FIZIKA II.

- **Vsebina:** Fizika I.: kinematika, dinamika, gravitacija, elastomehanika, mehanika tekočin, in toplota
  Fizika II.: elektromagnetno polje, električni tok, nihanje, valovanje, začetki moderne fizike in Fizikalni praktikum

(http://www.fkkt.um.si/sl/node/10)

(https://aips.um.si/PredmetiBP5/main.asp?Mode=prg&Zavod=17&Jezik=)

- **Literatura:**
  - A. Stanovnik: Fizika I in II (AS1, AS2)
  - D. Halliday, R. Resnick, Y. Walker: Fundamentals of Physics (HRW)
  - J. Strnad: Fizika, 1. in 2. del (JS1, JS2)

- **Internetna stran (http://fizika.fkkt.um.si/)**
  Rezultati so zaščiteni (u: xxx g: xxx)

- Pravilniki na univerzitetni spletni strani

(http://www.um.si/univerza/dokumentni-center/akti/Strani/studij-na-um.aspx)

Obveznosti:

- **Fizika I:**
  - pisni del izpita ali dva pisna testa (40%)
  - ustni del izpita (60%) – pogoj za pristop je opravljen pisni del
- **Fizika II:**
  - Fizikalni praktikum (30%) (vse vaje, delovni zvezek, pisni test)
  - pisni del izpita ali dva pisna testa (30%)
  - ustni del izpita (40%) – pogoji za pristop so opravljen izpit iz Fizike I ter opravljena Fizikalni praktikum in pisni del izpita

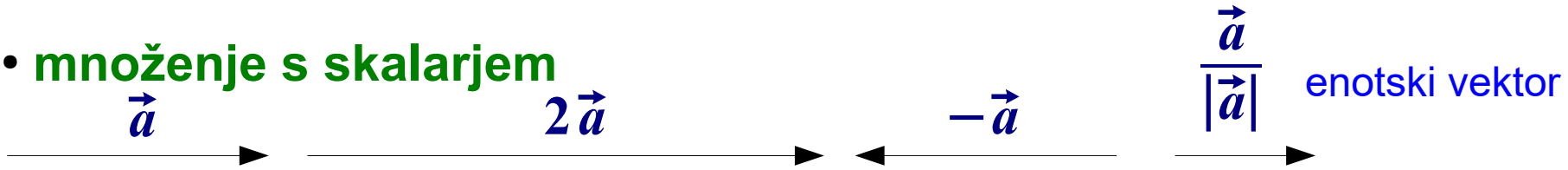
- **Veljavnost pisnih testov:**
  - ocena 50% ali več – tri izpitna obdobja
  - ocena 40%-49% – izpitno obdobje ki sledi zaključku predavanj pri predmetu (zimsko za Fiziko I in poletno za Fiziko II)

- **Za sporočanje po elektronski pošti uporabljajte svoj študentski elektronski naslov jaz.sem@student.um.si**

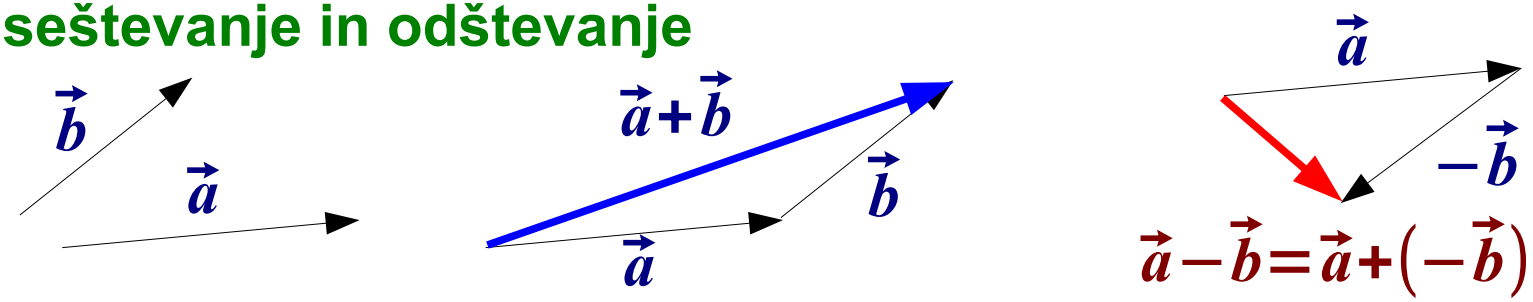
Vektorji

Vektor je količina, ki ima velikost in smer v prostoru

- množenje s skalarjem

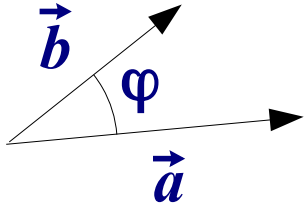


- seštevanje in odštevanje



- množenje dveh vektorjev
  - skalarni produkt

$$\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\varphi$$

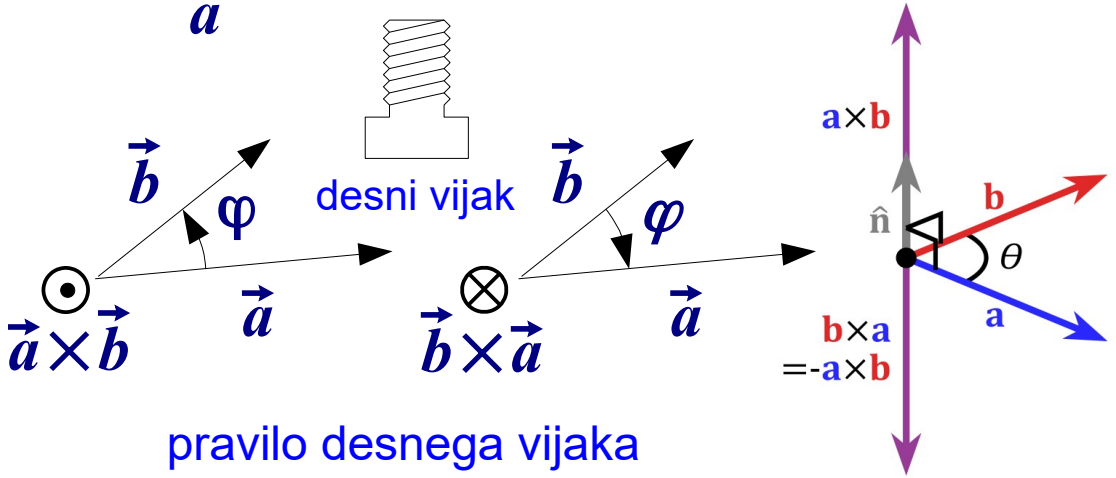


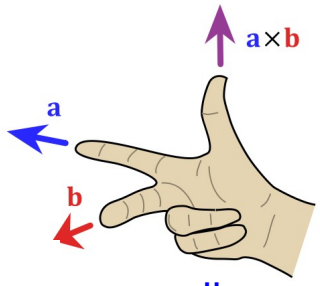
  - vektorski produkt

$$|\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}|\sin\varphi$$

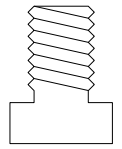
$$\vec{a}\times\vec{b}\perp\vec{a},\vec{b}$$

$$\vec{a}\times\vec{b}=-\vec{b}\times\vec{a}$$





pravilo desne roke



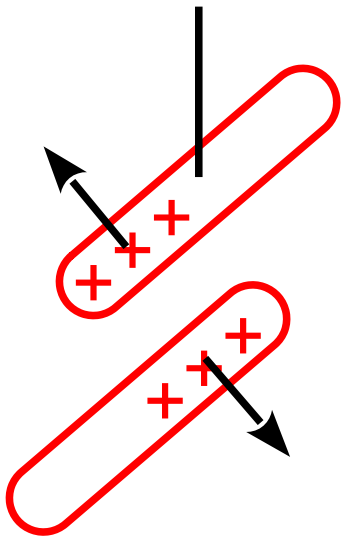
levi vijak

Stacionarno električno polje

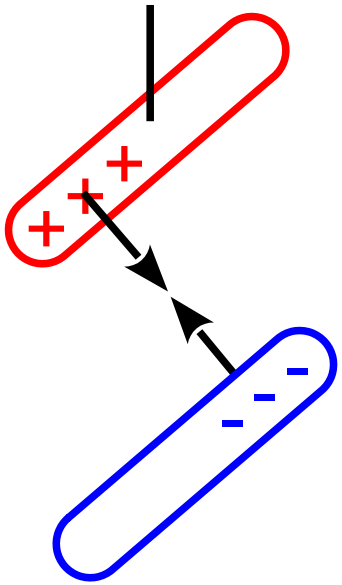
Električni naboj

$$q \quad [C\,(coulomb) = As]$$

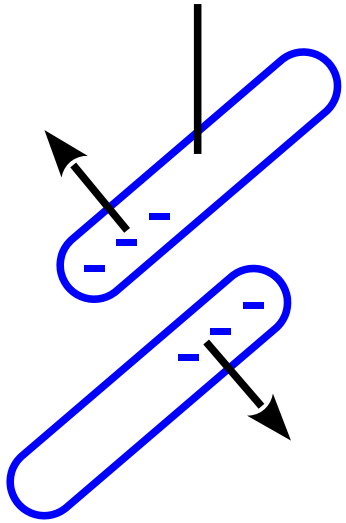
- naelektreni predmeti iz različnih materialov se lahko privlačijo ali odbijajo
- naelektreni predmeti iz iste snovi, ki jih naelektrimo na enak način, se odbijajo



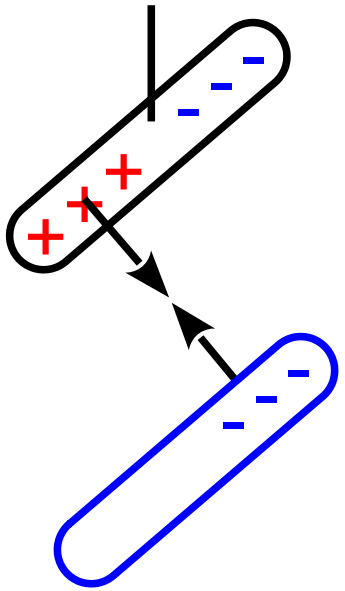
(steklo) odboj (steklo)



(steklo) privlak (plastika)



(plastika) odboj (plastika)

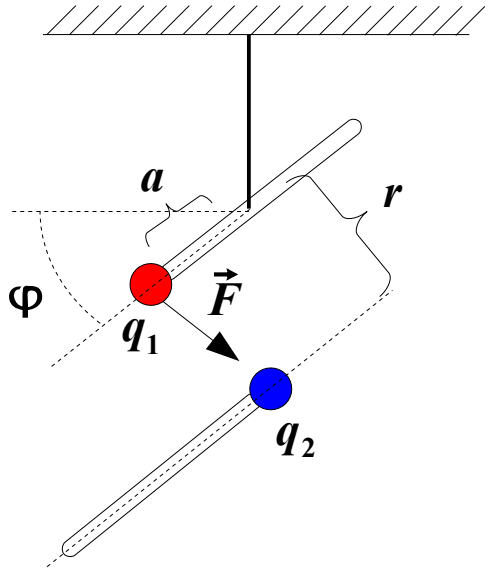


(kovina) privlak (plastika)

- predmet iz kovine pritegnejo naelektreni predmeti

Coulombov zakon

Sila med dvema nabojema je sorazmerna s produktom velikosti obeh nabojev in obratno sorazmerna s kvadratom razdalje med njima.



$$F \propto \frac{1}{r^2}$$
$$F \propto q_1 q_2$$

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

- influenčna konstanta (električna konstanta, dielektričnost vakuuma)

$$\epsilon_0 = 8{,}85 \cdot 10^{-12}\ \frac{As}{Vm} \qquad \left(c_0^2 = \frac{1}{\epsilon_0 \mu_0}\right)$$

- naboj je diskreten - osnovni naboj (Millikan) $e_0 = 1{,}6 \cdot 10^{-19}\ As$

- naboj se ohranja (primer: razpad nevtrona)

$$n \rightarrow p^+ + e^- + \overline{\nu}_e$$

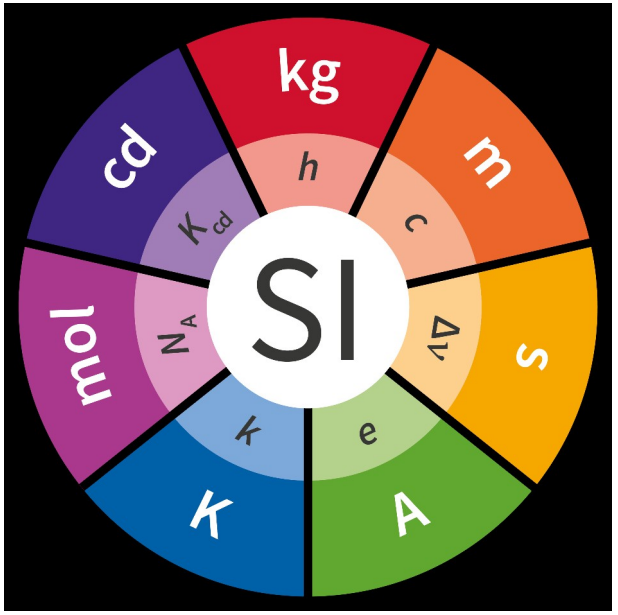
$$s : \Delta\nu_{Cs} = 9\,192\,631\,770\ s^{-1} \quad (^{133}Cs)$$

$$1\ s = \frac{9\,192\,631\,770}{\Delta\nu_{Cs}}$$

$$m : c = 299\,792\,458\ m\ s^{-1} \quad (c)$$

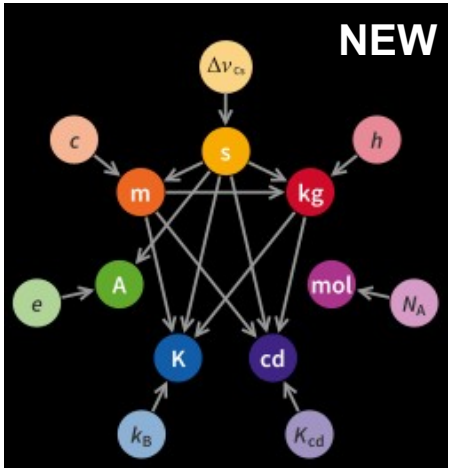
$$m = \frac{c\ s}{299\,792\,458} = \frac{c \cdot 9\,192\,631\,770}{\Delta\nu_{Cs} \cdot 299\,792\,458}$$

$$kg : h = 6{,}626\,070\,15 \cdot 10^{-34}\ kg\ m^2\ s^{-1} \quad (h)$$



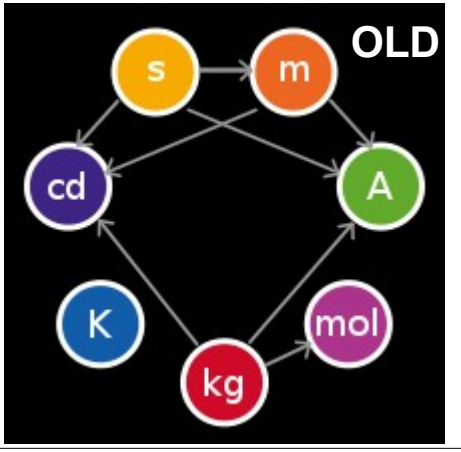
$$K : k = 1{,}380\,649 \cdot 10^{-23}\ J\ K^{-1} \quad (k)$$

$$1\ K = \frac{1{,}380\,649}{k} \cdot 10^{-23}\ kg\ m^2\ s^{-2} \quad (k)$$



- Enota za električni tok A (Amper) je določena z dogovorom o vrednosti osnovnega naboja (proton, elektron ...)

$$A : e_0 = 1{,}602\,176\,634 \cdot 10^{-19}\ A\ s \quad (e_0)$$



$$mol : N_A = 6{,}022\,140\,76 \cdot 10^{23}\ mol^{-1} \quad (N_A)$$

- primerjava električne in gravitacijske sile med dvema elektronoma

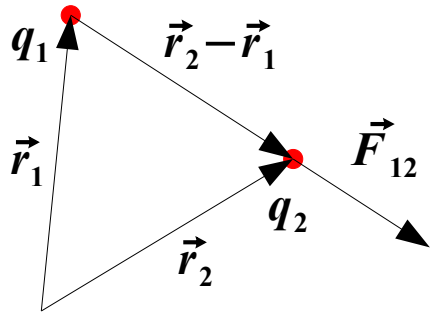
$$\frac{F_e}{F_g}=\frac{e_0^2}{4\pi\epsilon_0 \cancel{r^2}}\cdot\frac{\cancel{r^2}}{G m_e^2}\approx 10^{42}$$

$$q_e=-e_0=-1{,}6\cdot10^{-19}\,As \qquad \epsilon_0=8{,}85\cdot10^{-12}\,\frac{As}{Vm}$$

$$m_e=9\cdot10^{-31}\,kg \qquad G=6{,}67\cdot10^{-11}\,\frac{N\,m^2}{kg^2}$$

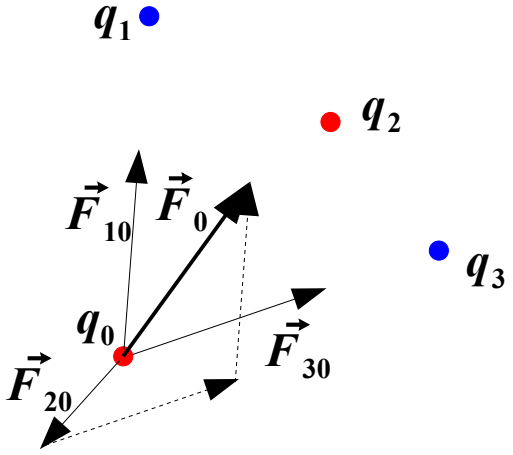
- vektorski zapis električne sile med dvema nabojema

$$\vec{F}_{12}=\frac{1}{4\pi\epsilon_0}\cdot\frac{q_1 q_2}{|\vec{r}_2-\vec{r}_1|^2}\cdot\frac{(\vec{r}_2-\vec{r}_1)}{|\vec{r}_2-\vec{r}_1|}$$

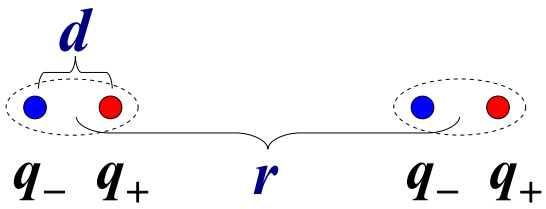


- načelo superpozicije (prisotnost ostalih nabojev ne vpliva na silo med dvema nabojema)

$$\vec{F}_0=\sum_i \vec{F}_{i0}=\sum_i \frac{1}{4\pi\epsilon_0}\cdot\frac{q\,q_i}{|\vec{r}_0-\vec{r}_i|^2}\cdot\frac{(\vec{r}_0-\vec{r}_i)}{|\vec{r}_0-\vec{r}_i|}$$



• primer: sila med dipoloma:



$q_+ = q$

$q_- = -q$

sila med sistemi, ki so navzven nevtralni pada z oddaljenostjo hitreje kot $1/r^2$.

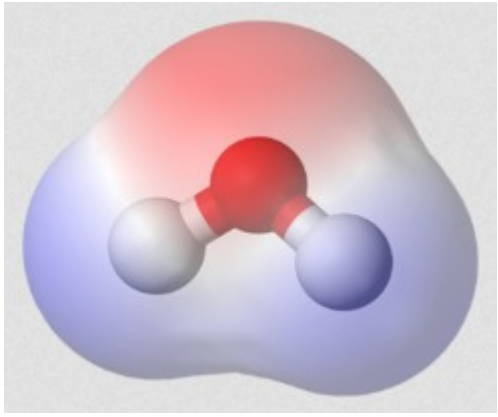
$$F \approx -\frac{6\,p_e^2}{4\pi\epsilon_0 r^4}; \quad r \gg d$$

$$F = F_{--} + F_{-+} + F_{+-} + F_{++} = \frac{qq}{4\pi\epsilon_0}\left(\frac{1}{r^2} - \frac{1}{(r+d)^2} - \frac{1}{(r-d)^2} + \frac{1}{r^2}\right) =$$

$$= \frac{q^2}{4\pi\epsilon_0}\left(\frac{2(r^2-d^2)^2 - r^2(r-d)^2 - r^2(r+d)^2}{r^2(r^2-d^2)^2}\right) =$$

$$= \frac{q^2}{4\pi\epsilon_0}\left(\frac{2r^4 - 4r^2d^2 + 2d^4 - r^4 + 2r^3d - r^2d^2 - r^4 - 2r^3d - r^2d^2}{r^2(r^2-d^2)^2}\right) =$$

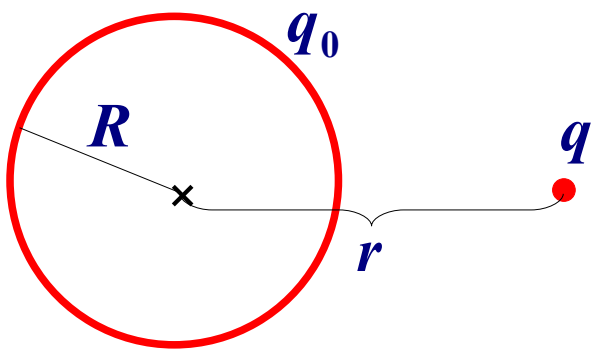
$$= \frac{q^2}{4\pi\epsilon_0}\left(\frac{-6r^2d^2 + 2d^4}{r^2(r^2-d^2)^2}\right) = \frac{qq}{4\pi\epsilon_0} \cdot \frac{-6r^2d^2\left(1 - \frac{d^2}{3r^2}\right)}{r^6\left(1 - \frac{d^2}{r^2}\right)^2} \approx \frac{-6q^2d^2}{4\pi\epsilon_0 r^4}$$



- izrek za krogelne lupine

Enakomerno nabita krogelna lupina deluje na točkast naboj izven nje z enako silo, kot če bi bil ves njen naboj zbran v njenem središču. Če se naboj nahaja znotraj krogelne lupine, je sila nanj enaka 0.

$$F = \begin{cases} 0 & ; r < R \\ \dfrac{1}{4\pi\epsilon_0} \cdot \dfrac{q_0 q}{r^2} & ; r \geqslant R \end{cases}$$

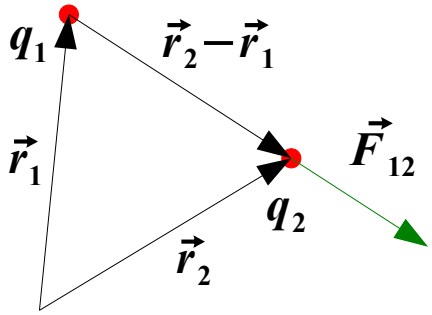


Jakost električnega polja

• prisotnost naboja spremeni lastnosti prostora in ga napolni z električnim poljem, ki posreduje silo med naboji

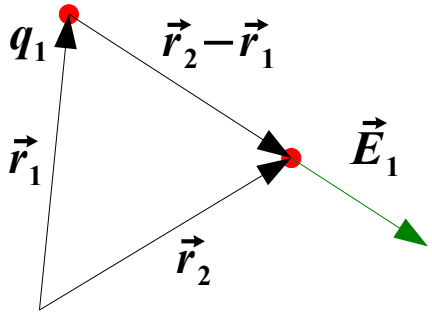
$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^2} \cdot \frac{(\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|} = q_2 \vec{E}_1(\vec{r}_2)$$

• električna poljska jakost, ki jo ustvari prvi naboj v točki, kjer se nahaja drugi



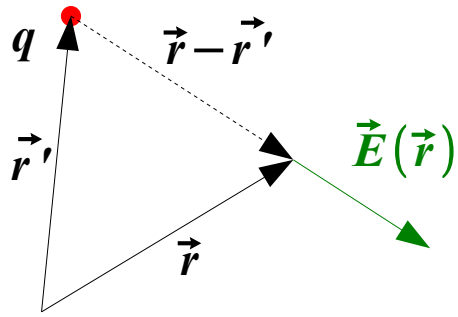
$$\vec{E}_1(\vec{r}_2) = \frac{\vec{F}_{12}}{q_2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1}{|\vec{r}_2 - \vec{r}_1|^2} \cdot \frac{(\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|}$$

$$\vec{E} = \frac{\vec{F}}{q} \qquad [\frac{V}{m} = \frac{N}{As}]$$

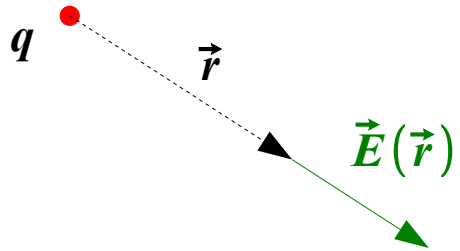


- električno polje točkastega naboja

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{|\vec{r}-\vec{r}\,'|^2} \cdot \frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|} = \frac{q}{4\pi\epsilon_0} \cdot \frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}$$

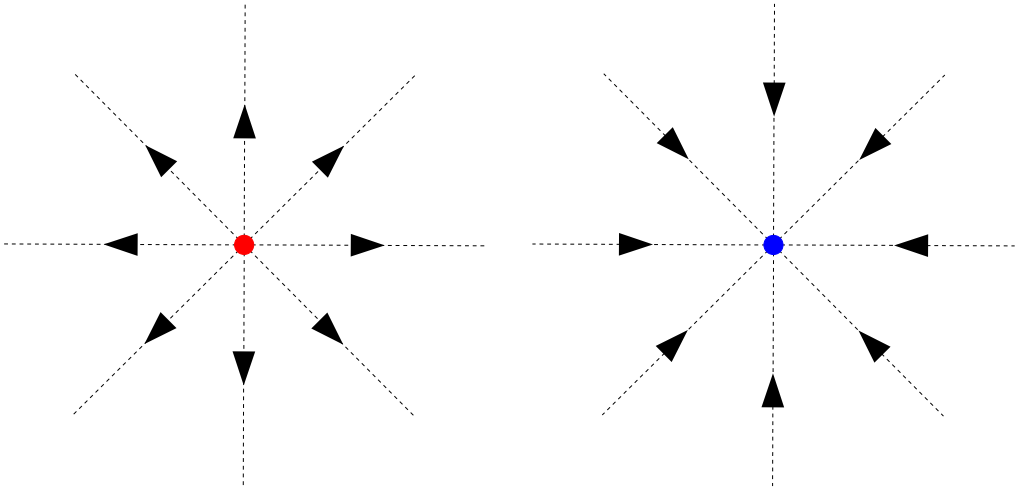


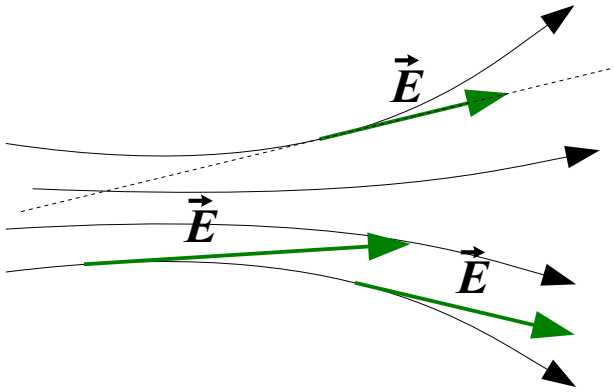
$$\vec{E}(\vec{r}) = \frac{q}{4\pi\epsilon_0 r^2} \cdot \frac{\vec{r}}{r} = \frac{q}{4\pi\epsilon_0} \cdot \frac{\vec{r}}{r^3}$$



$$\vec{r}\,' = \vec{0}$$

- silnice električnega polja

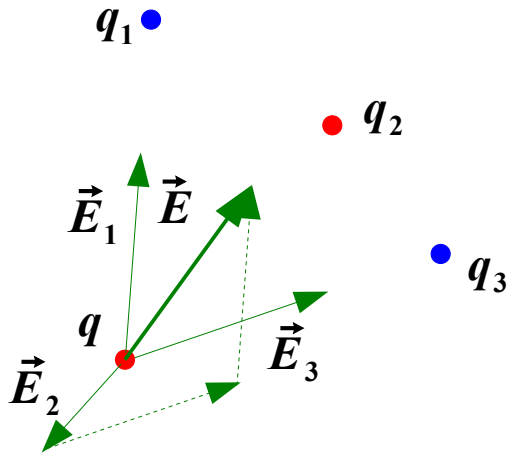




- električno polje diskretne porazdelitve nabojev (gruča točkastih nabojev)

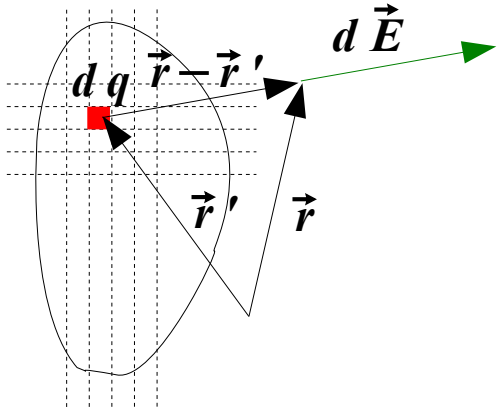
$$\vec{F}=\sum_i \vec{F}_i = q\sum_i \frac{q_i}{4\pi\epsilon_0}\cdot\frac{(\vec{r}-\vec{r}_i)}{|\vec{r}-\vec{r}_i|^3}=q\,\vec{E}(\vec{r})$$

$$\vec{E}(\vec{r})=\sum_i \vec{E}_i(\vec{r})=\sum_i \frac{q_i}{4\pi\epsilon_0}\cdot\frac{(\vec{r}-\vec{r}_i)}{|\vec{r}-\vec{r}_i|^3}$$



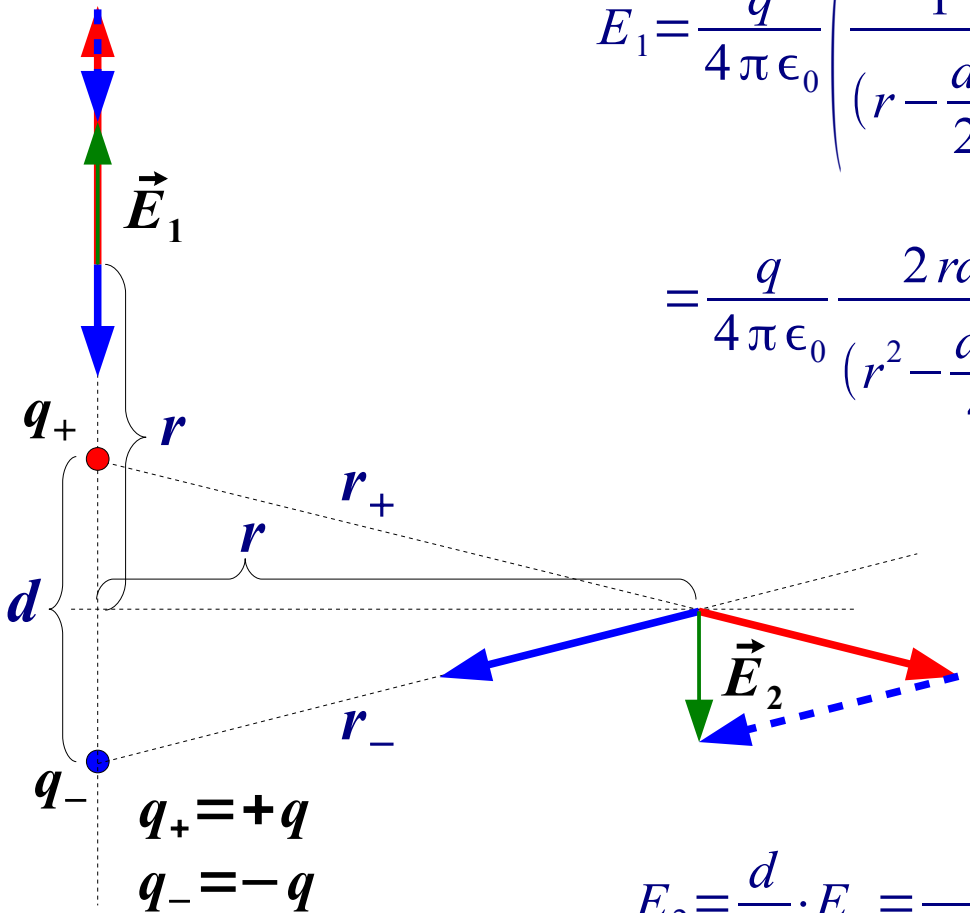
- električno polje zvezne porazdelitve nabojev

$$\vec{E}(\vec{r})=\frac{1}{4\pi\epsilon_0}\cdot\int\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\cdot dq$$



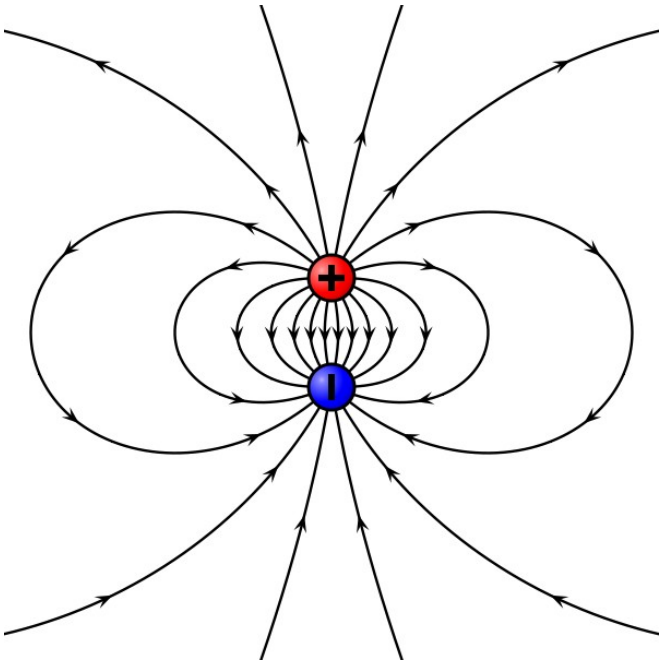
$$d\vec{E}(\vec{r})=\frac{1}{4\pi\epsilon_0}\cdot\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\cdot dq$$

