Speed and Decay of Cosmic Ray Muons

Nichols A. Romero and Mukund T. Vengalattore

Junior Physics Laboratory, Massachusetts Institute of Technology

Cambridge, Massachusetts 02139

(April 19, 1998)

ABSTRACT

An experiment has been performed that demonstrates the speed limit c and the phenomenon of time dilation predicted by special relativity. This was done by first measuring the mean velocity (v_μ) of cosmic ray muons and their lifetime at rest (τ_μ), and showing that the classical prediction of the range ($v_\mu \tau_\mu$) is much smaller than the observed muon ranges. The speed of cosmic-ray muons was determined by measuring the difference in the median time of flights between plastic scintillator paddles. Additionally, the mean lifetime of cosmic-ray muons at rest was determined by measuring the interval between start-stop electrical signals generated as these particles were brought to rest in a block of plastic scintillator. The experimentally determined values of the mean life, 2.19$\pm$0.02 μs, and their mean speed, 0.92$\pm$0.01 c, are in good agreement with the accepted values.

I. INTRODUCTION

This paper is a full report on the junior lab experiment: *Speed and Decay of Cosmic Ray Muons*. In this experiment, we study two of the consequences of special relativity: 1) the existence of a speed limit on particles by a measurement of the speed of cosmic-ray muons, and 2) time dilation by comparing the mean life of muons at rest with their inferred lifetime in motion ($v_\mu \approx 0.994c$).

This report has been partitioned into sections accordingly, each discussing a specific aspect of the experiment. When appropriate, sections have been divided to discuss the two distinct experiments involved, the speed distribution of cosmic-ray muons and the decay of muons at rest. Section II and section III discuss the theoretical background relevant to the experiment. Section II derives the relativistic kinematic effects of time dilation and Lorentz contraction from the Lorentz transformations. Additionally, the relativistic expression for the velocity is compared with that predicted by Newtonian mechanics. Section III discusses the phenomenon of muon production by cosmic rays and those aspects of the mechanics of muon decay relevant to this experiment. The reader unfamiliar with special relativity should refer to Appendix A where the basic terms are introduced and the Lorentz transformations are derived from the postulates of relativity. The experimental apparatus and details of its operation are discussed in section IV. Section V presents our experimental results and a discussion of the sources of systematic error.

II. RELATIVISTIC KINEMATICS

A. The Universal Speed Limit

The speed limit c can be deduced from Lorentz transformations as well as from Einstein's equation for the total energy of a body:

$$E = \gamma m_0 c^2 \tag{1}$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$ is the Lorentz factor and m_0 is the rest mass. Thus, the total energy E of a body is then simply the sum of its rest mass energy and it's kinetic energy (K).

$$E = m_0 c^2 + K \tag{2}$$

Using the the relativistic expression for momentum ($p = \gamma m_0 v$) the total energy can be written as,

$$E^2 = (m_0 c^2)^2 + (cp)^2 \tag{3}$$

(The reader is referred to [5] for a formal derivation). The velocity of particle can be derived using the expression for the relativistic momentum and Eq.(3).

$$v = \frac{c}{\sqrt{1 + \left(\frac{m_0 c}{p}\right)^2}} \tag{4}$$

The velocity vs. the kinetic energy of a muon ($m_0 c^2$=105.7 MeV) has been plotted below (Fig. 1). Notice that while the classical velocity approaches infinity as the kinetic energy approaches infinity, the relativistic velocity Eq.(4) is asymptotic to c in the limit of infinite kinetic energy.

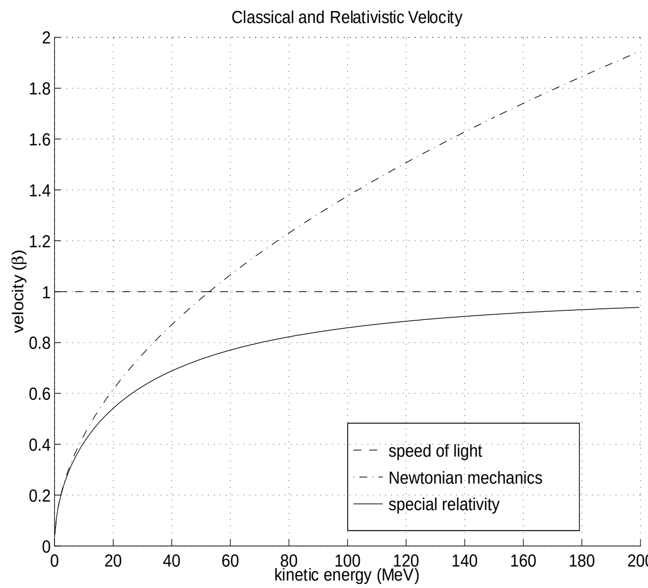

FIG. 1. There is *no* speed limit in Newtonian mechanics (*dot-dash line*) while c (*dashed line*) is the ultimate speed limit predicted by special relativity (*solid line*) ($\beta = v/c$).

In this experiment the cosmic-ray muons have kinetic energies on the order of 1 GeV. Our results on the measurement of the speed distribution of cosmic-ray muons, (section V), confirmed that the classically predicted speed of 4.3 c was **invalid** at these high energies [1].

B. Time Dilation

One of the consequences of special relativity is that *moving clocks run slower*. When dealing with relativity one needs to be careful about the words used to describe physical phenomena. Suppose a particle physicist in S(lab frame), sees the *birth* (Event 1) and *death* (Event 2) of a muon *moving* at 0.994 c. The physicist measures the time interval ($t_2 - t_1$) between these events using *two different stationary clocks* positioned accordingly at x_1 and x_2. Let's imagine that this scientist possesses extremely advanced technology and uses a relativistically fast craft (capable of matching the speed of the muon) to carry a *single clock* on board (positioned at x'_0) to measure the time interval ($t'_2 - t'_1$) corresponding to the

