February 21st 2012

Measuring atomic mass. For measurments, a two part machine is used. In the first part, once ionized atoms (ions) are accelerated with a known voltage, while in the second part their track is curved with a perpendicular magnetic field. For calibration, ions of isotope ^{12}C are used. Since natural carbon contains 1.1 % parts of isotope ^{13}C, an additional track will be observed with the original ^{12}C track. The measured ratio of the curvature of both tracks is $q=\sqrt{\frac{13.003355}{12}}=1.040967$. What is the binding energy of ^{13}C nucleus?

Atom mass	$m_a(^{12}C)=12u$
Nucleus mass	$m_j(^{12}C)=12u-12m_e$
Ion mass	$m_i(^{12}C^+)=12u-m_e$
Curvature r dependence on mass	$r=\frac{1}{B}\sqrt{\frac{2mU}{e_0}}$
Ion mass	$m_i(^{13}C^+)=q^2 m_i(^{12}C^+)$
Nucleus mass	$m_j(^{13}C)=m_i(^{13}C^+)-11m_e$
Binding energy	$W_v/c^2=m_j(^{13}C)-6m_p-7m_n$ [-0.1 u]
Proton mass	$m_p=1.007276$ u
Neutron mass	$m_n=1.008665$ u
Electron mass	$m_e=5.485799\times10^{-5}$ u
Atomic mass unit	$uc^2=931{,}494$ MeV

For mirrored nuclei the difference in binding energy arrises from different electrostatic energy of the nuclei. The nuclide $^{13}_{7}$N decays via β^+ decay to its mirrored counterpart $^{13}_{6}$C, and measurements show that the maximum kinetic energy of the created positron equals 1.2 MeV. What is the estimate on nuclide $^{13}_{7}$N or $^{13}_{6}$C radius based on this asumption?

Electric field in a sphere	$E(\mathbf{r})=\frac{Ze_0 r}{4\pi\varepsilon_0 R^3}\frac{\mathbf{r}}{r}$
Electric field outside of the sphere	$E(\mathbf{r})=\frac{Ze_0}{4\pi\varepsilon_0 r^2}\frac{\mathbf{r}}{r}$
Electrostatic potential in the sphere	$U(\mathbf{r})=U(0)-\frac{Ze_0 r^2}{8\pi\varepsilon_0 R^3}$
Electrostatic potential outside of the sphere	$U(\mathbf{r})=U(\infty)+\frac{Ze_0}{4\pi\varepsilon_0 r}$
Continuity at r=R	$U(0)=U(\infty)+\frac{3Ze_0}{8\pi\varepsilon_0 R}$
Arbitrary 0 of electrostatic potential	$U(\mathbf{r};r<R)=\frac{Ze_0}{8\pi\varepsilon_0 R^3}(3R^2-r^2)$
Electrostatic potential energy	$W_{ep}=\int_0^\infty \rho_e(\mathbf{r})U(\mathbf{r})dV$
For $\rho_e=\frac{Ze_0}{4\pi R^3}$, $r<R$; 0 otherwise	$W_{ep}=\frac{3}{20}\frac{Z^2e_0^2}{\pi\varepsilon_0 R}$
In mirrored nuclides	$\Delta W_{ep}=\frac{3}{20}\frac{e_0^2}{\pi\varepsilon_0 R}(2Z-1)$
Radius	$R=\frac{3}{20}\frac{e_0^2}{\pi\varepsilon_0}\frac{2Z-1}{\Delta W_{ep}}$ [6 fm]

February 28th 2012

What is the most stable number of protons for a given total number of nucleons (protons and neutrons) in the nuclide based on the semiempiric mass formula? Compare to the actual rations for ${}^{40}_{20}$Ca, ${}^{88}_{38}$Sr and ${}^{197}_{79}$Au! [$Z/A=0.5/(1+(w_2/4w_3)A^{2/3})$]

Scattering of electrons with kinetic energy of 300 MeV on a certain nucleus yields the first minimum in number of detected particles at 30° relative to the incoming current. What is the estimate on nucleus's radius, if you assume it being a homogenuous sphere?

$$\begin{aligned} F(s) &= \frac{3}{(sR_j)^3}\Big(\sin(sR_j) - (sR_j)\cos(sR_j)\Big) \\ \text{Min @ } sR_j &= (k - \frac{1}{2})\pi \text{ ; } k = 2, \ldots \\ sR_j &= \frac{2pcR_j}{\hbar c}\sin\theta/2 = \frac{3\pi}{2} \\ R_j &= \frac{3\pi}{2}\frac{\hbar c}{2pc\sin\theta/2} \qquad [6 \text{ fm}] \end{aligned}$$

Determine luminosity for an experiment, where a current of 1 mA of electrons focused to a 1 mm^2 spot, accelerated to an energy of 250 Mev hits a target, a thin 100 μm gold foil of gold, ${}^{197}_{79}$Au with a density of 19 g/cm^3! The intensity of the scattered electrons is measured with a detector with a cross-section of 5 cm^2 10 cm away from the target at the scattering angle of 30° relative to the incoming current. How frequent will be the hits in this sensor. Assume that the nucleus is point like!