- primer: električni dipol



$$E_1 = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{\left(r-\frac{d}{2}\right)^2} - \frac{1}{\left(r+\frac{d}{2}\right)^2}\right) =$$

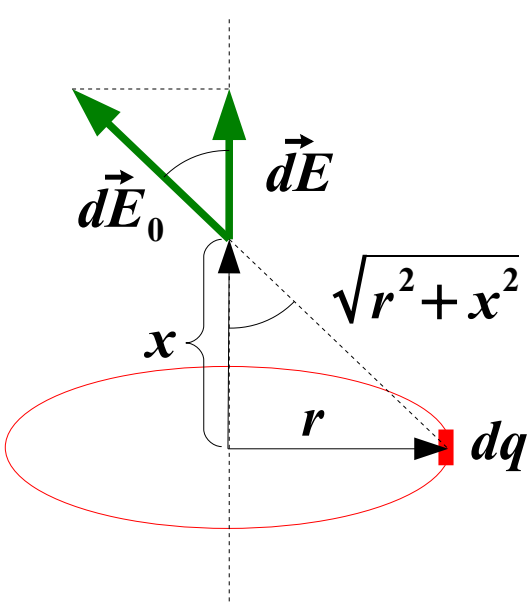
$$= \frac{q}{4\pi\epsilon_0}\frac{2rd}{\left(r^2-\frac{d^2}{4}\right)^2} \approx \frac{1}{4\pi\epsilon_0}\frac{2p_e}{r^3}$$

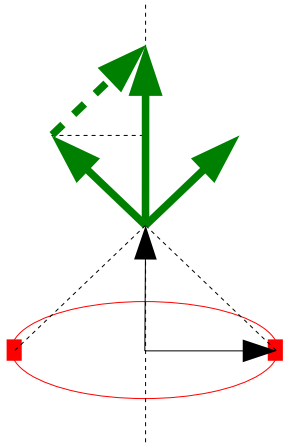


$$\frac{E_2}{E_+} = \frac{d}{r_+}$$

$$E_2 = \frac{d}{r_+}\cdot E_+ = \frac{q}{4\pi\epsilon_0}\cdot\frac{d}{r_+^3} = \frac{q}{4\pi\epsilon_0}\cdot\frac{d}{\left(r^2+\frac{d^2}{4}\right)^{\frac{3}{2}}} \approx \frac{1}{4\pi\epsilon_0}\frac{p_e}{r^3}$$

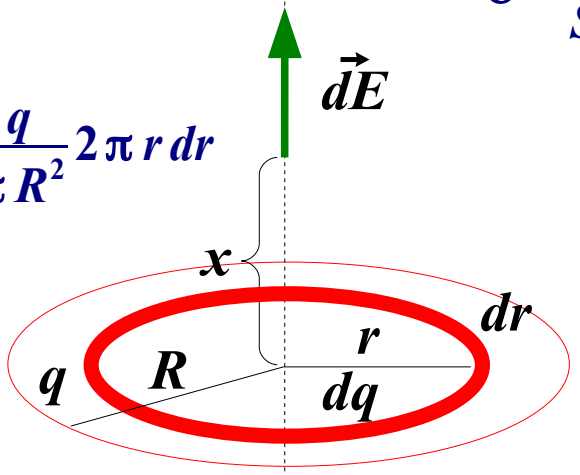
- primeri: tanek obroč, disk in neskončna ravnina





$$\sigma = \frac{q}{S} = \frac{q}{\pi R^2}$$

$$dq = \sigma\, dS = \frac{q}{\pi R^2} 2\pi r\, dr$$

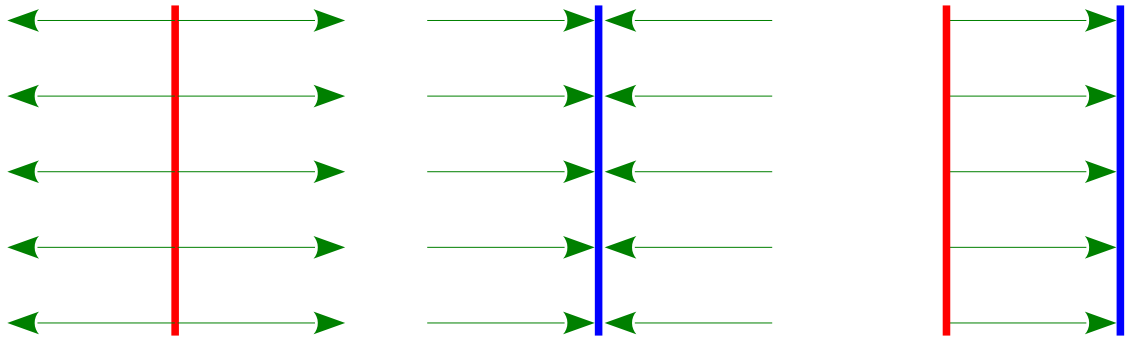


$$E = \frac{q}{4\pi\epsilon_0} \frac{x}{(x^2+r^2)^{\frac{3}{2}}}$$

$$E = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right)$$

- ko gre R proti neskončno disk preide v neskončno ravnino

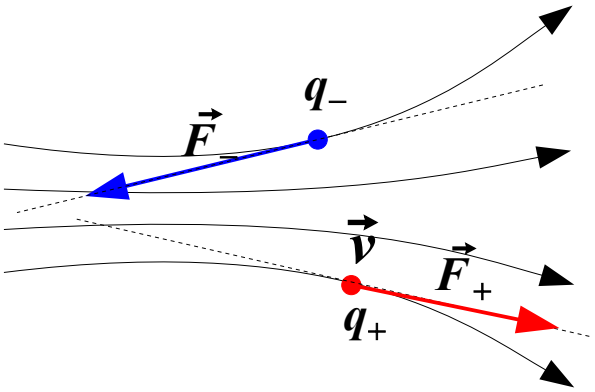
$$E = \frac{\sigma}{2\epsilon_0}$$



Naboj v električnem polju

- sila na naboj

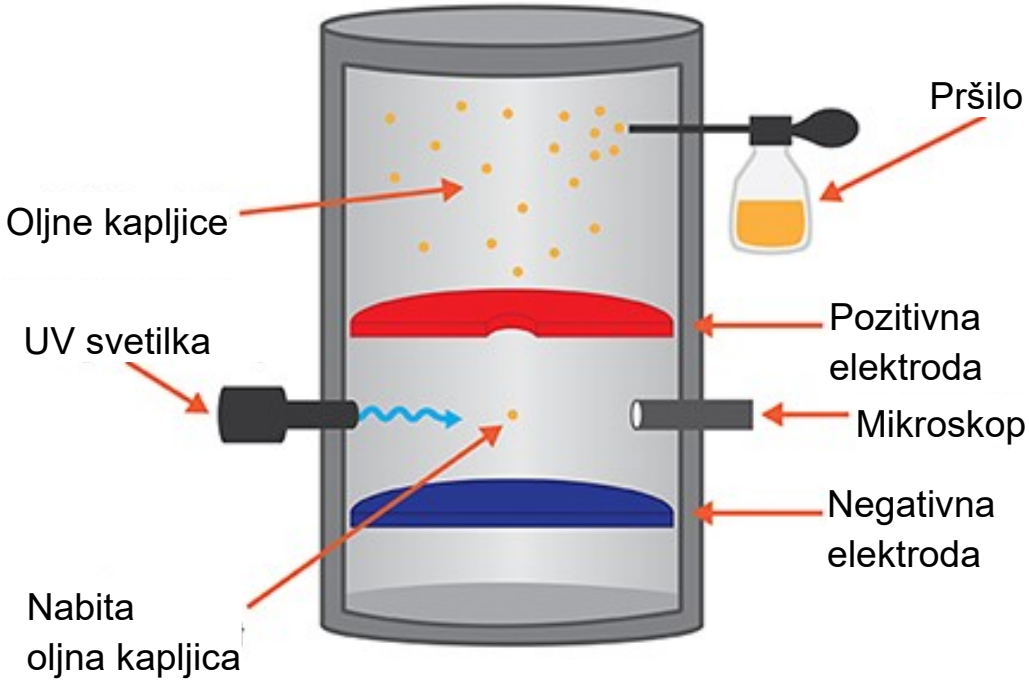
$$\vec{F}_e = q\vec{E}$$



$$\vec{F}_{EM} = q\vec{E} + q\vec{v}\times\vec{B}$$

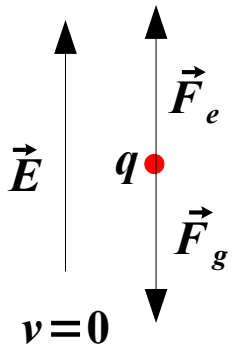
Lorentzova sila

- naboj v homogenem električnem polju
- Millikanov poskus – določitev osnovnega naboja



opazujemo nabito oljno kapljico

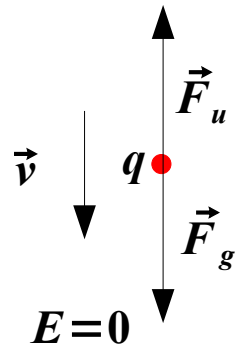
miruje



$$F_e = F_g$$

$$qE = mg$$

enakomerno pada

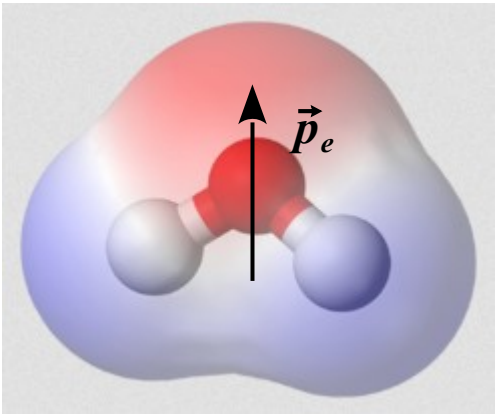


$$F_u = F_g$$

$$6\pi r\eta v = mg = \frac{4}{3}\pi r^3 \rho g$$

linearni zakon upora

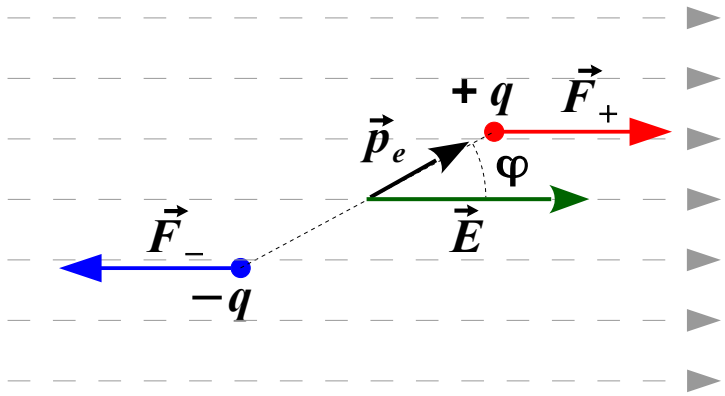
- električni dipol v homogenem električnem polju



$$\vec{F} = \vec{F}_+ + \vec{F}_- = q\vec{E} - q\vec{E} = 0$$

$$M = q\,E\,d\sin(\varphi) = p_e E \sin(\varphi)$$

- navor na električni dipol v električnem polju



$$\boxed{\vec{F} = 0 \quad \vec{M} = \vec{p}_e \times \vec{E}}$$

- energija električnega dipola v električnem polju

$$A_e = \int M\,d\varphi = -\int p_e E \sin\varphi\,d\varphi = -[p_e E(-\cos\varphi) - p_e E(-\cos\varphi')]$$

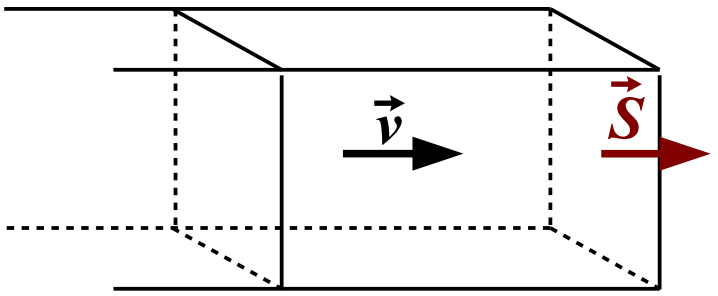
$$\Delta W_e = -A_e = (-p_e E\cos\varphi) - (-p_e E\cos\varphi') = W_e - W_e'$$

$$\boxed{W_e = -\vec{p}_e \cdot \vec{E}}$$

Pretok vektorskega polja – volumski pretok tekočine

- volumski pretok tekočine – pravokotno na ploskev

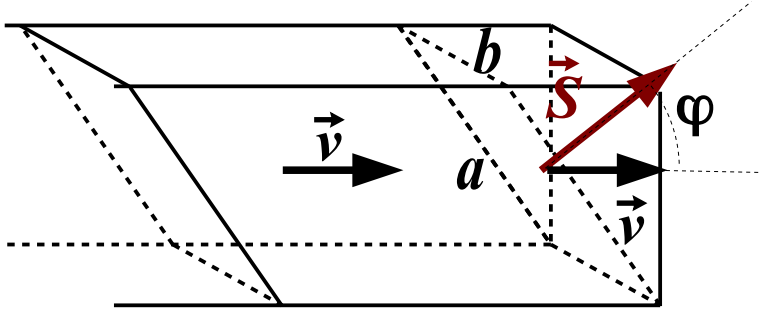
$$\Phi_V = \frac{\Delta V}{\Delta t} = \frac{v \cancel{\Delta t} S}{\cancel{\Delta t}} = vS$$



- volumski pretok tekočine – poševno skozi ploskev

$$\Phi_V = \frac{\Delta V}{\Delta t} = \frac{v \cancel{\Delta t}\, b\, a \cos(\phi)}{\cancel{\Delta t}} = vS\cos(\phi)$$

$$\Phi_V = \vec{v} \cdot \vec{S}$$

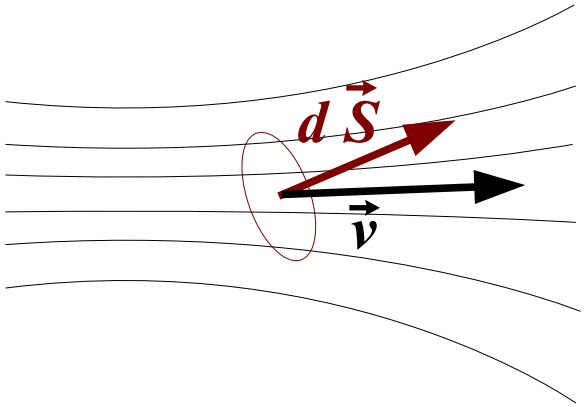


vektor ploskve je pravokoten na ploskev in velikost je enaka ploščini

- volumski pretok tekočine – splošni zapis

$$\Phi_V = \int \vec{v} \cdot d\vec{S}$$

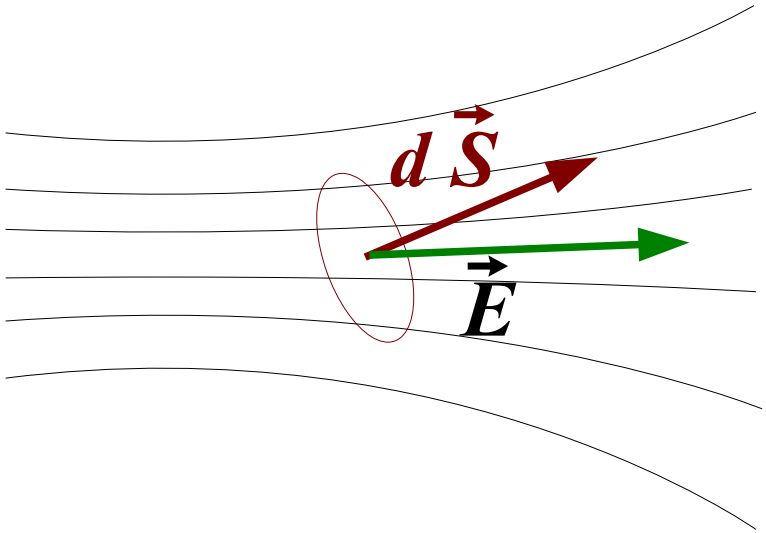
$$\Phi_m = \rho \int \vec{v} \cdot d\vec{S}$$



Zakon o električnem pretoku – Gaussov zakon

- električni pretok

$$d\Phi_e = \vec{D}\cdot d\vec{S} = \epsilon_0 \vec{E}\cdot d\vec{S}$$



$$\Phi_e = \int \vec{D}\cdot d\vec{S} = \epsilon_0 \int \vec{E}\cdot d\vec{S}$$

vakuum

Električni pretok skozi zaključeno ploskev je enak objetemu naboju.

$$\epsilon_0 \oint \vec{E}\cdot d\vec{S} = \oint \vec{D}\cdot d\vec{S} = q$$

- zaključena ploskev omejuje neko območje
- po dogovoru vektor površine kaže izven omejenega območja
- primeri: točkasti naboj, načelo superpozicije, krogelna lupina, neskončna ravnina ...

Elektrostatična potencialna energija

- električna (Coulombova) sila je konzervativna – delo je neodvisno od poti med začetno in končno točko

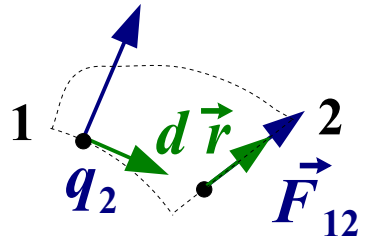
$$A_e = \int \vec{F}_e \cdot d\vec{r} = q \int \vec{E} \cdot d\vec{r}$$

- sprememba elektrostatične potencialne energije je enaka negativnemu delu električne sile

$$\Delta W_e = -A_e = -q \int \vec{E} \cdot d\vec{r}$$

- elektrostatična potencialna energija dveh točkastih nabojev

$$\Delta W_e = \frac{-q_1 q_2}{4\pi\epsilon_0} \int_{r'}^{r} \frac{dr}{r^2} = \frac{q_1 q_2}{4\pi\epsilon_0 r} - \frac{q_1 q_2}{4\pi\epsilon_0 r'}$$



$$W_e = \frac{q_1 q_2}{4\pi\epsilon_0 r}$$

Električni potencial

• električna potencialna energija naboja v električnem polju je enaka produktu naboja in električnega potenciala v točki, kjer se naboj nahaja

$$\Delta W_e = q \Delta V$$

$$\Delta V = \frac{\Delta W_e}{q} \quad [V]$$

$$\Delta V = \frac{\Delta W_e}{q} = -\frac{A_e}{q} = -\frac{q \int \vec{E} \cdot d\vec{r}}{q} = -\int \vec{E} \cdot d\vec{r}$$

- električna napetost med dvema točkama je enaka razliki potencialov v teh točkah

$$U = \Delta V = -\int_{\vec{r}\,'}^{\vec{r}} \vec{E} \cdot d\vec{r}$$

- izrek o električni napetosti po sklenjeni poti

$$\oint \vec{E} \cdot d\vec{r} = 0$$

- električni potencial:
  - točkastega naboja,
  - diskretne in
  - zvezne porazdelitve nabojev ...

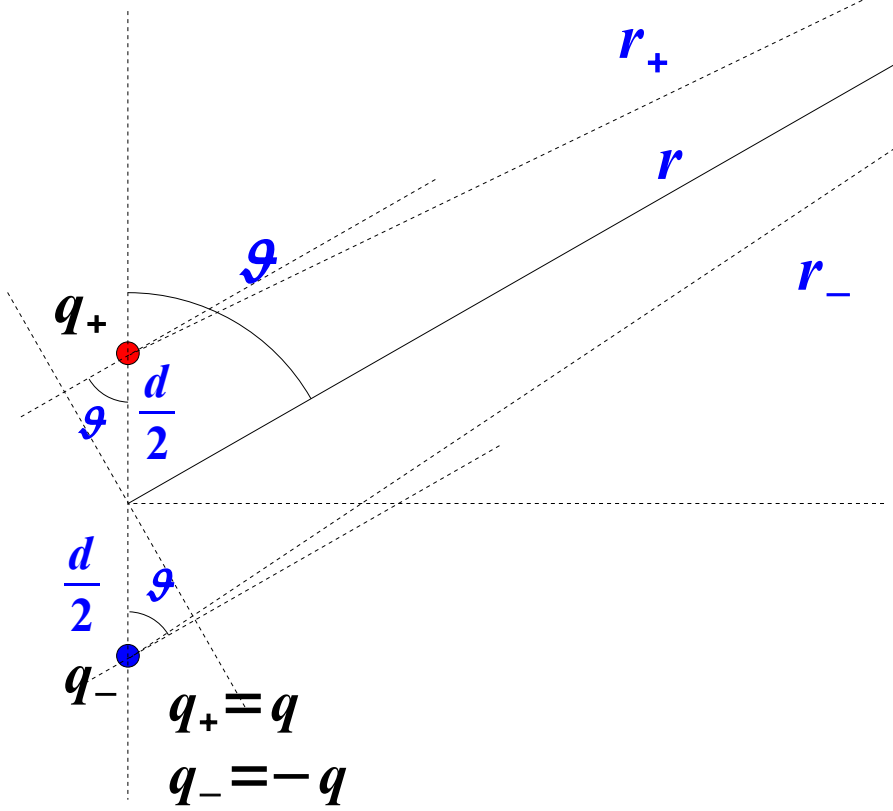
- izračun električne poljske jakosti iz znanega električnega potenciala

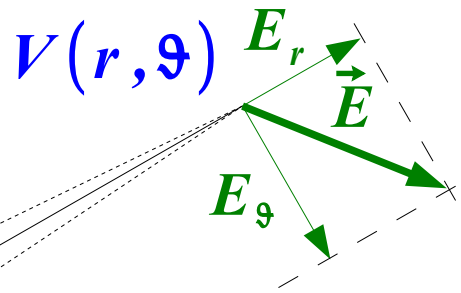
$$\vec{E} = -\left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right)$$

- primer: električni dipol

$$V = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_+} - \frac{1}{r_-}\right) = \frac{q}{4\pi\epsilon_0}\frac{r_- - r_+}{r_- r_+} =$$

$$= \frac{q}{4\pi\epsilon_0}\frac{d\cos\vartheta}{r^2 - \frac{d^2}{4}\cos^2\vartheta} \approx \frac{p_e\cos\vartheta}{4\pi\epsilon_0 r^2}$$





$$E_r = -\frac{\partial V}{\partial r}$$

$$E_r = \frac{2\,p_e\cos\vartheta}{4\pi\epsilon_0 r^3}$$

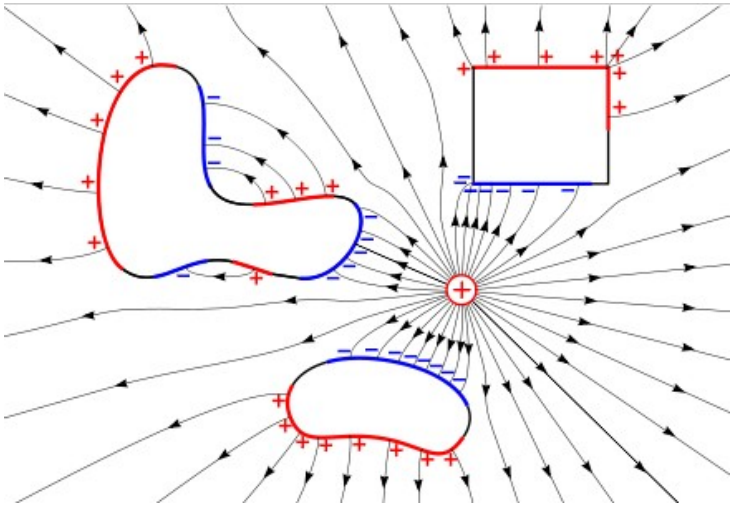
$$E_\vartheta = -\frac{dV}{r\,d\vartheta} = -\frac{1}{r}\frac{\partial V}{\partial\vartheta}$$

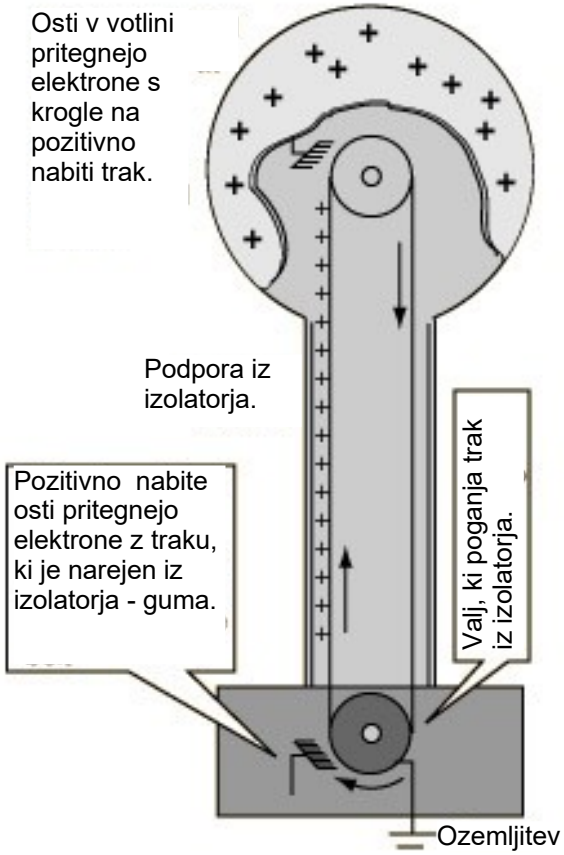
$$E_\vartheta = \frac{p_e\sin\vartheta}{4\pi\epsilon_0 r^3}$$

$$E = \sqrt{E_r + E_\vartheta} = \frac{p_e}{4\pi\epsilon_0 r^3}\sqrt{3\cos^2\vartheta + 1}$$

Prevodnik v zunanjem električnem polju

- v prevodniku so naboji (elektroni) gibljivi
- v prevodniku ni električnega polja (prerazporeditev nabojev)
- polje na površini prevodnika je pravokotno na površino (prerazporeditev nabojev)
- znotraj prevodnika je potencial konstanten (E=0)
  – površina je ekvipotencialna ploskev
- dodaten naboj, ki ga damo na prevodnik se razporedi po površini
- četudi ima prevodnik votlino, se naboj nabere le na zunanji površini
- elektrostatični generator – Van de Graaffov
- elektrostatična indukcija - influenca
- površinska gostota naboja





Kondenzator

- naboj, ki se nabere na ploščah kondenzatorja, je sorazmeren z napetostjo med ploščama

$$q = C\,U$$

$$C = \frac{q}{U} \quad \left[\frac{As}{V} = F\right] \text{ (farad)}$$

kapaciteta

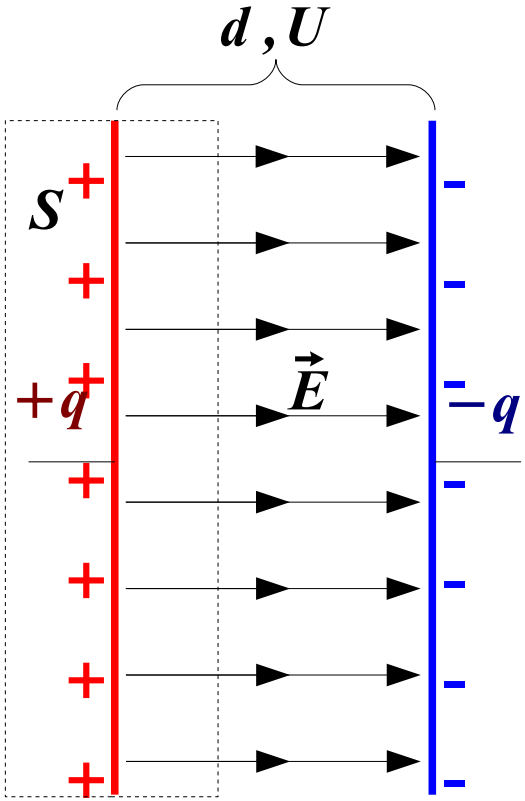
- ploščat kondenzator

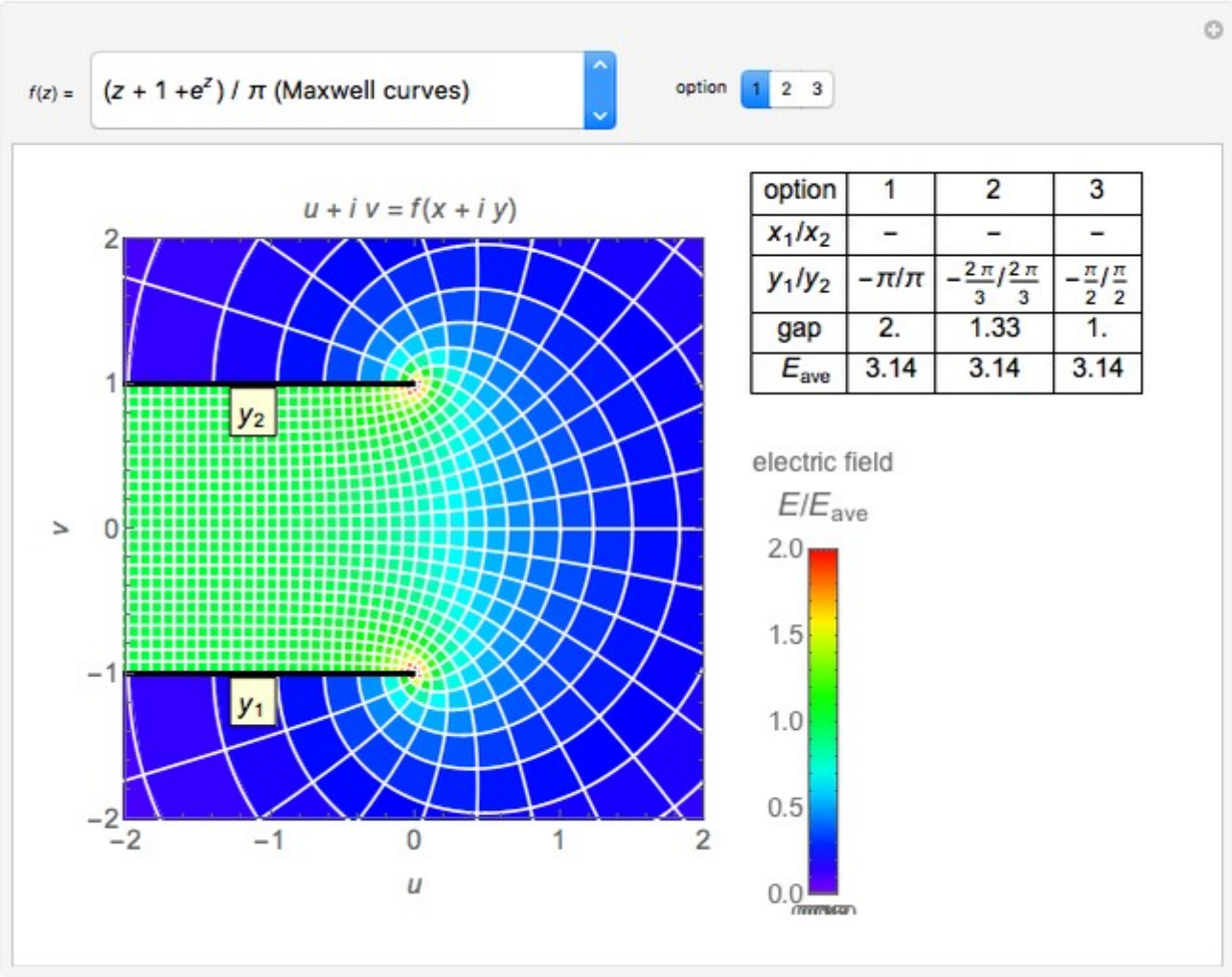
$$\epsilon_0 \oint \vec{E}\cdot d\vec{S} = q \Rightarrow \epsilon_0 E\,S = q$$

$$U = -\int_{\vec{r}'}^{\vec{r}} \vec{E}\cdot d\vec{r} \Rightarrow U = E\,d$$

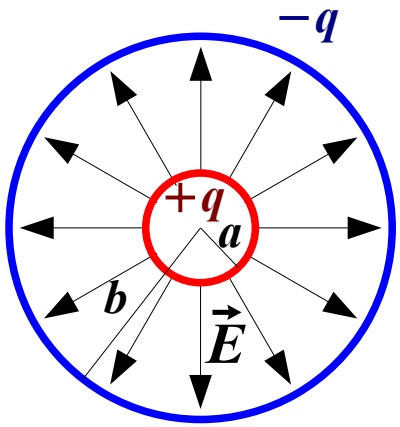
$$q = \frac{\epsilon_0 S}{d} U$$

$$C = \frac{\epsilon_0 S}{d}$$




f(z) =
(z + 1 +e^z) / π (Maxwell curves)
option
1
2
3
u + i v = f(x + i y)
y2
y1
v
u
2
1
0
−1
−2
−2
−1
0
1
2
option | 1 | 2 | 3
x1/x2 | – | – | –
y1/y2 | −π/π | −2π/3/2π/3 | −π/2/π/2
gap | 2. | 1.33 | 1.
Eave | 3.14 | 3.14 | 3.14
electric field
E/Eave
2.0
1.5
1.0
0.5
0.0