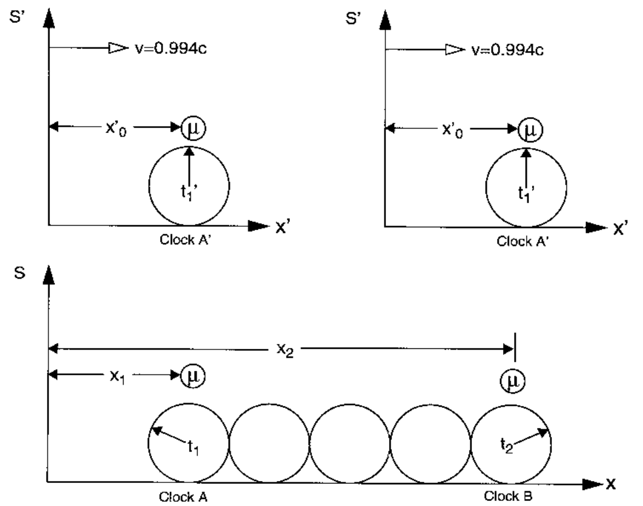

FIG. 2. Moving clocks run slow. The particle physicist in frame S sees a *single moving clock* (Clock A' fixed at x'_0 in frame S'). He/she has positioned *two different* clocks at x_1 and x_2 to measure the time $(t_2 - t_1)$ it takes the muon to travel this distance in frame S. When the clocks are compared $t_2 - t_1 > t'_2 - t'_1$.

lifetime of the muon in S'(rest frame). When the particle physicist compares the time interval measured by the *single moving clock* with the time difference recorded on the *two stationary clocks*, the former is consistently *shorter* (Refer to Fig. 2). How can this be explained in the context of special relativity?

Consider, the Lorentz transformation describing these events:

$$t_1 = \gamma\left(t'_1 + \frac{vx_0}{c^2}\right) \quad (Event\ 1) \tag{5}$$

$$t_2 = \gamma\left(t'_2 + \frac{vx_0}{c^2}\right) \quad (Event\ 2) \tag{6}$$

$$\tag{7}$$

Subtracting these two equations:

$$t_2 - t_1 = \gamma(t'_2 - t'_1)$$

Replacing the time variables with the conventional symbols used, we arrive at the equations for **time dilation**:

$$\tau = \gamma\tau_0 \tag{8}$$

τ_0 is known as the *rest time*.

In conclusion, a consequence of special relativity is that the lifetime of the muon is shortest as measured in the rest frame of the particle and always longer when measured from all other frames.

In 1941, M.I.T. physicists B. Rossi and D. B. Hall studied this time dilation phenomenon using cosmic-ray muons [2]. The M.I.T. Education Development Center also made a filmed version of their experiment in 1963 [3]. Some of their data appears in the table below.

TABLE I. Muon Decay At Rest

Elapsed time (μs)	No. of muons surviving
0	568
1	373
2	229
3	145
4	99
5	62
6	36
7	17
8	6

In the remake of this classic experiment, a count rate of 563 muons/hour was recorded at an altitude of 2000 m above sea level. Typical cosmic-ray muons traveling at a speed $\approx c$ would reach sea level in 6.5 μs. From this data, one would expect to find 25 counts/hr at sea level. However, when the muon count rate was measured at sea level it turned out to be 400 counts/hour! The data would imply that this result would be correct if the muons had been traveling for 0.7 μs except muons can only travel 210 m in this period of time. The answer must be the relativistic dilation of time, i.e. the lifetime τ of a cosmic-ray muon in *motion* (as measured in the lab frame) is a factor of about 9 larger than it's *rest* lifetime τ_0.

$$\left(\frac{\tau_0}{\tau}\right)^2 = 1 - \frac{v^2}{c^2}$$

$$\frac{1}{81} = 1 - \frac{v^2}{c^2}$$

Solving for v,

$$v \approx 0.994c$$

C. Lorentz Contraction

The first postulate of relativity states that there is no preferred inertial frame in physics. In the previous section, the Lorentz transformations were used to explain why the lifetime of the muon in *motion* as measured in the *lab frame* was longer than it's rest lifetime. However, the previous situation was examined from the frame of reference of the particle physicist. It seems like the first postulate has been discarded.

We now proceed to examine the *same* situation from the point of view of the muon as the *observer* in S (lab frame). In S, the planet Earth heads toward the muon at a relative velocity of $0.994c$. The muon measures the separation (l) between itself (x_1) and Earth (x_2) at the same time t_0,

$$x'_1 = \gamma(x_1 - vt_0) \tag{9}$$

$$x'_2 = \gamma(x_2 - vt_0) \tag{10}$$

Proceeding in a similar fashion as in the previous section,

$$x_2' - x_1' = \gamma(x_2 - x_1)$$

Replacing our position variables with the conventional symbols used ($l_0 = x_2' - x_1'$ and $l = x_2 - x_1$, we arrive at the equation for the **Lorentz contraction**:

$$l = \frac{l_0}{\gamma} \tag{11}$$

l_0 is often called the *rest length.*

Thus, another consequence of special relativity is that lengths are longest as measured in the rest frame of the "rod". Conversely "rods" appear to contract when they are in motion.

The principle of relativity should now be evident as it pertains to the phenomena at hand. While the particle physicist measured the *dilated* lifetime of the muon in motion as it traversed the rest-frame distance between the Earth and itself, the reciprocal event occurred in the *lab frame* of the muon. The Earth traveled a *contracted* distance during the muons *rest* lifetime.

III. COSMIC RAY MUON PRODUCTION AND DECAY

A. Muon Production by Cosmic Rays

Cosmic rays consist of positive ions, mainly protons with energies in the range of 10^{12} to 10^{18} eV. The isotropic nature of these rays implies that they are not solar in origin. Fermi's hypothesis to explain the origin of these rays began with the assumption that interstellar space is filled with ionized gases which tend to stream creating locally inhomogenous magnetic fields of weak strengths. A proton which collides head-on with such a region of inhomogeneity can gain energy by a ratio (v/c). Upon a sufficient number of such collisions, the magnetic fields will excite the proton to energies normally seen in cosmic rays. These primary cosmic ray particles interact with atmospheric nuclei prouducing showers of particles which can be divided into *hard* and *soft* components. The distinction is based on their ability to penetrate matter at the surface of the earth. The muon was first proposed by Yukawa as a quantum of the field acting as an intermediary of the nucear force, similar to the role played by the photon in electromagnetic interactions. Taking into account the extremely small range of the nuclear force, Yukawa calculated the mass of this particle to be around $200m_e$. In 1937, Neddermayer and Anderson discovered a particle with mass intermediate between that of a proton and an electron, while investigating the constituents of the hard component of cosmic rays. More accurate measurements of the mass of this particle put the value at $206.76m_e$.

