The Speed and Decay of Cosmic Ray Muons

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The speed of cosmic−ray muons was determined by measuring the difference in the media time of flights between plastic scintillator paddles and the mean lifetime of cosmic−ray muons at rest was determined by measuring the interval between start−stop electrical signals generated as these particles are brought to rest in a block of plastic scintillator. In good agreement with the accepted values, the mean speed of the muons was measured to be 0.98 (±0.02)c and their mean life 2.10 (±0.03) μ*s.*

Cosmic rays consist of positive ions, mainly protons with energies in the range of 10^{12} to 10^{18} eV. These primary cosmic ray particles interact with atmospheric nuclei to produce showers of particles, which can be divided into *hard* and *soft* components, the distinction being based on their ability to penetrate matter at the Earth's surface. The hard component consists of muons, while the soft includes photons, electrons and positrons. The soft component is created by strong interactions between a primary cosmic−ray particle and a atmospheric nucleus. This reaction produces neutral and charged pions. The former decay into photons, which produce a particle shower by pair−production, Compton scattering and bremsstrahlung. The latter decay into muons and neutrinos, thereby producing the hard component: $\pi^+ \rightarrow \mu^- + \upsilon_\mu$ and $\pi^- \rightarrow \mu^+ + \upsilon_\mu$.

The name *muon* is derived from mu meson, the former name of the particle. The muon was first observed in cosmic rays by Anderson and Neddermeyer in 1936, a year after the existence of a particle of about the same mass has been predicted by Yukawa as the intermediary of the nuclear force. However, the muon's behaviour did not conform to that of Yukawa's meson theory (which actually describes the pion π, discovered in 1947).

Maxwell's equations of electromagnetism have solutions in the form of waves that travel in a vacuum with the Universal velocity c, without regard to the motion of the source or observer of the waves. Einstein solved this contradiction, by modifying Galilean kinematics and Newtonian dynamics, instead using Lorentzian transformations. One of the consequences of special relativity developed by Einstein is that 'moving clocks run slower', that is there is apparent time dilation in a frame of reference moving with respect to another. A time interval in a moving frame, τ, is related to that in a rest frame, τ_o, by: $\tau = \gamma\, \tau_o$, where $\gamma = (1-v^2/c^2)^{-½}$ the Lorentz factor. MIT physicists Rossi and Hall tested this time dilation phenomenon by studying cosmic ray muons[i]. They showed that time dilation was required to account for the observed numbers of muons at sea−level, which was 16 times that expected without dilation.

In this experiment the speed of cosmic ray muons was measured using a time−of−flight technique between two plastic scintillator 'paddles'. Measurements were made over five distances, necessary because the delay time arising from the set up cannot be determined from a single configuration.

The top scintillation paddle was fixed and the middle paddle mounted on a platform whose height could be adjusted as shown in Figure 1. Each paddle was identical, being 40cm by 60cm in size. Cosmic−ray muons traversing either paddle generate scintillations that are collected by an optical funnel leading to a photomultiplier. These produce electrical pulses proportional to the intensity of the optical photons which reach it. The top detector generates a start pulse that, after passing through a constant fraction discriminator (CFD) initiates the time−to−amplitude converter (TAC). The middle detector creates a stop pulse which terminates the TAC. It generates a positive output pulse proportional to the time interval between the start and stop pulses which was recorded on a computer−based multi−channel analyser (MCA).

For small paddle separations it is possible that the travel time between paddles is of the order of the scintillation diffusing to the photomultiplier. Hence it was necessary to delay the stop pulse by approximately 70ns to ensure that each stop pulse reaches the TAC after the start pulse. A time calibrator was used to generate a series of precisely separated pulses to correctly calibrate the MCA channels to definite time intervals.

The measured time interval on the MCA, $t_i = t_o + d_i/v_i + \Delta t_i$; where: t_o is the characteristic delay for the apparatus, d_i is the slant distance travelled between the paddles by the i^{th} muon at velocity v_i, and Δt_i is the systematic error inherent in the apparent. The largest contribution to this error is due to the difference in diffusion times of the scintillation light to the two photomultipliers. Assuming that the refractive index of the scintillators ≈ 1.5, then, given the dimensions of the paddles, dispersion due to differences in diffusion time is approximately 1ns. It is reasonable to assume that the systematic error due to timing calibration is negligible.

The mean slant distances were assessed by integration over all possible paths traversable by the muons, weighted according to the empirical fits of the intensity of muons at sea level as a function of zenith angle[ii]. The dominating uncertainty was in the measurement of the separation of the paddles, assessed to be ±5cm, which was greater than those for the count rate, which were modelled by the Poisson distribution.

A plot of the distance travelled by the muons versus time is shown in Figure 2. The mean speed of the muons was determined by a Least Squares Fit to be 0.98 (±0.02) c. This compares well with the established value of 0.98c.

To obtain a measure of time dilation, the mean life time of muons at rest was measured. Bethe's formula [Equation 1] predicts that a muon will lose about 50 MeV before coming to rest, having travelled a distance of around 10cm. The electrons which are produced by the subsequent decay have energies around 20 MeV. Two photomultiplier tubes were used to detect both the gamma rays produced by the stopping of the muon then its decay, as shown in Figure 3. A discriminator level was chosen to avoid a significant count of accidental coincidences between random pulses while still not ignoring events of interest. The discriminated pulses were fed into a pair of constant fraction discriminators (CFD), then a coincidence circuit. The delayed start and stop pulses fed into the time−to−amplitude converter (TAC) and finally to the computer−based multi−channel analyser (MCA). The start pulse was delayed to avoid inhibiting the timing sequences by the simultaneous arrival of every pulse at the start and stop inputs. Delay of the start pulse ensures that the stop input was no longer activated when a timing sequence was initiated. As with the previous part, a time calibrator was used to generate pulses of known separation to test the linearity of the MCA and to calibrate the channel numbers to give definite times.

The exponentially observed decay curve of the muons is shown in Figure 4. This curve was fitted using the Levenberg−Marquardt algorithm for the model $\eta(t) = a.\exp(-t/\tau) + b$, taking care to exclude measurements for times in the first few tenths of a microsecond due to lag in the recovery of the CFD, after pulsing of the photomultiplier and the decay of negative muons that suffer loss by nuclear absorption. The experimentally determined value of the mean life, 2.10 ($\pm$0.03) μs, is in reasonable agreement with the established value of 2.21 ($\pm$0.01) μs. The discrepancy of 3.6 standard deviations (= 5%) from the established value may therefore be the result of some systematic error in the timing calibration, possibly due to an uncertainty > 5% in the pulse generator.

On the basis of our experimental results, in a time equal to the mean life of muons at rest, a typical high−energy muon will therefore travel 650km. High altitude observations[iii] show that the most of the muons that arrive at sea−level are created above 15km. It will therefore take such a muon 51μs to reach sea−level. The survival probability for such a length of time (~33 half−lives, if its life expectancy were the same of a muon at rest) is $\sim 10^{-10}$.

A muon travelling at the measured mean speed has a Lorentz factor $\gamma \approx 6.3$. Given time dilation, the survival probability of a muon created at 15km reaching sea−level increases to 0.026. This increased survival rate due to time dilation predicts the altitude dependence of cosmic−ray balloon experiments with far better accuracy than the non−relativistic calculation, and is good evidence for the existence of such relativistic phenomenon.

i B. Rossi, D.B. Hall, *Phys. Rev.*, 59, 223 (1941)

iiB. Rossi, *Rev. Mod. Phys*., 20, 537 (1948)

iii B. Rossi, *Cosmic Rays* (1964)