- valjast in krogelni kondenzator



$$C = 2\pi\epsilon_0 l \left(\ln\frac{b}{a}\right)^{-1}$$ valjasti

$$C = 4\pi\epsilon_0 \left(\frac{1}{a} - \frac{1}{b}\right)^{-1}$$ krogelni

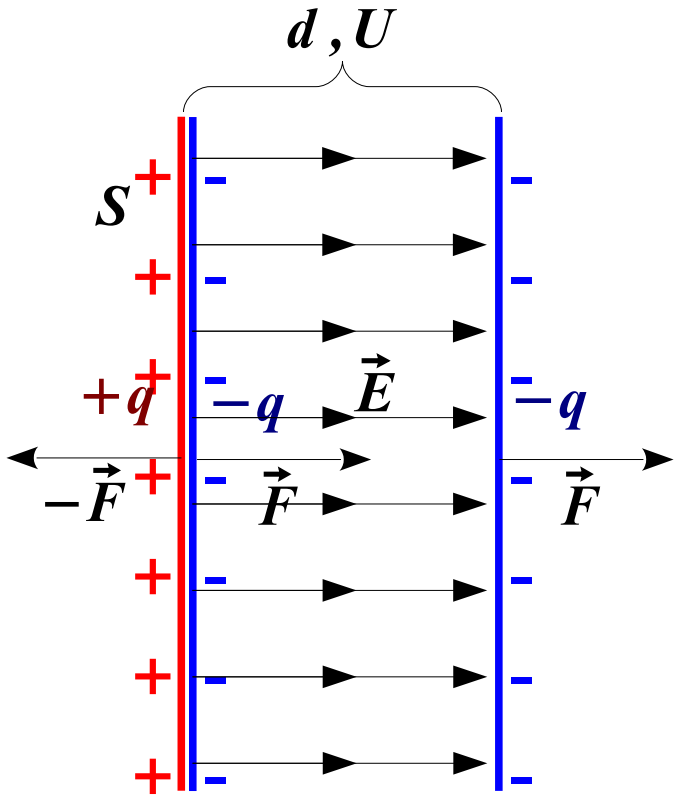
- sila med ploščama kondenzatorja

$$F = q E_1 = q\frac{q}{2\epsilon_0 S}$$

- energija kondenzatorja

$$\Delta W = F d = q\frac{q}{2\epsilon_0 S} d = \frac{1}{2}\frac{q^2}{C}$$

$$W_C = \frac{1}{2} C U^2 = \frac{1}{2}\frac{q^2}{C}$$

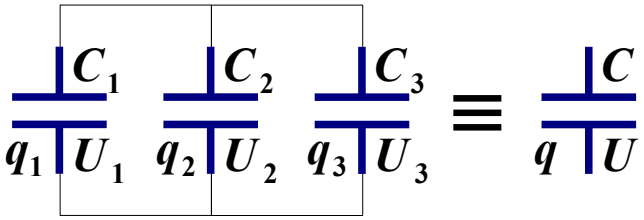


- gostota energije v električnem polju

$$w_e = \frac{1}{2}\epsilon\epsilon_0 E^2 = \frac{1}{2} ED$$

- nadomestna kapaciteta pri vzporedni in zaporedni vezavi

$$C = \sum_i C_i$$ vzporedna



$$U_1 = U_2 = U_3 = U$$

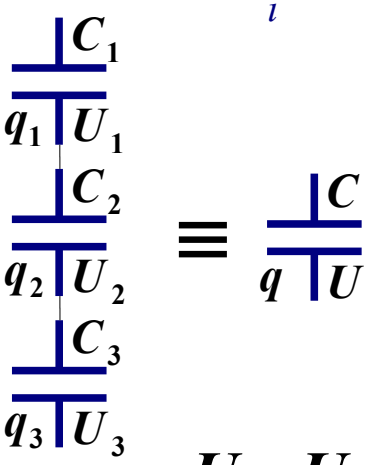
$$q_i = C_i U$$

$$q = q_1 + q_2 + q_3 = C_1 U + C_2 U + C_3 U$$

$$q = (C_1 + C_2 + C_3) U = C U$$

$$C = C_1 + C_2 + C_3$$

$$\frac{1}{C} = \sum_i \frac{1}{C_i}$$ zaporedna



$$q_1 = q_2 = q_3 = q$$

$$U_i = \frac{q}{C_i}$$

$$U = U_1 + U_2 + U_3 = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

$$U = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) q = \frac{q}{C}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Dielektrik v električnem polju

- Če postavimo dielektrik med plošči kondenzatorja, na katerem je konstanten naboj se napetost med ploščama zmanjša.

prazen $q = C_0 U_0$ → poln $q = C U$ $\qquad U = \frac{U_0}{\epsilon}$

- Če dielektrik izpolnjuje ves prostor med ploščama velja

$\frac{U_0}{U} = \frac{C}{C_0} = \epsilon$ dielektričnost $\qquad C = \epsilon C_0$

- med ploščama se zmanjša električna poljska jakost $U = Ed$

$$E = \frac{E_0}{\epsilon} = \frac{\sigma}{\epsilon \epsilon_0} = \frac{q}{S \epsilon \epsilon_0}$$

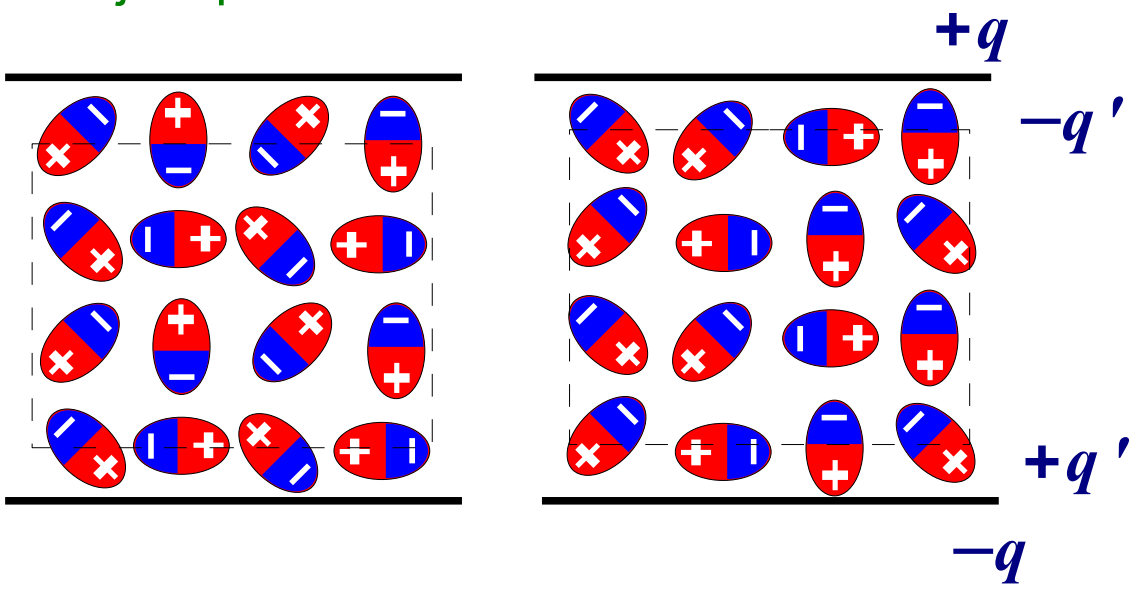
- vezan naboj q' in Gaussov zakon

$$\epsilon_0 \oint \vec{E} \cdot d\vec{S} = q - q' = \frac{q}{\epsilon}$$

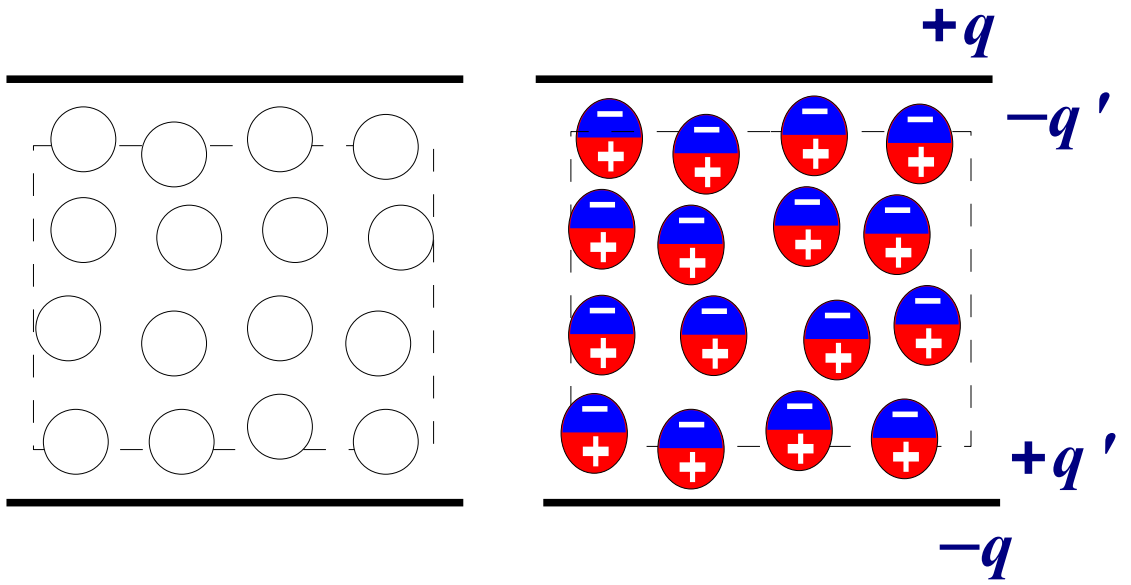
$$\epsilon_0 \oint \frac{\vec{E}_0}{\epsilon} d\vec{S} = \frac{q}{\epsilon}$$

$$\epsilon \epsilon_0 \oint \vec{E} \cdot d\vec{S} = \oint \vec{D} \cdot d\vec{S} = q$$

- polarizacija v polarnem sredstvu



- polarizacija v nepolarnem sredstvu

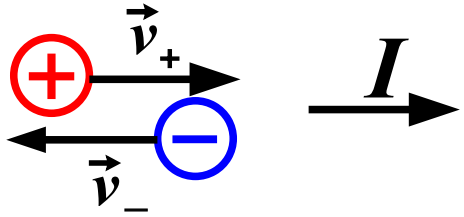


Električni tok

- električni tok pove, koliko naboja steče skozi izbrani presek (vodnika) v enoti časa

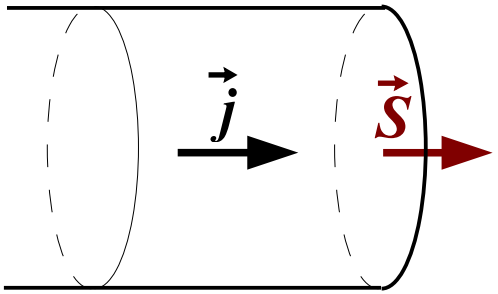
$$I = \frac{dq}{dt} \qquad q = \int_0^t I(t)\,dt$$

- smer toka je enaka smeri gibanja pozitivnih nabojev in nasprotno enaka smeri gibanja negativnih nabojev



- gostota električnega toka

$$j = \frac{dI}{dS} \qquad I = \int \vec{j} \cdot d\vec{S}$$



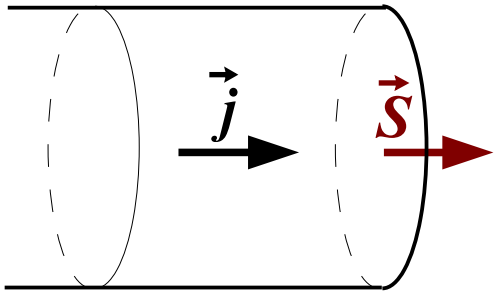
- gostota električnega toka v prevodniku
- potovalna hitrost – povprečna hitrost s katero se gibljejo nosilci naboja

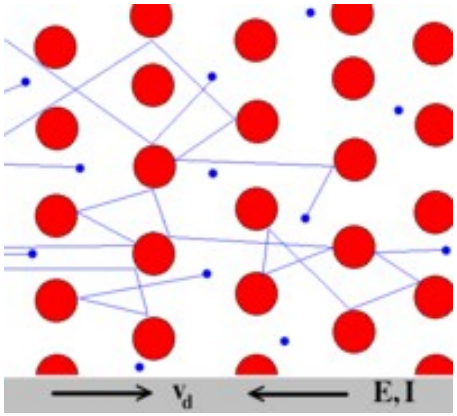
$$\Delta q = \Delta N\, e_0 = n \Delta V\, e_0 = nS\, v_d \Delta t\, e_0$$

$$\langle v \rangle \approx 1000\, km/s\,;\, v_d \approx 10\, \mu m/s$$

$$I = \frac{\Delta q}{\Delta t} = n e_0 v_d S$$

$$j = \frac{I}{S} = n e_0 v_d = \rho_e v_d$$

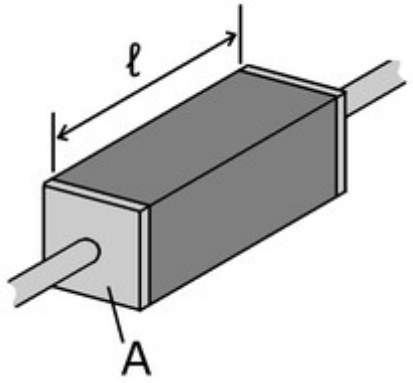




Upor in Ohmov zakon

- upornost je razmerje med napetostjo in tokom na nekem elementu

$$R = \frac{U}{I} \quad \left[\frac{V}{A} = \Omega\right] \text{ (Ohm)}$$



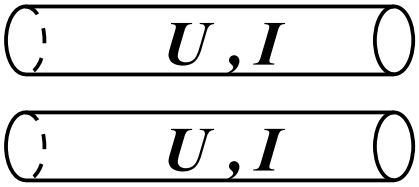
- če je upornost neodvisna od napetosti in toka na elementu, pravimo, da velja Ohmov zakon

$$U = R\, I$$

- specifična upornost (prevodnost)

$$R_0 = \frac{U}{I}$$

$$R = \frac{2U}{I} = 2\frac{U}{I} = 2R_0 \propto l$$



$$R = \frac{U}{2I} = \frac{1}{2}\frac{U}{I} = \frac{1}{2}R_0 \propto \frac{1}{S}$$

$$U = \rho\frac{l}{S}I \;\rightarrow\; \rho = R\frac{S}{l} \quad [\Omega m = 10^6\,\Omega\frac{mm^2}{m}]$$

$$\frac{U}{l} = \rho\frac{I}{S} \;\rightarrow\; \vec{E} = \rho\vec{j} \qquad \vec{j} = \sigma\vec{E} \;;\; \sigma = \frac{1}{\rho}$$

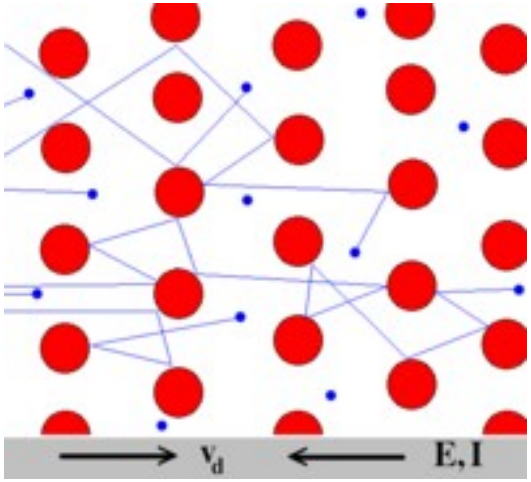
specifična upornost — specifična prevodnost

- mikroskopska slika Ohmovega zakona – gibljivost elektronov

$$a = \frac{F}{m} = \frac{e_0 E}{m} \qquad v_d \approx a\tau = \frac{e_0\tau}{m}E = \beta E$$

$$j = \rho^- v_d = \rho^- \frac{e_0\tau}{m}E = \frac{n e_0^2\tau}{m}E$$

$$\boldsymbol{\sigma = \frac{n e_0^2\tau}{m} \;;\; \rho = \frac{m}{n e_0^2\tau}}$$

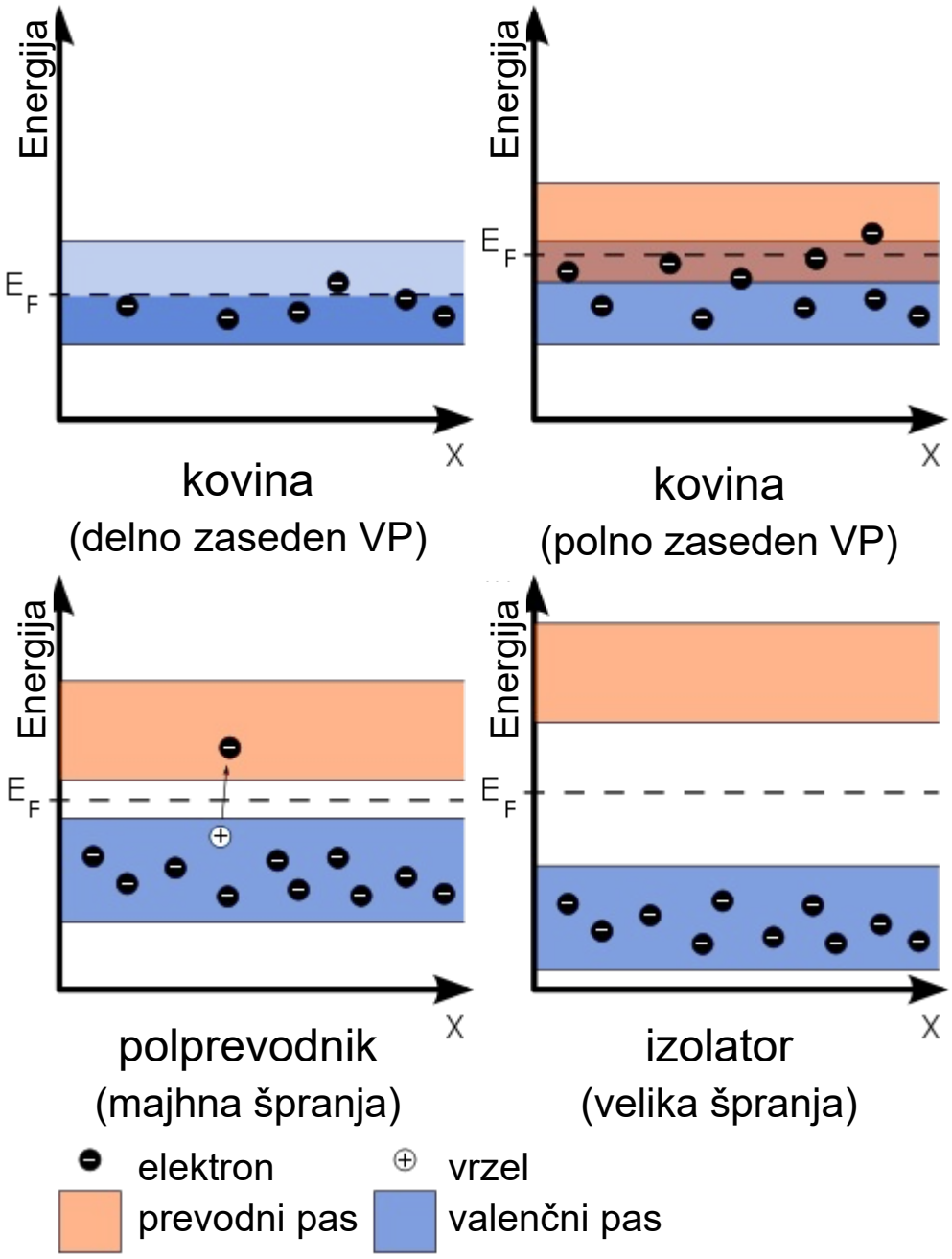


• prevodniki – imajo le delno izpolnjen valenčni pas ali pa se valenčni in prevodni pas prekrivata oz. je energijska špranja med njima majhna v primerjavi s tipično termično energijo kT

• polprevodniki – energijska špranja dovolj majhna v primerjavi s kT

• izolatorji - energijska špranja je precej večja od kT

• superprevodniki – pri nizkih temperaturah upornost izgine



- temperaturni koeficient specifičnega upora

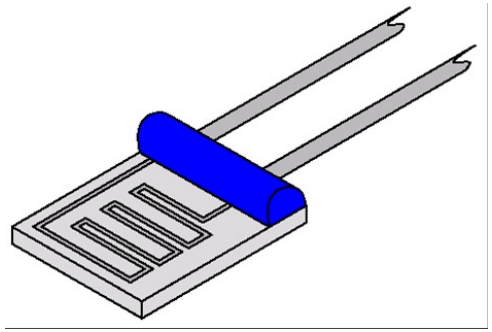
$$\Delta\rho = \alpha\,\rho_0\,\Delta T \;\rightarrow\; \alpha = \frac{1}{\rho_0}\frac{\Delta\rho}{\Delta T} \;;\; [K^{-1}]$$

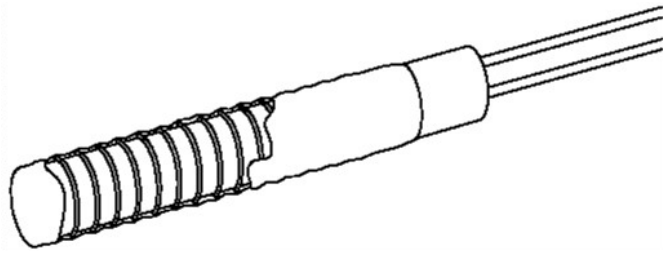
$$\rho = \rho_0 + \alpha\,\rho_0\,\Delta T = \rho_0\left(1+\alpha\,\Delta T\right)$$

	α [K^{-1}]	ρ [Ωm]
Ag	$3{,}8\times10^{-3}$	$1{,}6\times10^{-8}$
Cu	$4{,}0\times10^{-3}$	$1{,}7\times10^{-8}$
Au	$3{,}4\times10^{-3}$	$2{,}4\times10^{-8}$
Al	$3{,}9\times10^{-3}$	$2{,}6\times10^{-8}$
Pt	$3{,}9\times10^{-3}$	$10{,}6\times10^{-8}$
Konstantan*	8×10^{-6}	49×10^{-8}
Si	-75×10^{-3}	$6{,}4\times10^{3}$
Trda guma		10^{13}
Teflon		10^{23}-10^{25}

*Konstantan: 55% Cu, 45% Ni

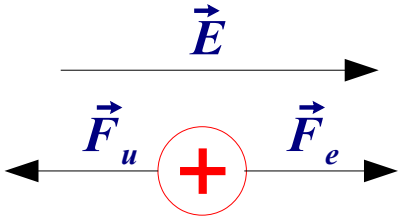
- uporovni termometer (RTD)
- PT100
  - R_0=100Ω @ 0°C,
  - αR_0=0,385Ω/1°C





$$R = R_0 + \alpha\,R_0\,\Delta T = R_0\left(1+\alpha\,\Delta T\right)$$

- tok v raztopinah



$$j = \rho_e v_d = z e_0 n v_d = z e_0 n \beta E$$

$$qE = 6\pi r \eta v_d$$

$$v_d = \frac{q}{6\pi r \eta} E = \beta E$$

$$j = j_+ + j_- = z_+ e_0 n_+ \beta_+ E + z_- e_0 n_- \beta_- E$$

- Faradayev naboj – velikost naboj enega mola protonov, elektronov ...

$$q_F = e_0 N_A$$

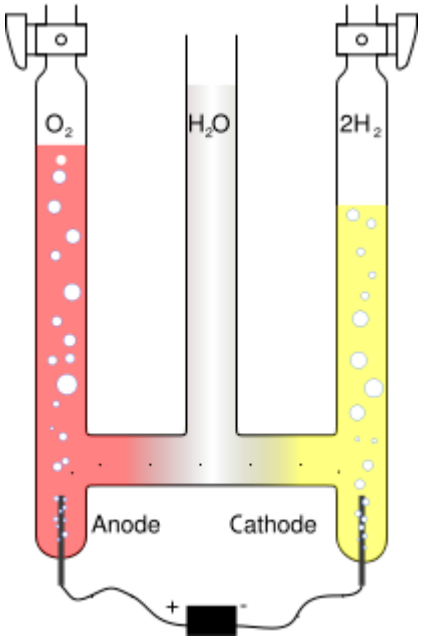
$$m = n_{mol} M = \frac{q}{z q_F} M = \frac{I t}{z q_F} M$$

$$2H^+ + 2e^- \rightarrow H_2$$

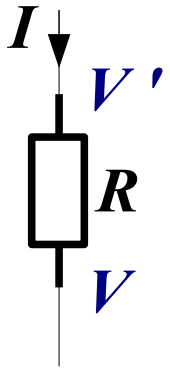
$$SO_4^{--} - 2e^- \rightarrow SO_4$$

$$2SO_4 + 2H_2O \rightarrow 2H_2SO_4 + O_2$$

elektroliza vode



• moč v električnem krogu

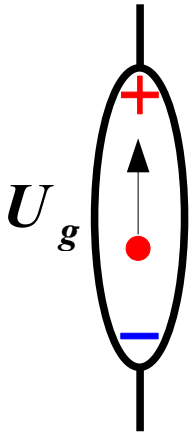


$$dW_e = V\,dq - V'\,dq = (V - V')\,dq = U\,dq$$

$$\left.\begin{aligned} dW &= U\,dq \\ dW &= P\,dt \end{aligned}\right\} \quad P = U\frac{dq}{dt} = U\,I$$

• gonilni člen

$$dA = dW_e = U_g\,dq$$

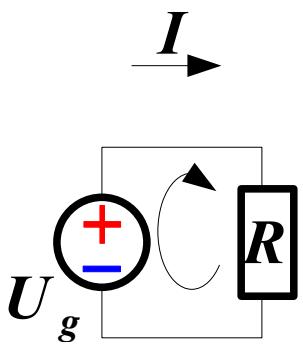


$$U_g = \frac{dA}{dq}$$

• za vzdrževanje stacionarnega toka moramo vzdrževati gonilno napetost
• v gonilnem členu se opravlja delo nad naboji, kar vzdržuje potencialno razliko – gonilno napetost
• gonilna napetost je enaka delu, ki ga opravi gonilni člen, na enoto prenesenega naboja

Tok po enostavnem električnem krogu

- energija, ki jo oddaja gonilni člen je enaka energiji ki jo prejema upornik



$$dW = U_g\, dq$$

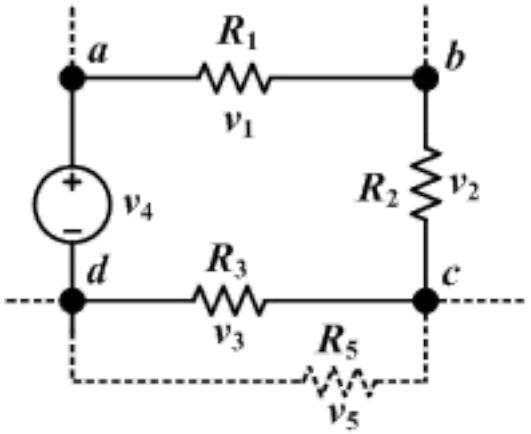
$$dW = P\, dt = U\, I\, dt = R\, I^2\, dt = R\, I\, dq$$

$$U_g\, dq = R\, I\, dq \;\rightarrow\; U_g - RI = 0$$

$$I = \frac{U_g}{R}$$

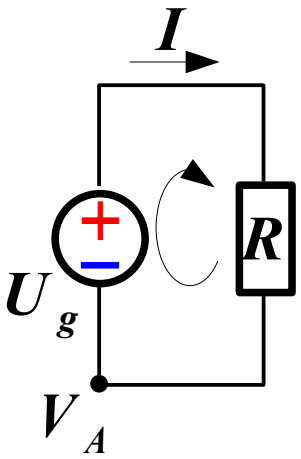
II. Kirchhoffov zakon

Vsota vseh sprememb potenciala (napetosti) v zaključenem električnem krogu je enaka 0.

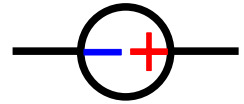


Je le poseben primer izreka o električni napetosti po sklenjeni poti.

$$\oint \vec{E} \cdot d\vec{r} = 0$$



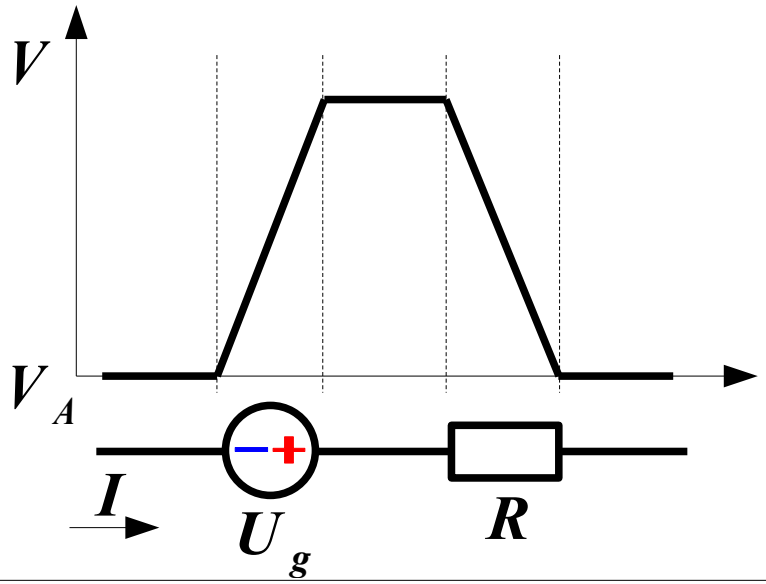
$$U_g - R\,I = 0 \;\rightarrow\; I = \frac{U_g}{R}$$

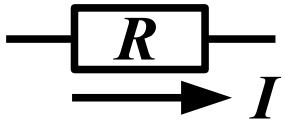


smer obhoda

$+U_g$

$-U_g$



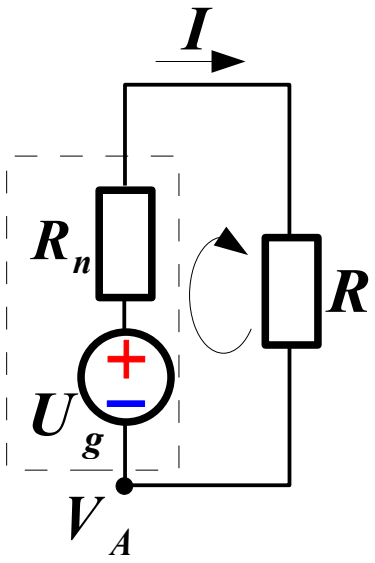


smer obhoda

$-RI$

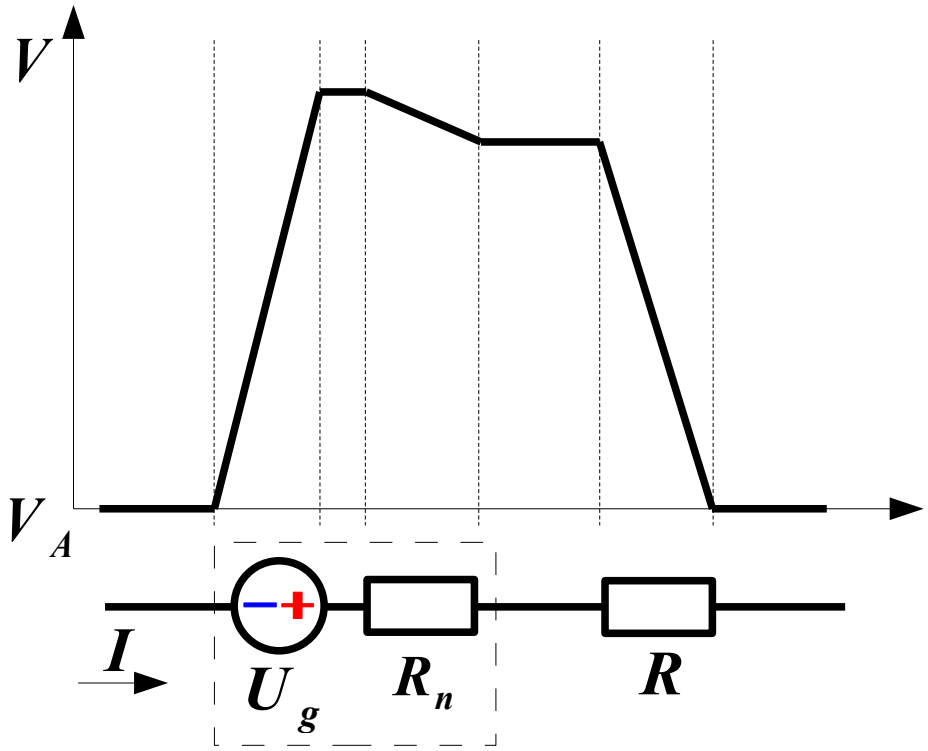
$+RI$

• notranja upornost gonilnega člena



$$U_g - R_n I - R I = 0$$

$$I = \frac{U_g}{R + R_n}$$



• potencialne razlike v električnem krogu – napetost med dvema točkama v el. krogu dobimo tako, da naredimo delni obhod od ene do druge točke in seštejemo napetosti (spremembe potenciala)

• moč gonilnega člena

$$U_g - R_n I - R I = 0 \quad / \cdot I$$

$$U_g I - R_n I^2 - R I^2 = 0$$

$$P' - P_n - P = 0$$

$$P = P' - P_n$$

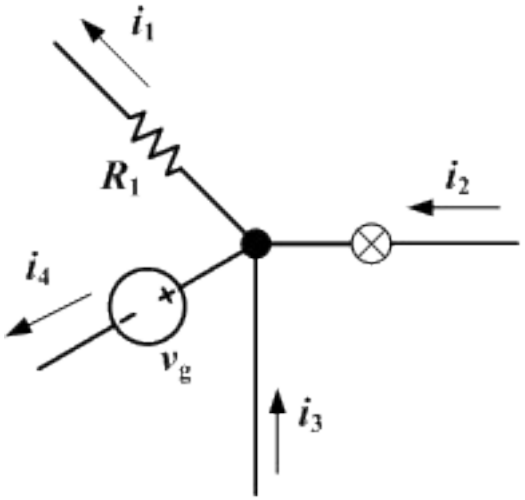
oddana moč = generirana moč – notranje izgube

I. Kirchhoffov zakon

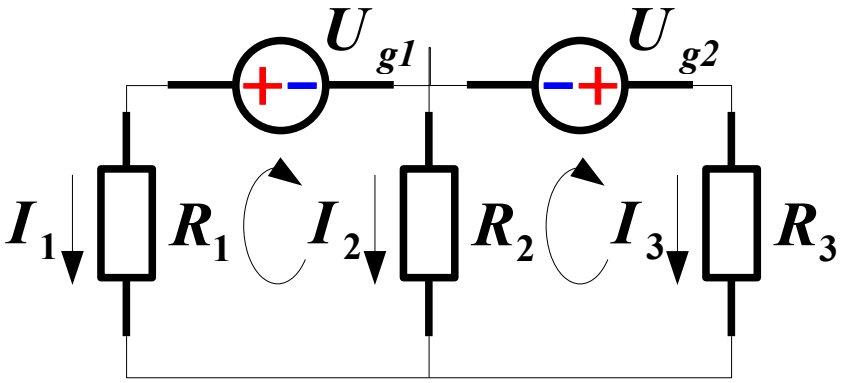
Vsota vseh tokov, ki pritekajo v neko vozlišče je enaka 0.