The hard component consists of muons while the soft component consists of photons, electrons and positrons.

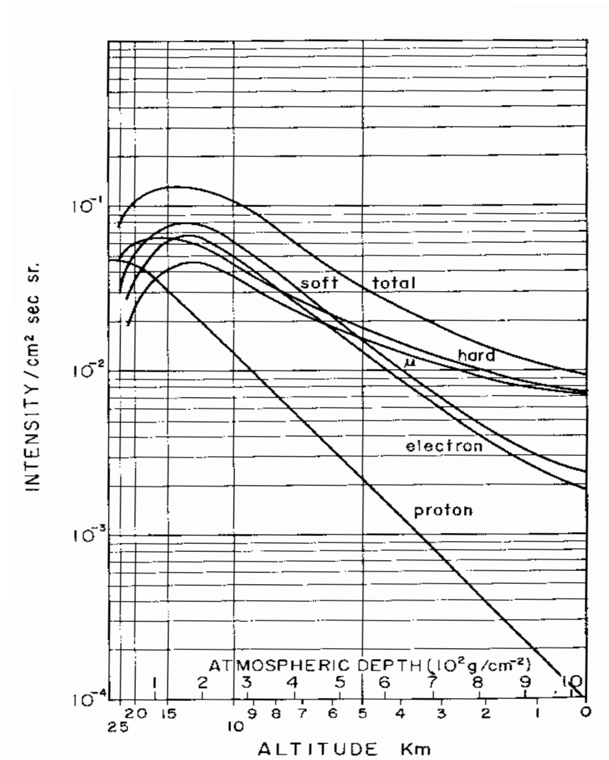

FIG. 3. The intensities of the various components of cosmic rays *(from Satio Hayakawa, Cosmic Ray Physics)*

The soft component is created due to strong interactions between a primary cosmic-ray particle and a nucleus. This reaction produces neutral pions and charged pions. The neutral pions then decay into photons which results in a particle shower due to pair production, Compton scattering and bremsstrahlung.

$$p +^{14} N \rightarrow \pi^0 \rightarrow \gamma \rightarrow e^+ + e^-$$

Charged pions decay into muons and neutrinos producing the hard component.

$$p +^{16} O \rightarrow n + \pi^+$$
$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$
$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

This process is the most common source of atmospheric muons although muons can also be produced by other decays like,

$$\Sigma^\pm \rightarrow \mu^\pm + \nu_\mu + n$$
$$\Lambda \rightarrow \mu^- + \bar{\nu}_\mu + p$$
$$K \rightarrow \mu + \nu_\mu$$

B. Muon decay

Muons are unstable particles with a mean life of 2.21 ± 0.01 μs. Free muons decay into electrons and neutral particles. The electron spectrum is a continuum ranging from 9 MeV to around 50 MeV. This implies that at least two other particles are required to conserve energy and momentum. The actual reaction is found to be

$$\mu^{\pm} \rightarrow e^{\pm} + \nu_{\mu} + \bar{\nu}_{e}$$

Of the two neutrinos, one is of the kind found in beta decay and the other is seen in pion decay. There are two choices of the neutrino pair. For instance, the neutrino pair for the decay of μ^{-} could be an electron antineutrino and a muon neutrino, or a muon antineutrino and an electron neutrino. The choice of an electron antineutrino and a muon neutrino is preferred since it associates the electron with the electron antineutrino as in beta decay. This decay reaction is of significance as none of the interacting particles are subject to strong interactions.

An important feature of this decay is the non-conservation of parity. Defining the Parity operator as,

$$\hat{P}\Psi(x,y,z) = \Psi(-x,-y,-z) \tag{12}$$

So the parity operator can be thought of as a inversion about the x-y plane, followed by a mirror reflection. We recognise two special cases,

$$\begin{aligned} \hat{P}\Psi &= +\Psi(even\ parity) \\ \hat{P}\Psi &= -\Psi(odd\ parity) \end{aligned} \tag{13}$$

The law of conservation of parity states that an isolated system with a well-defined parity will continue to have the same parity. This implies that any process that occurs in nature can also occur in a "mirror-image" world. The strong and electromagnetic interactions conserve parity.

Based on a suggestion by Lee and Yang, Garwin et al. showed that the emission of the positron and the neutrinos from a muon polarized with its spin aniparallel to the direction on motion, violates the conservation of parity since the positrons tend to travel parallel to the direction of the spin.

IV. EXPERIMENTS

A. Speed distribution of Cosmic-ray muons

A "direct" time-of-flight measurement of the speed of cosmic-ray muons was performed by measuring the arrival time of electrical pulses using two different scintillation detector separations (*top* 35.7±2 cm and *bottom* 338±2 cm). Measurements with two different separations were necessary because the delay time arising from the electronics cannot be ascertained by measurements with a single configuration. This point is discussed in further detail in section V. The *top* scintillation paddle is fixed near the ceiling of the room and the *middle* paddle is

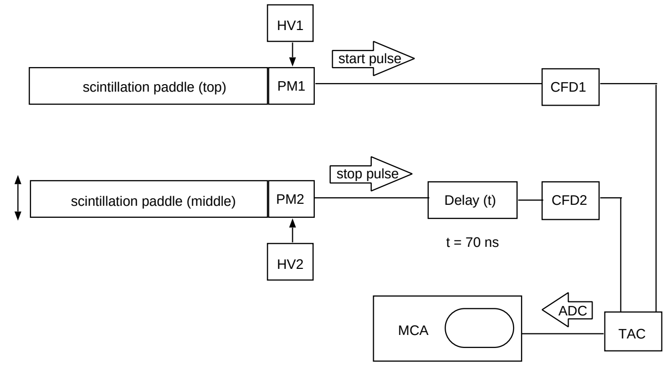

FIG. 4. Experimental arrangement for the speed distribution of cosmic ray muons

mounted on a platform whose height can be manually adjusted. **NOTE**: A *bottom* paddle was part of the experimental setup but was never used to collect data; it has *not* been included in Fig. 4. The parallel scintillation paddles (40 cm by 60 cm) have been rotated by 90° with respect to each other about their vertical axis.

