ANALOG SIGNAL PROCESSING

- Introduction
- 1- Electronic Signal Processing
- 2- Preamplifier & Shaper
- 3- Noise in electronic systems
- 4- General Formulation of Noise
- 5- Equivalent Noise Charge
- 6- ENC : Time Analysis
- Conclusion

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REFERENCES

Low-Noise Wide-Band Amplifiers in Bipolar and CMOS Technologies, Z.H. Chang, W. Sansen, Kluwer Academics Publishers

Low-Noise Techniques in Detectors, V. Radeka, Annual Review of Nuclear Particle Science 1988 28: 217-277

Introduction

In this lecture we will try to give an insight into standard signal processing, but having in mind the specific requirements of signal processing for particle physics.

- (Short) description of a typical “front-end” channel for physics detector
- Specific requirements about signal processing in particle physics
- Time domain/ frequency domain representation

Then we will look to the “noise” issue :

- Noise sources in electronics circuit and their model
- Example of Signal-to-noise ratio for simple circuits
- Signal-to-noise ratio formulation and relation to circuit parameters
- Doing analysis in time domain

Introduction

We will look at both frequency and time domain representations

Why?

• They give different insights into the circuit representations

• They end up into identical “models” (for signal/noise analysis for example)

•They are complementary

Introduction

What we will NOT cover in this lecture :

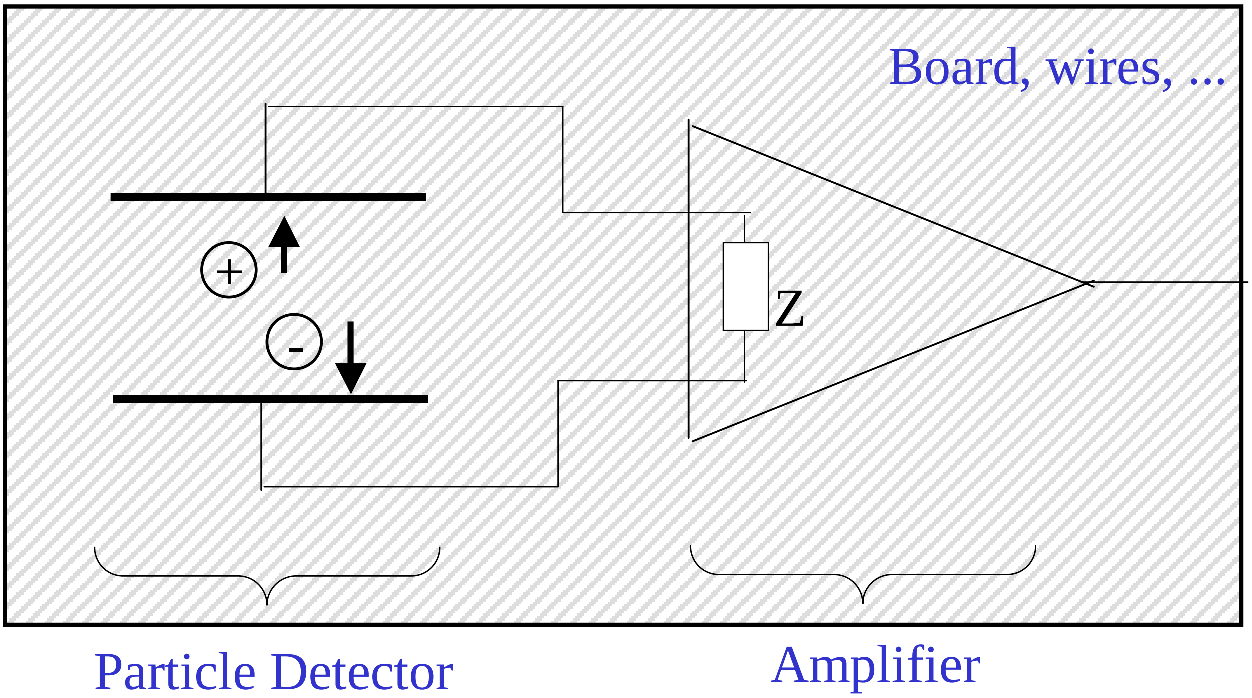
Detail representations of either detector systems or amplifier circuits.

Impedance calculation or some formulations for active components models (as for MOS or Bipolar transistors).

The above items, or a part of them, will be covered in other lectures of the present course ...

Introduction

Typical “front-end” elements



Particle detectors collect charges : ionizing gas detector, silicon detector :

a particle crossing the medium generates ionization + ions avalanche (gas detector) or electron-hole pairs (solid-state). Charges are collected on electrode plates (as a capacitor), building up a voltage.

Function is multiple :

signal amplification (signal multiplication factor)

noise rejection

signal “shape”

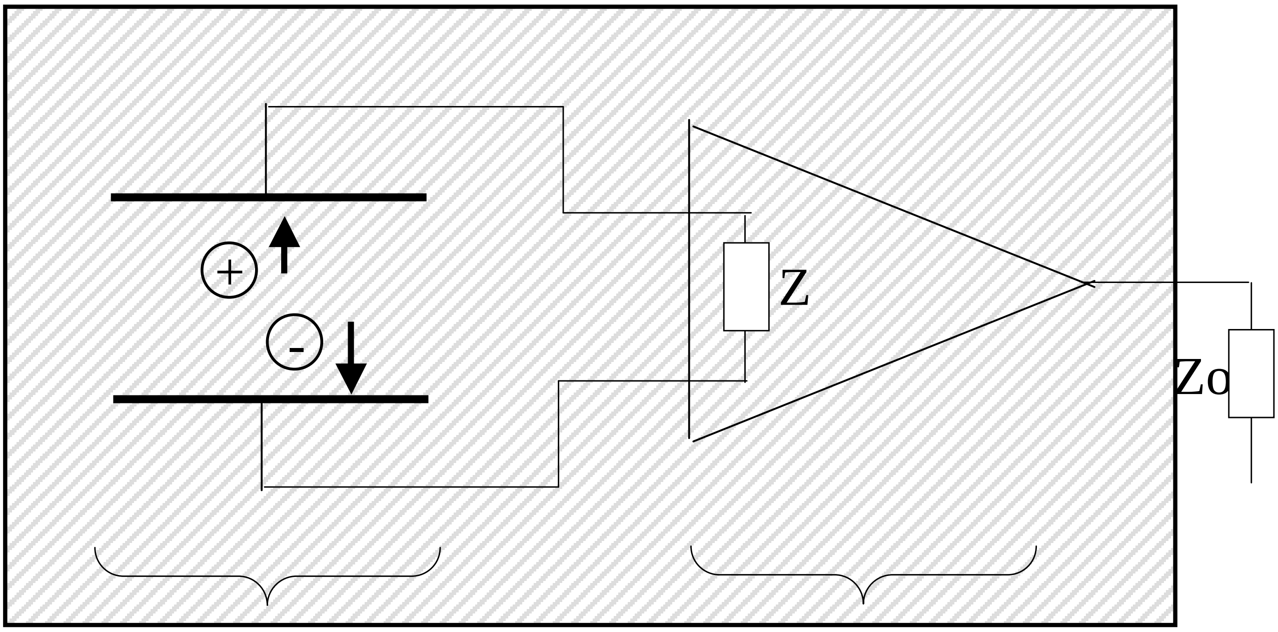
Final objectives :

amplitude measurement

time measurement

Introduction

Typical "front-end" elements



Particle Detector

Amplifier

Which problems to solve ?

Going from :

Tiny signals (Ex: 400uV collected in Si detector on 10pF)

Noisy environment

Collection time fluctuation

To :

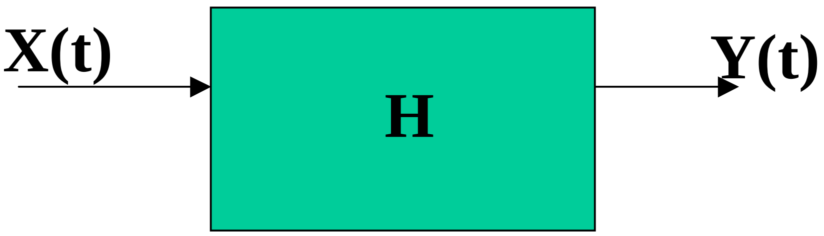
Large signals, accurate in amplitude and/or time

Affordable S/N ratio

Signal waveform and source impedance compatible with subsequent circuits

1-Electronic Signal Processing

Time domain :



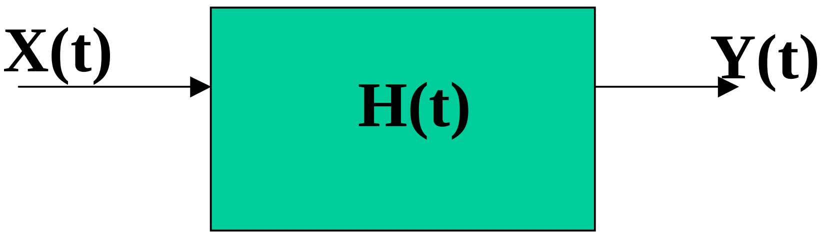
Electronic signals, like voltage, current or charge can be described in the time domain.
H in the above figure represents an object which modifies the (time) properties of the incoming signal X(t), so that we obtain another signal Y(t). H can be filter, transmission line, amplifier, resonator etc ...

As soon as the modifying object has linear properties

like :
if X1 ---> Y1 through H object
if X2 ---> Y2 through H object
then X1+X2 ---> Y1+Y2

the object H can be attached to a linear function of time H(t), such that the knowledge of X(t) and H(t) is enough to determine Y(t)

1-Electronic Signal Processing



In the Time domain, the relationship between X(t), H(t) and Y(t) is expressed by the following formula :

$$Y(t) = H(t)*X(t)$$

where

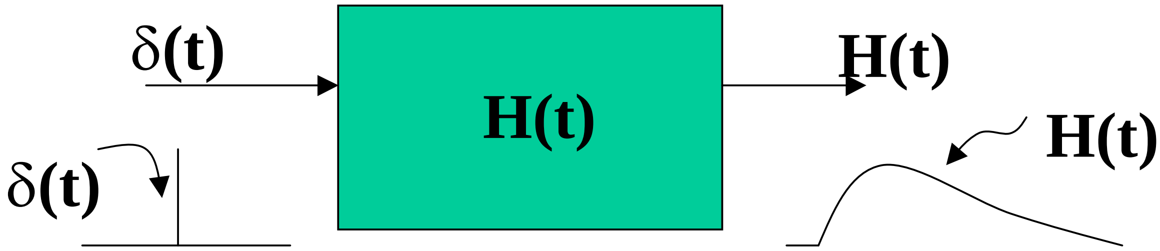
$$H(t) * X(t) = \int_{-\infty}^{\infty} H(u)X(t-u)du$$

This is the convolution function, that we can use to completely describe Y(t) from the knowledge of both X(t) and H(t)

This formula becomes easily very complicated ...

1-Electronic Signal Processing

<u>What is H(t) ?</u>



(Dirac function)

$$H(t) = H(t) * \delta(t)$$

If we inject a "Dirac" function to a linear system, the output signal is the characteristic function H(t)

H(t) is the transfer function in the time domain of the linear system H.

1-Electronic Signal Processing

Frequency domain :

The electronic signal X(t) can be represented in the frequency domain by the following transformation

$$x(f) = \int_{-\infty}^{\infty} X(t).\exp(- j2\pi ft) .dt$$

(Fourier Transform)

This is *not* an easy transform, unless we assume that X(t) can be described as a sum of "exponential" functions, of the form :

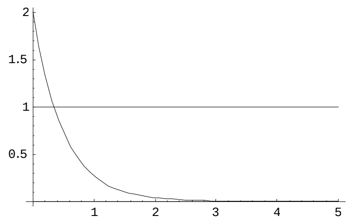
$$X(t) = \sum_{-\infty}^{\infty} c_k \exp(j 2\pi f_k t)$$

The conditions of validity of the above equation are precisely defined. We assume here that it applies for any signal (either periodic or not) that we will consider later on.

1-Electronic Signal Processing

Example :

$$X(t) = \exp(-at) \quad \text{For } (t > 0)$$



⇩

$$x(f) = \int_0^\infty \exp(-at) \,.\, \exp(-j2\pi f t)\,.dt$$

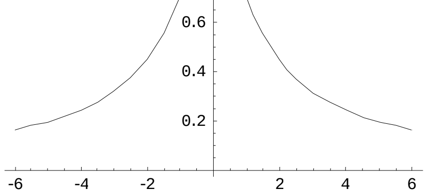
⇩

$$x(f) = \int_0^\infty \exp(-(a + j2\pi f)t)\,.dt$$

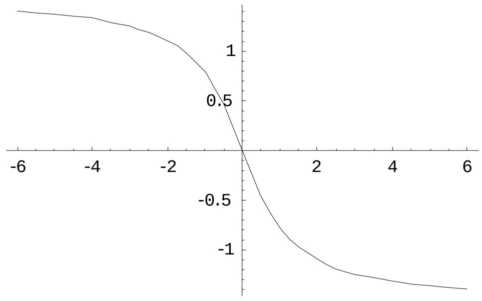
⇩

$$x(f) = \frac{1}{a + j2\pi f}$$

The "frequency" domain representation x(f) is using complex numbers.