Je posledica zakona o ohranitvi naboja.

$$-I_1+I_2+I_3-I_4=0$$



- sestavljen električni krog



$$R_1 I_1 - U_{g1} - R_2 I_2 = 0$$

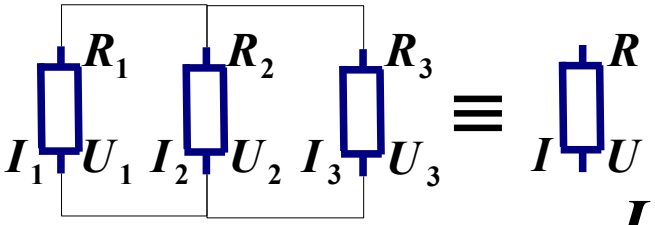
$$R_2 I_2 + U_{g2} - R_3 I_3 = 0$$

$$I_1 + I_2 + I_3 = 0$$

- zaporedna in vzporedna vezava uporov

$$\frac{1}{R}=\sum_i \frac{1}{R_i}$$ vzporedna

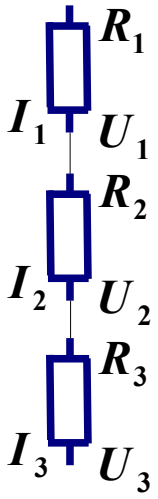
$$R=\sum_i R_i$$ zaporedna



$$U_1=U_2=U_3=U \qquad I_i=\frac{U}{R_i}$$

$$I=I_1+I_2+I_3=\frac{U}{R_1}+\frac{U}{R_2}+\frac{U}{R_3}$$

$$I=\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)U=\frac{U}{R}$$



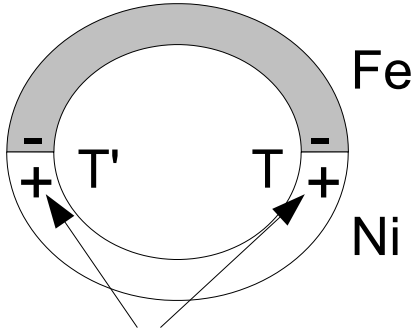
$$U_i=R_i I$$

$$I_1=I_2=I_3=I$$

$$U=U_1+U_2+U_3=R_1 I+R_2 I+R_3 I$$

$$U=\left(R_1+R_2+R_3\right)I=R\,I$$

- termoelement – termična gonilna napetost



kontaktna napetost

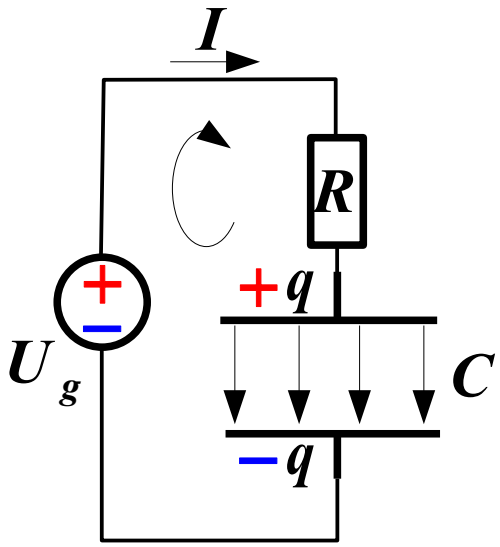
$$U_{gT}=k_T\left(T-T'\right)$$

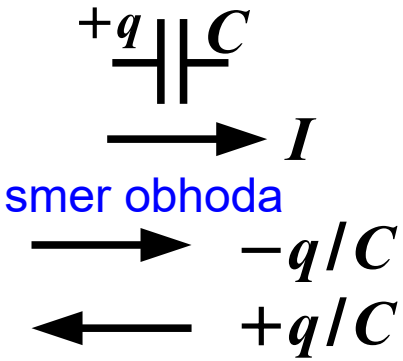
k_T – koeficient termične gonilne napetosti

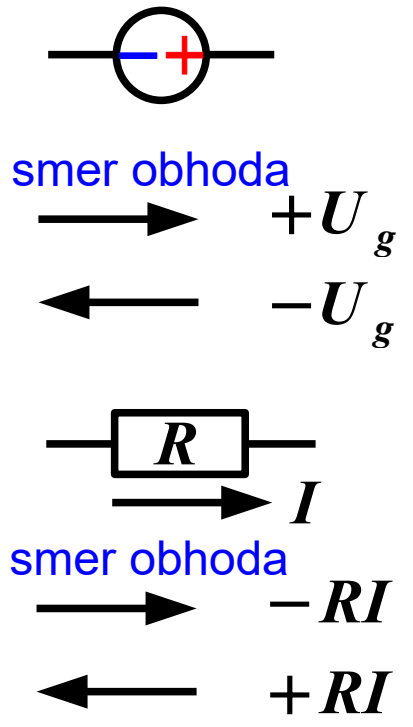
Seebeck – generator, Peltier - hladilnik

	μV/K
Fe-konstantan	50
Fe-Ni	32
Fe-Pt	16

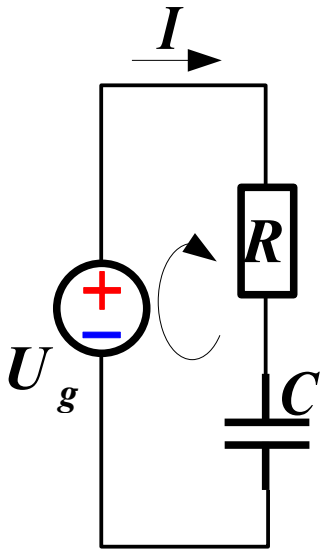
Električni krog s kondenzatorjem







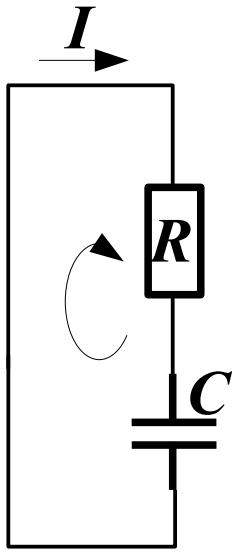
- polnjenje kondenzatorja



$$U_g - R\,I - \frac{q}{C} = 0$$

$$\frac{dq}{dt} = -\frac{1}{RC}\left(q - C\,U_g\right)$$

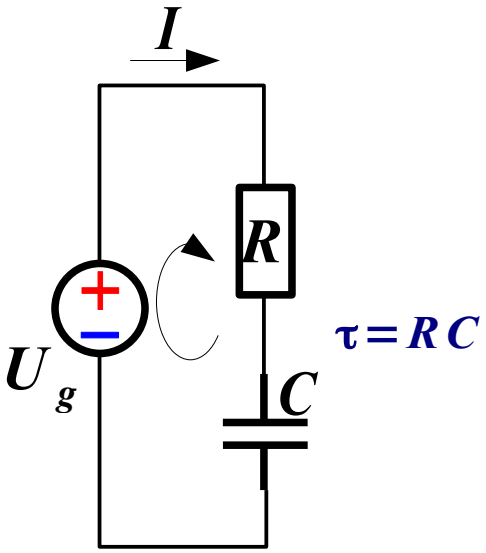
- praznjenje kondenzatorja



$$-R\,I - \frac{q}{C} = 0$$

$$\frac{dq}{dt} = -\frac{1}{RC}\,q$$

- polnjenje kondenzatorja



$$U_C(0)=0;\quad I(\infty)=0$$

$$U_R(0)=R\,I_0=U_g \Rightarrow I_0=\frac{U_g}{R}$$

$$U_C(\infty)=\frac{q_\infty}{C}=U_g \Rightarrow q_\infty=C\,U_g$$

$$U_g - R\,I - \frac{q}{C}=0$$

II. Kirchoffov zakon

$$U_g - R\frac{dq}{dt} - \frac{q}{C}=0$$

$$RC\frac{dq}{dt}+q-C\,U_g=0$$

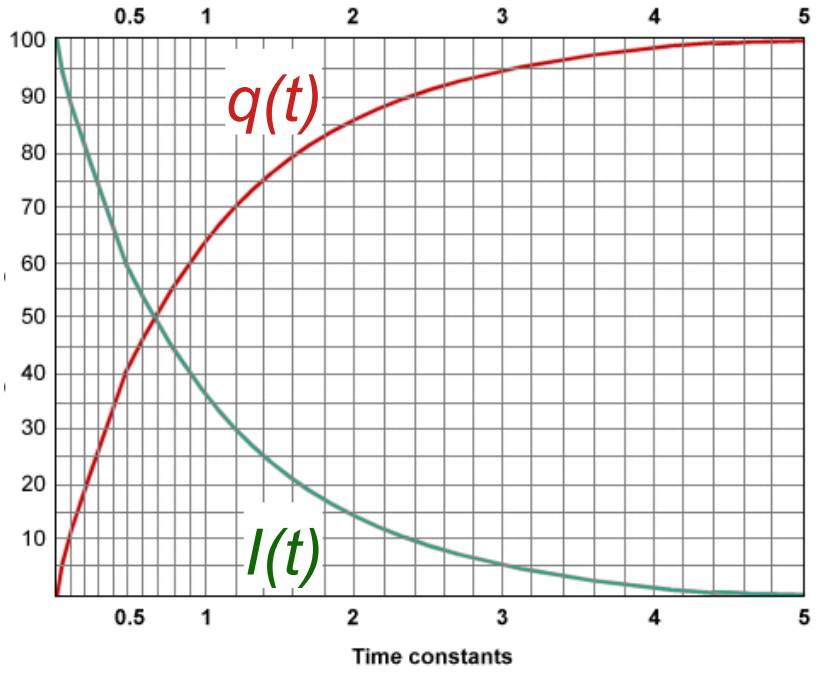
$$\frac{dq}{dt}=-\frac{1}{RC}(q-C\,U_g)$$

$$\int_0^q \frac{dq}{(q-q_\infty)}=-\int_0^t \frac{dt}{\tau}$$

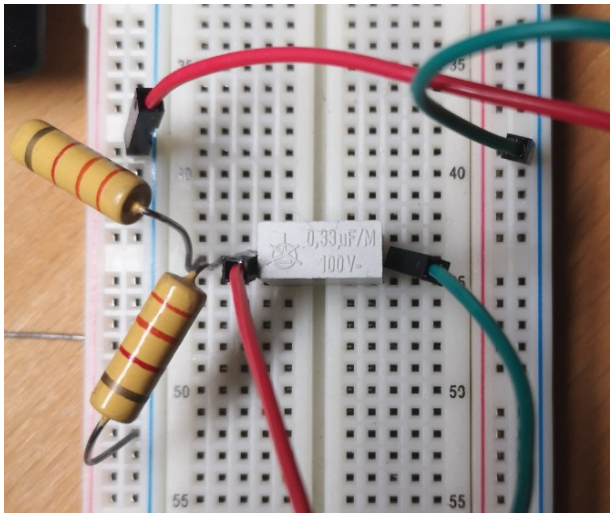
$$\ln\left(\left|\frac{q-q_\infty}{-q_\infty}\right|\right)=-\frac{t}{\tau}$$

$$q=q_\infty\left(1-e^{-\frac{t}{\tau}}\right)$$

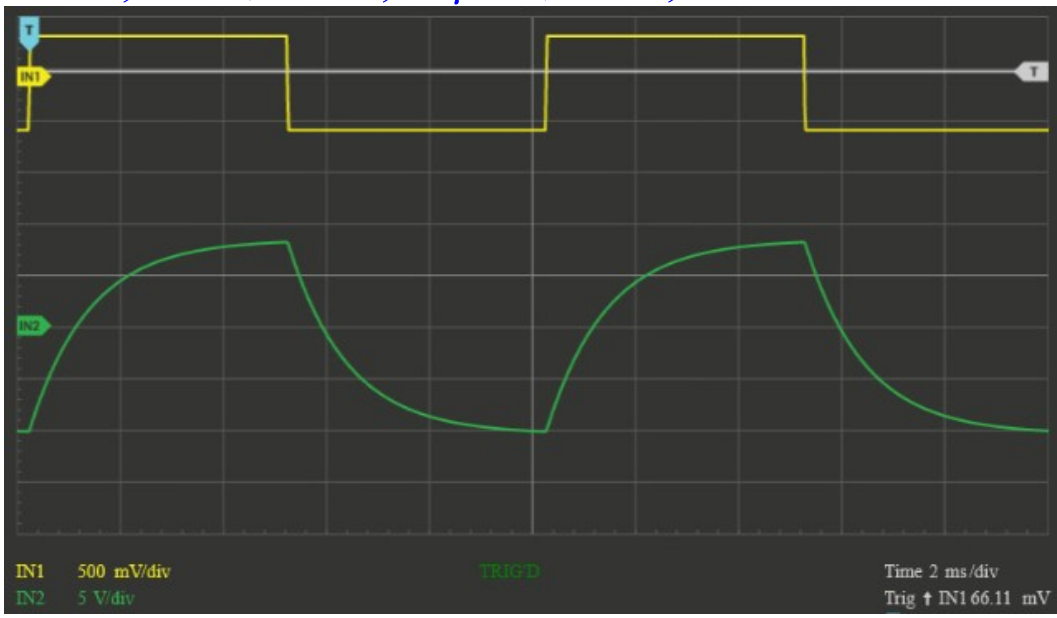
$$I=I_0\,e^{-\frac{t}{\tau}}$$



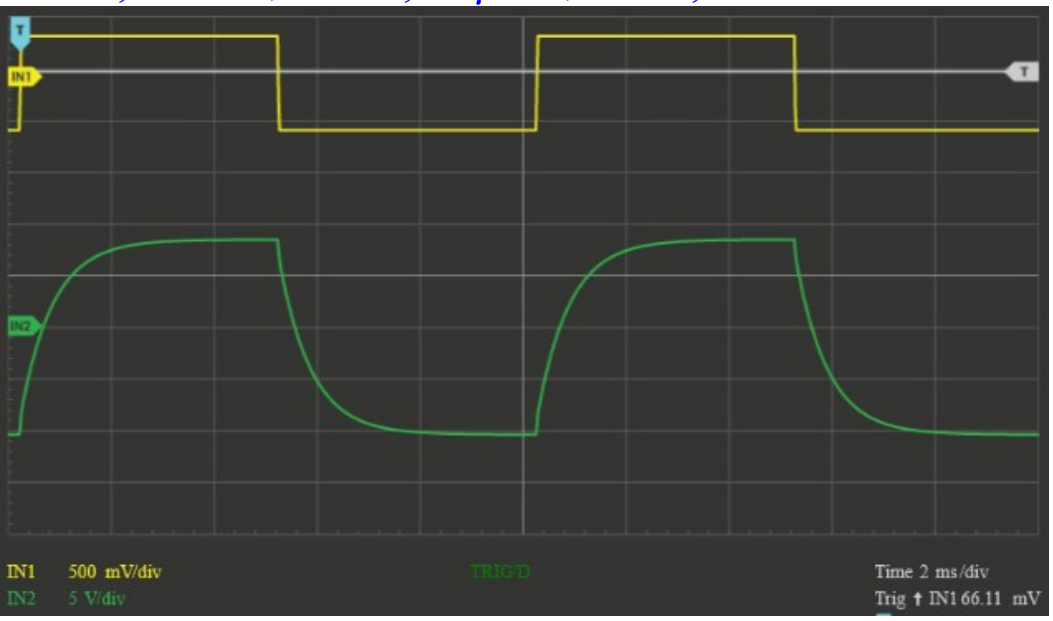
- časovni potek napetosti:
  - gonilni člen,
  - kondenzator



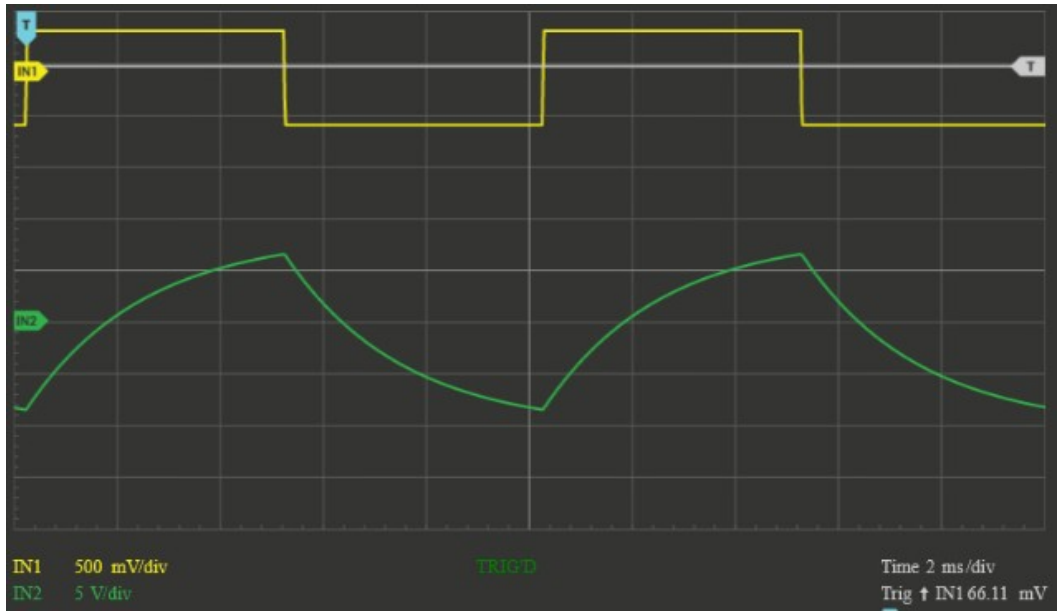
$R = 3{,}3\,k\Omega,\ C = 0{,}33\,\mu F,\ \tau \approx 1{,}1\,ms$



$R = 1{,}65\,k\Omega,\ C = 0{,}33\,\mu F,\ \tau \approx 0{,}55\,ms$



$R = 6{,}6\,k\Omega,\ C = 0{,}33\,\mu F,\ \tau \approx 2{,}2\,ms$

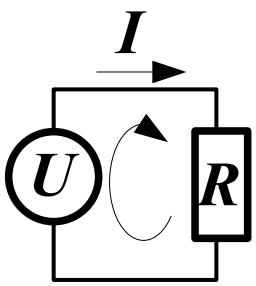


Merjenja v električnem krogu

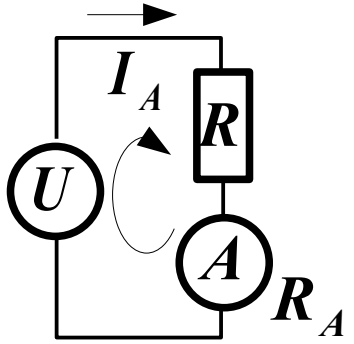
Ampermeter

- ampermeter vežemo zaporedno z elementom skozi katerega merimo tok
- upornost ampermetra mora biti precej manjša od upornosti elementa

$$R_A \ll R$$



$$U - R I = 0$$



$$U - R I_A - R_A I_A = 0$$

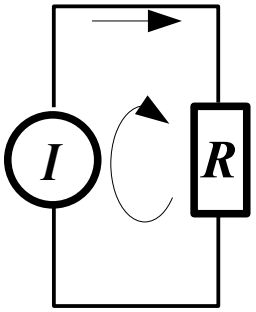
$$R I = R I_A + R_A I_A$$

$$I = I_A \left(1 + \frac{R_A}{R}\right)$$

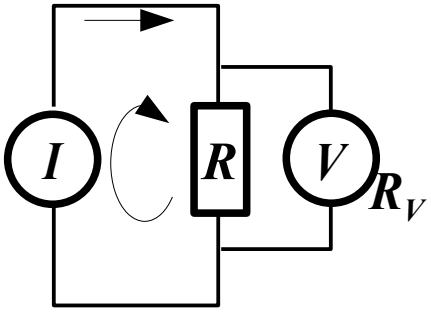
Voltmeter

- voltmeter vežemo vzporedno z elementom na katerem merimo napetost
- upornost voltmetra mora biti precej večja od upornosti elementa

$$R_V \gg R$$



$$U = R I$$

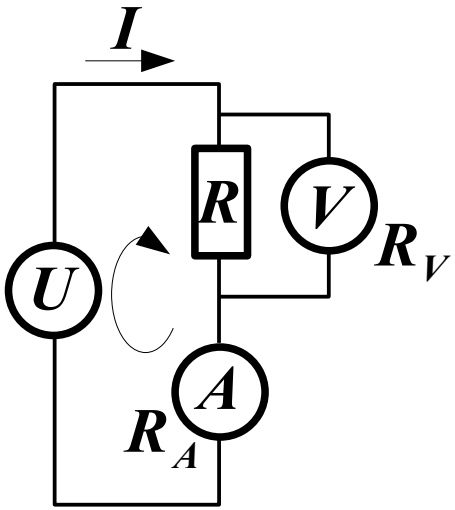


$$U_V = \frac{1}{\frac{1}{R} + \frac{1}{R_V}} I = \frac{1}{1 + \frac{R}{R_V}} R I$$

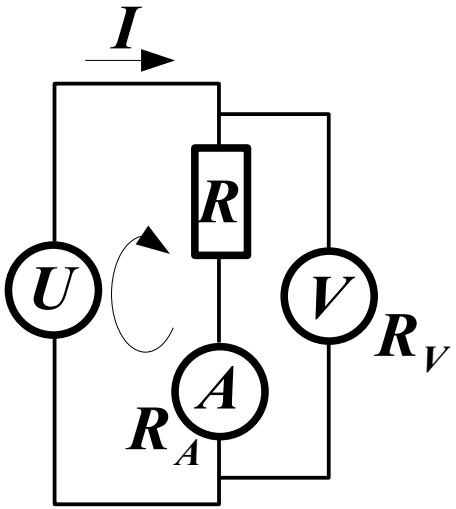
$$U_V = U \left(1 + \frac{R}{R_V}\right)^{-1}$$

Merjenje moči

• pri istočasnem merjenju toka in napetosti na elementu v električnem krogu lahko le enega izmerimo natančno



$$P_R = U_R I_R$$



$$U_R = U_V$$

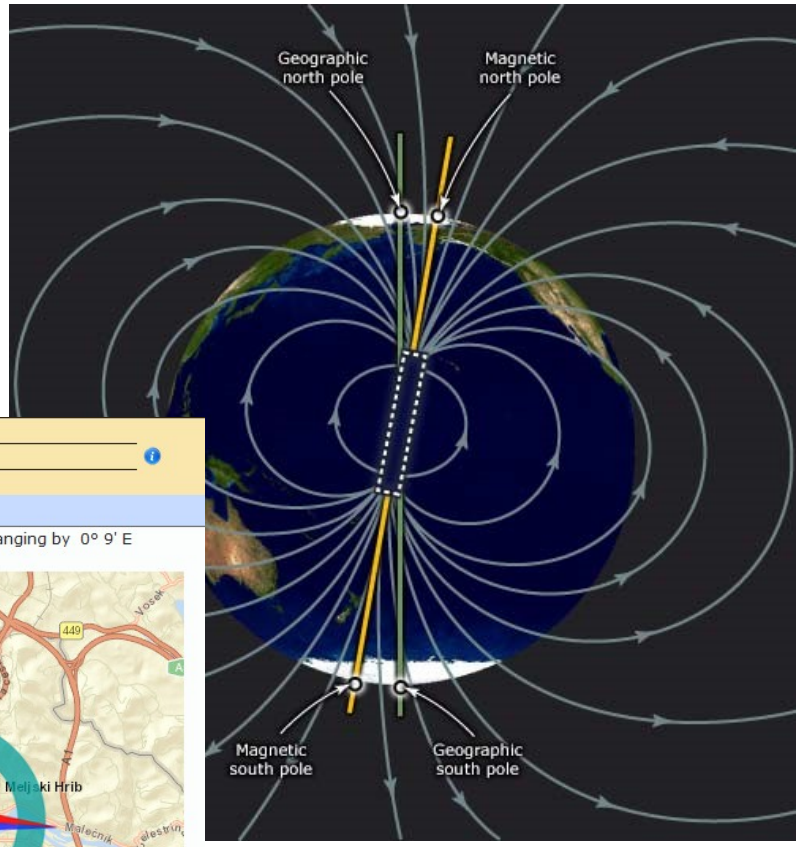
$$I_R = I_A - I_V = I_A - \frac{U_V}{R_V}$$

$$U_R = U_V - U_A = U_V - R_A I_A$$

$$I_R = I_A$$

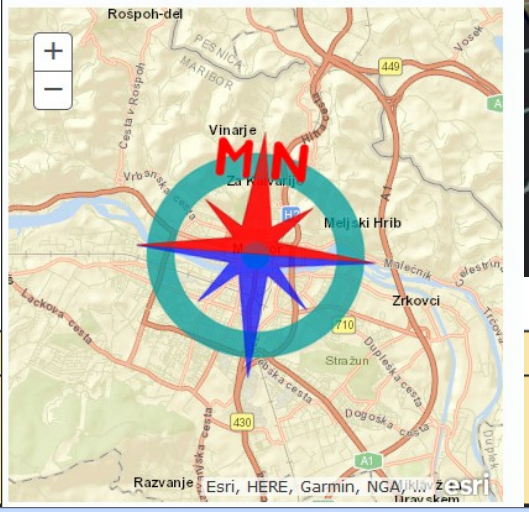
Magnetno polje trajnih magnetov in tokov

- v okolici trajnih magnetov in vodnikov s stacionarnimi tokovi delujejo sile ali navori na trajne magnete in vodnike
- v okolici je magnetno polje
- sile delujejo na telesa iz železa in nekaterih drugih kovin
- magnetnica – kompas in zemeljsko magnetno polje
- silnice magnetnega polja
- osnovni element – magnetni dipol



Model Used:	WMM-2020
Latitude:	46° 33' 31" N
Longitude:	15° 39' 3" E

Date	Declination
2021-02-24	4° 28' E ± 0° 22' changing by 0° 9' E per year

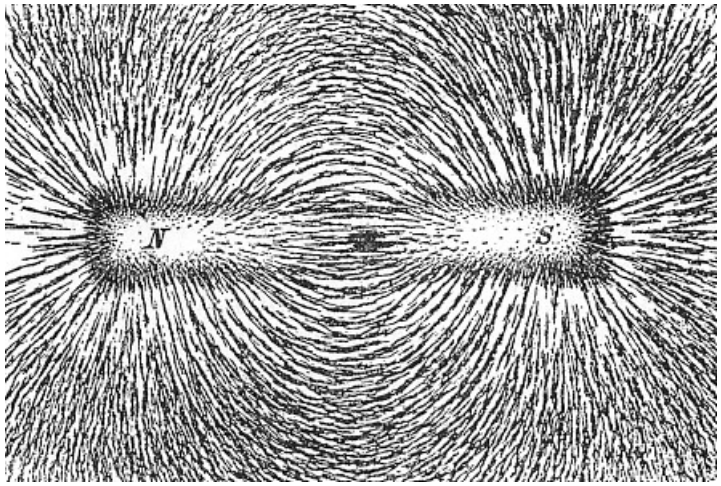


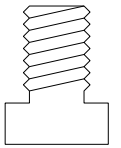
Model Used:	WMM-2020
Latitude:	46° 33' 27" N
Longitude:	15° 38' 44" E
Elevation:	0.3 km Mean Sea Level

Maribor

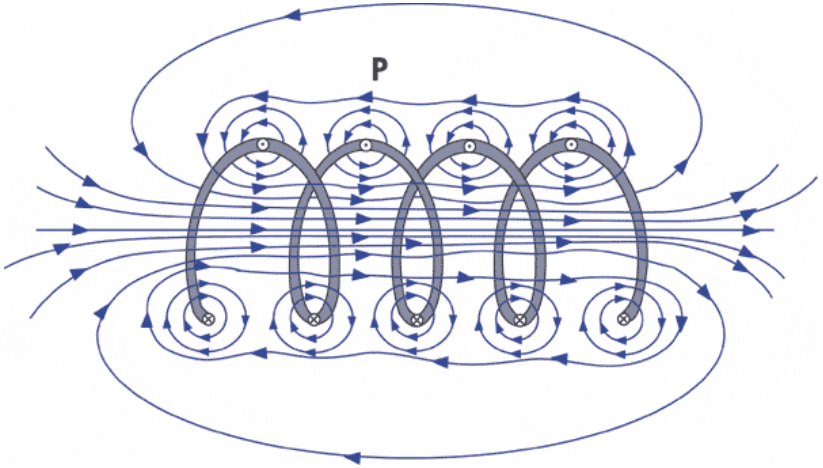
Date	Declination (+ E | - W)	Inclination (+ D | - U)	Horizontal Intensity	North Comp (+ N | - S)	East Comp (+ E | - W)	Vertical Comp (+ D | - U)	Total Field
2020-04-09	4° 20' 25"	63° 8' 49"	21,834.9 nT	21,772.2 nT	1,652.5 nT	43,126.5 nT	48,339.0 nT
Change/year	0° 8' 43"/yr	0° 1' 43"/yr	4.9 nT/yr	0.7 nT/yr	55.6 nT/yr	63.2 nT/yr	58.6 nT/yr
Uncertainty	0° 22'	0° 13'	128 nT	131 nT	94 nT	157 nT	145 nT

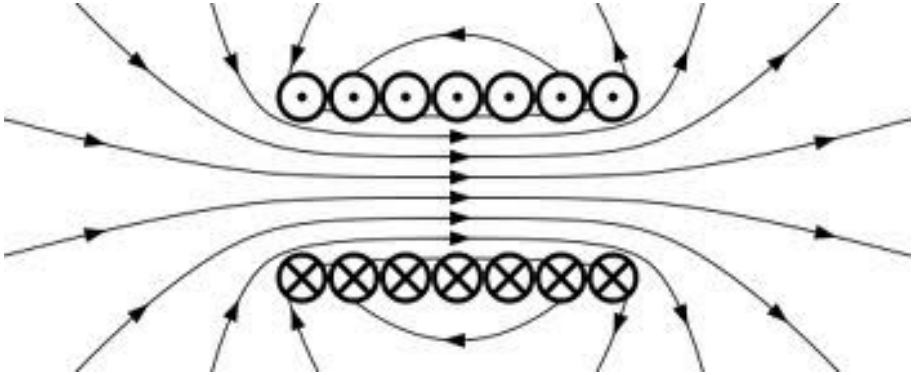
- silnice v okolici trajnega magneta, vodnika, tuljave in toroida

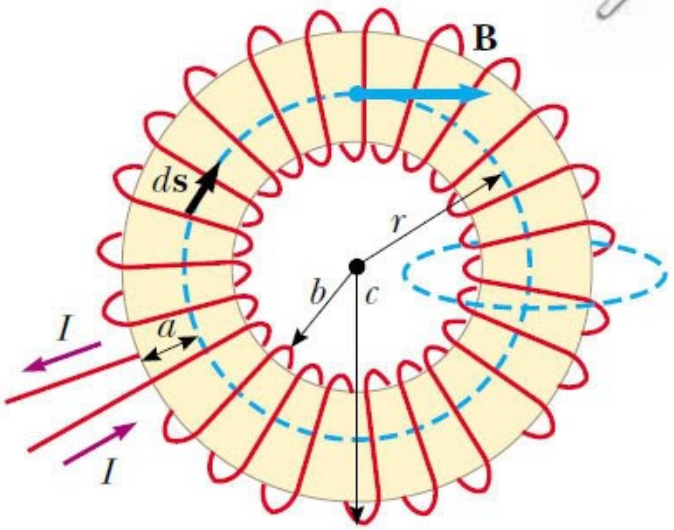


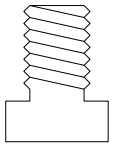


pravilo desnega vijaka





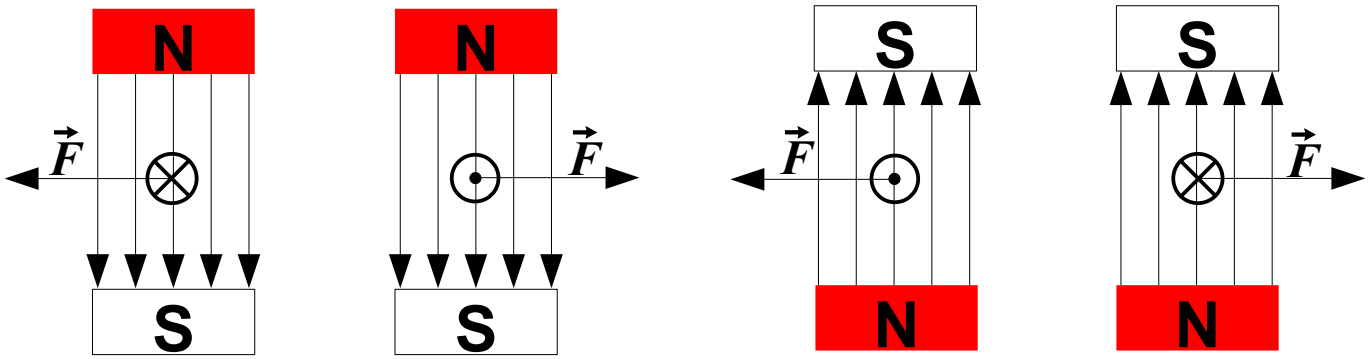




levi vijak

Sila na vodnik s tokom v magnetnem polju

• tanek raven vodnik in magnetno polje sta med seboj pravokotna – sila se pravokotna na oba



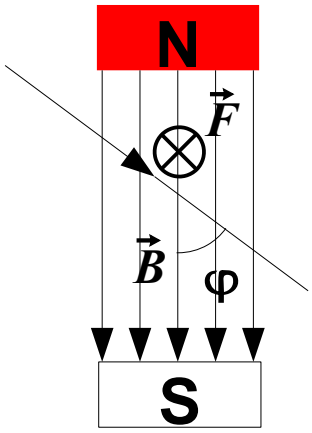
$$F = I\,l\,B \Rightarrow B = \frac{F}{I\,l} \quad \left[\frac{N}{Am} = \frac{Vs}{m^2} = T\right]$$ tesla