Cosmic-ray muons traversing a paddle will generate scintillations which are collected by an optical funnel which leads into the *photomultiplier* (PM). The PMs then generate electrical pulses proportional to the intensity of optical photons which reach it. The *top* detector generates a start pulse which initiates the *time-to-amplitude converter* (TAC). The *middle* detector generates a stop pulse which terminates the TAC, after suitable delay with additional coaxial cable. The reason the stop pulse is delayed is subtle. Consider that the cosmic-ray muons traverse the scintillation paddles from all directions. With a small enough paddle separation it is possible that the travel time between the paddles is on the order of scintillation diffusing to the PM. Thus, without delaying the *stop pulse*, it is possible to have the stop pulse reach the TAC before the start pulse. The TAC generates a positive output pulse proportional to the time interval between the start and stop pulses which is recorded by a *multichannel analyzer* (MCA). A *time calibrator* (TC) capable of generating a series of precisely separated negative pulses was used to determine the time interval corresponding to a given channel interval ($\Delta ch_i = \alpha \Delta t_i$, where α is a proportionality constant). Refer to Fig. 4

B. Decay of Muons at Rest

To obtain a measure of time dilation, the mean life of muons at rest are compared with the inferred limit of the lifetime of muons in motion using the measurements of muon velocities and the known variation of muon flux with altitude. A plastic scintillator is used to bring the muons to rest. A muon arrival and subsequent decay is detected by a coincidence requirement. From Bethe's for-

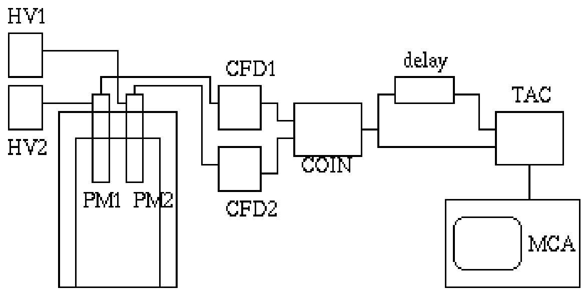

FIG. 5. Apparatus for detection of muon decay

mula (eq: C11), it can be seen that a muon loses around 50 MeV before it comes to rest having travelled a distance of around 10 cm. The electrons which are produced by the decay have energies around 20 MeV. Two photomultiplier tubes are used to eliminate the flood of gamma rays produced by these two events. An optimal discriminator level is required to avoid a large count of accidental coincidences between random pulses when the discriminator level is too low, and a low rate of decay events when the discriminator level is too high. A discriminator level of 0.8/sec was chosen. The schematic of the apparatus is shown. (Fig. 5) The pulses from the photomultiplier tubes are first fed into a pair of constant fraction discriminators (CFD). The outputs of the CFDs form inputs to the coincidence circuit. The delayed START pulse and the STOP pulse are then fed into the time-to-amplitude converter (TAC) and finally read on the MCA. The START pulse must be delayed to ensure that the STOP input is no longer activated when the pulse gets to the START input. Else, the TAC would never accept this input. This delay has the effect of shifting the actual time corresponding to each channel in the multi-channel analyzer (MCA) to the left by an appropriate interval. This does not affect the shape of the spectrum. A delay cable of length 10 m. was chosen which corresponds to a delay of around 3 ns.

C. Details of the Apparatus

1. Scintillation detectors

Organic scintillation counters were used to detect the muon decay events. Organic materials are better suited than inorganic crystals for this particular experiment both because of their smaller decay times and the large volumes involved. The scintillation process is a molecular phenomena in contrast to inorganic crystals. The intermolecular bonding primarily arises due to Van Der Waal's forces and the luminescence arises from the deexcitation of a molecule from its first excited electronic state. The molecule also shows various closely spaced vibrational levels in addition to the electronic levels. So, the absorption spectrum for an organic scintillator material shows several absorption peaks corresponding to transitions from various vibrational levels to the first excited state. The luminescence emission spectrum shows the same characteristics except at longer wavelengths.
A typical luminescence emission spectrum for an organic detector is shown. (The material was excited by ultravioleet light in this instance). Organic crystals also have high efficiencies (photons emitted per photons absorbed) of the order of 0.9 to 1.0(anthracene).

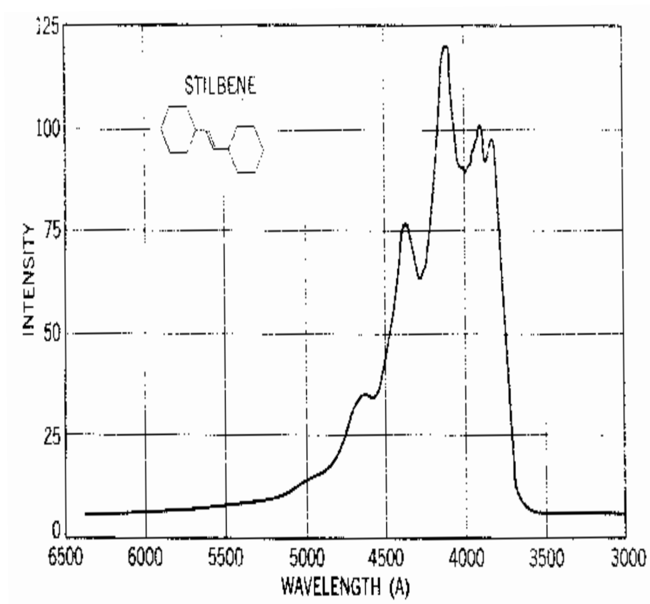

FIG. 6. Luminesence emission spectrum for a organic crystal (stilbene)