$|x(f)|$



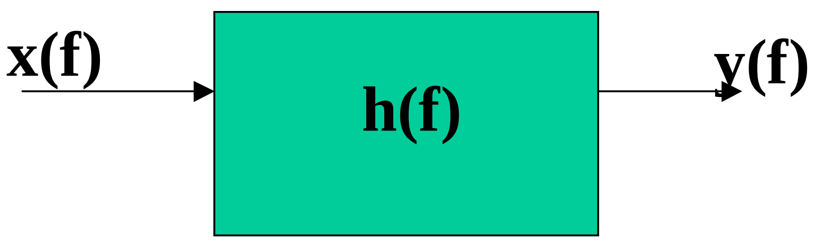
Arg(x(f))

1-Electronic Signal Processing

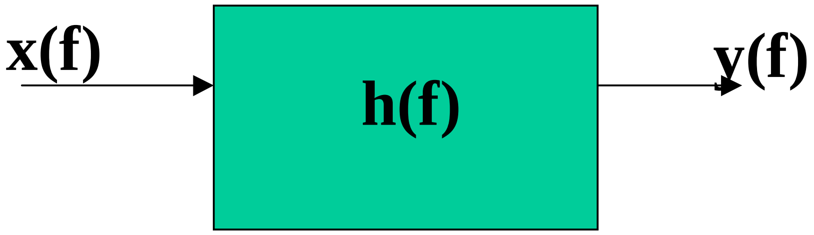
Some usual Fourier Transforms :

- $\delta(t)$ --> 1
- $\nu(t)$ --> $1/j\omega$
- e^{-at} --> $1/(a+ j\omega)$
- $t^{n-1}e^{-at}$ --> $1/(a+ j\omega)^n$
- $\delta(t)-a.e^{-at}$ --> $j\omega /(a+ j\omega)$

The Fourier Transform applies equally well to the signal representation X(t) ⟺ x(f) and to any linear system transfer function H(t) ⟺ h(f)



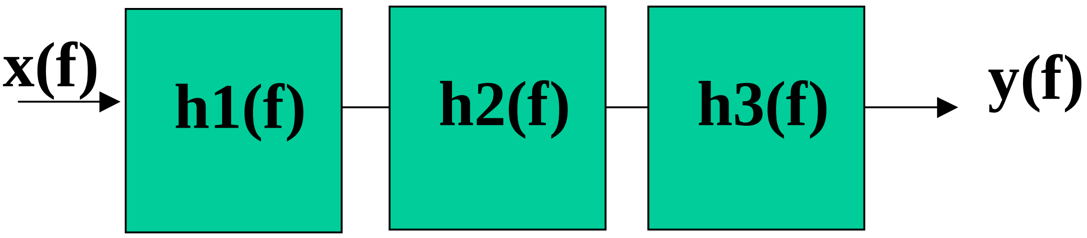
1-Electronic Signal Processing



With the frequency domain representation (signals and system transfer function mapped into frequency domain by the Fourier transform), the relationship between input, system transfer function and output becomes simple:

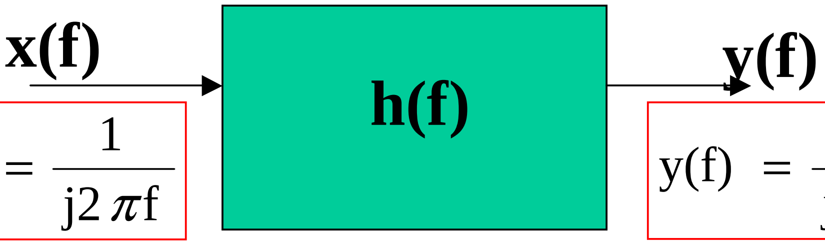
$$y(f) = h(f).x(f)$$

Example : cascaded systems



$$y(f) = h1(f).\ h2(f).\ h3(f).\ x(f)$$

1-Electronic Signal Processing

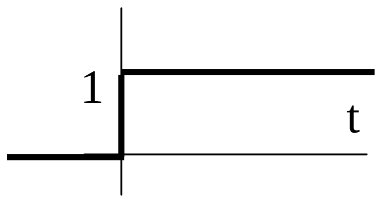


$$x(f) = \frac{1}{j2\pi f}$$

$$y(f) = \frac{1}{j2\pi f(1 + j2\pi f)}$$

$$h(f) = \frac{1}{1 + j2\pi f}$$

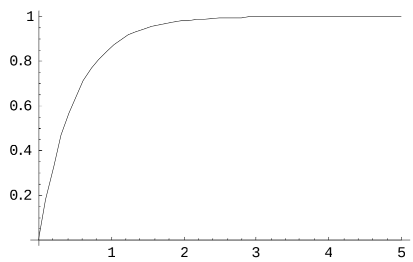
$$x(f) = \frac{1}{j2\pi f} \Longleftrightarrow X(t) = \nu(t)$$



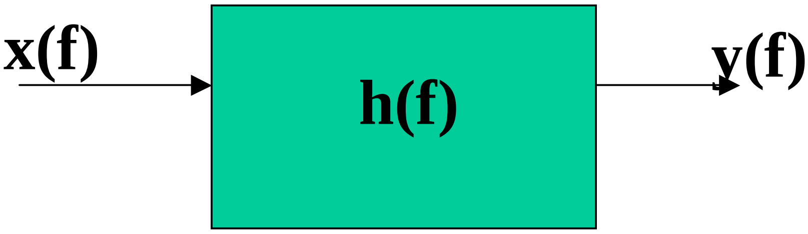
$$h(f) = \frac{1}{1 + j2\pi f}$$

Correspond to simple RC lowpass filter

$$y(f) = \frac{1}{j2\pi f(1 + j2\pi f)} \Longleftrightarrow Y(t) = 1 - \exp(-t)$$



1-Electronic Signal Processing



So, a way to proceed for doing time analysis when using frequency domain formulations is :

X(t) ----> x(f) (Fourier transform)
H(t) ----> h(f) (Fourier transform)

h(f) can also be directly formulated from circuit analysis

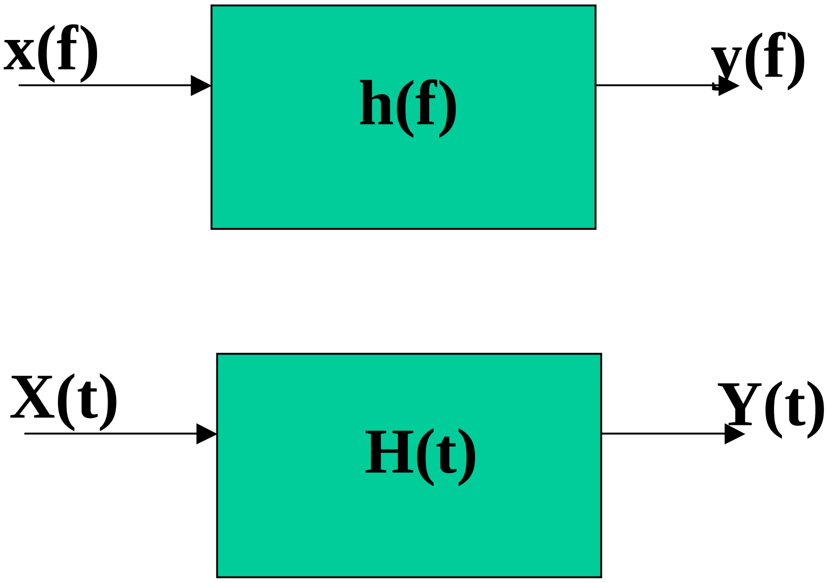
Apply $$y(f) = h(f).x(f)$$

then

y(f) ----> Y(t) (inverse Fourier Transform)

Fourier Transform	**Inverse Fourier Transform**
$h(f) = \int_{-\infty}^{\infty} H(t).e^{-j2\Pi.f.t}.dt$	$H(t) = \int_{-\infty}^{\infty} h(f).e^{j2\Pi.f.t}.df$

1-Electronic Signal Processing



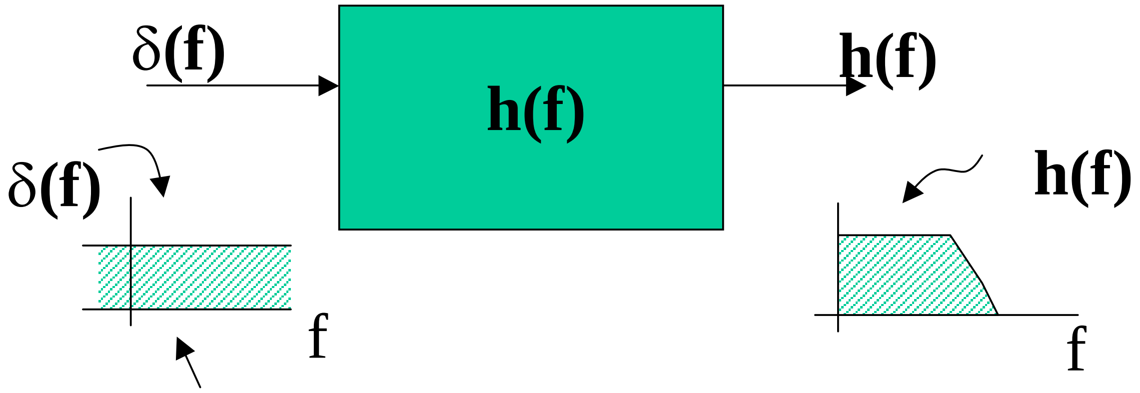
- THERE IS AN EQUIVALENCE BETWEEN TIME AND FREQUENCY REPRESENTATIONS OF SIGNAL or SYSTEM TRANSFER FUNCTION

- THIS EQUIVALENCE APPLIES ONLY TO A PARTICULAR CLASS OF SIGNALS or FUNCTIONS

- IN PARTICLE PHYSICS, FUNCTIONS OUTSIDE OF THIS CLASS CAN BE USED : IN SUCH A CASE ONLY THE TIME DOMAIN IS APPLICABLE

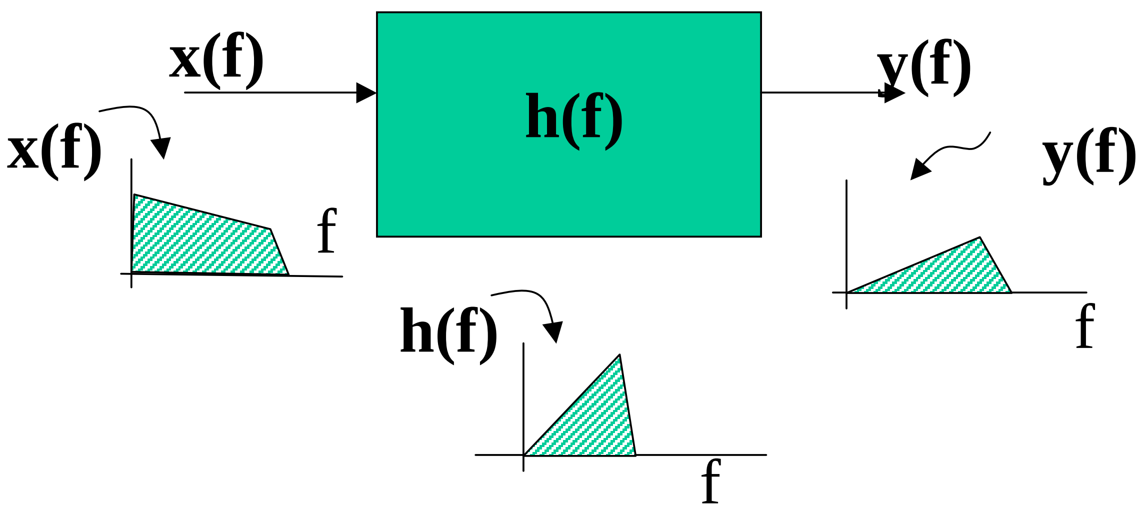
1-Electronic Signal Processing

$$y(f) = h(f).x(f)$$



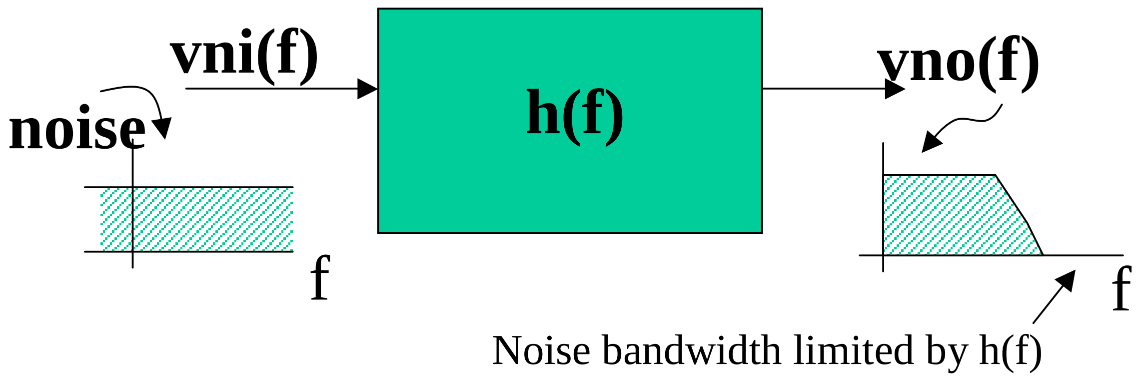
Dirac function frequency representation

In the frequency domain, a system (h) is a frequency domain "shaping" element. In case of h being a filter, it selects a particular frequency domain range. The input signal is rejected (if it is out of filter band) or amplified (if in band) or "shaped" if signal frequency components are altered.



1-Electronic Signal Processing

$$y(f) = h(f).x(f)$$



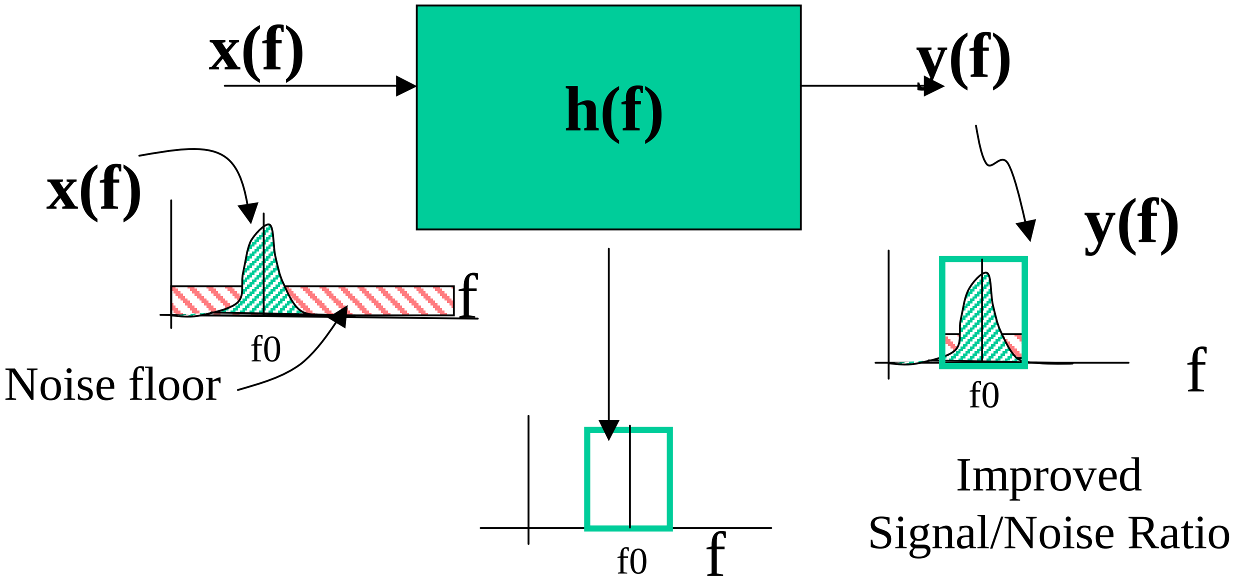
The "noise" is also filtered by the system h