• smer tankega ravnega vodnika je poševno glede na magnetno polje – sila je pravokotna na oba

$$\vec{l} \perp \vec{B} \Rightarrow F = I\,l\,B$$

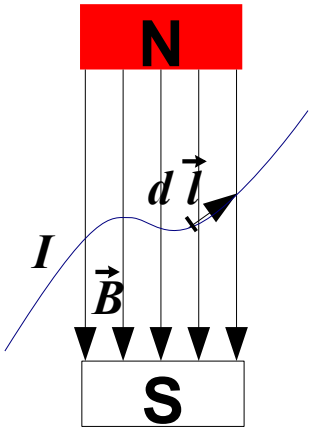
$$\vec{l} \parallel \vec{B} \Rightarrow F = 0$$

$$F = I\,l\,B\sin\varphi$$



$$\vec{F} = I\,\vec{l} \times \vec{B}$$

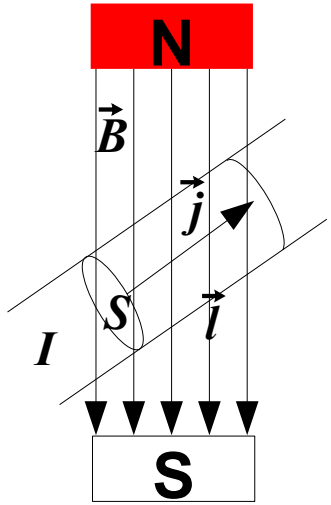
• tanek vodnik (ukrivljen ali nehomogeno polje) – razdelimo na kratke odseke, ki jih lahko obravnavamo kot ravne



$$d\vec{F} = I\, d\vec{l} \times \vec{B}$$

$$\vec{F} = I \int d\vec{l} \times \vec{B}$$

• debeli vodnik – razdelimo po preseku in dolžini vodnika na dele, ki jih lahko obravnavamo kot tanke in ravne



$$\vec{F} = I\,\vec{l} \times \vec{B} = j\,S\,\vec{l} \times \vec{B}$$

$$\vec{F} = S\,l\,\vec{j} \times \vec{B} = \vec{j} \times \vec{B}\,V$$

$$\vec{F} = \int \vec{j} \times \vec{B}\, dV$$

Sila na točkaste naboje – nosilce toka

$$j = \frac{I}{S} = \frac{q}{St} = \frac{v t S n e_0}{S t} = n v e_0$$

- sila na vodnik s tokom je posledica sile na posamezne naboje, ki sodelujejo pri toku

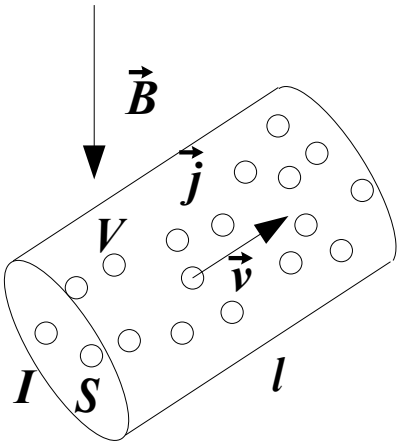
$$\vec{F} = \vec{j} \times \vec{B}\, V = n e_0 \vec{v} \times \vec{B}\, V$$

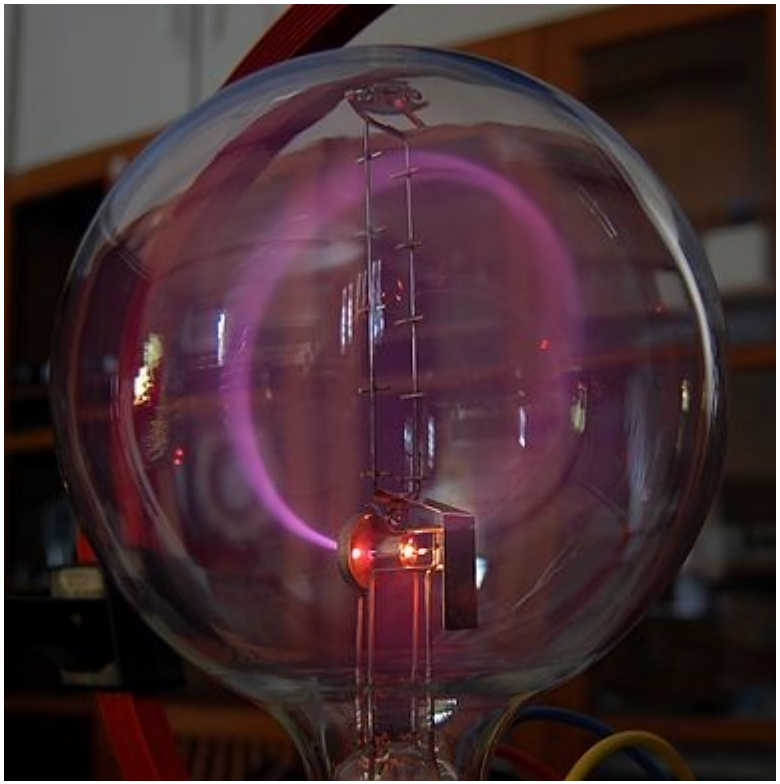
$$\vec{F} = N \vec{F}_0 = n V \vec{F}_0$$

$$\vec{F}_0 = q \vec{v} \times \vec{B}$$

$$\boxed{\vec{F}_{EM} = q \vec{E} + q \vec{v} \times \vec{B}}$$

Lorentzova sila

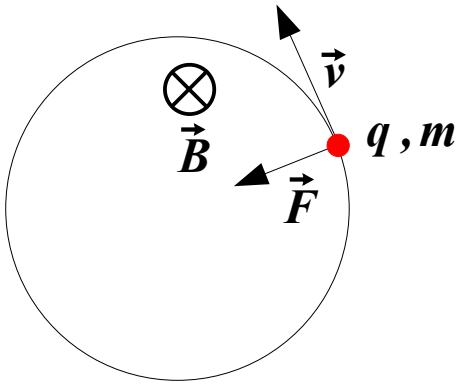




- v homogenem magnetnem polju se točkast naboj giblje po vijačnici

$$F = q v B = m \frac{v^2}{r}$$

$$r = \frac{G}{qB}$$



Hallov pojav

• posledica magnetna sile na naboje, ki sodelujejo pri toku, je električno polje, ki uravnovesi magnetno silo in ustvari el. napetost prečno na smer toka

$$\vec{F}_e + \vec{F}_m = 0$$

$$q E = q v_d B$$

$$U = E d = d v_d B$$

$$\boldsymbol{B = \frac{1}{d v_d} U \propto U}$$

merjenje magnetnega Polja – Hallova sonda

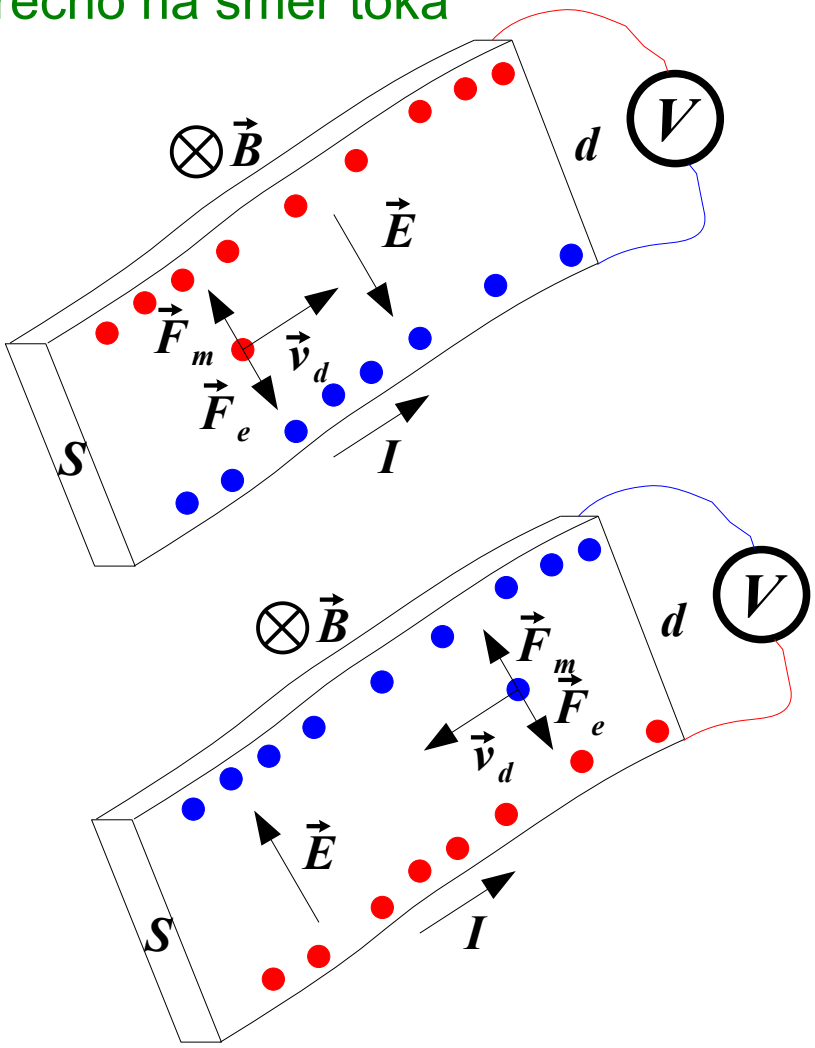
$$v_d = \frac{U}{B d}$$

merjenje povprečne hitrosti nosilcev toka

$$I = j S = n v_d e_0 S$$

$$n = \frac{I}{v_d e_0 S} = \frac{I B d}{U e_0 S}$$

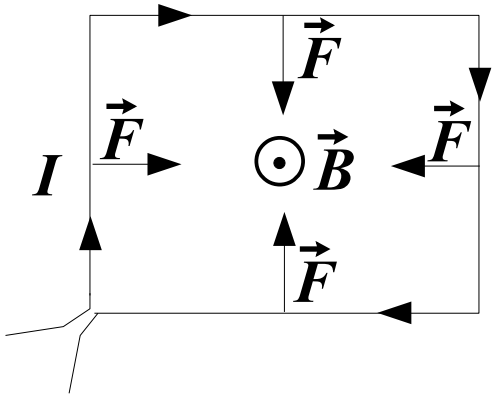
merjenje gostote nosilcev



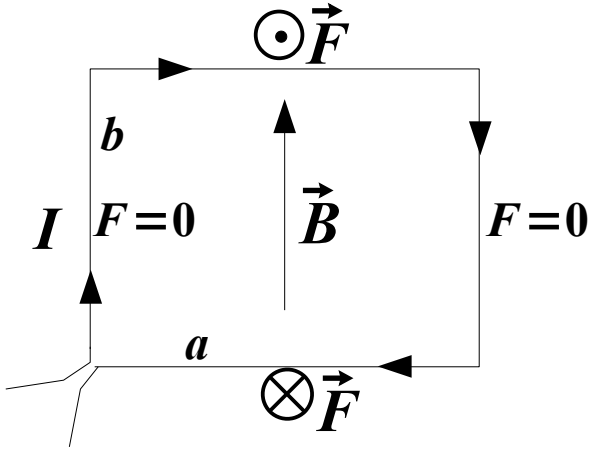
smer napetosti pove vrsto nosilca toka

Magnetni navor na tokovno zanko

- tokovna zanka v homogenem magnetnem polju



$$\sum \vec{F} = 0$$
$$\sum \vec{M} = 0$$



$$\sum \vec{F} = 0$$
$$\sum \vec{M} \neq 0$$
$$M = Fb = IaBb = ISB$$

$$M = ISB = p_m B$$

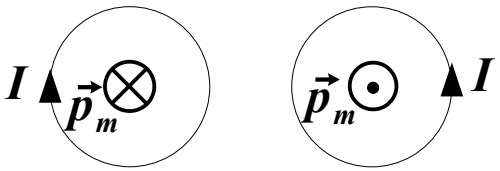
ena zanka

$$M = NISB = p_m B$$

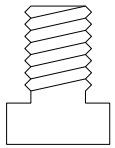
tuljava (N zank)

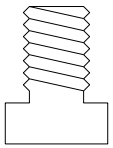
- magnetni dipolni moment $p_m [Am^2]$

$$\vec{p}_m = I \vec{S}$$



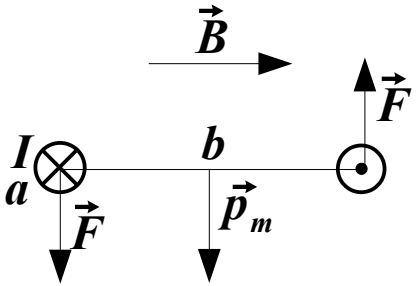
pravilo desnega vijaka

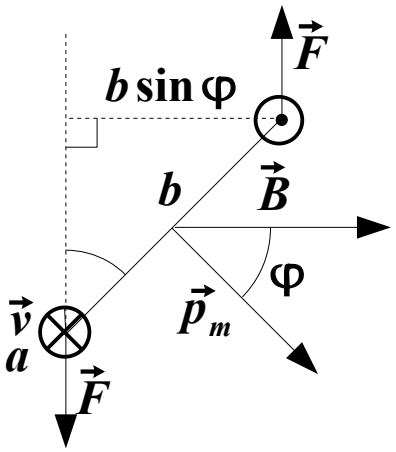




levi vijak

- navor na magnetni dipol v homogenem magnetnem polju





$$M = IaB\,b\sin(\varphi) = p_m B \sin(\varphi)$$

$$\sum \vec{F} = 0\,; \quad \vec{M} = \vec{p}_m \times \vec{B}$$

- energija dipola v homogenem magnetnem polju

$$A_m = \int M\,d\varphi = -\int p_m B \sin\varphi\,d\varphi = -[\,p_m B(-\cos\varphi) - p_m B(-\cos\varphi')]$$

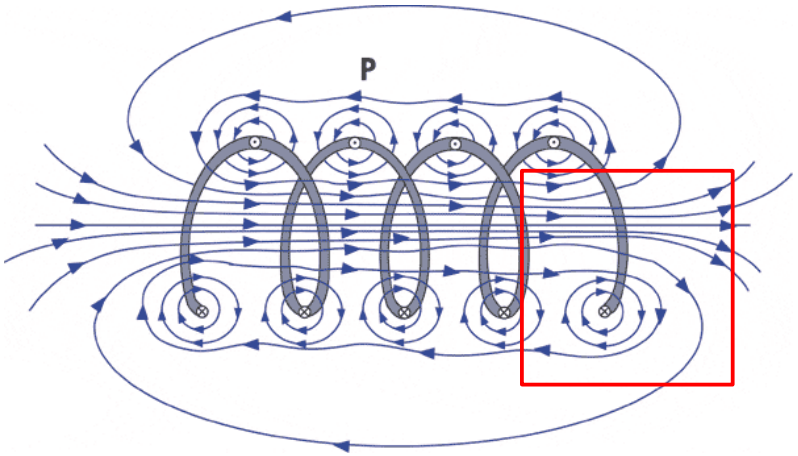
$$\Delta W_m = -A_m = (-p_m B\cos\varphi) - (-p_m B\cos\varphi') = W_m - W_m'$$

$$W_m = -\vec{p}_m \cdot \vec{B}$$

Zakon o magnetnem pretoku

$$\Phi_m = \int \vec{B} \cdot d\vec{S}$$

$$\oint \vec{B} \cdot d\vec{S} = 0$$

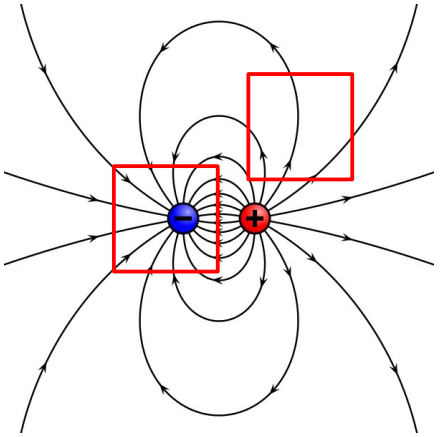


$$\Phi_e = \int \vec{D} \cdot d\vec{S}$$

električni pretok

$$\oint \vec{D} \cdot d\vec{S} = q$$

Gaussov zakon



Magnetni pretok skozi zaključeno ploskev je enak 0. (ni magnetih nabojev)

Izrek o magnetni napetosti po sklenjeni poti

• magnetna napetost – po zgledu električne

$$U_m = \int \vec{H} \cdot d\vec{r} = \frac{1}{\mu_0} \int \vec{B} \cdot d\vec{r}$$

$$U_e = -\int \vec{E} \cdot d\vec{r}$$

električna napetost

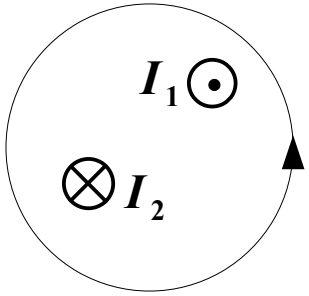
$$\oint \vec{E} \cdot d\vec{r} = 0$$

izrek o el. napetosti po sklenjeni poti

• magnetna napetost po sklenjeni poti je enaka objetemu električnemu toku.

$$\oint \vec{H} \cdot d\vec{r} = \frac{1}{\mu_0} \oint \vec{B} \cdot d\vec{r} = \sum_i I_i$$

• za smer toka velja pravilo desnega vijak (desne roke): tokove, ki tečejo v smer kamor bi lezel desni vijak, če ga vrtimo v smeri obhoda, štejemo pozitivno.

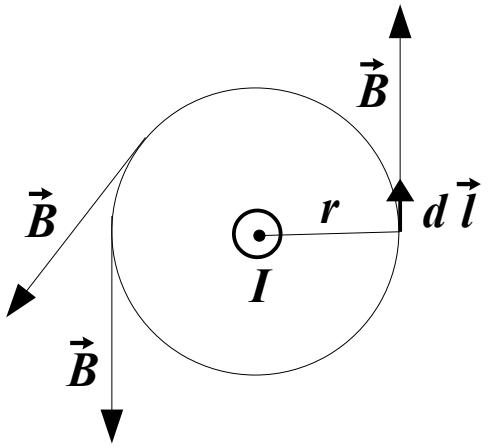


$$\frac{1}{\mu_0} \oint \vec{B} \cdot d\vec{r} = I_1 - I_2$$

• magnetno polje v okolici dolgega ravnega vodnika.

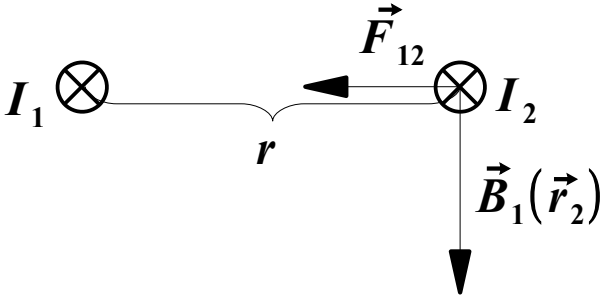
$$\frac{1}{\mu_0}\oint \vec{B}\cdot d\vec{l}=\frac{1}{\mu_0}\oint B\,dl=\frac{1}{\mu_0}B\oint dl=I$$

$$\frac{2\pi r B}{\mu_0}=I \Rightarrow \quad B=\frac{\mu_0 I}{2\pi r}$$



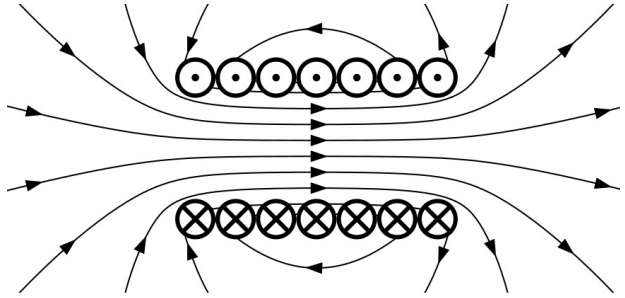
• definicija Ampera (stara) – enote električnega toka:

Po dveh vzporednih neskončnih ravnih vodnikih, ki sta oddaljena 1m, teče tok 1A, ko je sila na 1m vodnika enaka $2\text{x}10^{-7}$ N.



$$F_{12}=I_2|\vec{l}_2\times\vec{B}_1|=\frac{\mu_0 I_1 I_2 l_2}{2\pi r}$$

• primeri - magnetno polje:
  - v notranjosti debele žice
  - v notranjosti dolge tanke tuljave
  - v notranjosti toroida



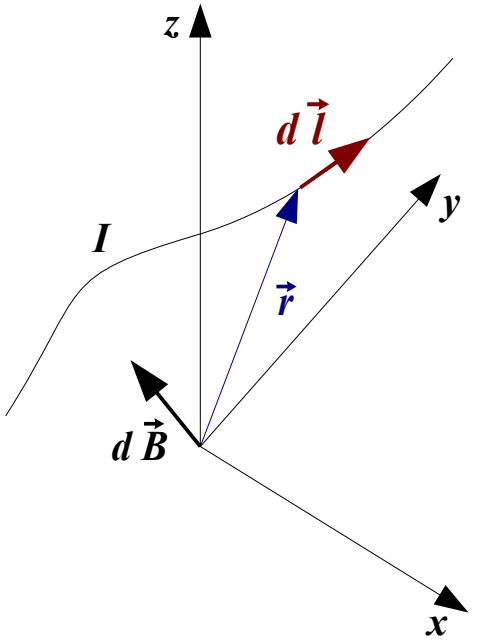
Biot-Savartov zakon

$$d\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \cdot dq \frac{(\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}$$

$$d\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \cdot I\, d\vec{l} \times \frac{(\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}$$

- če računamo polje v izhodišču $\vec{r} = 0$

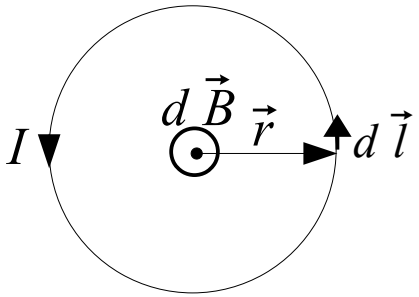
$$\vec{B}(\vec{0}) = \frac{\mu_0}{4\pi} I \int \frac{\vec{r} \times d\vec{l}}{r^3}$$



Primeri - magnetno polje:

- v sredini okrogle zanke
- na osi okrogle zanke
- v sredini tuljave
- v okolici ravnega vodnika

- v sredini okrogle zanke

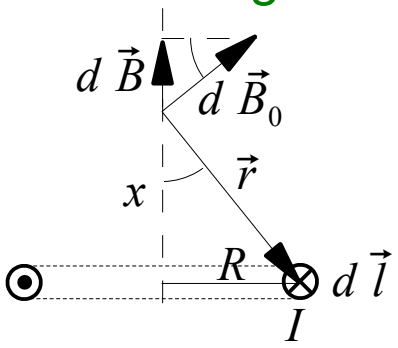


$$d\vec{B}=\frac{\mu_0}{4\pi}I\frac{\vec{r}\times d\vec{l}}{r^3}$$

$$dB=\frac{\mu_0}{4\pi}I\frac{r\,dl}{r^3}=\frac{\mu_0 I}{4\pi r^2}dl$$

$$B=\frac{\mu_0 I}{4\pi r^2}\int dl=\frac{\mu_0 I}{4\pi r^2}2\pi r=\frac{\mu_0 I}{2r}$$

- na osi okrogle zanke

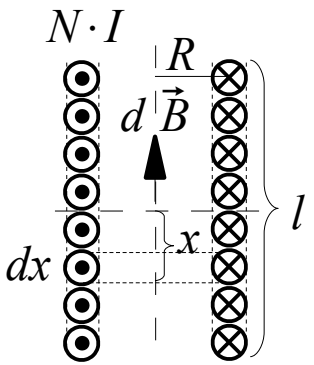


$$\frac{dB}{dB_0}=\frac{R}{r}=\frac{R}{\sqrt{x^2+R^2}}\qquad dB_0=\frac{\mu_0 I}{4\pi r^2}dl$$

$$\int dB=B=\frac{R}{\sqrt{x^2+R^2}}\cdot\frac{\mu_0 I}{4\pi\left(x^2+R^2\right)}\int dl=\frac{\mu_0 I}{4\pi}\cdot\frac{2\pi R^2}{\left(x^2+R^2\right)^{\frac{3}{2}}}$$

$$B\approx\frac{\mu_0}{4\pi}\cdot\frac{2p_m}{x^3},\quad x\gg R$$

- v sredini tuljave



$$dB=\frac{\mu_0 dI}{4\pi}\frac{2\pi R^2}{\left(x^2+R^2\right)^{\frac{3}{2}}},\quad dI=\frac{N\cdot I}{l}dx$$

$$B=\int_{-\frac{l}{2}}^{\frac{l}{2}}\frac{\mu_0 N I}{2l}\frac{R^2}{\left(x^2+R^2\right)^{\frac{3}{2}}}dx$$

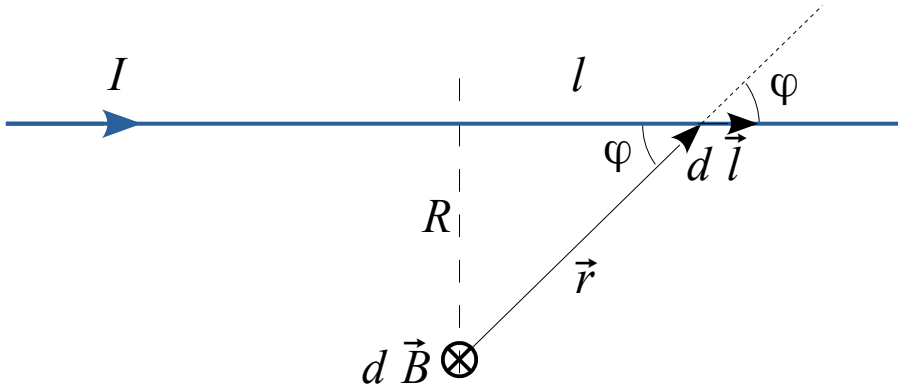
$$B=\frac{\mu_0 N I}{l}\frac{1}{\sqrt{1+\left(\frac{2R}{l}\right)^2}}\xrightarrow[R\ll l]{}\frac{\mu_0 N I}{l}$$

- neskončen raven vodnik

$$d\vec{B}=\frac{\mu_0 I}{4\pi}\frac{\vec{r}\times d\vec{l}}{r^3}$$

$$dB=\frac{\mu_0 I}{4\pi}\frac{r\sin\varphi\, dl}{r^3}=-\frac{\mu_0 I}{4\pi R}\sin\varphi\, d\varphi$$

$$B=-\frac{\mu_0 I}{4\pi R}\int_{\pi}^{0}\sin\varphi\, d\varphi=\frac{\mu_0 I}{4\pi R}\cos\varphi\Big|_{\pi}^{0}=\frac{\mu_0 I}{2\pi R}$$



$$l=R\,ctg\,\varphi,\quad dl=-\frac{R}{\sin^2\varphi}d\varphi$$

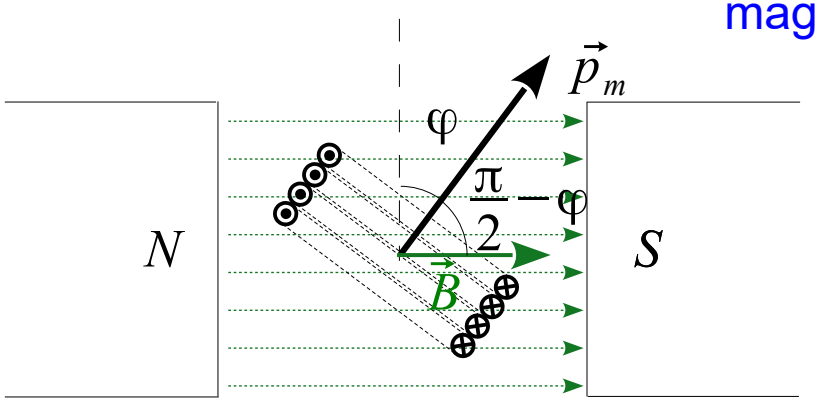
$$\sin\varphi=\frac{R}{r},\quad r=\frac{R}{\sin\varphi}$$

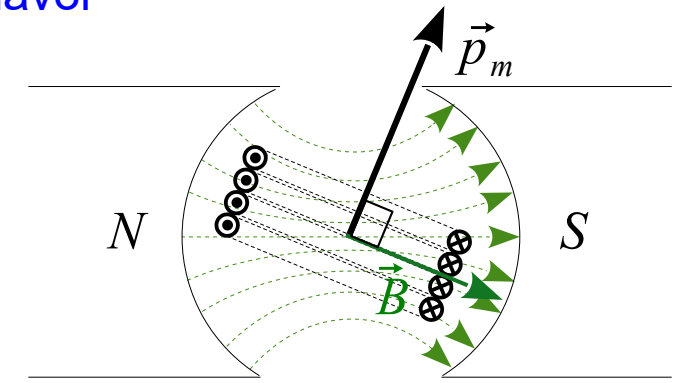
Galvanometer

- galvanometer na vrtljivo tuljavo

$$\vec{M} = \vec{p}_m \times \vec{B}$$

magnetni navor

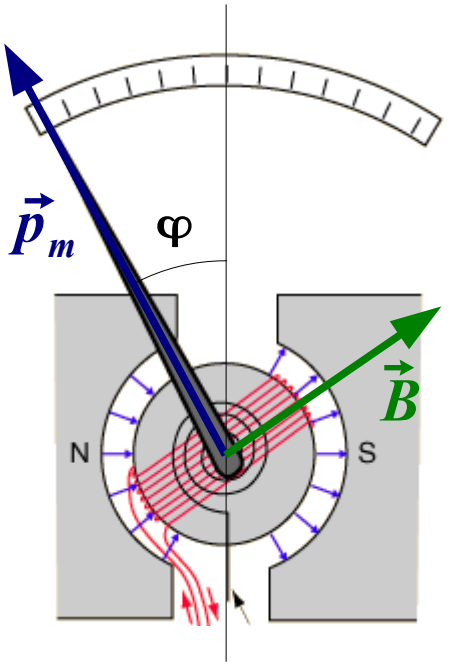




$$M = p_m B \sin\left(\frac{\pi}{2} - \varphi\right) = N\,I\,S\,B\cos\varphi \qquad M = p_m B = N\,I\,S\,B$$

$$M = D\varphi$$

navor polžaste vzmeti



- statična občutljivost C_0 – navora se izenačita

$$M = p_m B = N\,I\,S\,B$$

$$N\,I\,S\,B = D\varphi \;\Rightarrow\; I = \frac{D}{N\,S\,B}\varphi = C_0\varphi$$

• balistično merjenje, dinamična občutljivost – sunek magnetnega navora zavrti tuljavico, začetna kinetična energija se pretvori v energijo vzmeti

$$\int M\,dt = \Delta\Gamma$$

$$\int M\,dt = N\,S\,B\int I\,dt = N\,S\,B\,q = J\,\omega$$

$$\frac{1}{2}J\,\omega^2 = \frac{1}{2}D\,\varphi_0^2$$

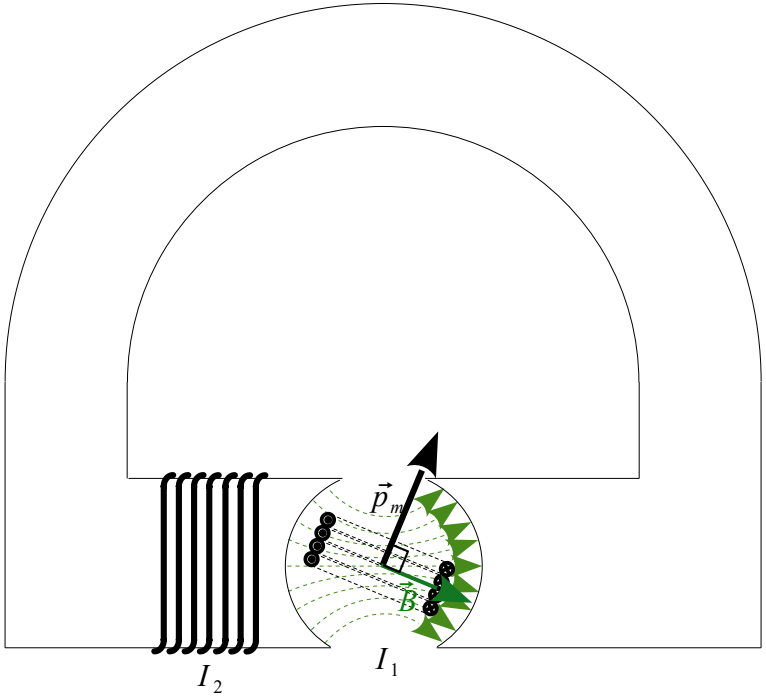
$$\int I\,dt = q = \frac{\sqrt{J\,D}}{N\,S\,B}\varphi_0 = C\,\varphi_0$$

$$J\,\omega = \sqrt{J\,D}\,\varphi_0$$

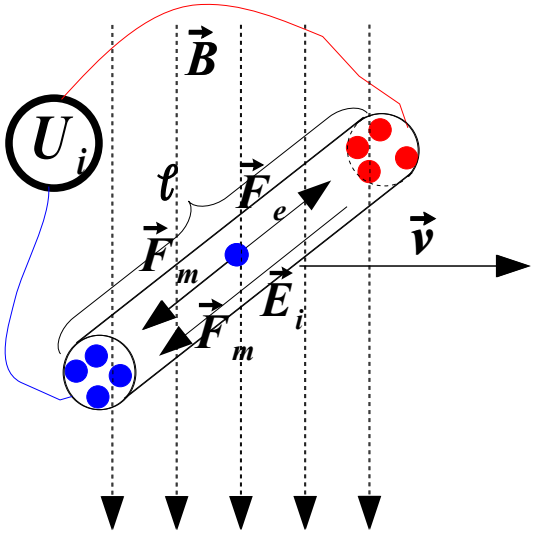
• elektrodinamični instrument

$$M = p_{m1}\,B_2 = N_1\,I_1\,S_1\,\frac{\mu_0\,N_2\,I_2}{l_2}$$

$$I_1 \cdot I_2 \propto \varphi$$



Indukcija – gibanje vodnika v magnetnem polju



$$\vec{F}_{EM} = q\vec{E} + q\vec{v}\times\vec{B}$$

- magnetna sila na gibljive naboje v prevodniku povzroči prerazporejanje nabojev in nastanek električnega polja, ki uravnovesi silo

$$\vec{F}_e + \vec{F}_m = 0 \;\Rightarrow\; F_e = F_m$$

$$\cancel{q}E_i = \cancel{q}vB$$

$$U_i = E_i l = vBl$$

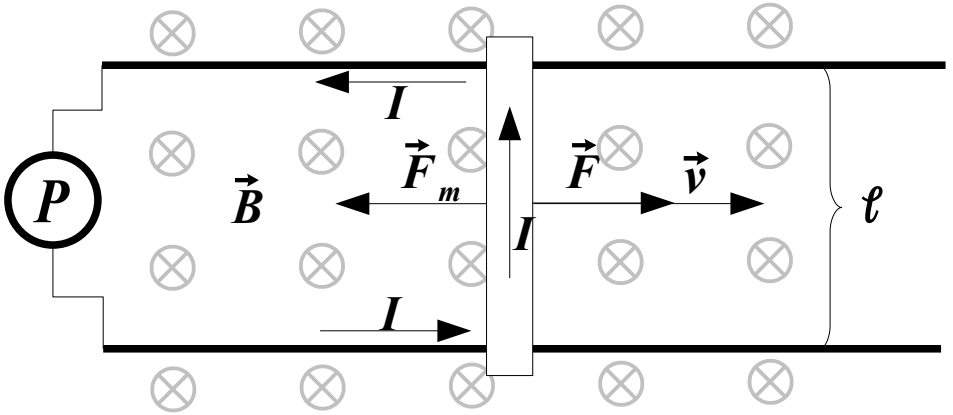
- Lentzovo pravilo: inducirana napetost požene tok tako, da magnetna sila na induciran tok nasprotuje gibanju vodnika.
- če električni krog sklenemo, inducirana napetost požene tok po zanki

$$\vec{F} = -\vec{F}_m \qquad \vec{F}_m = I\vec{l}\times\vec{B}$$

$$P = U_i I = \vec{F}\cdot\vec{v} = -I(\vec{l}\times\vec{B})\cdot\vec{v}$$

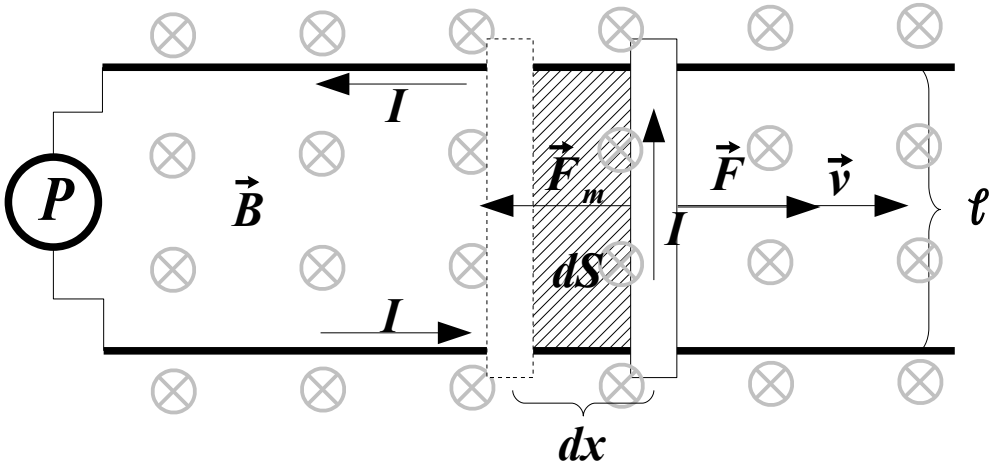
$$U_i = \vec{v}\cdot(\vec{B}\times\vec{l})$$

$$U_i = \int \vec{v}\cdot(\vec{B}\times d\vec{l})$$



Indukcija – spreminjanje magnetnega polja

• inducirano napetost lahko zapišemo tudi v obliki časovnega spreminjanja magnetnega pretoka zaradi spreminjanja ploščine zanke
• izkaže se, da ta zapis velja tudi za primer, ko je zanka pri miru in se spreminja magnetno polje



$$U_i = \frac{d\Phi_m}{dt}$$

$$U_i = v\,B\,l = \frac{dx}{dt}\,B\,l = \frac{B\,dS}{dt}$$

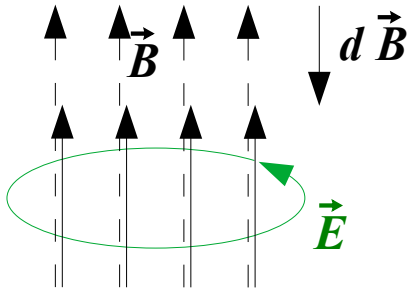
• Lentzovo pravilo: magnetno polje induciranega toka ima nasprotno smer kot prvotno polje, če magnetni pretok narašča, in enako smer, če magnetni pretok upada.