2. Coincidence circuit

A vital part of the circuitry is the coincidence circuit. This is mainly used for time of flight measurements and low-background counting. The principle here is that the pulse from one detector is used to trigger the gate, allowing the pulse from another detector to pass through, as long as the latter is within a set time. Bruno Rossi implemented a coincidence circuit for the first time, when he performed this experiment in 1943. He also invented the time-to-amplitude converter (TAC) used in this experiment. A simple implementation of such a coincidence circuit is shown.
The background rate limited by this circuit can be obtained as follows:
If the rates at the two detectors are n_1 and n_2 respectively, then, the gate will be "open" for a fraction $n_1\Delta\tau$ of the total analyzing time, where $\Delta\tau$ is the gate time. Then, the observed accidental rate will be $n_1 n_2\Delta\tau$. The gate time should be set to an appropriate value such that

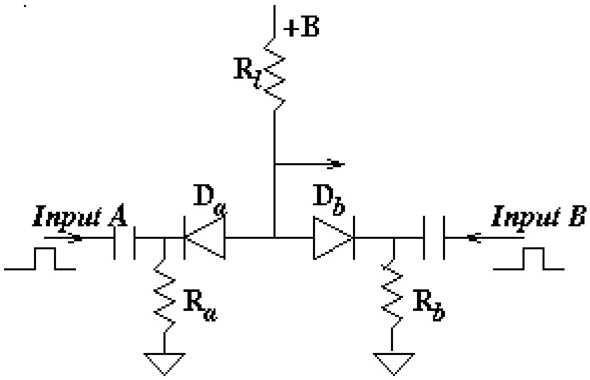

FIG. 7. A simple implementation of a coincidence circuit

the signal to noise ratio is low (< 0.01) without decreasing the count rate very much. ie:

$$\frac{R_0}{n_1 n_2 \Delta\tau} \gg 1 \tag{14}$$

where R_0 is the actual rate of events.

V. RESULTS AND DISCUSSION

A. Speed distribution of cosmic-ray muons

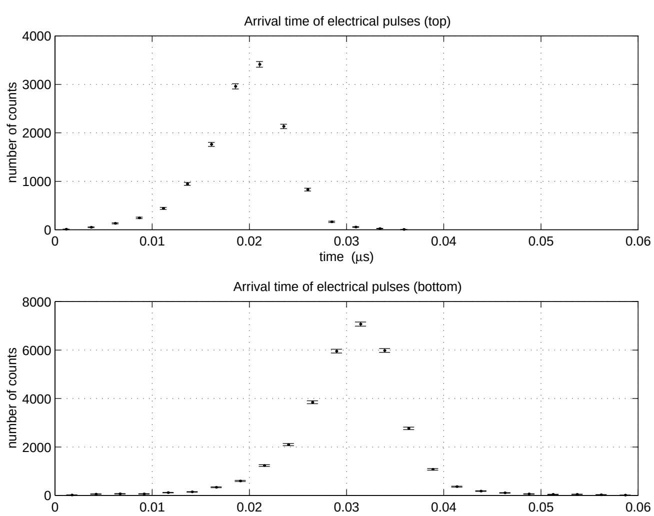

FIG. 8. Arrival time profile of electrical pulses in two different scintillation detector configurations. Detector separation *(Top)* 35.7 cm *(Bottom)* 338.0 cm

The data collected by the MCA integrated over several 25-channel intervals is shown in Fig. 8. Take note of the increase in the median arrival time Fig. 8(*bottom*) of the electrical pulses corresponding to the increased separation of the detectors. The peak has a finite width because of the momentum spectrum of cosmic-ray muons at sea level (refer to Appendix B) and because of the different path lengths traversed across the scintillation paddles by the muons.

The mean velocity $\langle v \rangle$ of the the cosmic-ray muons was extrapolated from the arrival time of the electrical pulses as follows. Each data point on the MCA ($ch_i = \alpha t_i$) corresponds to the arrival time of the electrical pulses and contains the following information

$$t_i = t_0 + \frac{d_i}{v_i} + \delta t_i \tag{15}$$

where t_0 is a delay-time constant for the apparatus (HV, TAC, etc) and d_i is the slant distance traveled between the scintillation detectors by the ith muon at the velocity v_i. The final term δt_i is the systematic error inherent to the experimental apparatus. *The largest contribution to the systematic errors is the variations in scintillation light travel time from the point of origin to the PM.* In the analysis that follows, we assume that the TAC and MCA make negligible contributions to the systematic error. Thus we continue working with the t_is instead of ch_is. The quantity of interest is the difference of the mean value of the arrival time of the pulses in the *top* $\langle t_t \rangle$ and *bottom* $\langle t_b \rangle$ configurations.

$$\langle t_t \rangle = t_0 + \left\langle \frac{d_t}{v_t} \right\rangle + \langle \delta t_t \rangle \tag{16}$$

$$\langle t_b \rangle = t_0 + \left\langle \frac{d_b}{v_b} \right\rangle + \langle \delta t_b \rangle \tag{17}$$

Subtracting these two equations,

$$\langle t_t \rangle - \langle t_b \rangle = \frac{\langle d \rangle}{\langle v \rangle}$$

Here we have made the assumption that 1) $\langle d/v \rangle = \langle d \rangle / \langle v \rangle$ and 2)$\langle \delta t_t \rangle - \langle \delta t_b \rangle = 0$. Thus, the equation below expresses *only* the random error in $\langle v \rangle$

$$\langle v \rangle = \frac{\langle d \rangle}{\langle t_t \rangle - \langle t_b \rangle} \tag{18}$$

The mean slant distances ($\langle d_t \rangle$ and $\langle d_b \rangle$) This calculation is discussed in detail in the following subsection.

The mean velocity of the cosmic-ray muons was determined to be 0.92±0.01 c which is in good agreement with the accepted value of 0.98 c. Refer to Appendix B for a calculation of the accepted value from the differential momentum spectrum of cosmic-ray muons at sea level.

B. Calculation of the Mean Slant Distance

The mean slant distances ($\langle d_t \rangle$ and $\langle d_b \rangle$) were calculated by integrating over all possible paths traversible by the muons between the scintillations paddles with two different separations ($d_t = 35.7$ cm and $d_b = 338$ cm).