Noise components (as we will see later on) are often "white noise", i.e. : constant distribution over all frequencies (as shown above)

So a filter h(f) can be chosen so that :

It filters out most of the noise components outside of the output signal frequency band

1-Electronic Signal Processing

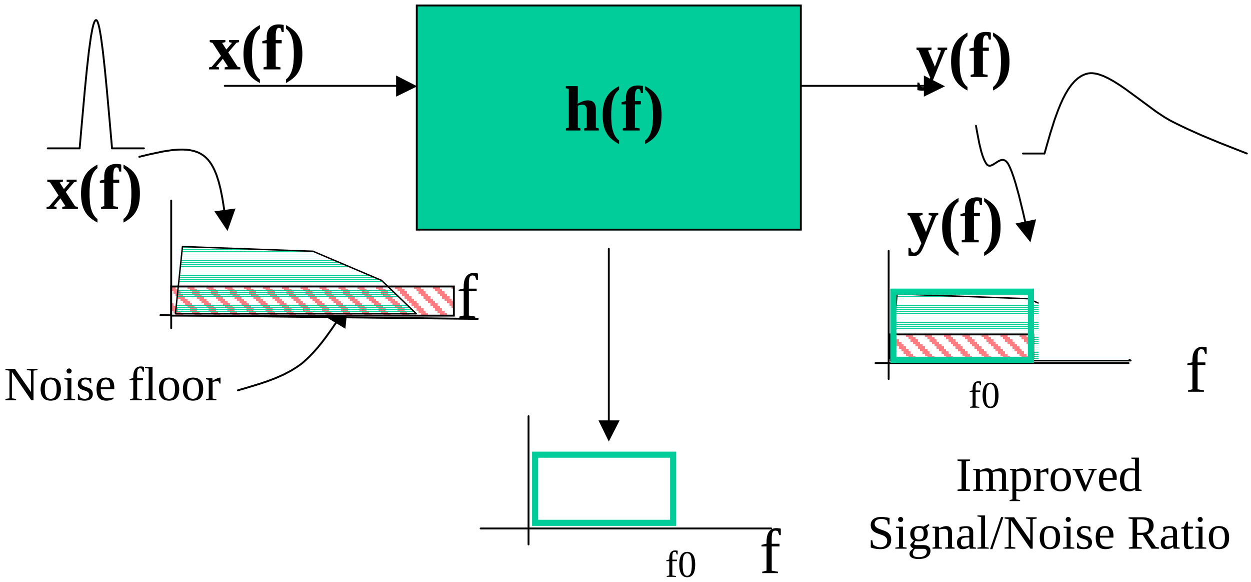


Example of signal filtering : the above figure shows a « typical » case, where the noise is filtered out.

In particle physics, the input signal, from the detector, is more like a random pulse. Therefore, its spectral representation is over a large frequency range.

The filter (shaper) provides a limitation in bandwidth, and the output signal shape is different from the input signal shape.

1-Electronic Signal Processing



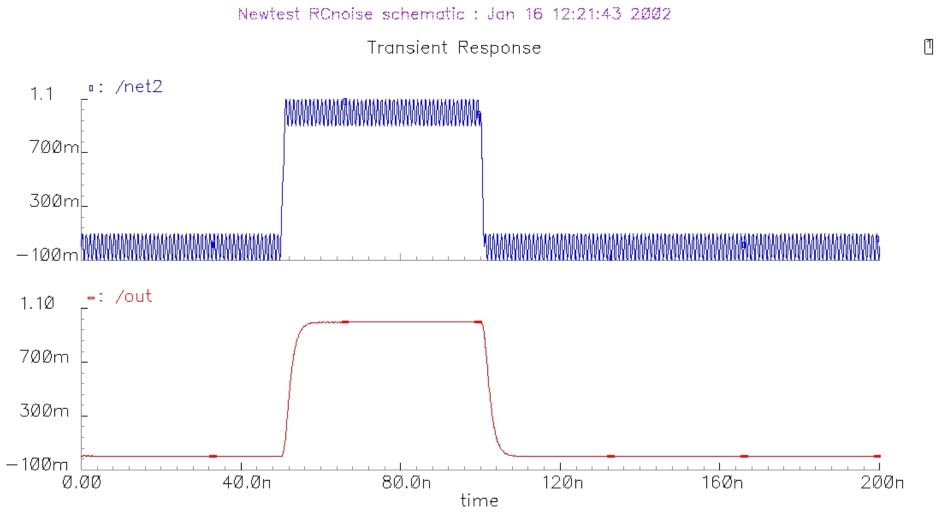
The output signal shape is determined, for each application, by the following parameters:

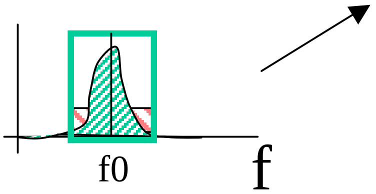
- Input signal shape (characteristic of detector)
- Filter (amplifier-shaper) characteristic

The output signal shape, different form the input detector signal, is chosen for the application requirements:

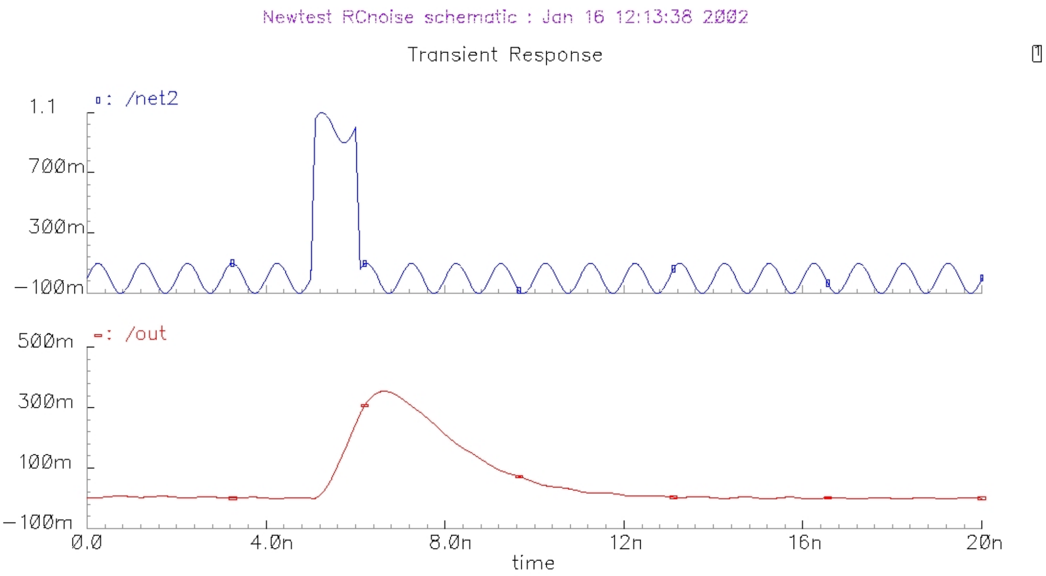
- Time measurement
- Amplitude measurement
- Pile-up reduction
- Optimized Signal-to-noise ratio

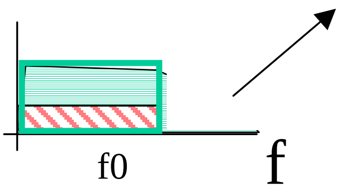
1-Electronic Signal Processing





Filter cuts noise. Signal BW is preserved



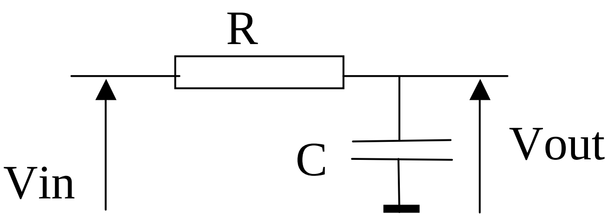


Filter cuts inside signal BW : modified shape

1-Electronic Signal Processing

SOME EXAMPLES OF SIGNAL SHAPERS ...

1-Electronic Signal Processing



Low-pass (RC) filter

$$Vout = \frac{Xc}{Xc + R} Vin \qquad Xc = \frac{1}{j2\Pi fC} = \frac{1}{j\omega C}$$

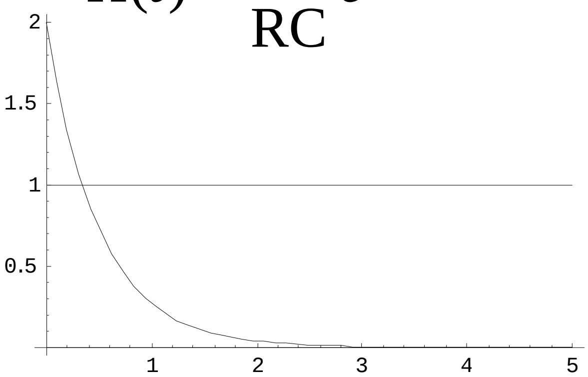
$$Vout = \frac{1}{1 + RCj\omega} Vin$$

Example RC=0.5

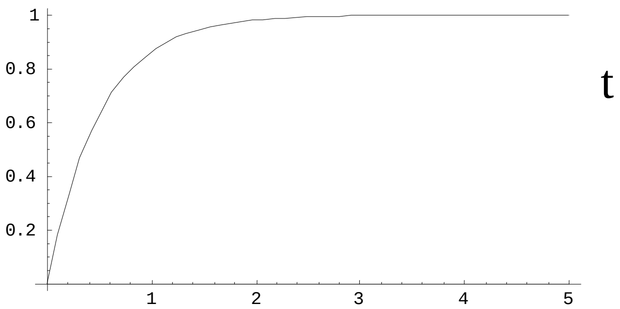
s=jω

Integrator time function

$$\mathrm{H}(t) = \frac{1}{\mathrm{RC}} e^{-t/RC}$$

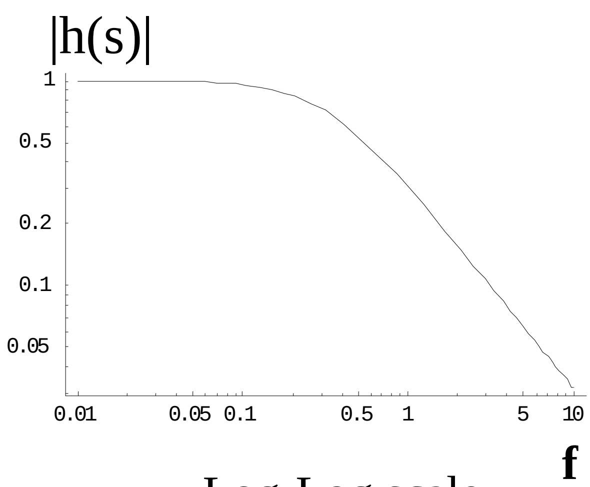


Step function response



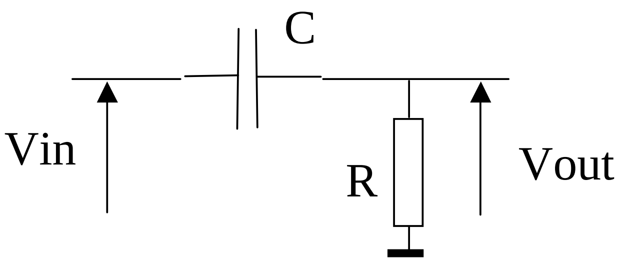
Integrator s-transfer function

h(s) = 1/(1+RCs)



Log-Log scale

1-Electronic Signal Processing



High-pass (CR) filter

$$Vout = \frac{R}{Xc + R} Vin \qquad Xc = \frac{1}{j2\Pi fC} = \frac{1}{j\omega C}$$

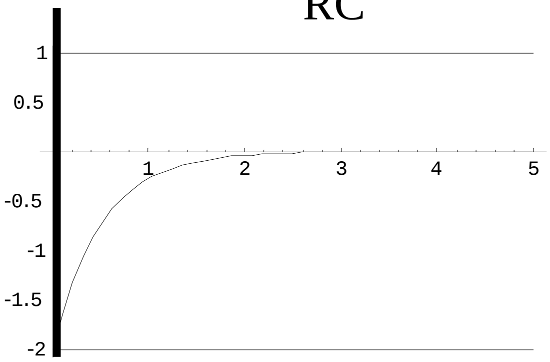
$$Vout = \frac{RCj\omega}{1 + RCj\omega} Vin$$

Example RC=0.5

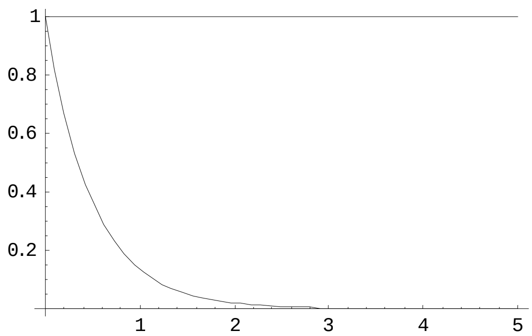
s=jω

Differentiator time function

$$\mathrm{H}(t) = \delta(t) - \frac{1}{\mathrm{RC}} e^{-t/RC}$$

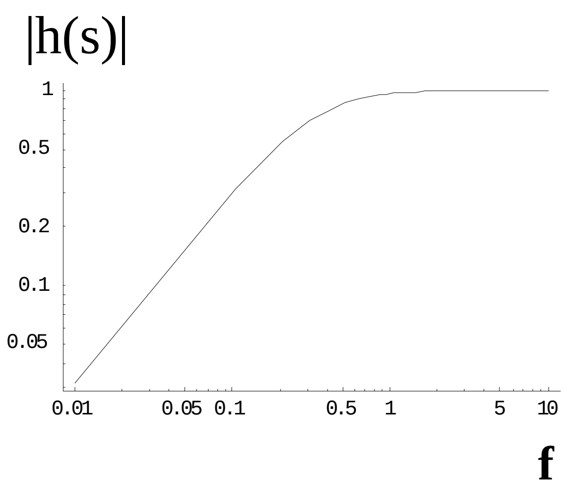


Step function response



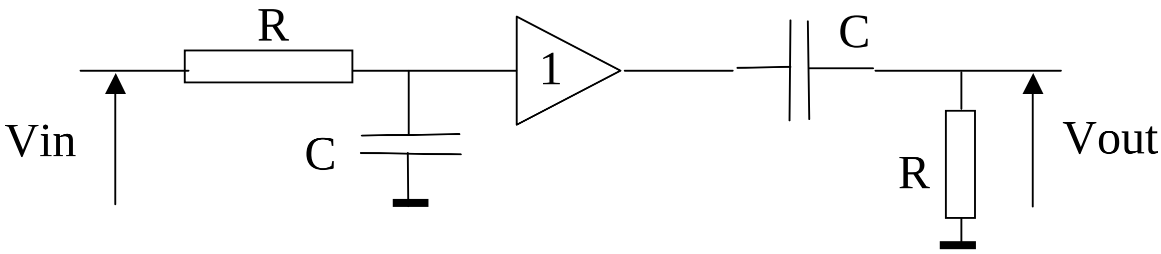
Differentiator s-transfer function

h(s) = RCs/(1+RCs)