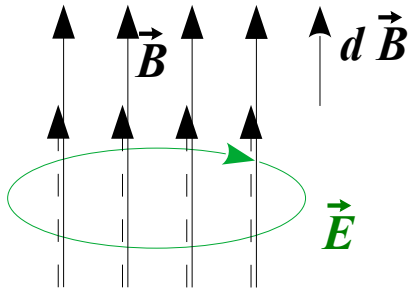
Indukcijski zakon

• inducirana napetost in električno polje se pojavita tudi, ko v prostoru ni vodnikov

$$-\oint \vec{E}\cdot d\vec{r} = 0$$

izrek o električni napetosti





$$-\oint \vec{E}\cdot d\vec{r} = U_i = \frac{d\Phi_m}{dt}$$

Maxwellove enačbe

- enačbe elektromagnetnega polja, ki izkazujejo simetrijo med električnimi in magnetnimi pojavi – glavna razlika je v manjkajočih magnetnih nabojih in njihovih tokovih

$$\oint \vec{D} \cdot d\vec{S} = q$$

Gaussov zakon

$$\oint \vec{B} \cdot d\vec{S} = 0$$

Zakon o magnetnem pretoku

$$\oint \vec{H} \cdot d\vec{r} = \frac{d\Phi_e}{dt} + \sum_i I_i$$

Amperov zakon

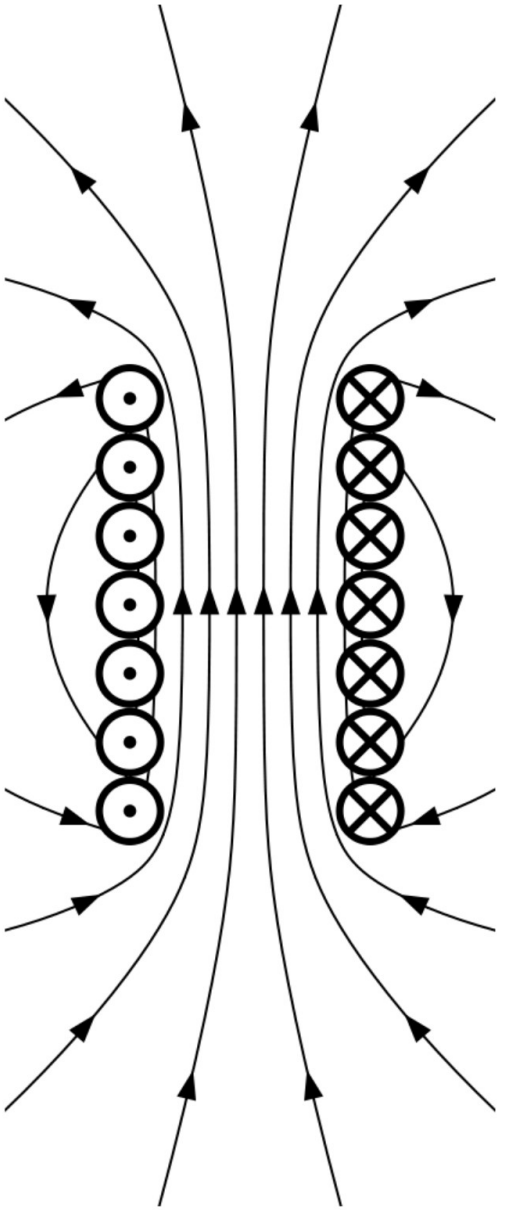
$$-\oint \vec{E} \cdot d\vec{r} = \frac{d\Phi_m}{dt}$$

Faradayev zakon - indukcijski zakon

$$\oint \vec{j} \cdot d\vec{S} = -\frac{dq}{dt}$$

ohranitev naboja, kontinuitetna enačba

Induktivnost tuljave



• lastna induktivnost tuljave povezuje tok po tuljavi z magnetnim pretokom v tuljavi, ki ga ta tok ustvari

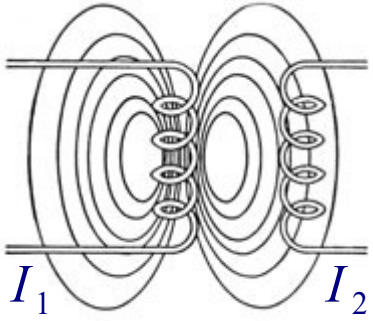
$$\Phi_m = N\,B\,S = \frac{N^2 \mu_0 I S}{l} = L I$$

$$\boldsymbol{\Phi_m = L I}$$

$$L = \frac{N^2 \mu_0 S}{l} \qquad L = \frac{N^2 \mu_0 S}{2\pi \overline{R}}$$

tuljava toroid

• vzajemna induktivnost – tok po eni tuljavi ustvari magnetno polje, ki ustvarja magnetni pretok skozi drugo tuljavo



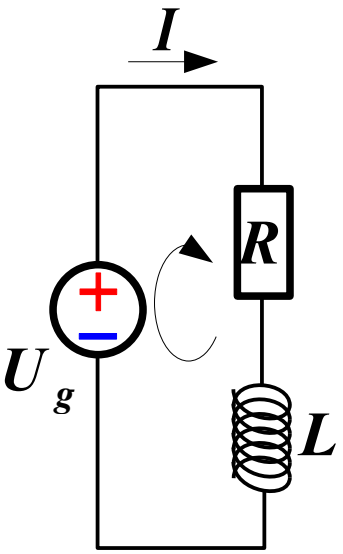
$$\Phi_{m,2} = L_{22} I_2 + L_{21} I_1$$

$$\Phi_{m,1} = L_{11} I_1 + L_{12} I_2$$

$$\boldsymbol{L_{12} = L_{21}}$$

Električni krog s tuljavo

- naraščanje toka v električnem krogu s tuljavo



$$U_g - R\,I - L\frac{dI}{dt} = 0$$

$$-R\left(I - \frac{U_g}{R}\right) = L\frac{dI}{dt}$$

$$-\frac{R}{L}dt = \frac{dI}{I - \frac{U_g}{R}}$$

$$-\int_0^t \frac{dt}{\tau} = \int_0^I \frac{dI}{I - I_\infty}$$

$$-\frac{t}{\tau} = \ln\left|\frac{I - I_\infty}{-I_\infty}\right|$$

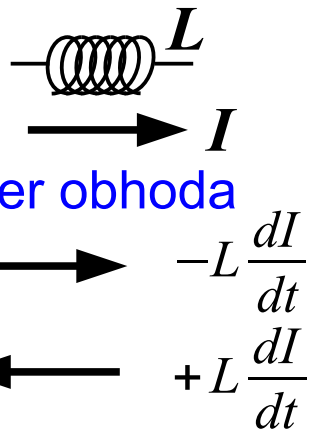
$$e^{-\frac{t}{\tau}} = \frac{I - I_\infty}{-I_\infty}$$

$$I = I_\infty\left(1 - e^{-\frac{t}{\tau}}\right)$$

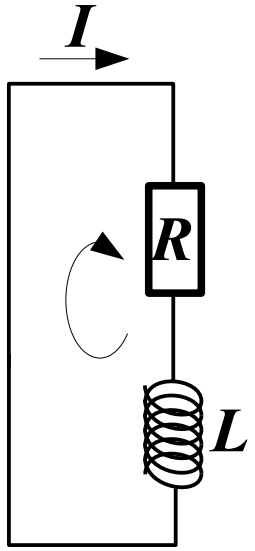
$$U_L = -L\frac{dI}{dt} = -U_g e^{-\frac{t}{\tau}}$$

$$\tau = \frac{L}{R}$$

$$I_\infty = \frac{U_g}{R}$$



- pojemanje toka v el. krogu s tuljavo



$$U_0 = I_0 R$$

$$-R\,I - L\frac{dI}{dt} = 0$$

$$I = I_0 e^{-\frac{t}{\tau}}$$

$$U_L = -L\frac{dI}{dt} = -U_0 e^{-\frac{t}{\tau}}$$

Energija tuljave

- električno delo generatorja se pretvori v energijo tuljave

$$A_e = \int_0^t P\,dt = \int_0^t U\,I\,dt = \int_0^t L\frac{dI}{dt}\,I\,dt = L\int_{I'}^{I} I dI = \frac{1}{2}L\,I^2 - \frac{1}{2}L\,I'^2$$

$$W_L = \frac{1}{2}L\,I^2 = \frac{1}{2}\frac{\Phi_m^2}{L}$$

Gostota energije magnetnega polja

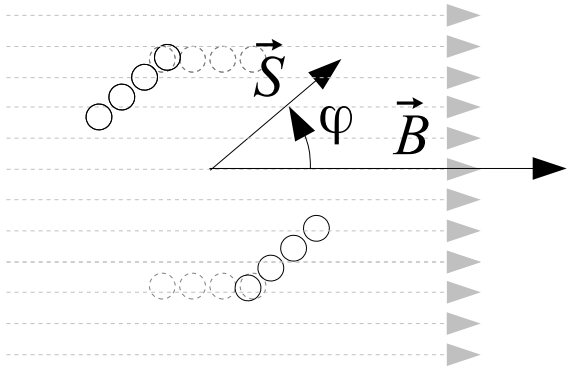
- energija tuljave je shranjena v magnetnem polju tuljave

$$W_L = \frac{1}{2}\frac{\Phi_m^2}{L} = \frac{1}{2}\frac{(NBS)^2}{L} = \frac{1}{2}\frac{N^2 B^2 S^2 l}{\mu_0 N^2 S} = \frac{1}{2}\frac{B^2 S l}{\mu_0} = \frac{1}{2}\frac{B^2}{\mu_0}V$$

$$w_m = \frac{W_L}{V} = \frac{1}{2}\frac{B^2}{\mu_0} = \frac{1}{2}H\,B$$

Generator

- pri vrtenju tuljave tuljave v magnetnem polju se v njej inducira sinusna izmenična napetost

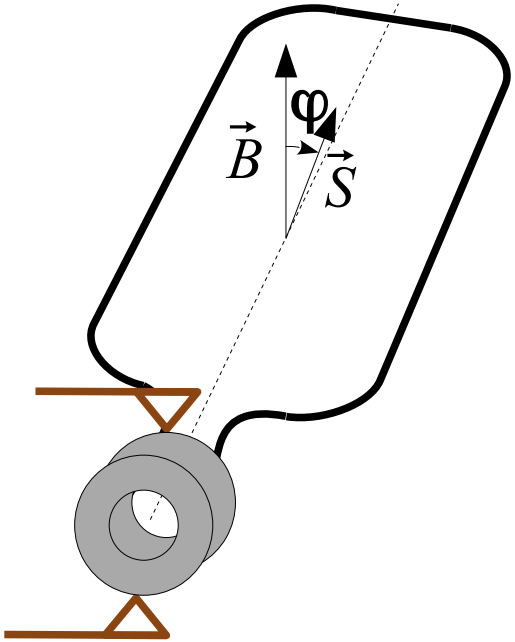


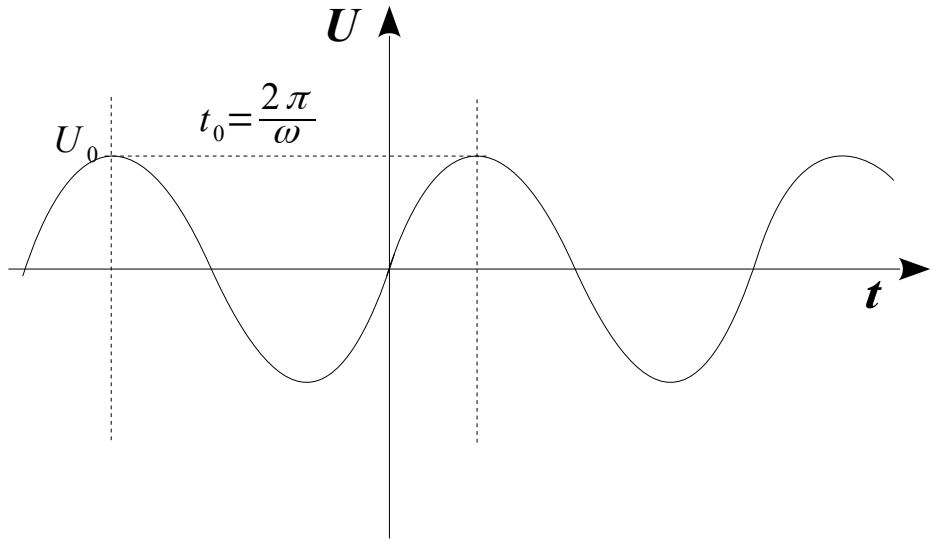
$$\Phi_m = \vec{B}\cdot\vec{S} = B\,S\cos\phi = B\,S\cos\omega t$$

$$U_i = \frac{d\,\Phi_m}{dt} = -N\,B\,S\,\omega\sin\omega t = -U_0\sin\omega t$$

$$U_i = U_0\cos(\omega t + \delta)$$

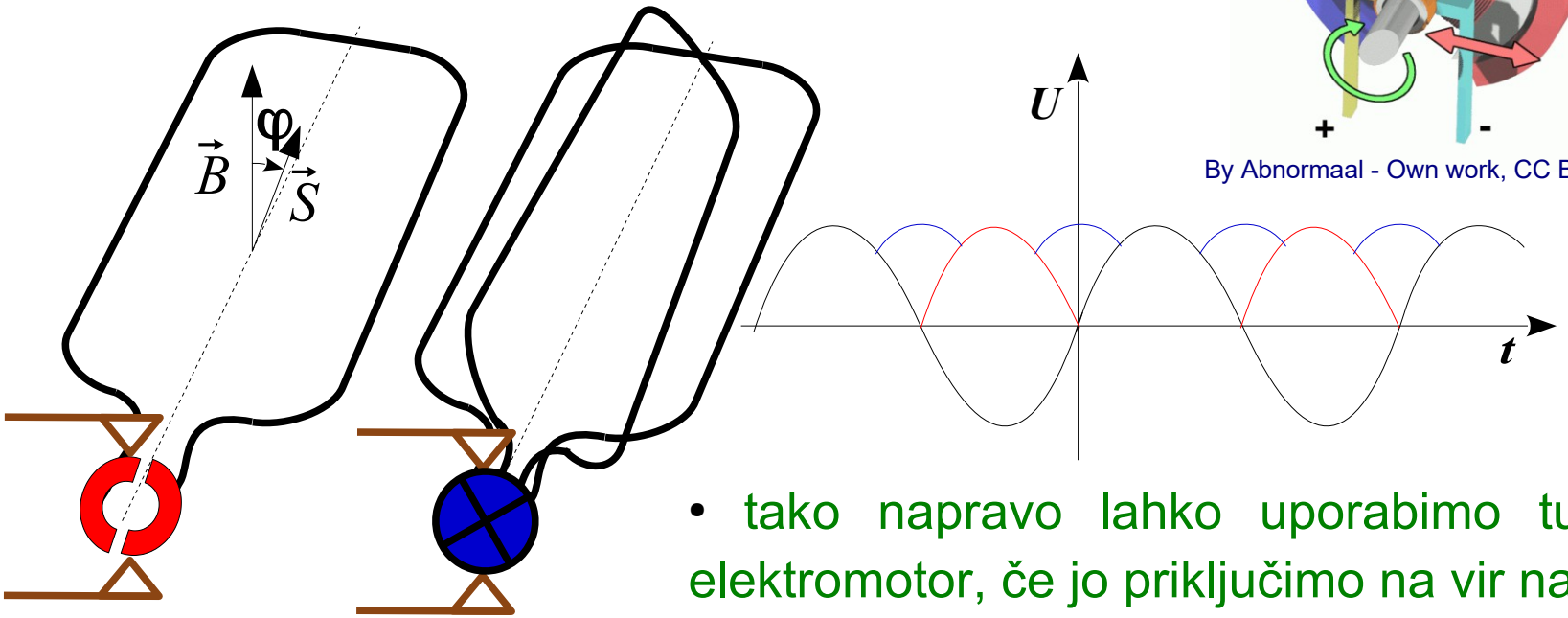
$$U_0 = N\,B\,S\,\omega$$

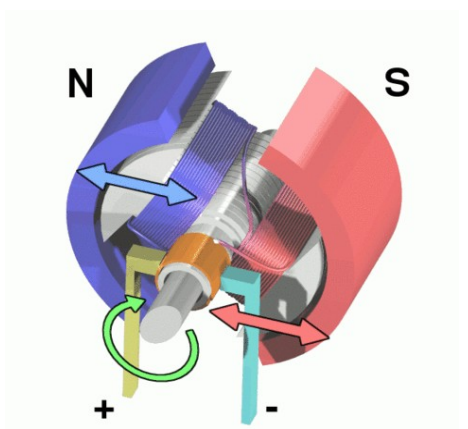




Dinamo stroj in elektromotor

• z menjavanjem polov generatorja pri vrtenju lahko generiramo enosmerno napetost





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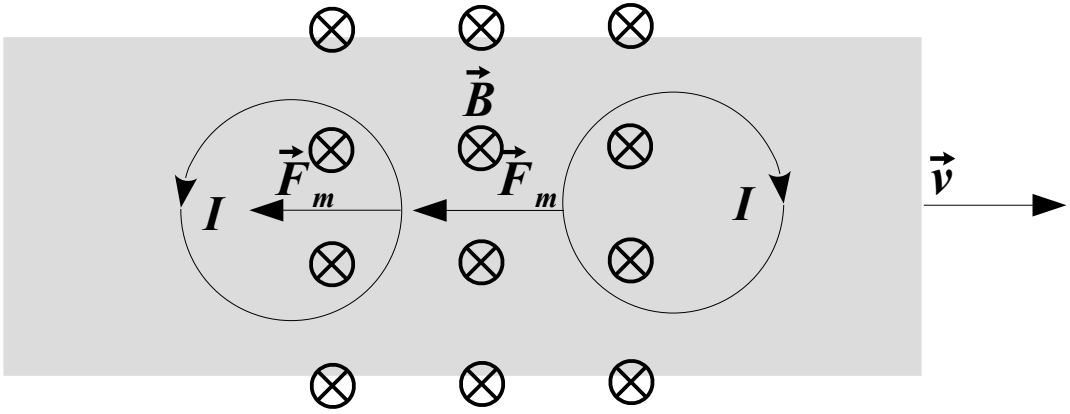
• tako napravo lahko uporabimo tudi kot elektromotor, če jo priključimo na vir napetosti

Vrtinčni tokovi



• vrtinčni tokovi se pojavijo v prevodniku, ki se premika po nehomogenem magnetnem polju – skozi rob območja s poljem

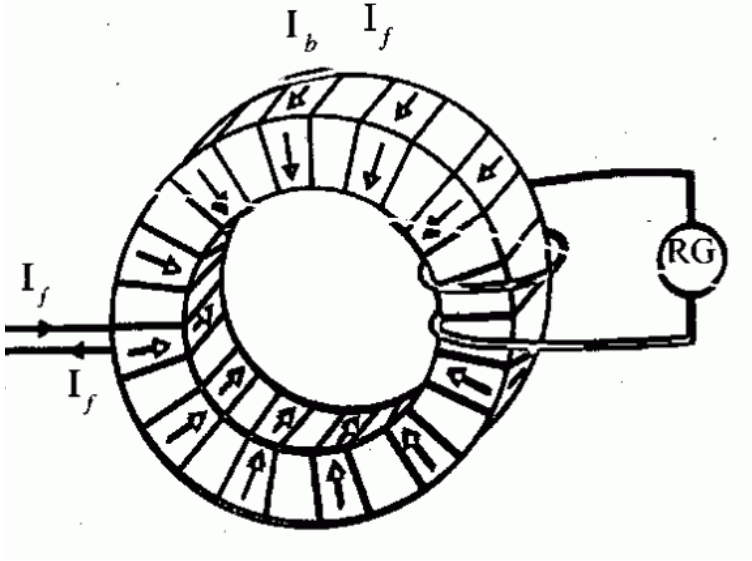
• magnetne zavore, dušenje nihanje ...

Snov v magnetnem polju

- snov v prostoru (tuljavi) spremeni zvezo med gosto in jakostjo magnetnega polja
- zvezo lahko opazujemo v toroidu z dvojnim navitjem, prvo spreminja polje (H) in drugo meri inducirano napetost (spremembo B)

$$\vec{B}_0 = \mu_0 \vec{H} \qquad \vec{B} = \vec{B}_0 + \vec{B}'$$

$$\Delta H = \frac{N_1 \Delta I}{2\pi \bar{R}} \qquad \Delta B = \frac{1}{N_2 S} \int U_i \, dt$$

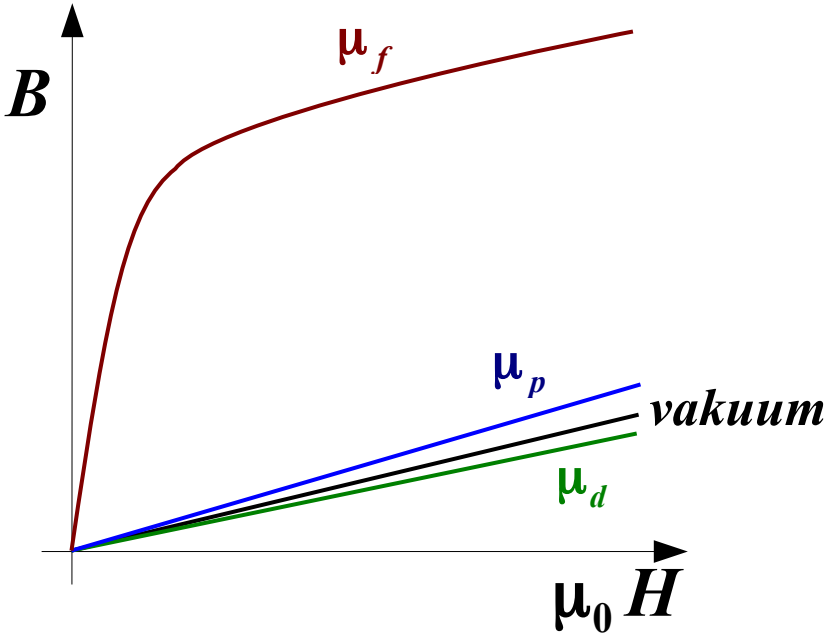


Rowlandov obroč

$$\mu = \frac{\Delta B}{\mu_0 \Delta H}$$

$$\vec{B} = \mu \mu_0 \vec{H}$$

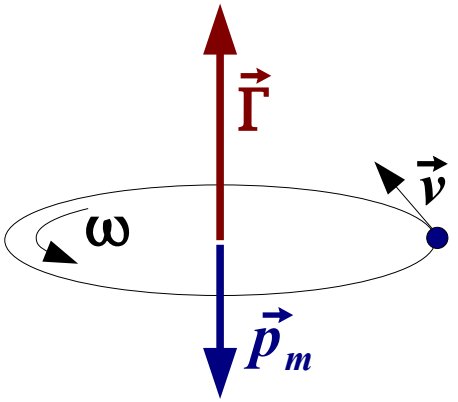
permeabilnost (relativna)



snov	μ
platina	1,00036
aluminij	1,000022
zrak	1,00000038
baker	0,999994
voda	0,999991
bizmut	0,99983

Tirni magnetni moment

- gibanje elektrona lahko ponazorimo s tokovno zanko



$$p_m = S\,I = S\frac{e_0}{t_0} = S\frac{e_0\omega}{2\pi}$$

$$p_m = \frac{\pi r^2 e_0 \omega}{2\pi}\frac{m}{m} = \frac{e_0}{2m}\Gamma$$

$$\vec{p}_T = -\frac{e_0}{2m}\vec{\Gamma}_T \qquad \Gamma_z = m_l \frac{h}{2\pi} = m_l \hbar$$

$$m_l = -l, -l+1 \ldots -1, 0, 1 \ldots l-1, l$$

$$p_{T,z} = -m_l p_B$$

$m_e = 9{,}1 \cdot 10^{-31}\ kg$

$e_0 = 1{,}6 \cdot 10^{-19}\ As$

$h = 6{,}63 \cdot 10^{-34}\ Js$

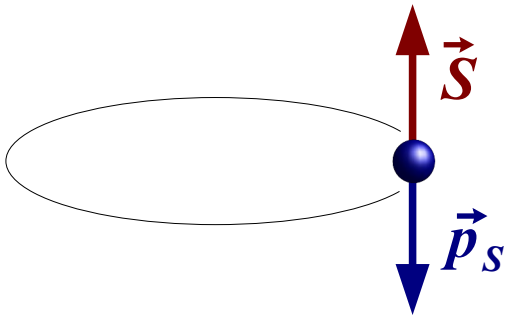
Planckova konstanta

$p_B = 9.27 \cdot 10^{-24}\ Am^2$

Bohrov magneton

Spinski magnetni moment

- lastni magnetni moment elektrona



$$\vec{p}_s = -\frac{e}{m}\vec{S} \qquad S_z = m_s \frac{h}{2\pi} = m_s \hbar$$

$$m_s = -\frac{1}{2}, \frac{1}{2}$$

$$p_{S,z} = \pm p_B$$

Paramagnetizem

$\mu > 1$

Material	χ [10^{-5}]
W	6,8
Cs	5,1
Al	2,2
Li	1,4
Mg	1,2
Na	0,72

- vsota tirnih in spinskih p_m je različna od nič, ko ni zunanjega polja
- v polju se magnetni momenti usmerjajo v smer polja, kar zveča gostoto magnetnega polja

$$\vec{B} = \vec{B}_0 + \vec{B}' = \mu_0 \vec{H} + \mu_0 \vec{M}$$

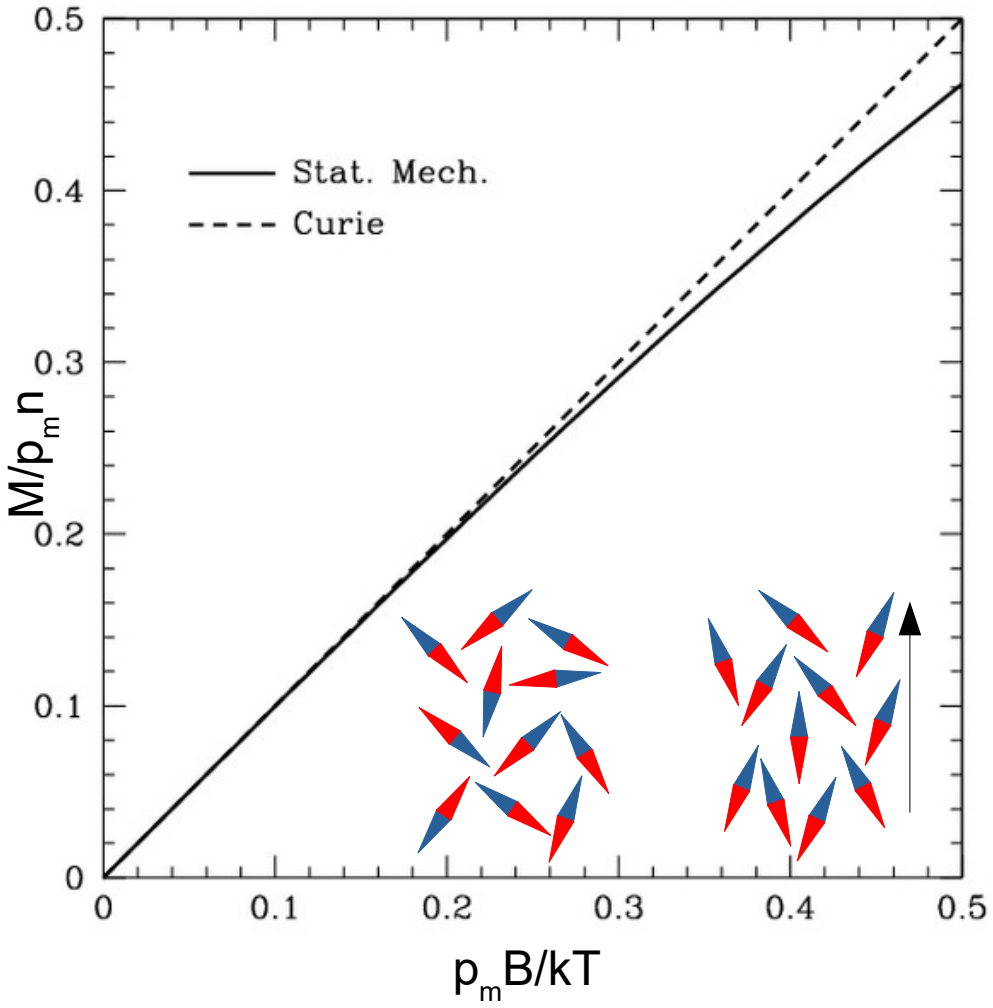
- magnetizacija – gostota magnetnega momenta

$$M = \frac{N \overline{p_m}}{V} = n \overline{p_m} = \chi H$$

magnetna susceptibilnost

$$\vec{B} = (1+\chi)\mu_0 \vec{H} = \mu \mu_0 \vec{H}$$

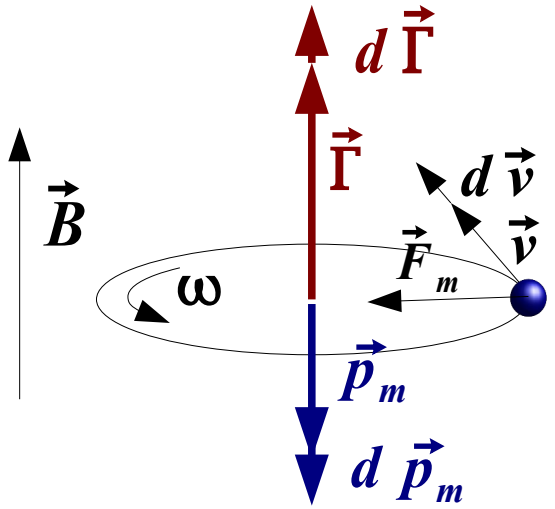
$$M \propto \frac{B_0}{T} \qquad M = \frac{C}{T} B_0$$