The integral below was evaluated using Mathematica:

$$\langle d_{t,b} \rangle = \frac{1}{N} \int_{-30}^{30} dx \int_{-20}^{20} dy \int_{-20}^{20} dx' \int_{-30}^{30} dy' R \cos^2 \phi \quad (19)$$

NOTE: The limits of integration reflect the 90° orientation (about the z-axis) of the *top* paddle with respect to the *middle* paddle. R is the distance between a point on the *top* scintillation paddle ($x', y', z' = d_{t,b}$) and a point on the *middle* scintillation paddle ($x, y, z = 0$),

$$R = \sqrt{(x - x')^2 + (y - y')^2 + d_{t,b}^2}$$

the integral has been weighted with $\cos^2 \phi$ due to empirical fits of the intensity of muons at sea level as a function of zenith angle (ϕ) [4]

$$\cos^2 \phi = \frac{d_{t,b}^2}{(x - x')^2 + (y - y')^2 + d_{t,b}^2}$$

and N is a normalization constant equal to:

$$N = \int_{-30}^{30} dx \int_{-20}^{20} dy \int_{-20}^{20} dx' \int_{-30}^{30} dy' cos^2 \phi$$

The mean slant distances were determined to be $\langle d_t \rangle = 43.7$ cm and $\langle d_b \rangle = 339.3$ cm.

C. Decay of Muons at Rest

The exponential decay curve that was seen on the MCA is shown. (Fig. 9) The decay curve was fit us-

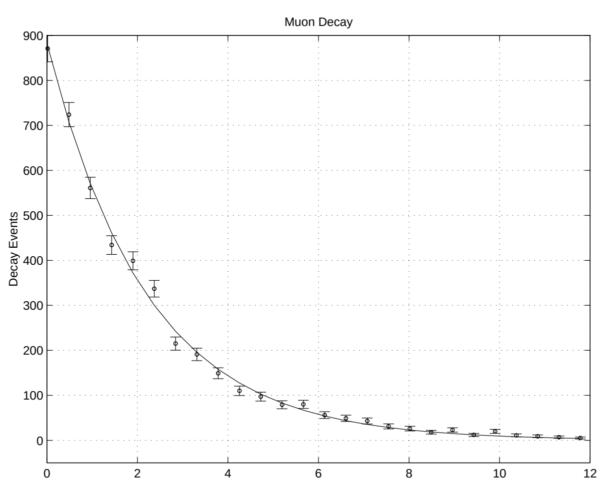

FIG. 9. Mean Life of muons at rest

ing the model

$$\eta(t) = a e^{t/\tau_m} + b \quad (20)$$

where τ_m is the mean lifetime of free muons, a and b are parameters which depend on the total counts per channel and the background counts per channel respectively. A curvefit obtained by least squares following the Levenberg-Marquardt algorithm gave the following fit

$$\eta(t) = 870.5 e^{0.4554t} \quad (21)$$

with a χ^2 of 30 which is much smaller than the total number of counts (4570).This gives the value for the mean life to be 2.19 ± 0.02 μs which is well correlated with the established value of 2.21 ± 0.01 μs. The background counts are negligible because of (i) the short duration of the experiment ($\approx$ 12hr) and (ii) the very low value of the discriminator rate. The theoretical value of the background counts per channel ρ_b can be calculated as,

$$\begin{aligned} \rho_b &\approx \rho_d^2 (\Delta\tau) T \quad (22) \\ &\approx (0.8)^2 (0.64 \times 10^{-6})(12 \times 3600) \\ &\approx 0.02 \texttt{ counts/ch} \end{aligned}$$

where ρ_d is the discriminator level, $\Delta\tau$ is the width of the channel and T is the total analyzing time. This value is in correlation with the data where the decay curve is seen to converge asymptotically to zero.

The measured value is expected to be slightly smaller because of the fact that μ^- particles comprise about 45 % of the muon flux. The μ^- particles have a reduced mean life compared to μ^+ particles because of the finite probability with which a μ^- particle can be captured into atomic orbitals and subsequently interact with nuclei. Accurate measurements have put the value of the ratio of the positive muon to negative muon flux, $\frac{j_{\mu^+}}{j_{\mu^-}}$ at 1.251 ± 0.003, and the lifetime of a negative muon in at atomic orbital at around $1.5\mu s$.

There are other factors which contribute to an increase in the background rate such as

1. The muon flux peaks in intensity around 1 GeV. This means that most of the muons trigger the START pulse but pass through the scintillator without coming to rest.

2. The decay electrons may not have enough energy to trigger the discriminator. They may also escape from the scintillator before they deposit enough energy to trigger the discriminator.

These factors would not introduce a systematic error as they do not affect the shape of the spectrum. More so in this case where the background rate is negligible.

Exact time-base calibration and better values of the analyzing rate to discriminator rate ratio could yield better results.

ACKNOWLEDGMENTS

The authors would like to acknowledge valuable discussions with Prof. George Clark and Dr. Jordan Kirsch. The authors thank Dr. Alan Levine for his helpful comments and suggestions. The assistance of Rebecca Christianson and Jake Kendall is appreciated.

APPENDIX A: DERIVATION OF THE LORENTZ TRANSFORMATIONS

It was Albert Einstein's concern that Maxwell's equations were not invariant under the Galilean transformations that lead him to formulate The Special Theory of Relativity. The correct inertial frame transformations, those of H. A. Lorentz, can be deduced from the *two postulates of relativity*: 1) the laws of physics are the same in all inertial systems. 2) The speed of light in free space has the same value c in all inertial systems.