$$L = N_{\text{tarča}}\frac{dn_{\text{curek}}}{dSdt} = \frac{\rho N_A S d}{M}\frac{I}{e_0 S} = \frac{\rho N_A I d}{M e_0} \qquad [3,6\times 10^{37}/\text{m}^2\text{s}]$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{k}{2pc}\right)^2\frac{1}{\sin^4\theta/2} = \frac{1}{4}\left(\frac{Ze_0^2\hbar c}{4\pi\varepsilon_0\hbar cE}\right)^2\frac{1}{\sin^4\theta/2} = \left(\frac{197\text{MeVfm}}{\alpha E}\right)^2\frac{1}{4\sin^4\theta/2}$$

$[0.0018 \text{ fm}^2/\text{ster} = 18\mu\text{barn/ster}]$

$$\frac{d\Gamma}{d\Omega} = L\frac{d\sigma}{d\Omega} \qquad [6,5\times 10^4/\text{s}]$$

$$\Delta\Omega = \frac{S}{r^2} \qquad [5\times 10^{-2}]$$

$$\Gamma = \int_{\Delta\Omega}\frac{d\Gamma}{d\Omega}d\Omega \approx \frac{d\Gamma}{d\Omega}\Delta\Omega \qquad [3,3 \text{ kHz}]$$

Show that the angular part of the wave function, $Y_{lm}(\theta,\phi)$ is an eigensolution of the angular part of the Laplace operator ∇^2 with an eigenvalue -l(l+1).

In a certain nucleus the emission spectrum allows us to determine that the energy difference between a state with the last nucleon in the state $2d_{3/2}$ and the state $2d_{5/2}$ equals to 1.35 MeV. Based on this fact we can determine the strength of the spin-orbit coupling, written as an energy contribution $W_s = -2\eta\hat{l}\hat{s}$. What is the strength of the coupling η?

$$\begin{aligned}
\hat{j} &= \hat{l} + \hat{s} \\
\hat{j}^2 &= \hat{l}^2 + 2\hat{l}\hat{s} + \hat{s}^2 \\
2\hat{l}\hat{s} &= \hat{j}^2 - \hat{l}^2 - \hat{s}^2 \\
2d_{3/2} &: \; j = 3/2 \; ; \; l = 2 \; ; \; s = 1/2 \\
2d_{5/2} &: \; j' = 5/2 \; ; \; l = 2 \; ; \; s = 1/2 \\
\Delta W &= \eta\Big(j'(j'+1) - j(j+1)\Big) = 5\eta \\
\eta &= \frac{\Delta W}{5} \qquad [0.27 \text{ Mev}]
\end{aligned}$$

In the same nucleus the difference between states of the last nucleon $2f_{7/2}$ and $2d_{5/2}$ equals to 6.3 MeV. Assuming a harmonic nuclear potential $V(r) = -V_0 + \frac{1}{2}m_N\omega^2 r^2$, determine parameter ω, measuring the slope of the potential. Include spin-orbit effects!

$$\begin{aligned}
W_{n_r lj} &= (2(n_r - 1) + l + 3/2)\hbar\omega - 2\eta\hat{l}\hat{s} \\
2f_{7/2} &: \; n_r = 2 \; ; \; l = 3 \; ; \; j = 7/2; \\
2d_{5/2} &: \; n'_r = 2 \; ; \; l' = 2 \; ; \; j' = 5/2; \\
\Delta W &= \Big(2(n_r - n'_r) + l - l'\Big)\hbar\omega - \Big(j(j+1) - l(l+1) - j'('j+1) - l'(l'+1)\Big)\eta = \\
&= \hbar\omega - \eta \\
\hbar\omega &= \Delta W + \eta \qquad [6,6 \text{ MeV}]
\end{aligned}$$

With a know parameter ω an estimate on the well depth V_0 can be determined. For values from the previous exercise and a 200 nucleon nucleus (giving estimate on the nucleus size) determine the potential value at r=0!

$$R_j = r_0 A^{1/3} \qquad r_0 = 1.1 \text{ fm}$$

$$\begin{aligned}
V(R_j) = 0 \;\rightarrow\; V_0 &= -\frac{1}{2}m_{p,n}\omega^2 R_j^2 = -\frac{1}{2}\frac{m_{p,n}c^2\hbar^2\omega^2 r_0^2 A^{2/3}}{\hbar^2 c^2} = \\
&= -\frac{1}{2}\frac{938 \text{ MeV} \cdot (6.6 \text{ MeV})^2 \cdot 1,2 \text{ fm}^2 200^{2/3}}{(197\text{MeVfm})^2} \qquad [\text{-21.6 MeV}]
\end{aligned}$$