Diamagnetizem

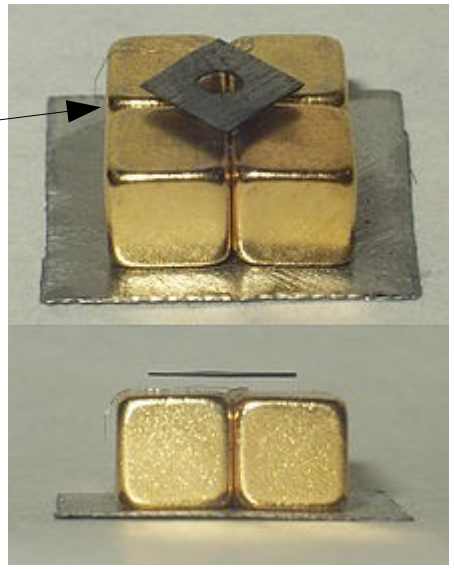
$\mu < 1$

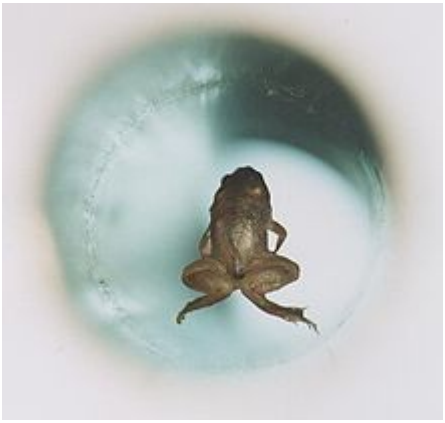
- vsota tirnih in spinskih magnetnih momentov (p_m) je nič, ko ni zunanjega polja
- v polju se inducirajo magnetni momenti, v nasprotni smeri, kar zmanjša gostoto mag. polja



$\chi = \mu - 1$ magnetna susceptibilnost

Material	χ [10^{-5}]
superprevodnik	-10^5
pirolitski ogljik	-40,0
Bi	-16,6
Hg	-2,9
Ag	-2,6
C (diamant)	-2,1
Pb	-1,8
C (grafit)	-1,6
Cu	-1,0
H_2O	-0,91

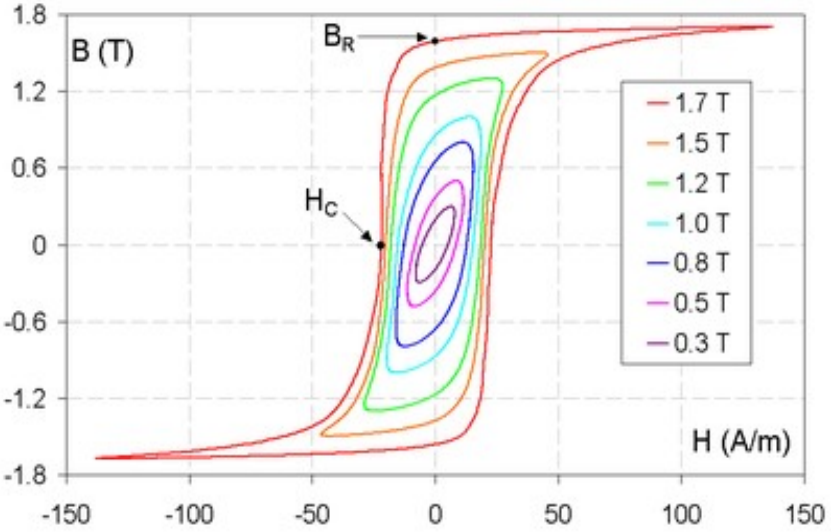




Feromagnetizem

$\mu \gg 1$

- spini sosednjih atomov se usmerijo v isto smer – močan efekt
- histerezna krivulja
- remanentna gostota (B_R) in koercitivna jakost (H_C)
- feromagnetne lastnosti izginejo nad Curiejevo temperaturo



Material	Curie temp. [K]
Co	1388
Fe	1043
Fe_2O_3	948
$NiOFe_2O_3$	858
Ni	627
MnSb	587
CrO_2	386
MnAs	318
Gd	292
Tb	219
Dy	88
EuO	69

- dušilka – tuljava s feromagnetnim jedrom

$$L = \bar{\mu} L_0$$

- histerezne izgube – ploščina histerezne zanke

$$A = V \oint H \, dB$$

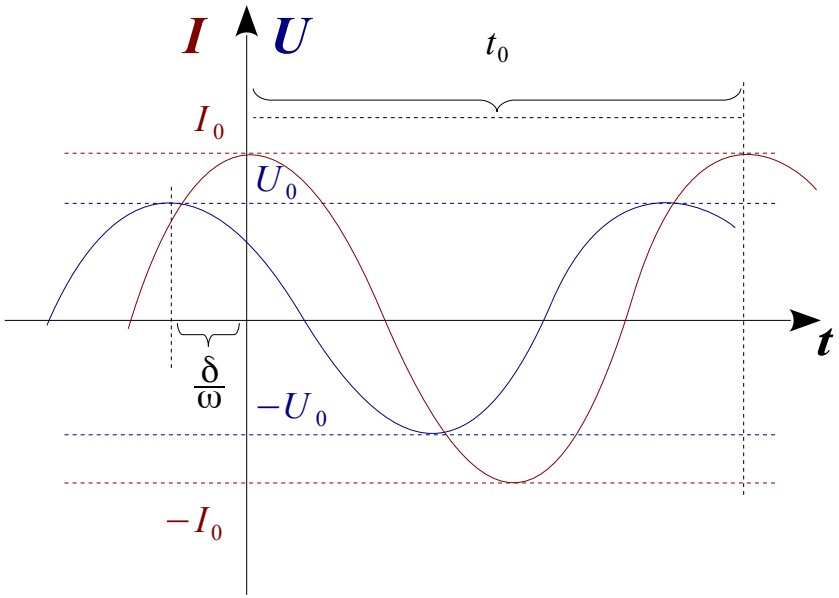
$$A = \int P \, dt = \int U \, I \, dt = \int \frac{d\Phi_m}{dt} I \, dt = \int I \, d\Phi_m = \int I \, d(SNB) = S l \int \frac{N I}{l} dB = V \int H \, dB$$

Sinusni izmenični tok

- zapis toka in napetosti, napetost je fazno zamaknjena

$$I(t)=I_0\cos(\omega t)$$

$$U(t)=U_0\cos(\omega t+\delta)$$



Trenutna in povprečna moč

- trenutna moč

$$P(t)=U(t)I(t)=U_0\cos(\omega t+\delta)I_0\cos(\omega t)$$

- povprečna moč

$$A=\int_{t'}^{t}P(t)\,dt=\overline{P}\,\Delta t \;\Rightarrow\; \overline{P}=\frac{1}{\Delta t}\int_{t'}^{t}P(t)\,dt$$

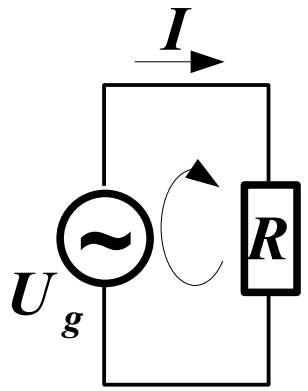
$$\overline{P}=\overline{U(t)I(t)}=U_0I_0\overline{\cos(\omega t+\delta)\cos(\omega t)}=$$

$$=U_0I_0\frac{1}{2}\overline{(\cos(2\omega t+\delta)+\cos(\delta))}=\frac{1}{2}U_0I_0\cos(\delta)=U_{ef}I_{ef}\cos(\delta)$$

$$I_{ef}=\sqrt{\frac{1}{\Delta t}\int_{t'}^{t}I^2(t)\,dt}=\frac{I_0}{\sqrt{2}}$$

$$U_{ef}=\sqrt{\frac{1}{\Delta t}\int_{t'}^{t}U^2(t)\,dt}=\frac{U_0}{\sqrt{2}}$$

Sinusni izmenični tok skozi upornik



$$U_g = U_0 \cos(\omega t + \delta)$$

$$I = I_0 \cos(\omega t)$$

- enako kot pri enosmernem toku zapišemo Kirchhoffova zakona

$$U_g - RI = 0$$

II. Kirchhoffov zakon

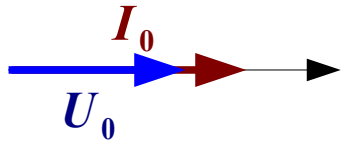
Vsoti sinusnih in kosinusnih členov morata biti ločeno enaki 0!!!

$$U_0 \cos(\omega t + \delta) - RI_0 \cos(\omega t) = 0$$

$$U_0 \cos(\omega t)\cos(\delta) - U_0 \sin(\omega t)\sin(\delta) - RI_0 \cos(\omega t) = 0$$

$$\left.\begin{aligned} U_0 \sin(\delta) &= 0 \\ U_0 \cos(\delta) - RI_0 &= 0 \end{aligned}\right\} \quad \frac{U_0}{I_0} = \begin{matrix} \delta = 0 \\ Z_R = R \end{matrix}$$

upornost

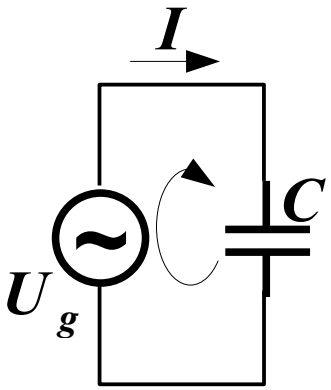


kazalci

- napetost in tok nihata sočasno

$$\overline{P} = U_{ef} I_{ef} \cos(\delta) = U_{ef} I_{ef}$$

Sinusni izmenični tok skozi kondenzator



$$U_g = U_0 \cos(\omega t + \delta)$$
$$\dot{U}_g = -\omega U_0 \sin(\omega t + \delta)$$
$$I = I_0 \cos(\omega t)$$

$$U_g - \frac{q}{C} = 0 \quad \Rightarrow \quad \dot{U}_g - \frac{1}{C} I = 0$$

II. Kirchhoffov zakon

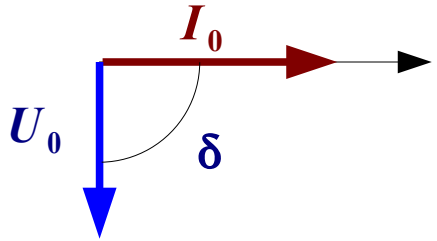
$$-\omega U_0 \sin(\omega t + \delta) - \frac{1}{C} I_0 \cos(\omega t) = 0$$

$$-\omega U_0 \sin(\omega t)\cos(\delta) - \omega U_0 \cos(\omega t)\sin(\delta) - \frac{1}{C} I_0 \cos(\omega t) = 0$$

$$-\omega U_0 \cos(\delta) = 0$$
$$-\omega U_0 \sin(\delta) - \frac{1}{C} I_0 = 0$$

$$\frac{U_0}{I_0} = Z_C = \frac{1}{\omega C}$$

$$\delta = -\frac{\pi}{2}$$



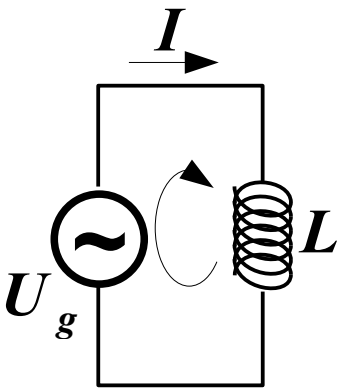
kazalci

- napetost zaostaja za tokom za četrt nihaja

kapacitanca

$$\overline{P} = U_{ef} I_{ef} \cos(\delta) = 0$$

Sinusni izmenični tok skozi tuljavo



$$U_g = U_0 \cos(\omega t + \delta)$$
$$I = I_0 \cos(\omega t)$$
$$\dot{I} = -\omega I_0 \sin(\omega t)$$

II. Kirchhoffov zakon

$$U_g - L\dot{I} = 0$$

$$U_0 \cos(\omega t + \delta) + \omega L I_0 \sin(\omega t) = 0$$

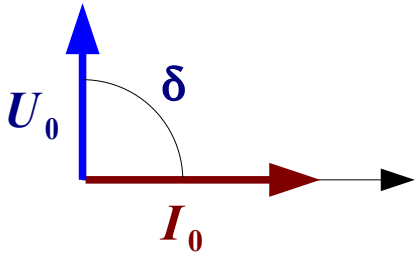
$$U_0 \cos(\omega t)\cos(\delta) - U_0 \sin(\omega t)\sin(\delta) + \omega L I_0 \sin(\omega t) = 0$$

$$-U_0 \sin(\delta) + \omega L I_0 = 0$$
$$U_0 \cos(\delta) = 0$$

$$\delta = \frac{\pi}{2}$$

$$\frac{U_0}{I_0} = Z_L = \omega L$$

induktanca

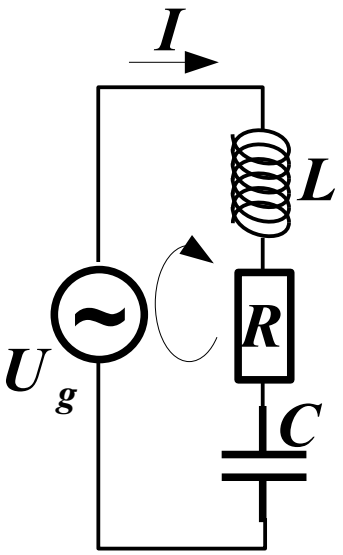


kazalci

- napetost prehiteva tok za četrt nihaja

$$\overline{P} = U_{ef} I_{ef} \cos(\delta) = 0$$

Zaporedna vezava - RCL



II. Kirchhoffov zakon

$$U_g - L\dot{I} - RI - \frac{1}{C}q = 0$$

$$\dot{U}_g - L\ddot{I} - R\dot{I} - \frac{1}{C}I = 0$$

$$U_g = U_0\cos(\omega t + \delta)$$

$$\dot{U}_g = -\omega U_0 \sin(\omega t + \delta)$$

$$I = I_0\cos(\omega t)$$

$$\dot{I} = -\omega I_0 \sin(\omega t)$$

$$\ddot{I} = -\omega^2 I_0 \cos(\omega t)$$

$$-\omega U_0 \sin(\omega t + \delta) + \omega^2 L I_0 \cos(\omega t) + \omega R I_0 \sin(\omega t) - \frac{1}{C} I_0 \cos(\omega t) = 0$$

$$-\omega U_0 \sin(\delta)\cos(\omega t) - \omega U_0 \cos(\delta)\sin(\omega t) + \omega^2 L I_0 \cos(\omega t) +$$

$$+\omega R I_0 \sin(\omega t) - \frac{1}{C} I_0 \cos(\omega t) = 0$$

$$U_0\cos(\delta) = R I_0$$

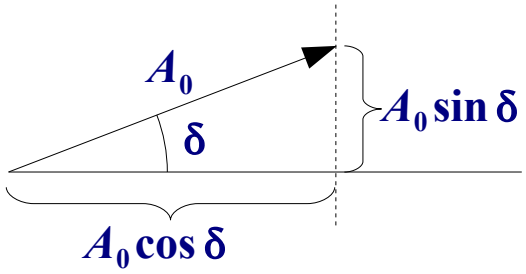
$$U_0\sin(\delta) = \omega L I_0 - \frac{1}{\omega C}$$

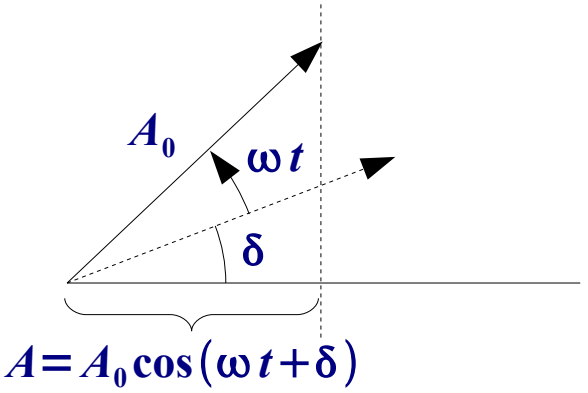
$$\tan(\delta) = \frac{\omega L - 1/\omega C}{R}$$

$$\frac{U_0}{I_0} = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

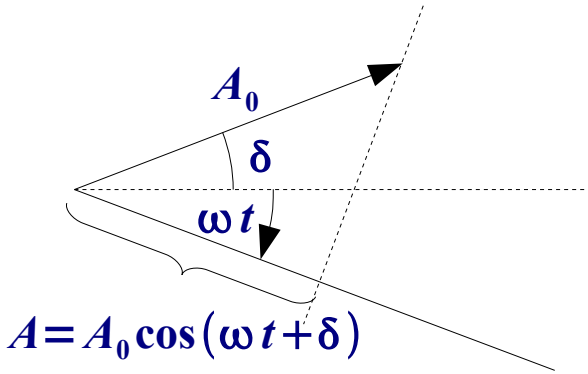
Kazalčni diagram

• pri seštevanju sinusnih količin, ki nihajo z enako frekvenco si lahko pomagamo s kazalčnim diagramom





vrtenje kazalca

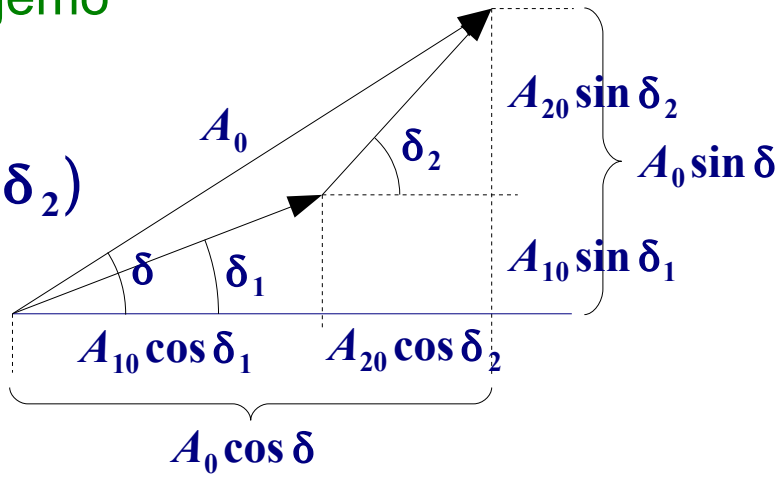


vrtenje osi

• kazalec ponazarja sinusno količino z amplitudo in faznim zamikom

• kazalec vsote sinusnih količin dobimo tako, da seštejemo njihove kazalce → skupna amplituda in fazni zamik

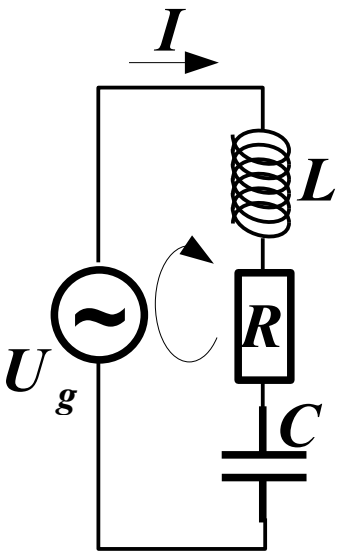
$$A_0 \cos(\omega t + \delta) = A_{01} \cos(\omega t + \delta_1) + A_{02} \cos(\omega t + \delta_2)$$



$$\tan(\delta) = \frac{A_{01} \sin(\delta_1) + A_{02} \sin(\delta_2)}{A_{01} \cos(\delta_1) + A_{02} \cos(\delta_2)}$$

$$A_0 = \sqrt{(A_{01} \cos(\delta_1) + A_{02} \cos(\delta_2))^2 + (A_{01} \sin(\delta_1) + A_{02} \sin(\delta_2))^2}$$

- obravnava zaporedne vezave RCL s kazalci

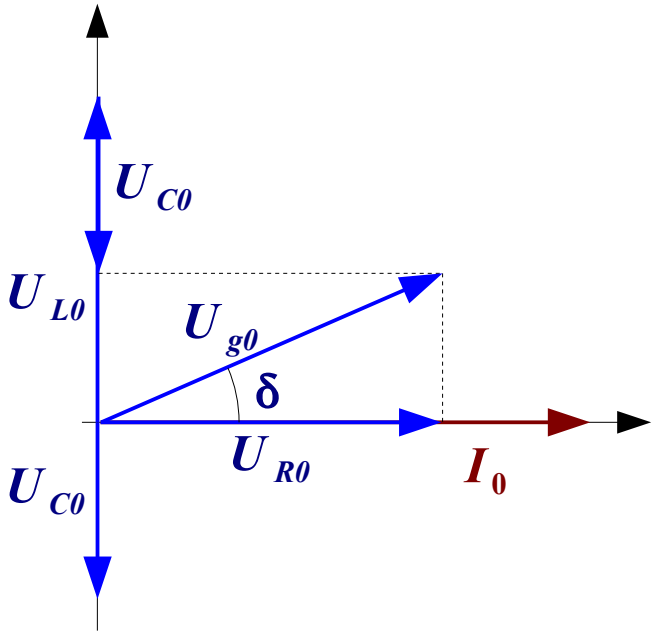


II. Kirchhoff zakon

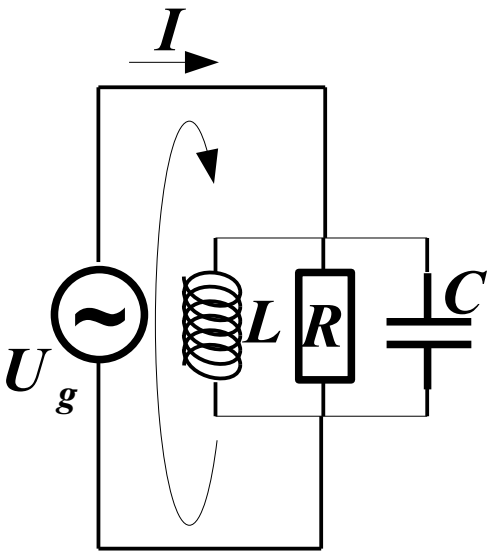
$$U_g = U_L + U_R + U_C$$

$$\tan\delta = \frac{U_{L0} - U_{C0}}{U_{R0}} = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$



- obravnava vzporedne vezave RCL s kazalci

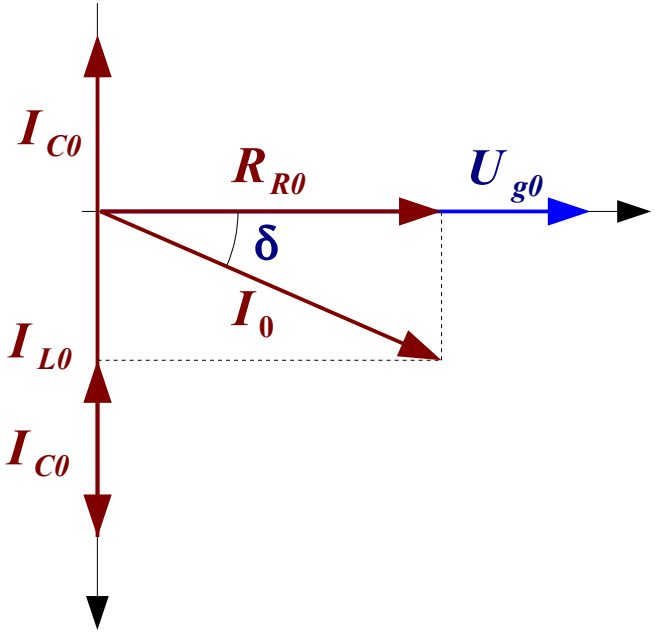


I. Kirchhoffov zakon

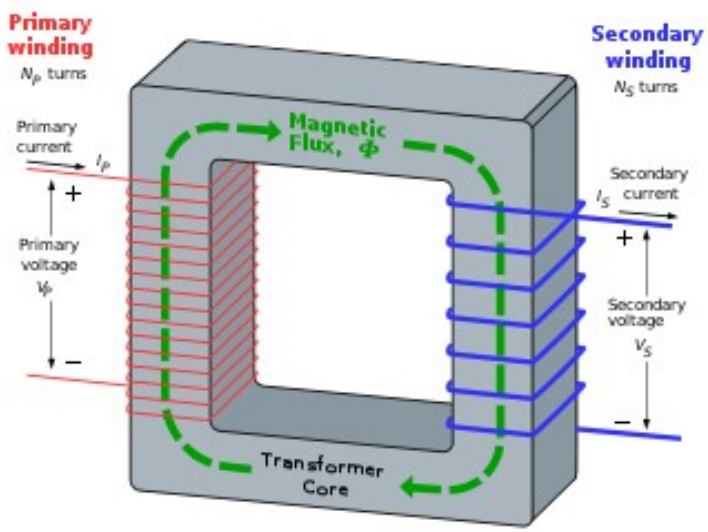
$$I = I_L + I_R + I_C$$

$$\tan\delta = \frac{I_{L0} - I_{C0}}{I_{R0}} = \frac{\frac{1}{\omega L} - \omega C}{\frac{1}{R}}$$

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C\right)^2}$$



Transformator



• magnetni pretok skozi zanko primarnega in sekundarnega navitja je enak

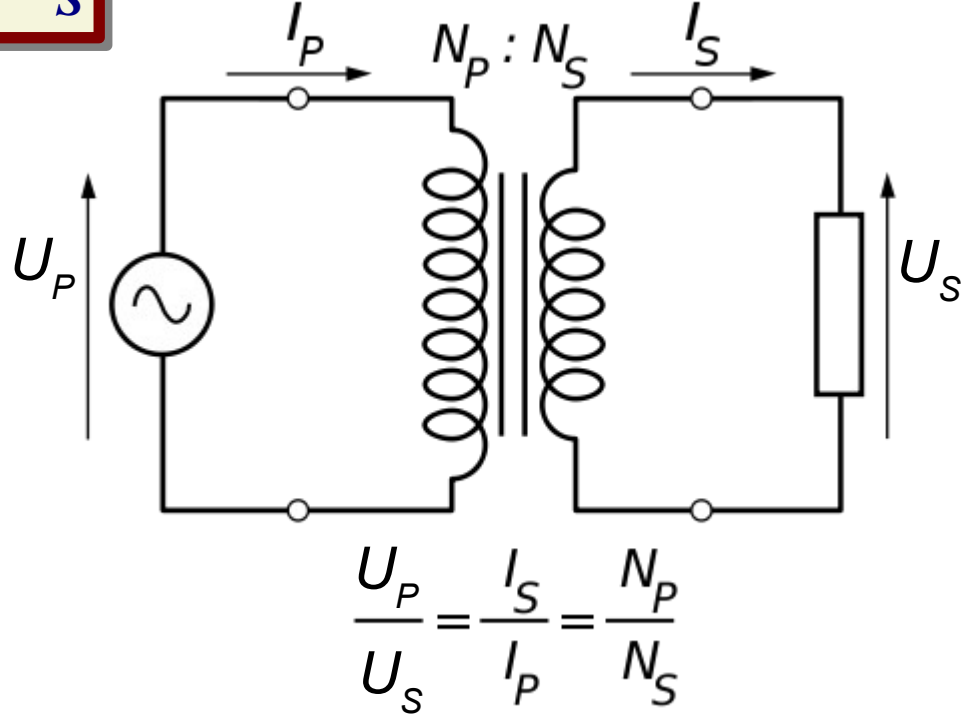
$$\Phi_{m,P} = N_P \Phi_m \qquad \Phi_{m,S} = N_S \Phi_m$$

$$\left.\begin{aligned} U_P &= \frac{d\Phi_{m,P}}{dt} = N_P \frac{d\Phi_m}{dt} \\ U_S &= \frac{d\Phi_{m,S}}{dt} = N_S \frac{d\Phi_m}{dt} \end{aligned}\right\} \quad \boxed{\frac{U_P}{U_S} = \frac{N_P}{N_S}}$$

• moč sekundarnega navitja je enaka moči primarnega navitja (idealni transformator)

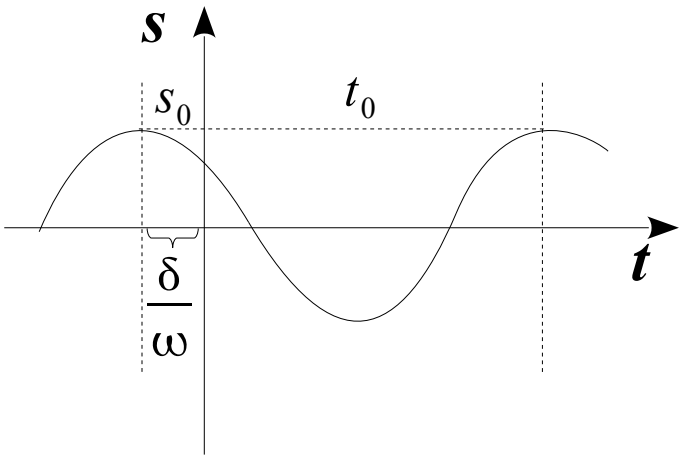
$$P_P = U_P I_P = U_S I_S = P_S$$

$$\boxed{\frac{I_P}{I_S} = \frac{U_S}{U_P} = \frac{N_S}{N_P}}$$



Nihanje

- nihanje je ponavljajoče gibanje



Sinusno (harmonično) nihanje

$$s(t)=s_0\cos(\omega_0 t+\delta)$$

$$v(t)=\frac{ds}{dt}=-\omega_0 s_0\sin(\omega_0 t+\delta)$$

$$a(t)=\frac{dv}{dt}=-\omega_0^2 s_0\cos(\omega_0 t+\delta)=-\omega_0^2 s$$

$$\omega_0=2\pi\nu=\frac{2\pi}{t_0}$$

- nihajni čas - t_0
- frekvenca - ν
- krožna frekvenca - ω
- amplituda - s_0
- fazni zamik - δ

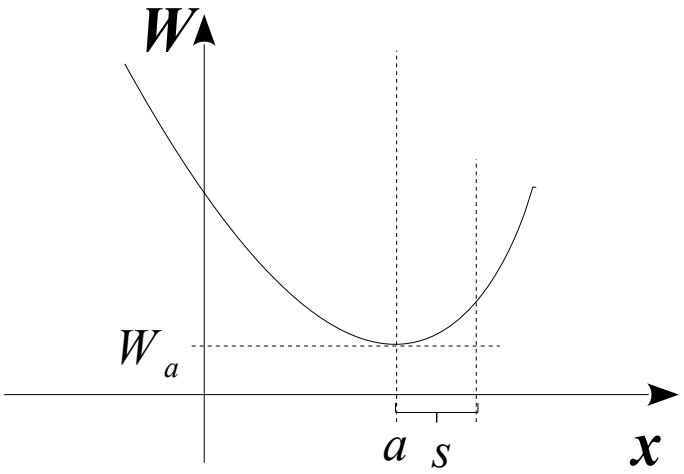
Enačba nihanja – sila pri nihanju

- z uporabo II Newtonovega zakona ugotovimo, kakšna sila je potrebna za takšno gibanje

$$F=m\,a=-\omega_0^2 m\,s=-k\,s$$

- sila je sorazmerna z odmikom in nasprotno usmerjena

• v prvem redu je v večini primerov nihanje okrog stabilne ravnovesne lege sinusno



$$dW_p = -dA_k = -F\,ds \;\Rightarrow\; F = -\frac{dW_p}{ds}$$

$$s = x - a$$

$$W(a+s) = W(a)\frac{s^0}{0!} + W'(a)\frac{s^1}{1!} + W''(a)\frac{s^2}{2!} + W'''(a)\frac{s^3}{3!} + \ldots$$

$$W(a+s) = W_a + W'(a)s + \frac{1}{2}W''(a)s^2 + \frac{1}{6}W'''(a)s^3 + \ldots$$

$W'(a) = 0$ nihanje v okolici minimuma → prvi odvod potencialne energije je 0

$$W''(a) = k$$

$W(s) = W_a + 0 + \frac{1}{2}k\,s^2 + \frac{1}{6}W'''(a)s^3 + \ldots$ člene višjih redov zanemarimo

$$F = -\frac{dW}{ds} = -k\,s - \frac{1}{2}W'''(a)s^2 + \ldots \approx -k\,s$$

• sila je v prvem redu sorazmerna z odmikom

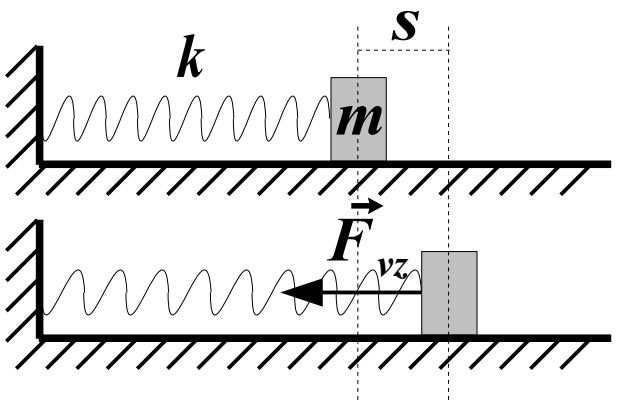
Vzmetno nihalo

- telo pritrjeno na vijačno vzmet

$$\sum F = F_{vz} = -ks = ma = m\ddot{s}$$

$$\ddot{s} + \frac{k}{m} s = 0 \qquad s(t) = s_0 \cos(\omega_0 t + \delta)$$

$$-\omega_0^2 s + \frac{k}{m} s = 0 \;\Rightarrow\; \omega_0 = \sqrt{\frac{k}{m}}$$