In general, a *group* of inertial frame transformation equations connect an *event* in one inertial frame S characterized by a set of space-time coordinates (x, y, z, t) to the *same event* in another inertial frame S' described by (x', y', z', t') . To simplify the algebra, we arbitrarily chose the relative velocity of S to S' to be along the common x-x' axis. Our goal is to derive a functional relationship of the form x'=$x'(x, y, z, t)$, y'=$y'(y, x, z, t)$, z'=$z'(x, y, z, t)$ and t'=$t'(x, y, z, t)$. First, we argue that the *uniformity* of space-time requires that these equations have certain *symmetries*. The transformation equations should have a *linear* dependence on the variables, (x, y, z, t). In their most general form, the transformation equations are [5]:

$$x' = \alpha_{11}x + \alpha_{12}y + \alpha_{13}z + \alpha_{14}t \tag{A1a}$$
$$y' = \alpha_{21}x + \alpha_{22}y + \alpha_{23}z + \alpha_{24}t \tag{A1b}$$
$$z' = \alpha_{31}x + \alpha_{32}y + \alpha_{33}z + \alpha_{34}t \tag{A1c}$$
$$t' = \alpha_{41}x + \alpha_{42}y + \alpha_{43}z + \alpha_{44}t \tag{A1d}$$

The following arguments should convince the reader that if these equations were anything *but* linear then we would violate the first postulate of relativity and the uniformity of space-time. Consider the following *gedanken* experiment in which the endpoints of a rod of unit length[1] in S have coordinates $(1, 0, 0, t)$ and $(2, 0, 0, t)$. Furthermore, suppose that the equations above now have a quadratic dependence, i.e. replace the variables in Eq.(A1a)-(A1d) as follows $x \to x^2, y \to y^2, z \to z^2, t \to t^2$. The length in S', $x'_1 - x'_0 = \alpha_{11}(x_1^2 - x_0^2)$. In this case, the length in S' is $3\alpha_{11}$. Instead, suppose the endpoints of the rod are located at $(2, 0, 0, t)$ and $(3, 0, 0, t)$. Now, the length in S' is $5\alpha_{11}$. The rod's length is a function of it's position in space! Thus, if our inertial frame transformation equations are nonlinear the first postulate of special relativity would be violated.[2]

Because the relative velocity of S' with respect S is along the common x'-x, there is nothing to uniquely distinguish the other axes. In order to preserve the isotropic nature of space-time, we conclude that $y' = y$ and $z' = z$. (I refer the reader to [6] for a more rigorous proof). Additionally, by this same symmetry, t' *must* be independent of y and z. To reiterate, the following coefficients have been determined: $\alpha_{21} = \alpha_{23} = \alpha_{24} = \alpha_{31} = \alpha_{32} = \alpha_{34} = 0$. Now consider a particle at rest at the origin of $S', (0, 0, 0, t')$. In frame S, the particle must move along the x-axis with speed v, $(vt, 0, 0, t)$. Using Eq. (A1a):

$$0 = \alpha_{11}vt + \alpha_{12}y + \alpha_{13}z + \alpha_{14}t$$
$$0 = (\alpha_{11}v + \alpha_{14})t + \alpha_{12}y + \alpha_{13}z$$

Equating like coefficients:

$$\alpha_{14} = -\alpha_{11}v$$
$$\alpha_{12} = \alpha_{13} = 0$$

Thus far, our original equations Eq.(A1a-A1d) have simplified to:

$$x' = \alpha_{11}(x - vt) \tag{A2a}$$
$$y' = y \tag{A2b}$$
$$z' = z \tag{A2c}$$
$$t' = \alpha_{41}x + \alpha_{44}t \tag{A2d}$$

Another *gedanken* experiment is needed to arrive at the two missing coefficients. A light beam is emitted along the positive x-x' axis. By the *second postulate*, the light travels at the same velocity in *both* frames:

$$x = ct$$
$$x' = ct'$$

Substituting these expressions into Eq.(A1a) and it's inverse transformation[3]:

$$ct' = \alpha_{11}(c - v)t$$
$$ct = \alpha_{11}(c + v)t'$$

Eliminating t and t' between these equations:

$$c^2 = \alpha_{11}^2(c^2 - v^2)$$

Solving for α_{11} and letting $\gamma = \alpha_{11}$ so to stay with convention:

[1] The measurement of a length is the difference in the position between two events, in this case both endpoints of the rod measured *simultaneously* in S.

[2] Arguments similar to those used for position can also be used to show that a nonlinear time dependence would violate the *uniformity* of space-time.

[3] I have made use of an inverse transformation; Eq.(A2a) with the following substitution: $x' \to x, y' \to y, z' \to, t' \to t$, and $v \to -v$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{A3}$$

Using Eq.(A2a) and it's inverse transformation:

$$x' = \gamma(x - vt) \tag{A4a}$$
$$x = \gamma(x' + vt') \tag{A4b}$$

It's a simple matter to derive the time transformation Eq.(A2d) using Eq.(A4a-A4b) and elementary algebraic manipulations:

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

In conclusion, starting with the most general form of the inertial frame transformations and then using the postulates of relativity, we have derived the **Lorentz Transformations**:

$$x' = \gamma(x - vt) \tag{A5a}$$
$$y' = y \tag{A5b}$$
$$z' = z \tag{A5c}$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \tag{A5d}$$

Furthermore, to abide by convention, let S be the *lab frame* of the *observer* and S' be the *rest frame* of the *event.*

APPENDIX B: MOMENTUM SPECTRUM OF COSMIC-RAY MUONS

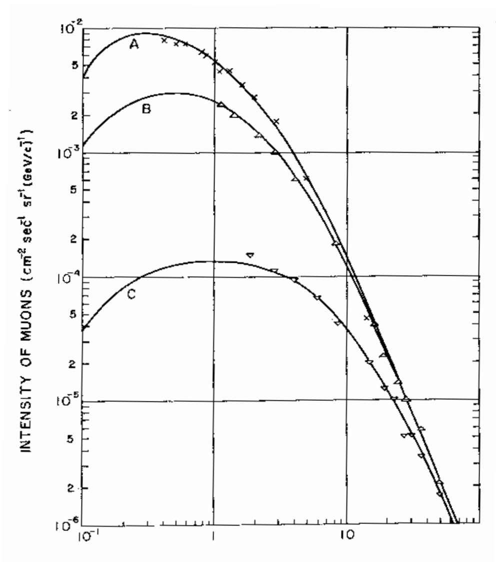

FIG. 10. Intensity of cosmic-ray muons at (A) 1000 m, (B) sea level and (C) sea level at 68°.