Log-Log scale

1-Electronic Signal Processing



Combining one low-pass (RC) and one high-pass (CR) filter :

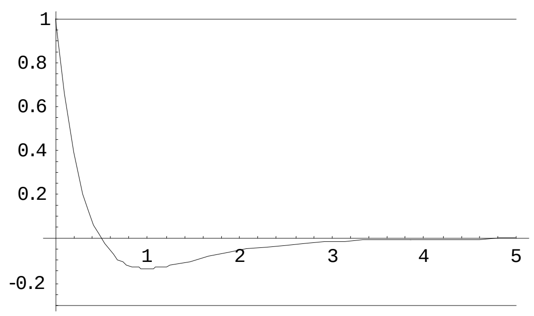
$$Vout = \frac{RCj\omega}{(1+RCj\omega)^2} Vin$$

Example RC=0.5

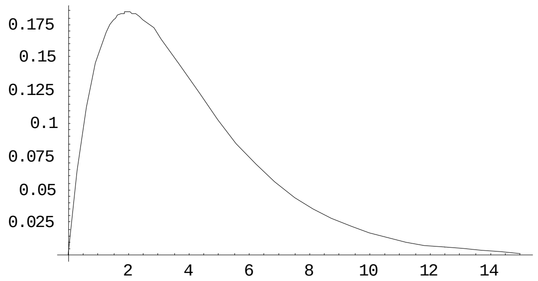
s=jω

CR-RC time function

$$H(t) = (1 - t/RC)e^{-t/RC}$$



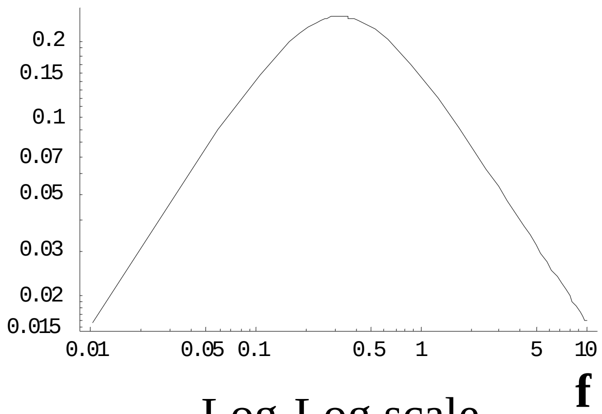
Step function response



CR-RC s-transfer function

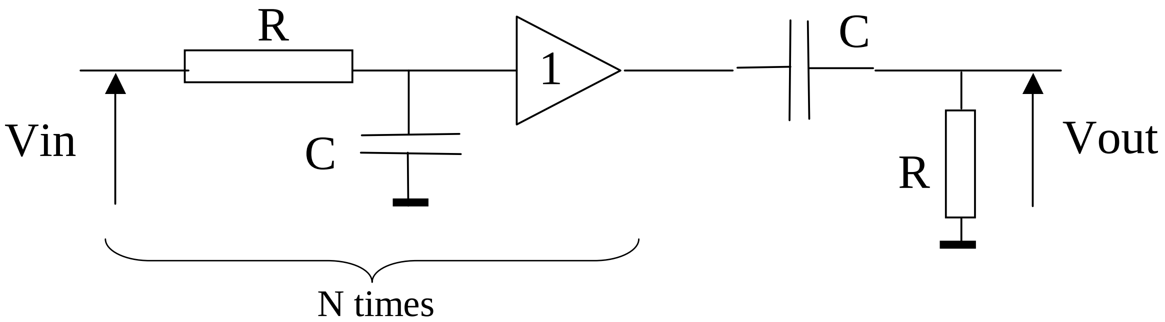
$$h(s) = RCs/(1+RCs)^2$$

|h(s)|



Log-Log scale

1-Electronic Signal Processing



Combining n low-pass (RC) and one high-pass (CR) filter :

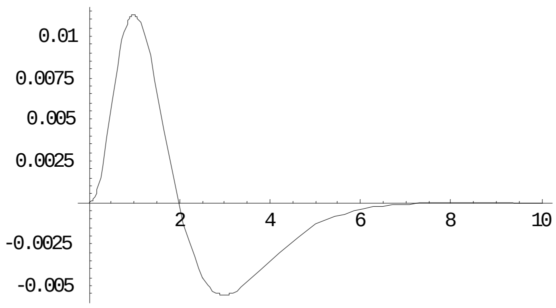
$$Vout = \frac{RCj\omega}{(1 + RCj\omega)^n} Vin$$

Example RC=0.5, n=5

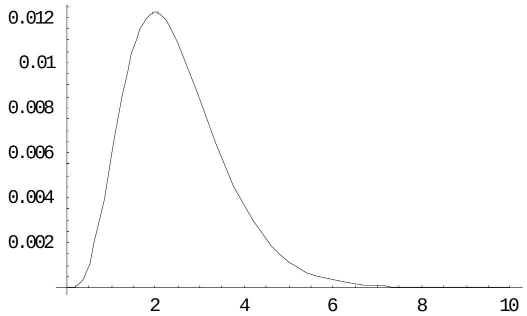
s=jω

CR-RC⁴ time function

$$\mathrm{H}(t) = (4 - t/RC).t^3 e^{-t/RC}$$

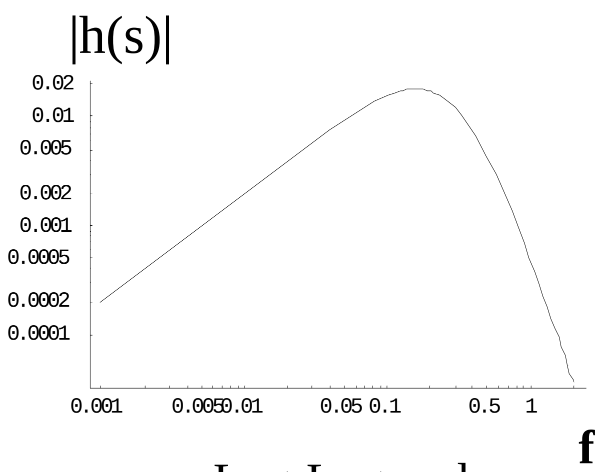


Step function response



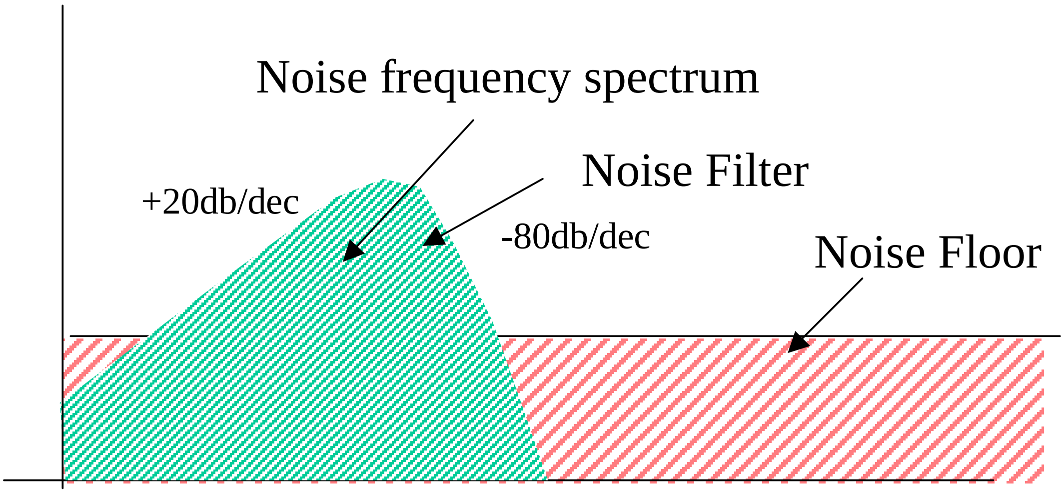
CR-RC⁴ s-transfer function

$$h(s) = RCs/(1+RCs)^5$$



Log-Log scale

1-Electronic Signal Processing

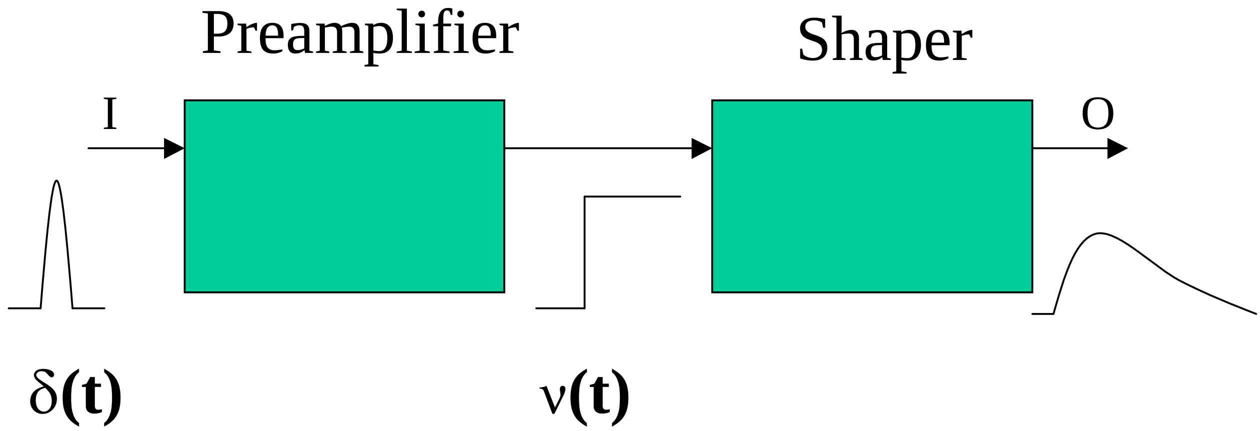


$$h(s) = RCs/(1+RCs)^5$$

The shaper limits the noise bandwidth. The choice of the shaper function defines the noise power visible at the output.

Thus, it defines the signal-to-noise ratio

2-Preamplifier & Shaper

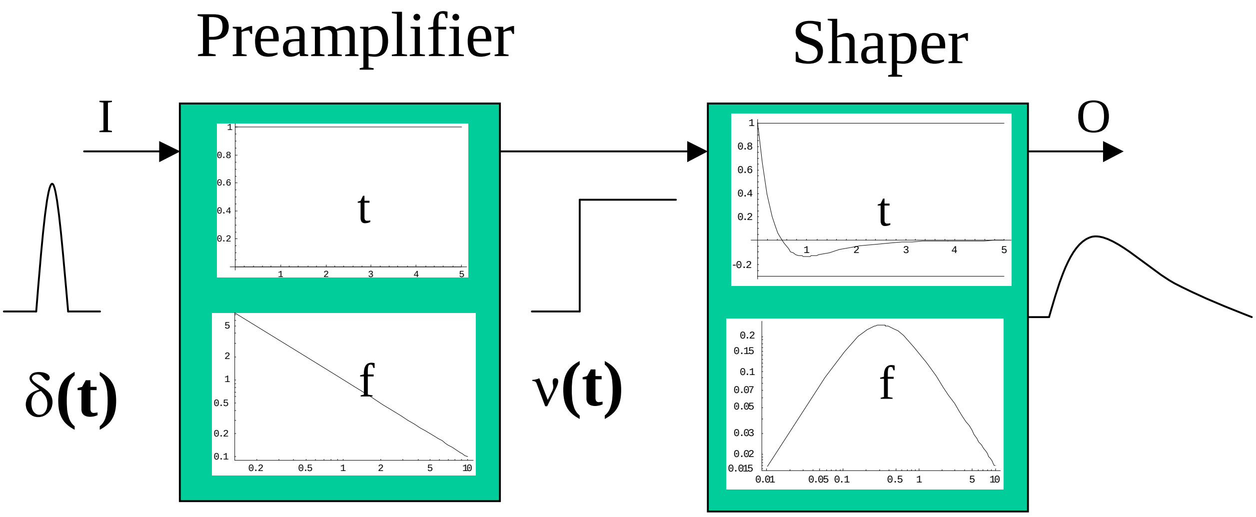


What are the functions of the preamplifier and the shaper (in an ideal world) ?

- Preamplifier : an ideal integrator : it detects an input charge burst Q $\delta(t)$. The output is a voltage step Q/Cf.ν(t). It has a large signal gain such that the noise of the subsequent stage (shaper) is negligible.

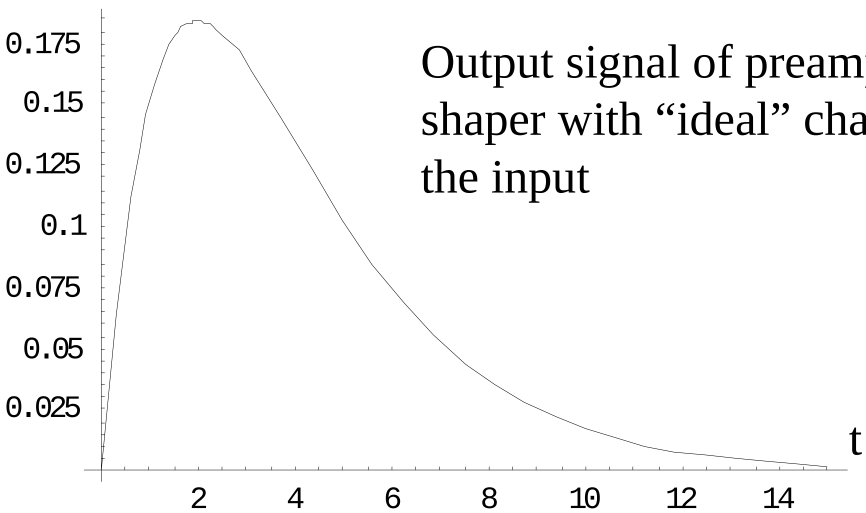
- Shaper : a filter with : characteristics fixed to give a predefined output signal shape, and rejection of (input) noise components outside of the useful output signal band.

