERA-B RICH

To demonstrate the efficiency of the method, we will consider a particular example of the RICH counter in the HERA-B experiment [3,4]. Events were generated by the GEANT program [5] and tracks of charged particles with all the relevant information written to a file. This has served as an input to the Monte Carlo simulation of the HERA-B RICH detector.

In the analysis of the simulated RICH data all those detected photons were taken into consideration for each measured track within the event, whose Cherenkov angles were within $\pm 3\sigma_\theta$ around each of the possible particle hypotheses. The iteration algorithm was repeated twenty times. The

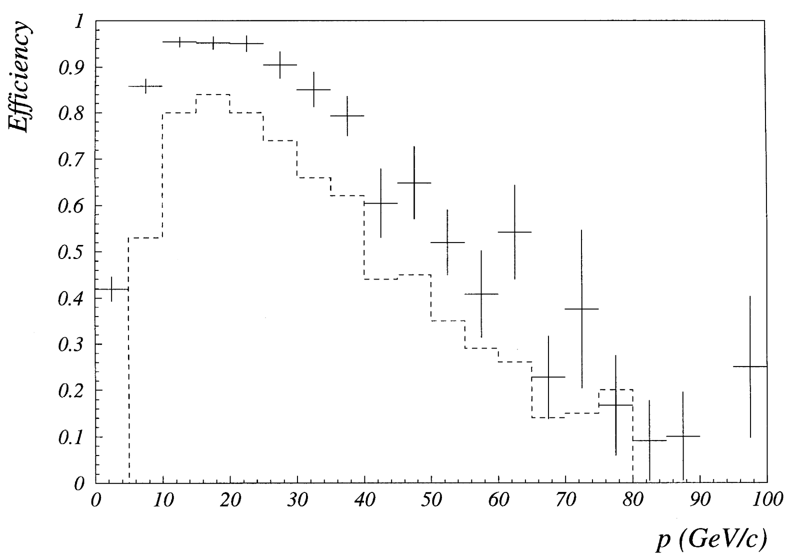

Fig. 7. The momentum-dependent efficiency for resolving a kaon from a pion, extracted from Fig. 6 under the requirement that the probability for a pion to be identified as a kaon amounts to 5%. For comparison, we show as a dashed histogram the efficiency of the pion–kaon separation without performing the iteration procedure.

shape of the background was determined from the Cherenkov angle histograms of electrons after the last iteration, and turned out to be well described by a cubic function in Cherenkov angle θ. We note that for pions, kaons and protons we have not considered hypotheses that can be excluded by using information from other detector components (electrons and muons).

The pion–kaon separation is shown in Fig. 6, where the logarithm of the ratio of likelihoods $\log L^{\pi}/L^{K}$ is plotted against the particle momentum. Of course, the efficiency and fake probability depend on the cuts in the likelihood ratio applied to separate pions from kaons. By fixing the pion fake probability to a certain value, a momentum dependent cut on the likelihood ratio is defined. In Fig. 7 the efficiency of the method is shown for a fixed pion fake probability of 5%.

It is interesting to compare the result with the performance of the usual method, in which the likelihood functions are calculated without the iteration procedure. As can be seen from Fig. 7, the method described in the present work certainly improves the performance of the RICH counter when many overlapped rings are expected in a single event.

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