The accepted value of the mean velocity $\langle v\rangle$ of cosmic-ray muons at seal level was calculated by numerical integration of the quantity:

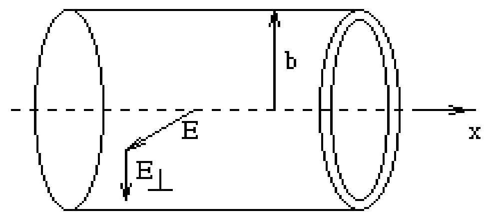

FIG. 11. Integral over a Gaussian pillbox

$$\langle v\rangle = \frac{\int_0^\infty vI(p)dp}{\int_0^\infty I(p)dp} \tag{B1}$$

where v is given by Eq.(4) and $I(p)$ is the differential momentum spectrum of muons at sea level.

APPENDIX C: ENERGY LOSS OF CHARGED PARTICLES IN A MEDIUM

Charged particles travelling through a medium can lose energy by ionization or excitation of the atoms in the medium due to inelastic collisions with electrons, or Rutherford scattering with the nuclei. The most significant of these processes is ionization. Heavy particles like protons, alpha particles and muons behave in a different manner compared to light particles like electrons. Heavy particles lose energy in a continuous manner by ionization until their kinetic energy approaches zero. It is only after this stage that they decay or interact with the nuclei. So, these particles can be associated with a definite range which varies from medium to medium. Electrons, on the other hand, undergo drastic accelerations due to their high e/m ratio producing a shower of photons in the process. To obtain an expression for the rate of energy loss of a charged particle in a medium, consider a particle of mass of charge ze travelling through a medium with N electrons per unit volume. Assume that the velocity of the particle is so large that the electrons can be considered to be at rest relative to the particle. In this scenario, the momentum imparted to the electrons will be in a direction perpendicular to the trajectory of the charged particle. Calling this momentum $\Delta p_\perp$, we have,

$$\begin{aligned}\Delta p_\perp &= \int_{-\infty}^{\infty} e\varepsilon_\perp dt = \int_{-\infty}^{\infty} e\varepsilon_\perp \frac{dx}{v} \\ &= ze^2 \int_{-\infty}^{\infty} \frac{1}{r^2}\cos\Theta \frac{dx}{v}\end{aligned} \tag{C1}$$

The integral can be calculated using Gauss's theorem, Refer to Fig.11

$$\Phi = \int \varepsilon(2\pi b)dx = 4\pi ze \tag{C2}$$

$$\Delta p_\perp = \frac{e}{v}\left(\frac{4\pi z e}{2\pi b}\right) = \frac{2ze^2}{bv} \qquad \text{(C3)}$$

So, the energy transferred to the electron,

$$\delta E = \frac{(\Delta p_\perp)^2}{2m} = \frac{2}{m}\left(\frac{ze^2}{bv}\right)^2 \qquad \text{(C4)}$$

The energy loss per unit length is then

$$-\frac{\partial E}{\partial x} = 2\pi N \int b\frac{(\Delta p_\perp)^2}{m} db \qquad \text{(C5)}$$

$$= 4\pi N\frac{z^2e^4}{mv^2}\int_{b_{min}}^{b_{max}}\frac{db}{b} \qquad \text{(C6)}$$

$$= 4\pi N\frac{z^2e^4}{mv^2}\log\left(\frac{b_{max}}{b_{min}}\right) \qquad \text{(C7)}$$

The above expression is also called the "stopping power" of the medium. The values for b_{min} and b_{max} are not 0 and infinity as there are physical limits which impose restrictions on their allowed values. The restriction on b_{max} is imposed by the adiabatic principle of quantum mechanics which states that a transition between two states of a system cannot be brought about by a time dependant perturbation if the time scale of the perturbation is comparable to the period of the system. The time dependant perturbation in this case is the coulombic interaction between the charged particle and the electron, and its time scale is given by the time for which the charged particle is in the vicinity of the electron, *ie.* $\delta t = b/v$. With relativistic corrections, this becomes

$$b_{max} \leq \frac{v}{\bar{\nu}(1-\beta^2)^{1/2}} \qquad \text{(C8)}$$

The lower limit on b_{min} comes from the fact that an electron's position cannot be localized to within a length given by its De Broglie wavelength. Therefore,

$$b_{min} \geq \left(\frac{\hbar}{p}\right)(1-\beta^2)^{1/2} \qquad \text{(C9)}$$

With these values of b_{min} and b_{max}, the expression for the rate of energy loss becomes,

$$-\frac{\partial E}{\partial x} = \frac{4\pi z^2 e^4}{mv^2}N\left[\log\left(\frac{mv^2}{\hbar\bar{\nu}(1-\beta^2)}\right)\right] \qquad \text{(C10)}$$

A more sophisticated analysis by Hans Bethe gives the expression,

$$-\frac{\partial E}{\partial x} = \frac{4\pi z^2 e^4}{mv^2}N\left[-\beta^2 + \log\left(\frac{mv^2}{I(1-\beta^2)}\right)\right] \qquad \text{(C11)}$$

where $I = \hbar\bar{\nu}$ is the ionization energy of the medium.

[1] G. Clark and J. Kirsch, *Junior Physics Laboratory: The Speed and Decay of Cosmic-Ray Muons*, M.I.T., Cambridge, Mass., 1997.

[2] B. Rossi and D. B. Hall, *Phys. Rev.*, **59**, 223 (1941).

[3] Film, *Time Dilation–An Experiment with μ-Mesons* by D. H. Frisch and J. H. Smith, Education Development Center, Newton, Mass., 1963

[4] B. Rossi, *Rev. Mod. Phys.*, **20**, 537 (1948).

[5] R. Resnick and D. Halliday, *Basic Concepts in Relativity* (MacMillan, New York, 1992).

[6] Robert Resnick, *Introduction to Special Relativity* (Wiley, New York, 1968).

[7] R. E. Hall, D. A. Lind and R. A. Ristinen, *A Simplified Muon Lifetime Experiment for the Instructional Laboratory*, *American Journal of Physics*, **38**, 1196-1200, (Oct, 1970).

[8] Kowalski, *Nuclear Electronics* (Springer-Verlag)

[9] Emilio Segre, *Nuclei and Paricles* (Wiley, New York)

[10] Satio Hayakawa, *Cosmic Ray Physics* (Wiley, New York)

[11] A. C. Melissinos, *Experiments in Modern Physics* (Academic Press, 1966)