2-Preamplifier & Shaper



Ideal Integrator   CR_RC shaper

T.F. from I to O $= 1/s \times RCs/(1+RCs)^2$

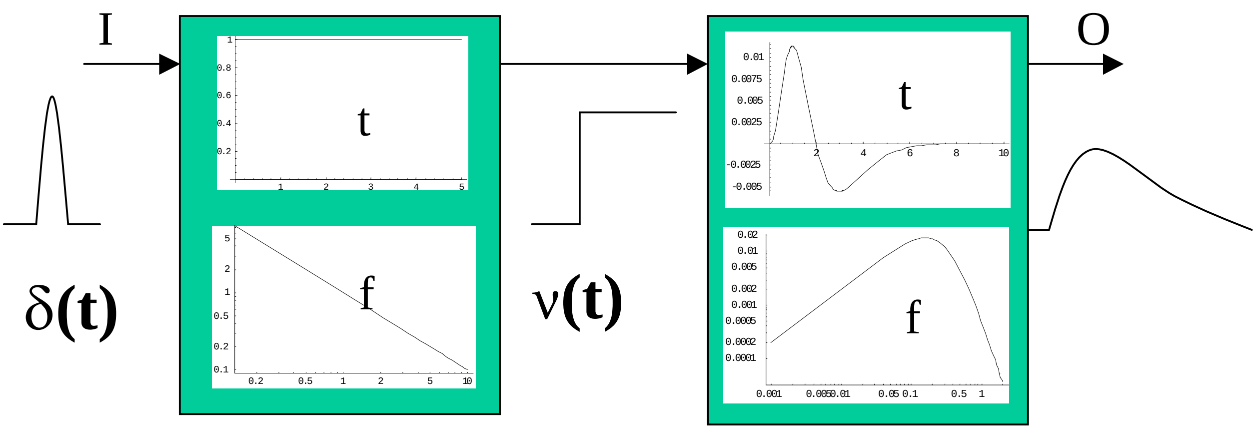


Output signal of preamplifier + shaper with “ideal” charge at the input

$o(s) = RC/(1+RCs)^2$ $O(t) = t\frac{1}{RC}e^{-t/RC}$

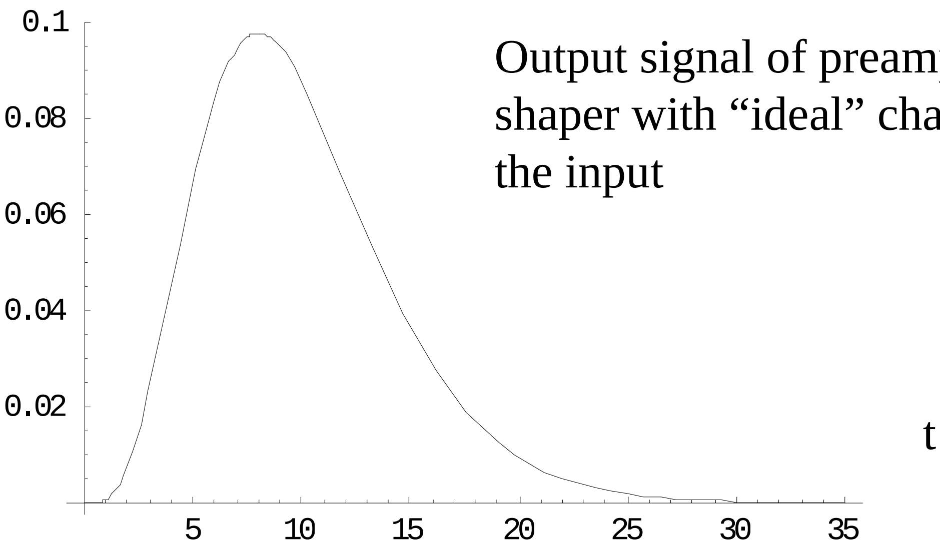
2-Preamplifier & Shaper

Preamplifier   Shaper



Ideal Integrator   CR_RC shaper

T.F. from I to O = 1/s x RCs /(1+RCs)^5

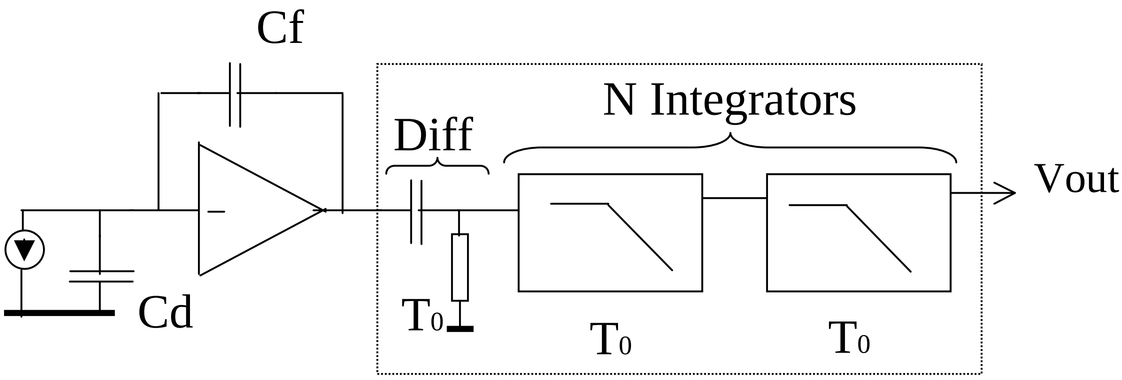


Output signal of preamplifier + shaper with “ideal” charge at the input

o(s) = RC/(1+RCs)^5 $O(t) = t^4 \frac{1}{RC^4} e^{-t/RC}$

2-Preamplifier & Shaper

Basic scheme of a Preamplifier-Shaper structure



Semi-Gaussian Shaper

$$Vout(s) = Q/sCf \,.\, [sT_0/1+ sT_0].[A/1+ sT_0]^n$$

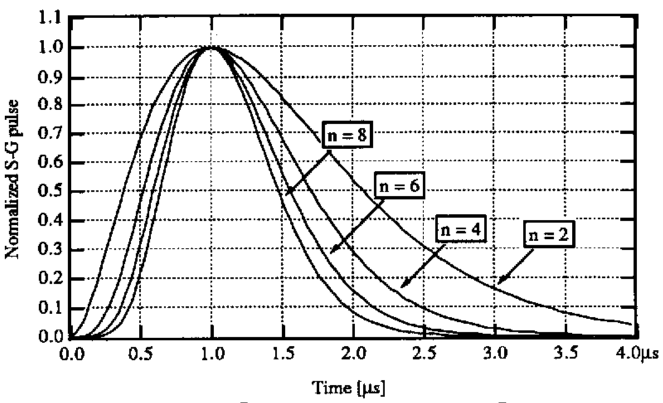
By inverse Laplace transform

$$Vout(t) = [QA^n\, n^n /Cf\, n!].[t/Ts]^n.e^{-nt/Ts}$$

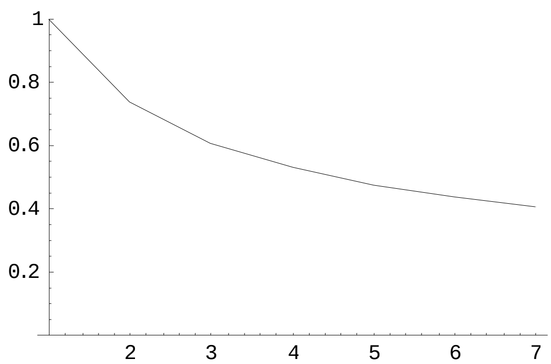
Peaking time Ts = nT_0

The Output voltage at the peak is given by :

$$Voutp = QA^n\, n^n /Cf\, n!e^n$$

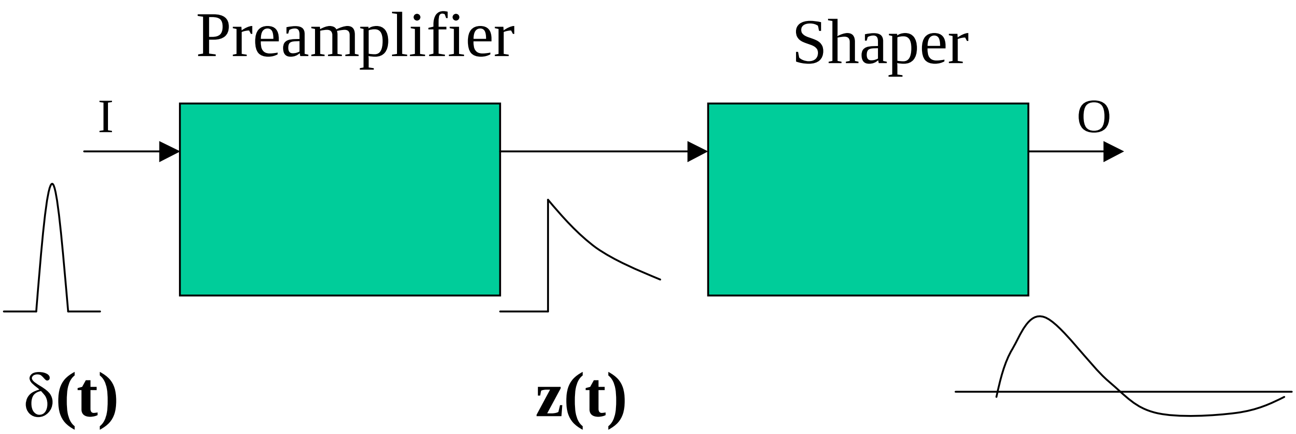


Vout shape vs. n order, normalized to 1



Vout peak vs. n

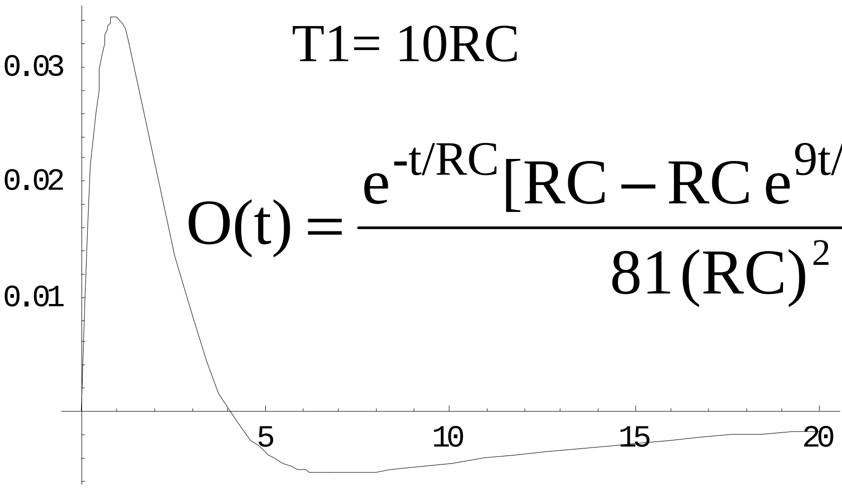
2-Preamplifier & Shaper



Non-Ideal Integrator CR_RC shaper

T.F. from I to O $1/(1+T1s)$ x $RCs/(1+RCs)^2$

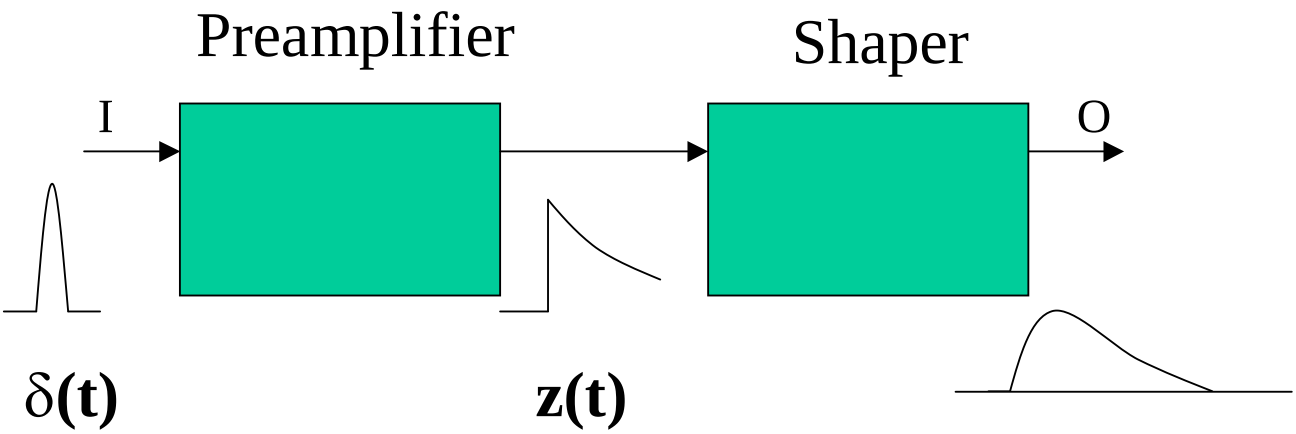
T1= 10RC



$$O(t) = \frac{e^{-t/RC}[RC - RC\,e^{9t/10RC} + 9t]}{81\,(RC)^2}$$

$$o(s) = RCs/(1+10RCs)(1+RCs)^2$$

2-Preamplifier & Shaper

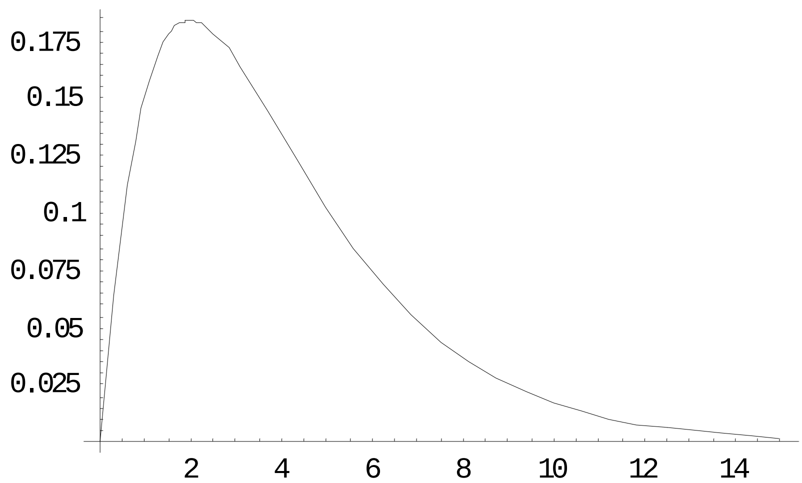


Non-Ideal Integrator

CR-RC Shaper
with pz Cancellation

T.F.
from I to O $1/(1+T1s) \quad x \quad (1+T1s)/(1+RCs)^2$

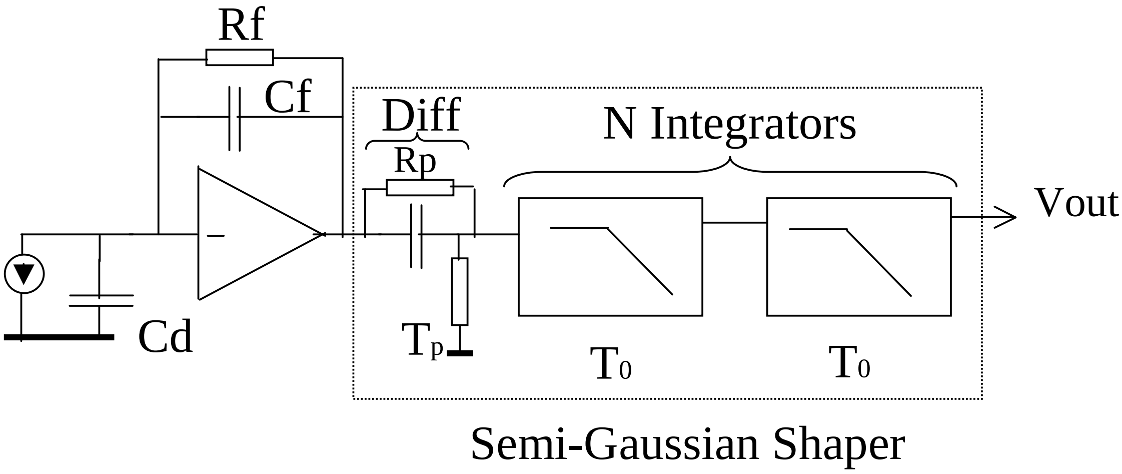
Pole-Zero Cancellation



$o(s) = RC/(1+RCs)^2$ $\quad O(t) = t\frac{1}{RC}e^{-t/RC}$

2-Preamplifier & Shaper

Basic scheme of a Preamplifier-Shaper structure
with pole-zero cancellation



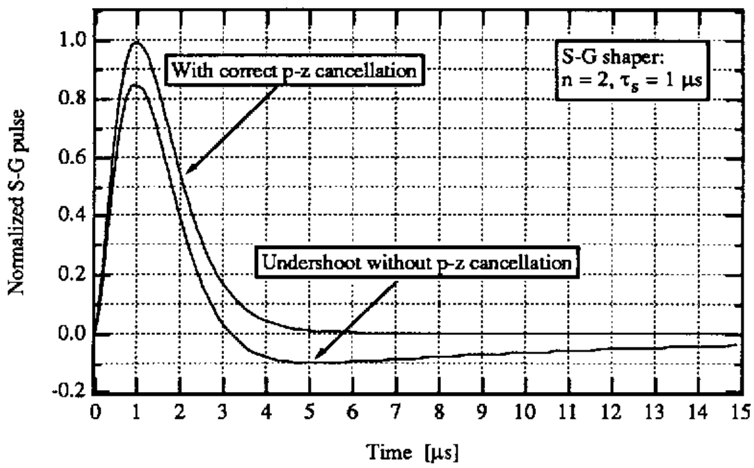
$$Vout(s) = Q/(1+sTf)Cf \,.\, [(1+sT_p/1+ sT_0].[A/1+ sT_0]^n$$

By adjusting Tp such that Tp = Tf, we obtain the same shape as with a perfect integrator at the input

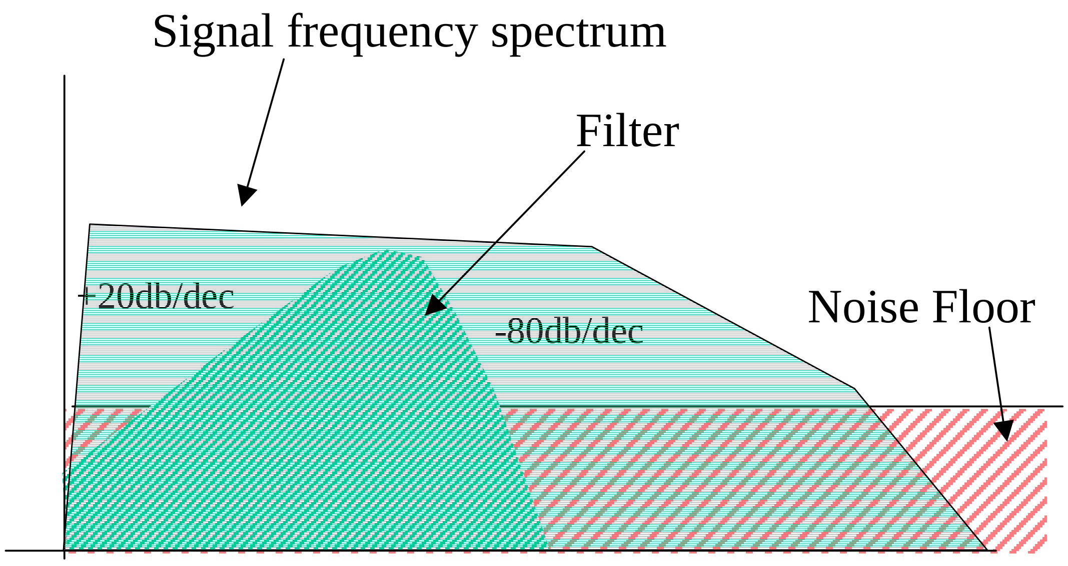
$$Vout(t) = [QA^n\, n^n /Cf\, n!].[t/Ts]^n.e^{-nt/Ts}$$

$Ts = nT_0$

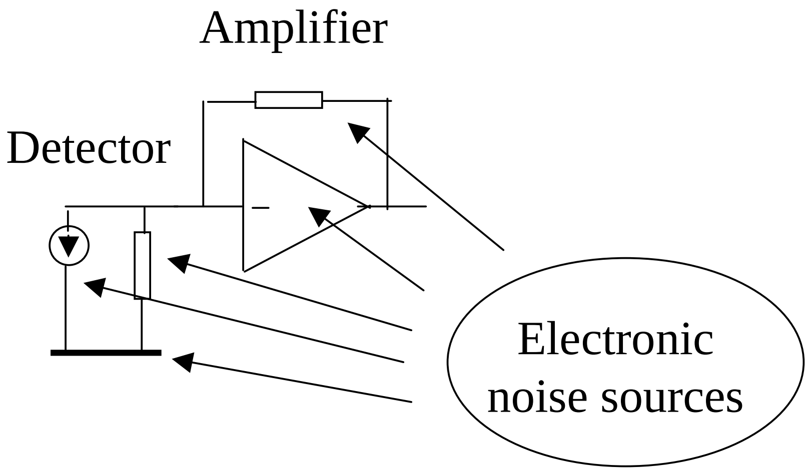
$$Voutp = QA^n\, n^n /Cf\, n!e^n$$



3-NOISE in Electronic System



3-NOISE in Electronic System



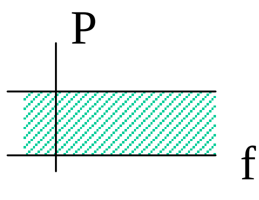
FOR ANY CURRENT, i.e. N CARRIERS MOVING IN A MATERIAL WITH A VELOCITY V,
THERE ARE FLUCTUATIONS :

$$\langle di \rangle^2 = \left(\frac{ne}{l} \langle dv \rangle \right)^2 + \left(\frac{ev}{l} \langle dn \rangle \right)^2$$

Term in <dv>^{2} (velocity fluctuation) = thermal noise

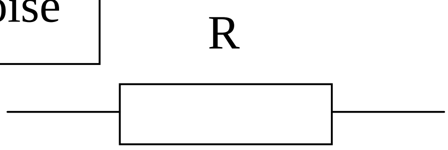
Term in <dn>^{2} (number fluctuation) = shot noise
or 1/f noise

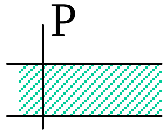
Thermal noise and shot noise are « white »,
i.e the noise power spectrum is constant



3-NOISE in Electronic System

Thermal Noise



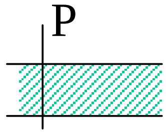


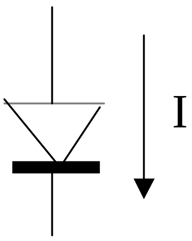
$$\langle v^2 \rangle = 4kTR.\Delta f$$ or $$\langle i^2 \rangle = 4kTR^{-1}.\Delta f$$

K = Boltzmann constant ($1.381\ 10^{-23}$ VC/K)
T = Temperature
@ ambient temperature (300K): 4kT = $1.66\ 10^{-20}$ VC

Shot Noise

Present where carrier transportation occurs across two media like a diode





$$i_{shot}^2 = 2qI\Delta f$$

q is the charge of one electron ($1.602\ 10^{-19}$ C)

3-NOISE in Electronic System

<u>Some examples :</u>

Thermal noise in resistor

$$\langle v^2 \rangle = 4kTR.\Delta f$$

For R=100 Ohm

$$\langle v^2 \rangle = 1.28nV/\sqrt{Hz}$$

For 10KHz-100MHz bandwidth : $\langle v^2 \rangle = 12.88\mu V_{rms}$

Rem : 0-100MHz bandwidth gives : $\langle v^2 \rangle = 12.80\mu V_{rms}$

For R=1 MOhm

For 10KHz-100MHz bandwidth : $\langle v^2 \rangle = 1.28mV_{rms}$

As a reference of signal amplitude, consider the mean peak charge deposited on 300um Silicon detector : 22000 electrons, ie ~4fC. If this charge was deposited instantaneously on the detector capacitance (10pF), the signal voltage would be Q/C= 400µV

3-NOISE in Electronic System

Some examples :

Thermal noise in diode

$$i_{shot}^2 = 2qI\Delta f$$

"I" can be the leakage current of a reverse biased diode.
With $I = I_{leak} = 1\mu A$

$$i_{shot}^2 = 56.6\,pA/\sqrt{Hz}$$

For 10KHz-100MHz bandwidth :

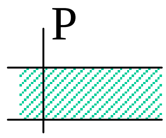
$$\left\langle i^2 \right\rangle = 5.65nA_{rms}$$

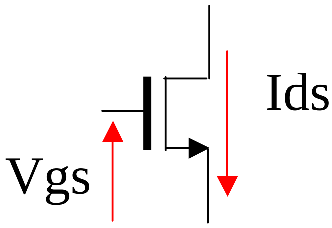
By comparing the two expressions $i_{shot}^2 = 2qI\Delta f$ and $\left\langle i_R^2 \right\rangle = \frac{4kT}{R}.\Delta f$, there is an equivalent "R" for the shot noise current $4kT/R_{eq} \equiv 2qI_{leak}$

For $I_{leak} = 1\mu A$ $\quad R_{eq} = 51.8Kohms$

3-NOISE in Electronic System

Thermal Noise in a MOS Transistor





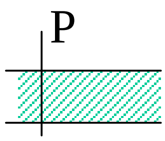
Transconductance

$$gm = \frac{\Delta I_{DS}}{\Delta V_{GS}}$$

$$i = gm.v$$

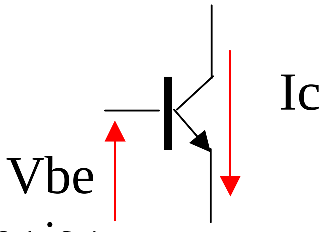
$$\left\langle v_G^2 \right\rangle = 4kT\frac{2}{3}gm^{-1}.\Delta f \quad \text{or} \quad \left\langle i_d^2 \right\rangle = 4kT\frac{2}{3}gm.\Delta f$$

Noise in a BIPOLAR Transistor



In a bipolar transistor, minority carriers crossing the B-E barrier form the collector current Ic :

$$i_{col}^2 = 2qIc\Delta f$$



The transconductance in bipolar transistor is :

$$gm = qIc / kT$$

$$\left\langle i_{col}^2 \right\rangle = 2kTgm.\Delta f \quad \text{thus} \quad \left\langle v_a^2 \right\rangle = 2kTgm^{-1}.\Delta f$$

3-NOISE in Electronic System

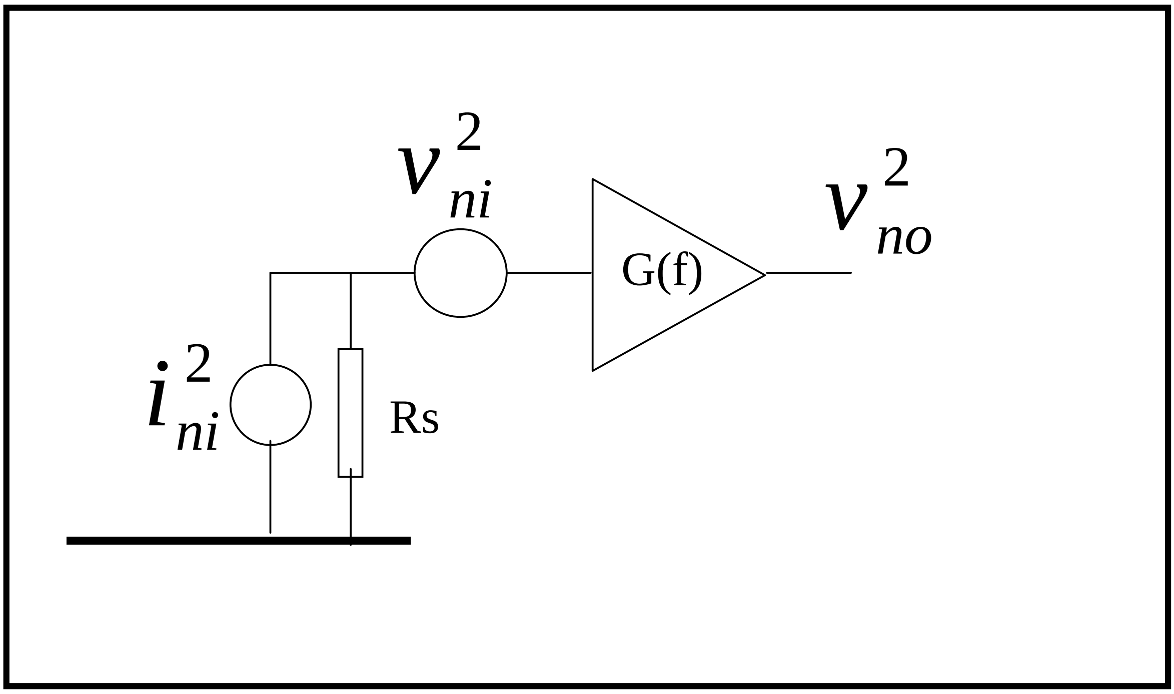
1/f Noise

General formulation

$$\left\langle v_f^2 \right\rangle = \frac{A}{f^{\alpha}}.\Delta f$$

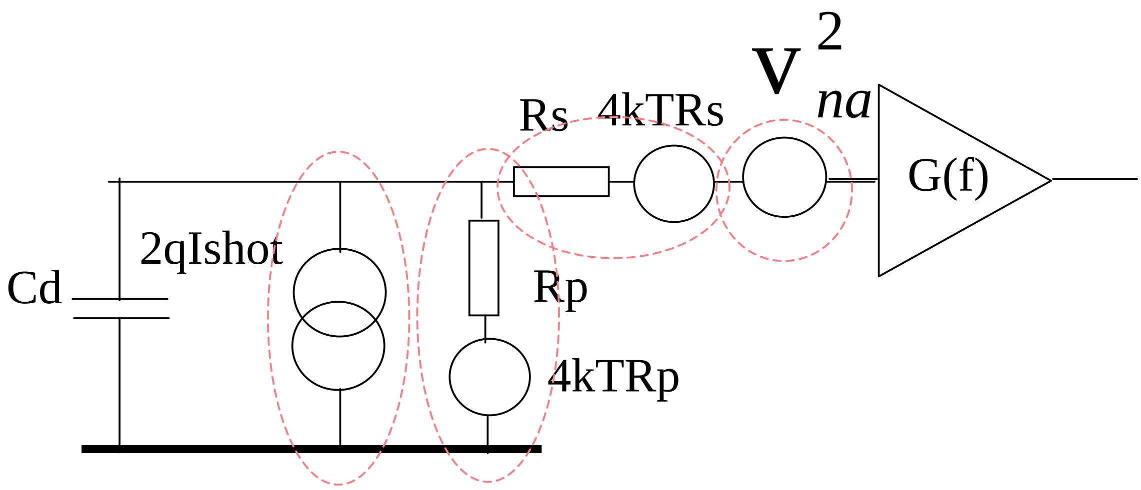
1/f noise is present in all conduction phenomena. Physical origins are multiple. It is very weak in resistors. It is very strong for MOS transistors.

4-General Formulation of Noise with Amplifier



4-General Formulation of Noise for Charge Amplifier

Equivalent Circuit for Noise Analysis



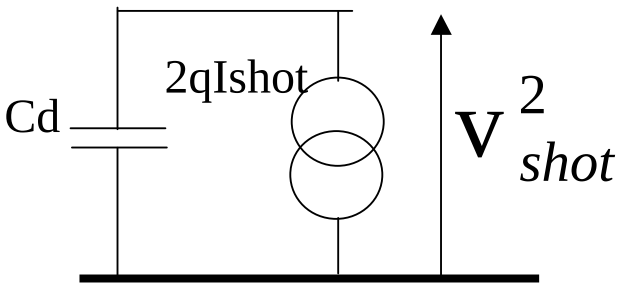
• All noise contributions are calculated in terms of noise voltage appearing at the input of the amplifier

• Noise sources are from detector elements and from the amplifier.

4 noise sources are considered here :

1. Ishot current in diode (base current of bipolar input device, leakage current in Si Detector element)
2. Rp noise, (any) resistance in parallel to the input
3. Rs noise, (any) resistance in series with the input
4. V^2_{na} equivalent input noise of input transconductance amplifier

4-Ileak Noise Source Formulation

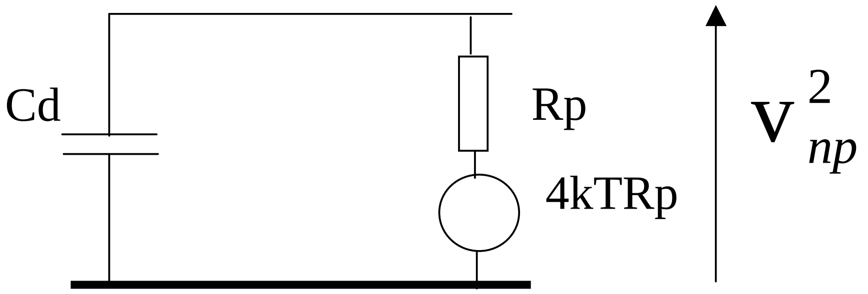


Under the assumption that Rp is much higher than the equivalent impedance of the detector capacitance in the frequency range of interest :

$$v_{sh}^2 = i_{sh}^2 \cdot \frac{1}{(j\omega Cd)^2} . \Delta f$$

$$v_{sh}^2 = 2qI_{shot} \cdot \frac{1}{(j\omega Cd)^2} . \Delta f$$

4-Rp Noise Source Formulation



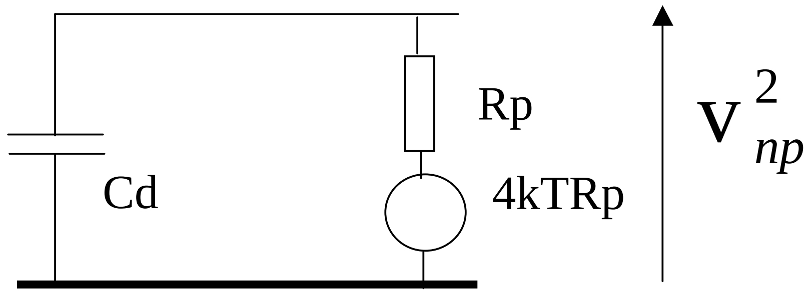
4kTRp is a voltage source loaded by the network made of the capacitance Cp and resistance Rp

$$v^2_{np} = 4kTRp \cdot \frac{1}{(1 + j\omega RpCd)^2} . \Delta f$$

v^2_{np} is the contribution of the noise of the parallel resistance to the noise at the input node of the amplifier

$$v^2_{np} = 4kT \frac{1}{(j\omega Cd)^2} \cdot \frac{1}{Rp} . \Delta f$$

4-Comment on kT/C Noise



Consider the full noise power of the above system
(integrate the noise spectral density over all frequencies)

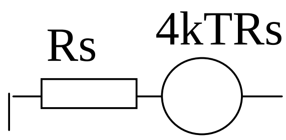
$$v^2_{tot} = \int_0^{\infty} 4kTRp \cdot \frac{1}{(1 + j\omega RpCd)^2} . d\omega$$

$$= \frac{kT}{Cd}$$

Often referred as the "kT/C" noise, it is the expression of the total noise power across a resistor when shunt by a capacitor. The total noise is independent of Rp. At constant Cd, increasing Rp means an increase of the voltage noise density, but a diminution of the system (RC) bandwidth.

With a shaper, the noise power is further limited by the shaper noise bandwidth, and total noise is below the kT/C value.

4-Rs Noise Source Formulation

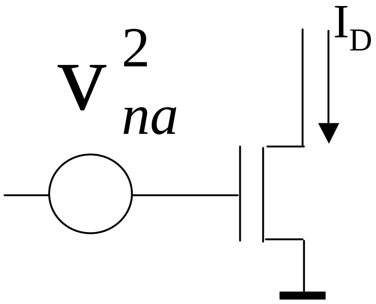


4kTRs is directly seen as a voltage noise source at the input node of amplifier (assuming Rs is small compared to Rp, or to the impedance of the detector capacitance)

$$v_{ns}^2 = 4kTRs\,.\Delta f$$

v_{ns}^2 is the contribution of the noise of the series resistance to the noise at the input node of the amplifier

4-V^2_{na} Noise Source Formulation



$$i_d^2 = 4kT\frac{2}{3}gm$$

I_d^2 is the current noise in the channel of the input MOS transistor.

With $gm = \dfrac{\Delta I_{DS}}{\Delta V_{GS}}$

or $i = gm.v$

For a MOS transistor

$$gm = 2\sqrt{k\frac{W}{L}I_d}$$

$$v^2_{na} = 4kT\frac{2}{3}gm^{-1}.\Delta f$$

V^2_{na} is the contribution of the noise of the input transistor to the noise at the input node of the amplifier

5-Equivalent Noise Charge

Noise at Detector Input :

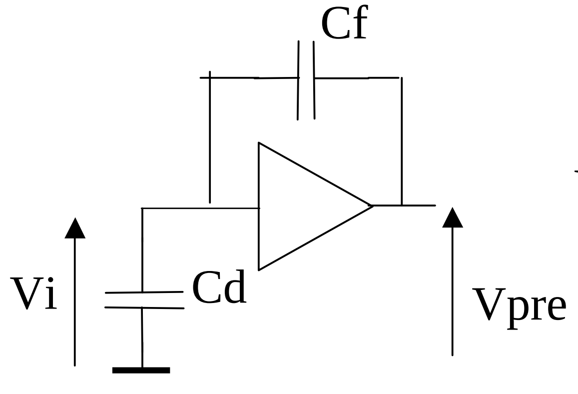
$$V_{ni}^2 = V_{sh}^2 + V_{np}^2 + V_{ns}^2 + V_{na}^2$$

Ishot — Parallel Resistance — Series Resistance — Input Transconductance

Noise voltage at output of Preamplifier and Shaper :

$$V_{no}^2 = \int_0^\infty (V_{sh}^2 + V_{np}^2 + V_{ns}^2 + V_{na}^2).P^2(s).G^2(s)ds$$

P(s) is the preamplifier voltage gain



$$Vpre = Vi.\left(\frac{C_d}{C_f}\right)$$

5-Equivalent Noise Charge

Noise voltage at output of shaper :

$$\mathrm{V}_{\mathrm{no}}^2 = \int_0^\infty (\mathrm{V}_{\mathrm{sh}}^2 + \mathrm{V}_{\mathrm{np}}^2 + \mathrm{V}_{\mathrm{ns}}^2 + \mathrm{V}_{\mathrm{na}}^2).\left[\frac{Cd}{Cf}\right]^2 .G^2(s)ds$$

For a CR-RC shaper :

$$G(s) = \frac{\tau s}{(1+\tau s)^2} \qquad s = j\omega \qquad \tau = RC$$

and :

$$\mathrm{V}_{\mathrm{no}}^2 = \int_0^\infty \left[\frac{2qI_{shot}}{\omega^2 C_f^2} + \frac{4kT}{R_p \omega^2 C_f^2} + \left(4kTRs + V_{na}^2\right)\left(\frac{C_d^2}{C_f^2}\right)\right] .\left[\frac{\omega^2\tau^2}{2\pi.(1+\omega^2\tau^2)^2}\right] d\omega$$

5-Equivalent Noise Charge

$$V_{no}^2 = \frac{1}{8C_f^2}\left(\frac{4kT}{R_p} + 2qI_{shot}\right)\tau + \left(\frac{C_d}{C_f}\right)^2 \left(4kTRs + V_{na}^2\right)\frac{1}{\tau}$$

This is the expression of the noise at the OUTPUT of the preamplifer-shaper CR-RC

Expression of the signal « peak » amplitude at the output :

At the output of Charge preamplifier:

$$\text{Vsi} = \text{Vi}\,\frac{Qs}{C_f}$$

5-Equivalent Noise Charge

For the simple case with T1=T2=τ
(CR and RC time constants are equal) :

$$V_{so} = V_{si}\frac{1}{e}$$

$$V_{so} = \frac{Q_s}{C_f}\frac{1}{e}$$

The ratio of Noise Power versus Signal Power at the output is given by :

$$\frac{V_{no}^2}{V_{so}^2}$$

(Expression for noise power at the output of the shaper) :

$$V_{no}^2 = \frac{1}{8C_f^2}\left[\left(\frac{4kT}{R_p} + 2qI_{shot}\right)\tau + \left(\frac{C_d}{C_f}\right)^2 \left(4kTRs + V_{na}^2\right)\frac{1}{\tau}\right]$$

5-Equivalent Noise Charge

For the simple case with T1=T2=τ
(CR and RC time constants are equal) :

$$\frac{V_{no}^2}{V_{so}^2} = \frac{1}{Q_s^2}\frac{e^2}{8}\left[\left(\frac{4kT}{R_p} + 2qI_{shot}\right)\tau + \left(4kTRs + V_{na}^2\right)\frac{C_d^2}{\tau}\right]$$

If the input signal is 1 electron (Qs=q), then this ratio expresses the noise power at the output in terms of « number of equivalent electrons at the input». This is the

EQUIVALENT NOISE CHARGE (ENC)

$$ENC^2 = \frac{e^2}{8q^2}\left[\left(\frac{4kT}{R_p} + 2qI_{shot}\right)\tau + \left(4kTRs + V_{na}^2\right)\frac{C_d^2}{\tau}\right]$$

Expression in (electrons r.m.s.)^2

5-Equivalent Noise Charge

EQUIVALENT NOISE CHARGE (ENC)

$$ENC^2 = \frac{e^2}{8q^2}\left[\left(\frac{4kT}{R_p} + 2qI_{shot}\right)\tau + \left(4kTRs + V_{na}^2\right)\frac{C_d^2}{\tau}\right]$$

Rp : Resistance in parallel at the input

Rs : Resistance in series with input

Ishot : Shot noise current (bipolar base current)

Vna^2 : Noise voltage of input transistor

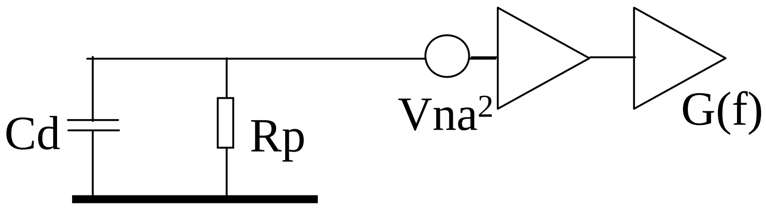
τ : Shaping time constant

Cd : Capacitance at the input

5-Equivalent Noise Charge

EQUIVALENT NOISE CHARGE (ENC)

A very simple case, with an input MOS transistor of transconductance gm, no leakage current, no resistance in series:



$$ENC^2 = Fp.\frac{4kT}{q^2 R_p}\tau + Fs.\frac{4kT}{q^2}\frac{2}{3}gm^{-1}\frac{C_d^2}{\tau}$$

Fp and Fs are factors which depend on the choice of the shaper. For the simple RC-CR with equal time constants, Fp=Fs, and are close to 1 ($e^2/8$).

The Parallel Noise (in electrons r.m.s.) is proportional to the square root of time constant

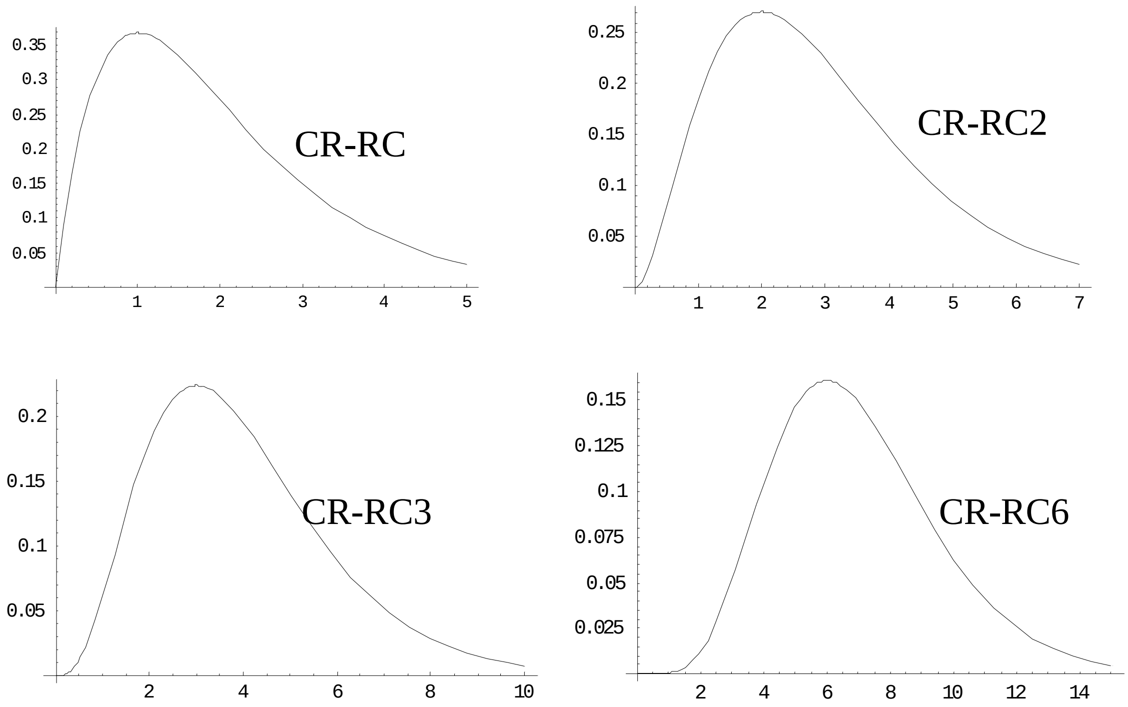
The Series Noise (in electrons r.m.s.) is inversely proportional to the square root of time constant, and proportional to the input capacitance.

5-Noise Figures

The Fp and Fs factors depend on the CR-RC shaper order n

n	1	2	3	4	5	6	7
Fs	0.92	0.84	0.95	0.99	1.11	1.16	1.27

n	1	2	3	4	5	6	7
Fp	0.92	0.63	0.51	0.45	0.40	0.36	0.34



5-ENC : Generalisation

Noise voltage at output of shaper :

$$V_{no}^2 = \int_0^{\infty} (V_{sh}^2 + V_{np}^2 + V_{ns}^2 + V_{na}^2).\left[\frac{Cd}{Cf}\right]^2 .G^2(s)ds$$

For a shaper with CR-RCn form :

$$G(s) = \left[\frac{s\tau}{1+s\tau}\right].\left[\frac{A^n}{(1+s\tau)^n}\right]$$

The signal amplitude at the shaper output is determined for one electron charge at the input as :

$$Vout = \frac{qA^n.n^n}{C_f.n!.e^n}$$

5-ENC : Generalisation

Series Noise

Series Noise (Channel Thermal noise) :

The voltage noise at the output of the preamplifier-shaper is given by combination of :

The channel noise expression

$$Vna^2 = 4kT\frac{2}{3}(gm)^{-1}$$

and the expression

$$\mathrm{V}_{\mathrm{no}}^2 = \int_0^\infty (\mathrm{V}_{\mathrm{na}}^2).\left[\frac{Cd}{Cf}\right]^2 .G^2(s)ds$$

Integration gives :

$$\mathrm{V}_{\mathrm{no}}^2 = \frac{8}{3}kT(gm)^{-1}\left(\frac{C_d}{C_f}\right)^2 \frac{A^{2n} nB\left(\frac{3}{2}, n-\frac{1}{2}\right)}{4\pi\tau_s}$$

with $\tau_s = n\tau$!

5-ENC : Generalisation

Series Noise

The Equivalent ENCs is given by the ratio of voltage noise output to the signal amplitude for one electron charge at the input

$$ENC_s^2 = \frac{8}{3}\frac{kT}{q^2}(gm)^{-1}C_d^{\;2}\,\frac{1}{\tau_s}\cdot\underbrace{\frac{nB\left(\frac{3}{2},n-\frac{1}{2}\right)}{4\pi}\left(\frac{n!^2\,e^{2n}}{n^{2n}}\right)}_{Fs}$$

n	1	2	3	4	5	6	7
Fs	0.92	0.84	0.95	0.99	1.11	1.16	1.27

The « Series Noise » (either resistance in series at the input, or transconductance of the input device) is proportional to the input capacitance (detector capacitance).

It is inversely proportional to the square root of the shaper time constant.

5-Exemple of Series Noise Contribution

$$ENC_s^2 = \frac{8}{3}\frac{kT}{q^2}(gm)^{-1}C_d^{\ 2}\frac{1}{\tau_s}.\underbrace{\frac{nB\left(\frac{3}{2}, n-\frac{1}{2}\right)}{4\pi}\left(\frac{n!^2\, e^{2n}}{n^{2n}}\right)}_{Fs}$$

Numerical values for modern « typical » conditions :

gm = 10^{-3} Siemens
t = 25ns
Fs = 0.84 (n=2)
Cd= 20pF

The noise contribution of the input transconductance device (with the above conditions) is calculated as :

ENC = 2400 electrons @ 20pF input capacitance

or

ENC = 120el/pF

5-ENC : Generalisation

Parallel Noise

Parallel Noise (Shot noise or Parallel resistance) :

The voltage noise at the output of the preamplifier-shaper is given by the combination of :

The parallel resistance noise expression

$$v_{np}^2 = 4kT \frac{1}{(j\omega Cd)^2} \cdot \frac{1}{Rp}$$

and the expression

$$V_{no}^2 = \int_0^\infty (V_{np}^2) . \left[\frac{Cd}{Cf}\right]^2 . G^2(s) ds$$

Integration gives :

$$V_{no}^2 = 4kTRp^{-1} \left(\frac{1}{C_f}\right)^2 . \tau_s . \frac{A^{2n} B\left(\frac{1}{2}, n+\frac{1}{2}\right)}{4\pi n}$$

with $\tau_s = n\tau$

5-ENC : Generalisation

Parallel Noise

The equivalent ENC is given by the ratio of voltage noise output to the signal amplitude for one electron charge at the input

$$ENC_p^2 = 4\frac{kT}{q^2}(Rp)^{-1}.\tau_s.\underbrace{\frac{B\left(\frac{1}{2}, n+\frac{1}{2}\right)}{4\pi n}\left(\frac{n!e^{2n}}{n^{2n}}\right)}_{Fp}$$

n	1	2	3	4	5	6	7
Fp	0.92	0.63	0.51	0.45	0.40	0.36	0.34

The « Parallel Noise » (either resistance in parallel at the input, or shot noise current) is only dependent on the shaper τ characteristic.

It is proportional to the square root of the shaper time constant.

5-Exemple of Parallel Noise Contribution

$$ENC_p^2 = 4\frac{kT}{q^2}(Rp)^{-1}.\tau_s.\underbrace{\frac{B\left(\frac{1}{2},n+\frac{1}{2}\right)}{4\pi n}\left(\frac{n!e^{2n}}{n^{2n}}\right)}_{Fp}$$

Numerical values for modern « typical » conditions :

Rp = 100Kohms
t = 25ns
Fp = 0.63 (n=2)

The noise contribution due to the resistance in parallel to the input (with above conditions) is calculated as :

ENCp = 320 electrons

5-1/f Noise Contribution

The 1/f noise contribution at the output of shaper is :

$$\mathrm{V}_{\mathrm{no}}^2 = \int_0^{\infty} \mathrm{V}_{\mathrm{nf}}^2 \cdot \left[\frac{Cd}{Cf}\right]^2 .G^2(s)ds$$

$$\left\langle v_f^2 \right\rangle = \frac{A}{f^{\alpha}}.\Delta f$$

With α=1:

$$\mathrm{V}_{\mathrm{no}}^2 = \frac{K_f}{C_{ox}^2 WL}\left(\frac{C_D}{C_f}\right)^2 \frac{A^{2n}}{2n}$$

And, by ratioing with the output signal for one electron charge :

$$\mathrm{E}NC_f^2 = \frac{K_f}{q^2 C_{ox}^2 WL}.C_D{}^2 \cdot \frac{1}{2n} \cdot \frac{n!^2\, e^{2n}}{n^{2n}}$$

5-Exemple of Contribution of 1/f Noise

$$ENC_f^2 = \frac{K_f}{q^2 C_{ox}^2 WL} . C_D{}^2 . \underbrace{\frac{1}{2n} . \frac{n!^2 \, e^{2n}}{n^{2n}}}_{F_{1/f}}$$

Numerical values for modern CMOS technologies :

Kf = 5.10^{-27} C^2/m^2
Cox = 3fF/um^2
WL = 2000 um^2
Cd= 20pF

The 1/f noise contribution is calculated as :

ENC = 123el @ 20pF input capacitance

or

ENC = 6el/pF

The 1/f noise contribution is independent of the shaping time constant. The $F_{1/f}$ factor is close to 3.5 and depends weakly on n, the order of CR-RCn shaper.
The 1/f noise contribution is therefore only dependent on the input transistor charcateristics (Kf, α, Cox, WL), and the input capacitance. However, the contribution is small for modern technologies.

5-ENC Noise Optimisation

From the preceding formulation we have found :

• **Parallel noise** (contribution of parallel resistances, shot noise current with equivalence 2qI vs. $4kT(Rp)^{-1}$)

ENC^2 Form : **$4kT\ .Fp.(Rp)^{-1}.\tau$**

•**Series noise** (contribution of series resistances, input transconductance device as $2/3gm^{-1}$ equivalent Rs)

ENC^2 Form : **$4kT\ .Fs.(Rs) \cdot 1/\tau.\ Cd^2$**

•**1/f noise** (contribution of 1/f noise from input transistor)

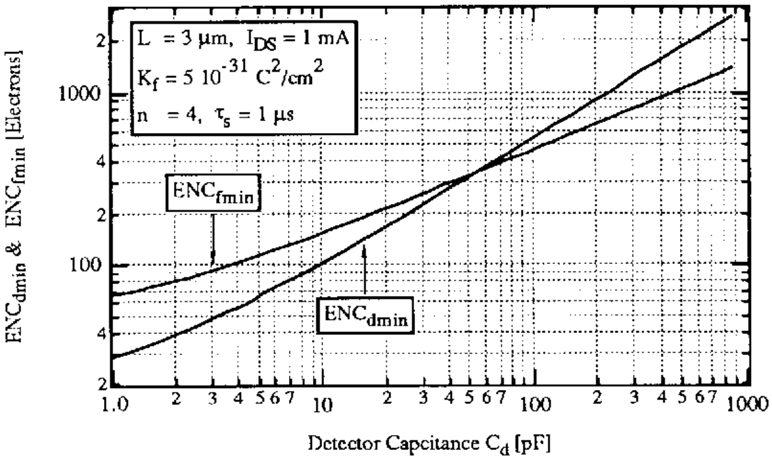
ENC^2 Form : **$Kf/(Cox^2WL).F_{1/f}.Cd^2$**

The total noise:

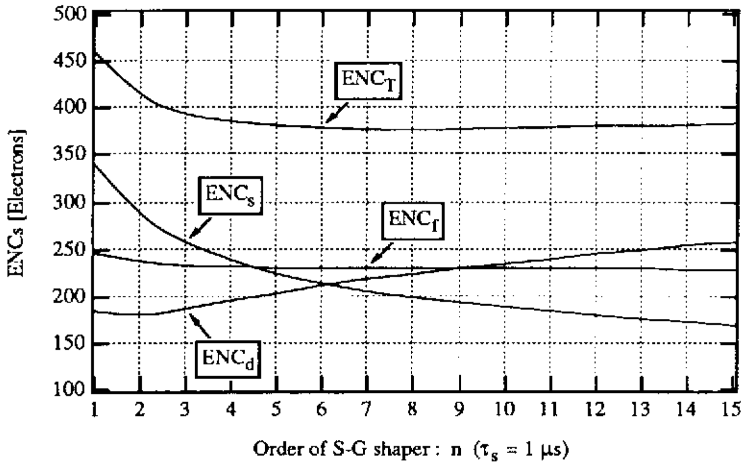
$$ENC_t = \sqrt{ENC_p^2 + ENC_s^2 + ENC_{1/f}^2}$$

For given technological parameters it is dependent on τ, n, Cd and gm of the input transistor.

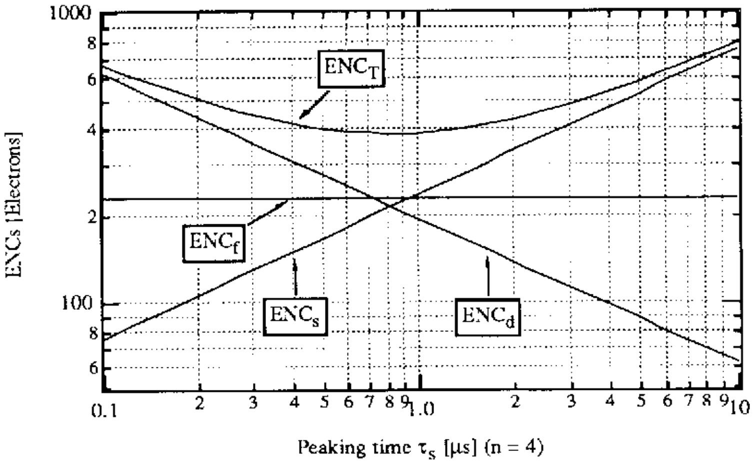
5-ENC Noise Optimisation



ENC versus Cd



ENC versus n



ENC versus τ

5-ENC Noise Optimisation

We conclude here our discussion on the ENC calculation, using the frequency domain representation.

What we have done :

- Analyse different sources of noise at the input node of a charge amplifier
- Formulate the transfer function for a very simple front-end circuit (charge amplifier + shaper)
- Evaluate Signal and Noise power at the front-end output
- Obtain a "generic" ENC formulation of the form :

$$ENC^2 = Fp.\underbrace{\frac{4kT}{q^2 R_p}\tau}_{\text{Parallel noise}} + Fs.\underbrace{\frac{4kT}{q^2} R_s \frac{C_d^2}{\tau}}_{\text{Series noise}}$$