Energija nihanja

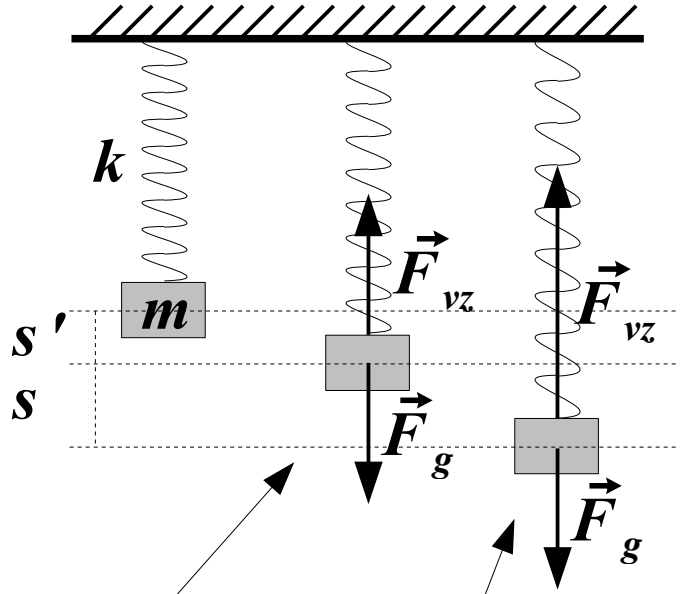
- kinetična in potencialna energija se izmenjujeta
- celotna mehanska energija se ohranja

$$W_{pr} = \frac{1}{2} k s_0^2 \cos^2(\omega_0 t + \delta)$$

$$W_k = \frac{1}{2} m \omega_0^2 s_0^2 \sin^2(\omega_0 t + \delta)$$

$$W_{pr,max} = \frac{1}{2} k s_0^2 = \frac{1}{2} m \omega_0^2 s_0^2 = W_{k,max}$$

$$W_m = W_{pr,max} = W_{k,max}$$

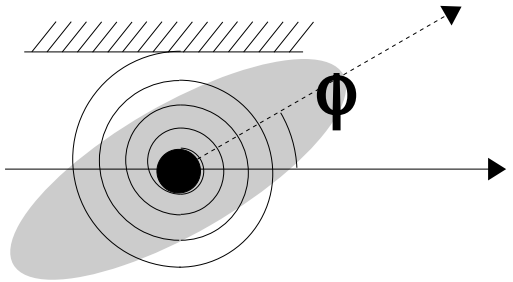


$$F_g - k s' = 0$$

$$F_g - k(s + s') = m\ddot{s}$$

$$F_g - k s' - k s = m\ddot{s}$$

Sučnostno nihalo



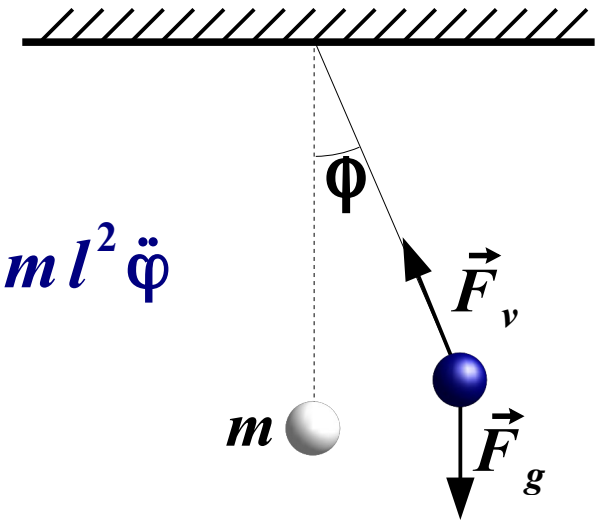
- telo pritrjeno na polžasto vzmet

$$\sum M = M_D = -D\varphi = J\alpha = J\ddot{\varphi}$$

$$\ddot{\varphi} + \frac{D}{J}\varphi = 0$$

$$-\omega_0^2\varphi + \frac{D}{J}\varphi = 0 \;\Rightarrow\; \omega_0 = \sqrt{\frac{D}{J}} \qquad \varphi(t) = \varphi_0 \cos(\omega_0 t + \delta)$$

Matematično nihalo



- točkasto telo obešeno z lahko nitko → poseben primer fizičnega nihala

$$\sum M = M_g = -F_g l \sin\varphi = -mgl\sin\varphi = J\alpha = ml^2\ddot{\varphi}$$

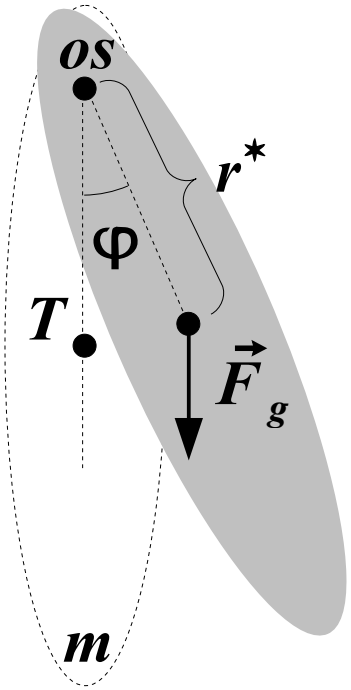
$$\ddot{\varphi} + \frac{g}{l}\sin\varphi = 0$$

$$\ddot{\varphi} + \frac{g}{l}\varphi \approx 0$$

$$-\omega_0^2\varphi + \frac{g}{l}\varphi = 0 \;\Rightarrow\; \omega_0 = \sqrt{\frac{g}{l}} \qquad \varphi(t) = \varphi_0 \cos(\omega_0 t + \delta)$$

Fizično nihalo

- togo telo vrtljivo okrog vodoravne osi, ki ne gre skozi osišče



$$\sum M = M_g = -F_g r^* \sin\varphi = -m g r^* \sin\varphi = J\alpha = J\ddot{\varphi}$$

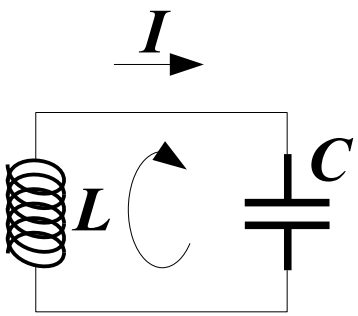
$$\ddot{\varphi} + \frac{m g r^*}{J}\varphi \approx 0$$

$$-\omega_0^2\varphi + \frac{m g r^*}{J}\varphi = 0 \;\Rightarrow\; \omega_0 = \sqrt{\frac{m g r^*}{J}}$$

$$\varphi(t) = \varphi_0 \cos(\omega_0 t + \delta)$$

Električni nihajni krog

- idealni električni nihajni krog: LC



$$-L\dot{I}-\frac{1}{C}q=0$$

$$\ddot{I}+\frac{1}{LC}I=0$$

$$I(t)=I_0\cos(\omega_0 t)$$

$$\omega_0=\sqrt{\frac{1}{LC}};\quad t_0=2\pi\sqrt{LC}$$

$$\omega_0 L=\frac{1}{\omega_0 C}\qquad U_0=\omega_0 L I_0=\frac{1}{\omega_0 C}I_0$$

$$U_L=-L\dot{I}=L I_0\omega_0\sin(\omega_0 t)=U_0\sin(\omega_0 t)$$

- energija električnega nihajnega kroga

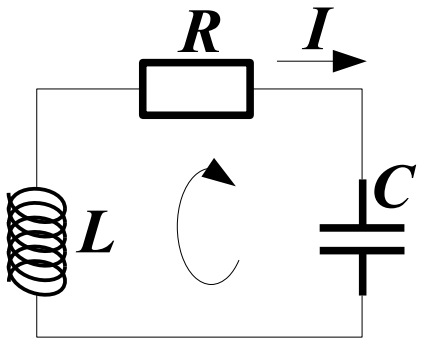
$$W_m=\frac{1}{2}L I^2=\frac{1}{2}L I_0^2\cos^2(\omega_0 t)$$

$$W_e=\frac{1}{2}C U^2=\frac{1}{2}C U_0^2\sin^2(\omega_0 t)$$

$$W=\frac{1}{2}L I_0^2=\frac{1}{2}C U_0^2$$

Dušeno nihanje

- dušeni električni nihajni krog: LRC



$$-L\dot{I}-RI-\frac{1}{C}q=0 \;\Rightarrow\; \ddot{I}+\frac{R}{L}\dot{I}+\frac{1}{LC}I=0$$

$$I(t)=I_0 e^{-\beta t}\cos(\omega t)$$

ω_0^2 | $I=I_0 e^{-\beta t}\cos(\omega t)$

$\frac{R}{L}$ | $\dot{I}=I_0 e^{-\beta t}[-\beta\cos(\omega t)-\omega\sin(\omega t)]$

| $\ddot{I}=I_0 e^{-\beta t}[\beta^2\cos(\omega t)+2\beta\omega\sin(\omega t)-\omega^2\cos(\omega t)]$

$$\omega_0^2-\beta\frac{R}{L}+\beta^2-\omega^2=0 \;\Rightarrow\; \omega=\sqrt{\omega_0^2-\beta^2}$$

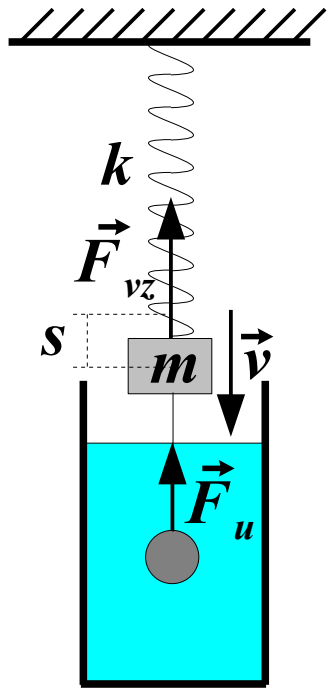
$$-\omega\frac{R}{L}+2\beta\omega=0 \;\Rightarrow\; 2\beta=\frac{R}{L}$$

$$\ddot{I}+2\beta\dot{I}+\omega_0^2 I=0 \qquad I(t)=I_0 e^{-\beta t}\cos(\omega t) \qquad \omega=\sqrt{\omega_0^2-\beta^2}$$

$$2\beta=\frac{R}{L};\quad \omega_0=\sqrt{\frac{1}{LC}};\quad \omega=\sqrt{\omega_0^2-\beta^2}$$

- dušeno vzmetno nihalo – linearni upor

$$\sum F = -Kv - ks = ma$$



$$\ddot{s} + \frac{K}{m}\dot{s} + \frac{k}{m}s = 0 \qquad s(t) = s_0 e^{-\beta t}\cos(\omega t)$$

ω_0^2 | $\frac{K}{m}$

$$s = s_0 e^{-\beta t}\cos(\omega t)$$
$$\dot{s} = s_0 e^{-\beta t}[-\beta\cos(\omega t) - \omega\sin(\omega t)]$$
$$\ddot{s} = s_0 e^{-\beta t}[\beta^2\cos(\omega t) + 2\beta\omega\sin(\omega t) - \omega^2\cos(\omega t)]$$

$$\omega_0^2 - \beta\frac{K}{m} + \beta^2 - \omega^2 = 0 \Rightarrow \omega = \sqrt{\omega_0^2 - \beta^2}$$

$$-\omega\frac{K}{m} + 2\beta\omega = 0 \Rightarrow 2\beta = \frac{K}{m}$$

$$\ddot{s} + 2\beta\dot{s} + \omega_0^2 s = 0 \qquad s(t) = s_0 e^{-\beta t}\cos(\omega t) \qquad \omega = \sqrt{\omega_0^2 - \beta^2}$$

Vsiljeno nihanje

$$s_v(t)=s_0\cos(\omega t)$$

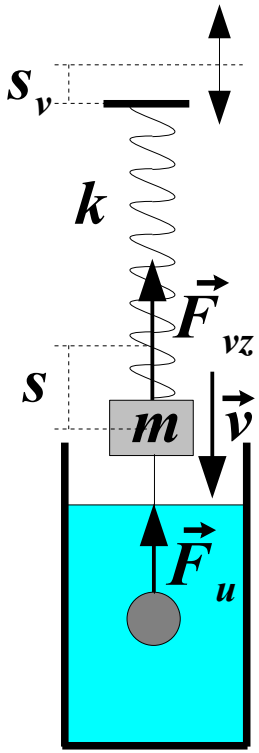
$$\sum F=-Kv-k(s-s_v)=ma$$

$$\ddot{s}+\frac{K}{m}\dot{s}+\frac{k}{m}s=\frac{k}{m}s_v$$

$$\ddot{s}+2\beta\dot{s}+\omega_0^2 s=\omega_0^2 s_v$$

$$s(t)=s_1\cos(\omega t-\delta)+s_0'\mathrm{e}^{-\beta t}\cos(\omega' t+\delta')$$

zamre po dolgem času



- rešitev je kombinacija vsiljenega in lastnega nihanja
- obravnavamo samo člen vsiljenega nihanja, prestavimo δ

$$s(t)=s_1\cos(\omega t);\quad s_v(t)=s_0\cos(\omega t+\delta)$$

$$-\omega^2 s_1\cos(\omega t)-2\beta\omega s_1\sin(\omega t)+\omega_0^2 s_1\cos(\omega t)=\omega_0^2 s_0\cos(\delta)\cos(\omega t)-\omega_0^2 s_0\sin(\delta)\sin(\omega t)$$

$$-2\beta\omega s_1=-\omega_0^2 s_0\sin(\delta)$$

$$-\omega^2 s_1+\omega_0^2 s_1=\omega_0^2 s_0\cos(\delta)$$

$$\tan(\delta)=\frac{2\beta\omega}{(\omega_0^2-\omega^2)}=\frac{bx}{(1-x^2)}$$

$$\frac{s_1}{s_0}=\frac{\omega_0^2}{\sqrt{(\omega_0^2-\omega^2)^2+(2\beta\omega)^2}}=\frac{1}{\sqrt{(1-x^2)^2+(bx)^2}}$$

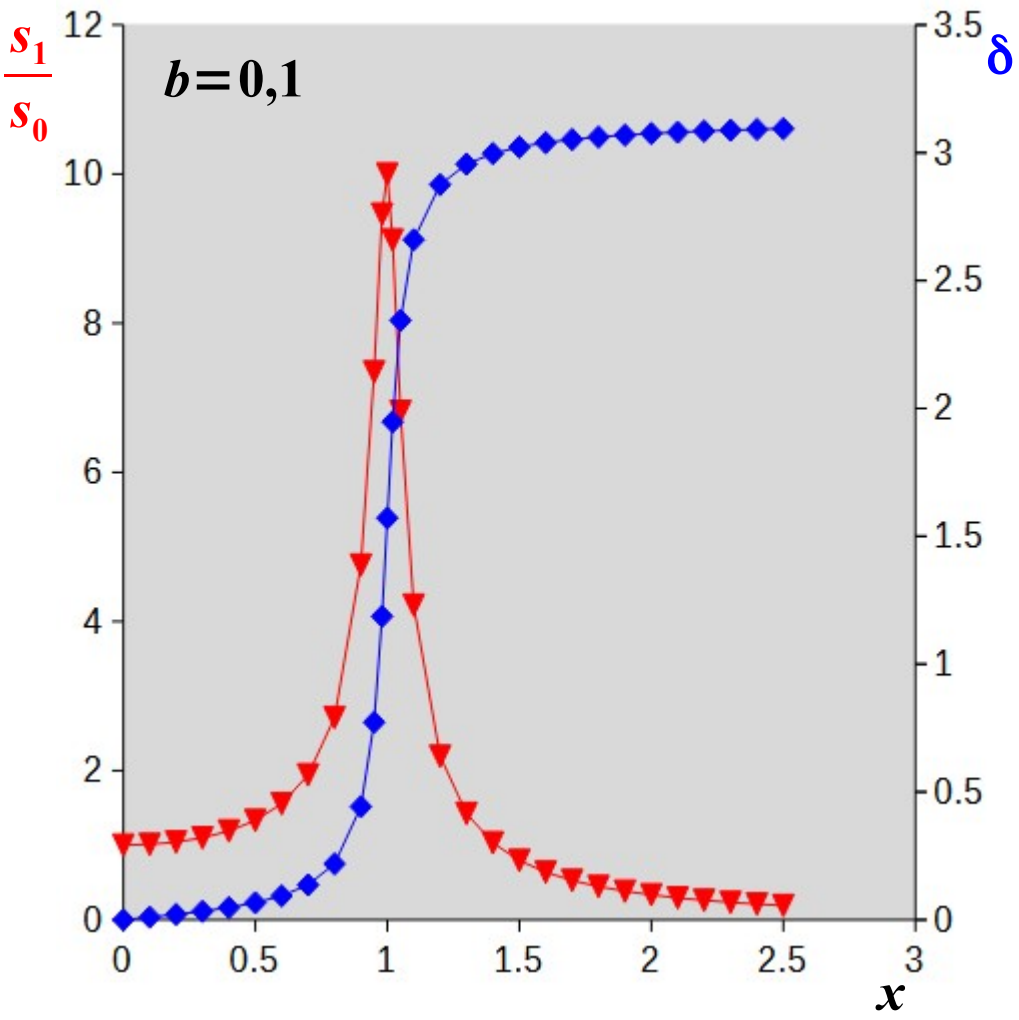
$$x=\frac{\omega}{\omega_0}\quad b=\frac{2\beta}{\omega_0}$$

• resonančna krivulja – maksimalni odziv, ko je frekvenca vsiljevanja enaka lastni (gugalnica) $x=\frac{\omega}{\omega_0}=1$ $b=\frac{2\beta}{\omega_0}$

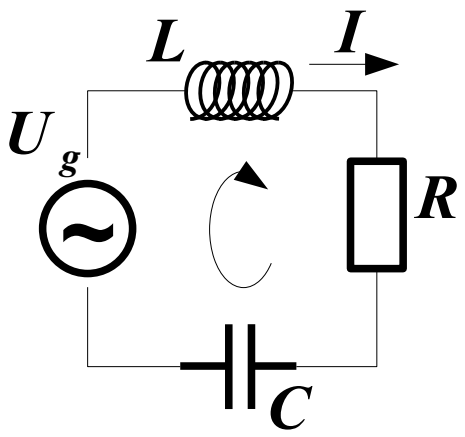
$$\frac{s_1}{s_0}=\frac{1}{\sqrt{(1-x^2)^2+(bx)^2}}=\frac{1}{b}$$

• v resonanci nihalo zaostaja za četrt nihaja za vsiljevalnim nihanjem

$$\tan(\delta)=\frac{2\beta\omega}{(\omega_0^2-\omega^2)}=\frac{bx}{(1-x^2)}$$



- vsiljeno nihanje električnega nihajnega kroga: U_g + LRC



$$U_g - L\dot{I} - RI - \frac{1}{C}q = 0$$

$$\ddot{I} - \frac{R}{L}\dot{I} - \frac{1}{LC}I = \dot{U}_g$$

$$\ddot{I} - 2\beta\dot{I} - \omega_0^2 I = \dot{U}_g$$

$$I_0 = \frac{U_0}{Z} = \omega C U_0 \frac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(\frac{2\beta}{\omega_0}\frac{\omega}{\omega_0}\right)^2}}$$

$$x = \frac{\omega}{\omega_0}$$

$$b = \frac{2\beta}{\omega_0}$$

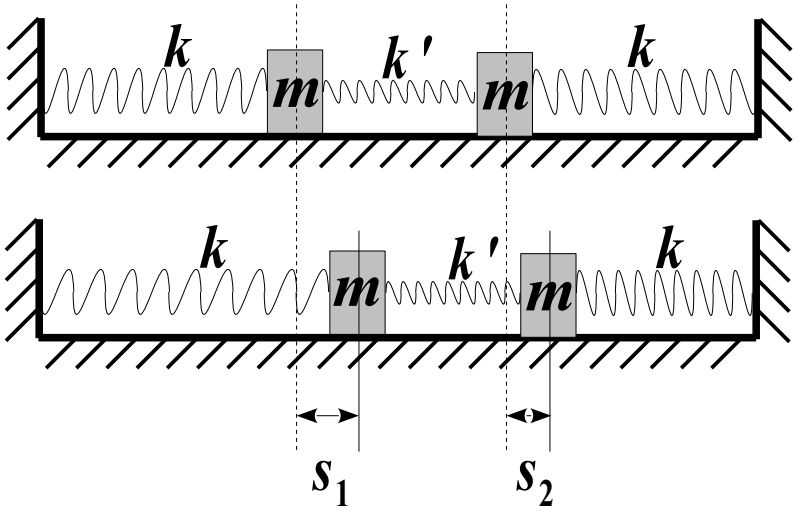
$$\frac{U_{0C}}{U_0} = \frac{1}{\sqrt{(1-x^2)^2 + (bx)^2}}$$

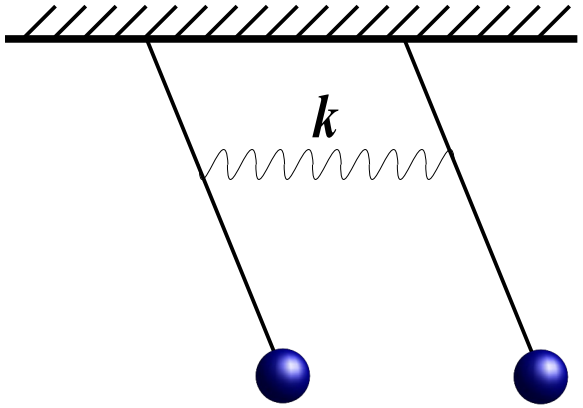
$$I_0 = \frac{U_0}{Z} = \frac{U_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{\omega C U_0}{\sqrt{(R\omega C)^2 + (\omega^2 LC - 1)^2}} = \frac{\omega C U_0}{\sqrt{\left(\frac{R}{L}\omega LC\right)^2 + (\omega^2 LC - 1)^2}}$$

$$2\beta = \frac{R}{L};\quad \omega_0 = \sqrt{\frac{1}{LC}};$$

Nihanje sestavljenih nihal

- nihal sestavljena iz več enostavnih nihal ne nihajo sinusno, razen ko jih zanihamo na poseben način – lastno nihanje
- sestavljeno nihalo ima toliko lastnih načinov nihanja, kot je enostavnih nihal, ki ga sestavljajo





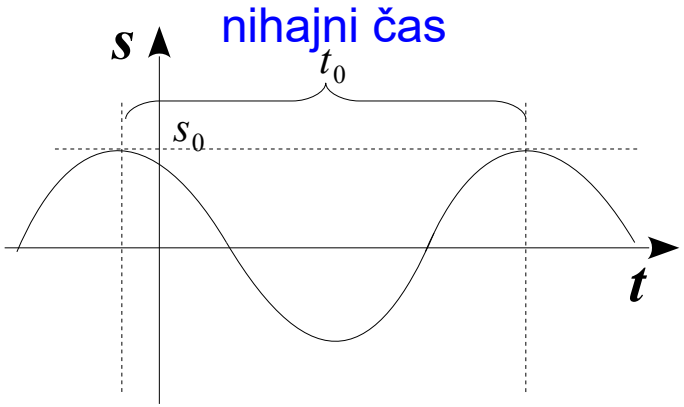
$$\left.\begin{aligned} -k s_1 - k'(s_1 - s_2) &= m\ddot{s}_1 \\ -k s_2 + k'(s_1 - s_2) &= m\ddot{s}_2 \end{aligned}\right\} \quad \begin{aligned} -k(s_1 + s_2) &= m(\ddot{s}_1 + \ddot{s}_2) \\ -k(s_1 - s_2) - 2k'(s_1 - s_2) &= m(\ddot{s}_1 - \ddot{s}_2) \end{aligned}$$

$$s_I = s_1 + s_2: \quad m\ddot{s}_I + k s_I = 0$$
$$s_{II} = s_1 - s_2: \quad m\ddot{s}_{II} + (k + 2k')s_{II} = 0$$

VALOVANJE

- odmik sredstva pri določeni legi se sinusno spreminja s časom - niha

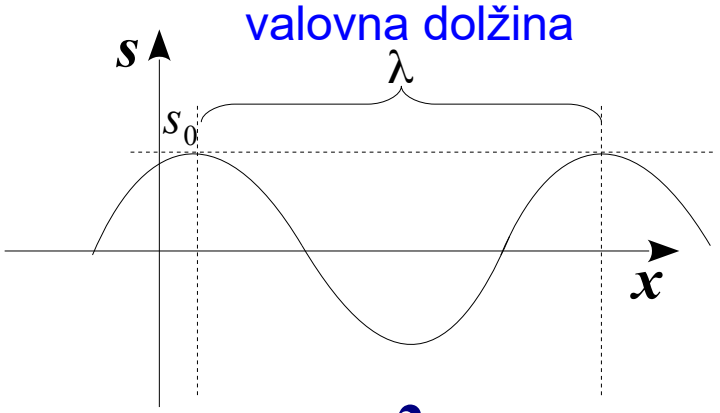
$$s(0,t)=s_0\cos\left(2\pi\frac{t}{t_0}+\delta\right)=s_0\cos\left(\omega t+\delta\right)$$



$$\omega=\frac{2\pi}{t_0}$$

- odmik sredstva v nekem trenutku se sinusno spreminja s krajem - oblika vala

$$s(x,0)=s_0\cos\left(2\pi\frac{x}{\lambda}+\delta\right)=s_0\cos\left(k\,x+\delta\right)$$



$$k=\frac{2\pi}{\lambda}$$

- oboje združimo v potujoči val

$$s(x,t)=s_0\cos\left(\omega t-k\,x+\delta\right)$$

odmik — amplituda — krožna frekvenca — valovni vektor — fazni zamik

Hitrost širjenja valovanja

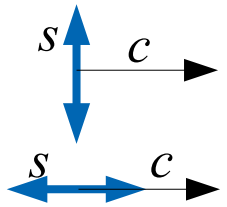
• hitrost širjenja valovanja je hitrost premikanja mesta s konstantno fazo – premik za eno valovno dolžino v enem nihajnem času

$$c = \frac{\lambda}{t_0} = \frac{\omega}{k}$$

$$\omega t - k x + \delta = konst \;\Rightarrow\; \omega = k\frac{dx}{dt} = kc \;\Rightarrow\; c = \frac{\omega}{k}$$

$$\omega t + k x + \delta = konst \;\Rightarrow\; \omega = -k\frac{dx}{dt} = kc \;\Rightarrow\; c = -\frac{\omega}{k}$$

• glede na smer širjenja valovanja je lahko odmik:
  • pravokoten – transverzalno valovanje
  • Vzporeden – longitudinalno valovanje

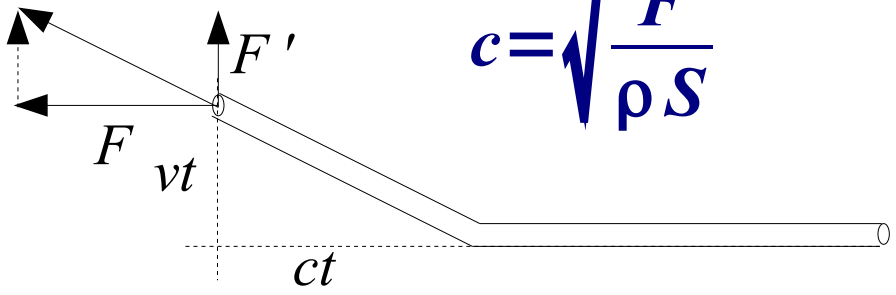


• hitrost širjenja motnje po napeti struni

$$\frac{F'}{F} = \frac{vt}{ct}$$

$$F' t = m v = \rho S ct v$$

$$F\frac{v}{c} = \rho S c v \;\Rightarrow\; c^2 = \frac{F}{\rho S}$$



$$c = \sqrt{\frac{F}{\rho S}}$$

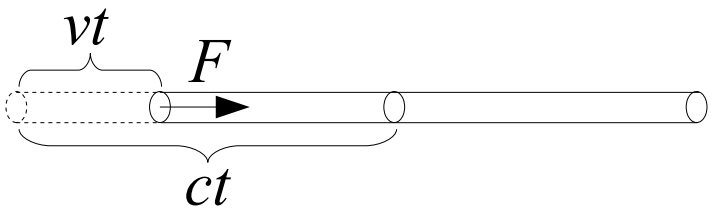
- hitrost širjenja motnje po prožni palici

$$\frac{F}{S}=E\frac{\Delta l}{l}=E\frac{vt}{ct}$$

$$F\,t=m\,v=\rho\,S\,ct\,v$$

$$S\,E\frac{v}{c}=\rho\,S\,c\,v \;\Rightarrow\; c^2=\frac{E}{\rho}$$

$$c=\sqrt{\frac{E}{\rho}}$$



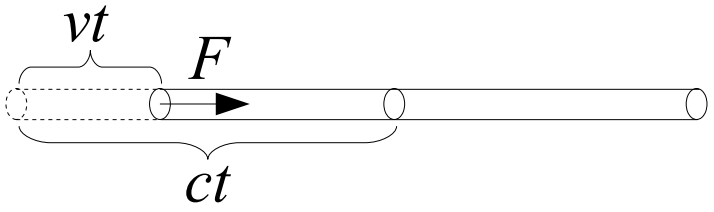
- hitrost širjenja motnje po stolpcu plina (hitrost zvoka)

$$\frac{\Delta V}{V}=-\chi_S\,\Delta p=-\chi_S\frac{F}{S} \;\Rightarrow\; F=\frac{S}{\chi_S}\frac{vt}{ct}$$

$$F\,t=m\,v=\rho\,S\,ct\,v$$

$$\frac{S}{\chi_S}\frac{v}{c}=\rho\,S\,c\,v \;\Rightarrow\; c^2=\frac{1}{\chi_S\rho}$$

$$c=\sqrt{\frac{1}{\chi_S\rho}}=\sqrt{\frac{\kappa\,p}{\rho}}$$



$$\frac{\Delta V}{V}=-\chi_S\,\Delta p \;\Rightarrow \chi_S=-\frac{1}{V}\frac{dV}{dp}=\frac{1}{\kappa\,p}$$

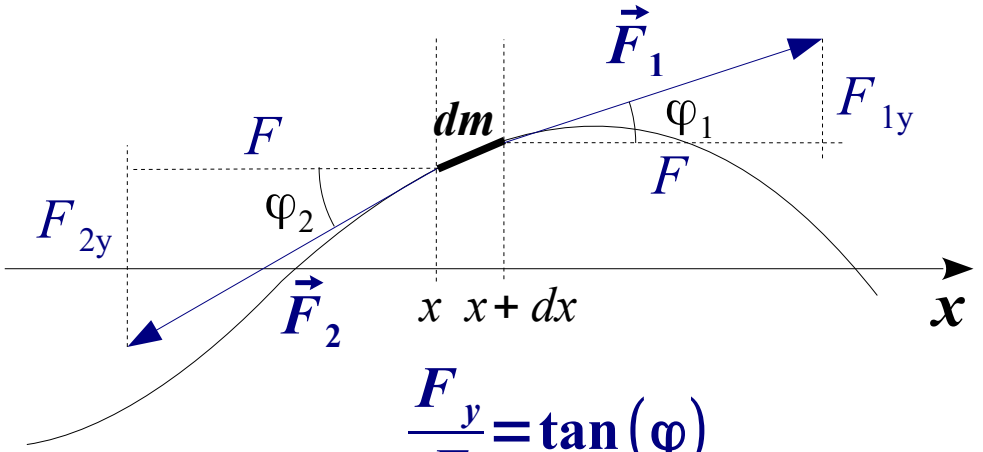
$$p\,V^{\kappa}=konst.\;\;/\frac{d}{dp}$$

$$V^{\kappa}+p\,\kappa\,V^{\kappa-1}\frac{dV}{dp}=0 \;\Rightarrow\; \frac{dV}{dp}=-\frac{V}{\kappa\,p}$$

adiabatna stisljivost

- enačba za nihanje strune

$$F_{1y}+F_{2y}=dm\,a_y=dm\,\frac{\partial^2 s}{\partial t^2}$$



$$\frac{F_y}{F}=\tan(\varphi)$$

II. Newtonov zakon za kratek odsek strune

$$F_{1y}=F\tan(\varphi_1)=F\,\frac{\partial s}{\partial x}(x+dx)$$

$$F_{2y}=-F\tan(\varphi_2)=-F\,\frac{\partial s}{\partial x}(x)$$

$$F\left(s'(x+dx)-s'(x)\right)=\rho S\,dx\,\ddot{s}$$

$$F\,\frac{\left(s'(x+dx)-s'(x)\right)}{dx}=\rho S\,\ddot{s}$$

$$F\,s''=\rho S\,\ddot{s}$$

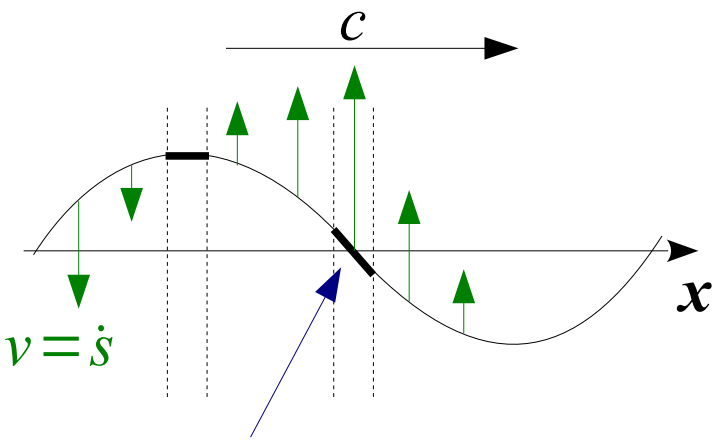
$$\frac{\partial^2 s}{\partial x^2}=\frac{\rho S}{F}\,\frac{\partial^2 s}{\partial t^2}=\frac{1}{c^2}\,\frac{\partial^2 s}{\partial t^2}$$

$$s(x,t)=f(x-ct)$$

$$s''=f''(x-ct)$$

$$\ddot{s}=c^2 f''(x-ct)=c^2 s''$$

Energija valovanja



maksimalna hitrost in razteg →
maksimalni kinetična in prožnostna energija

- povprečna gostota kinetične energije

$$dW_K = \frac{1}{2} dm\, v^2 = \frac{1}{2} dm\, \omega^2 s_0^2 \sin^2(\omega t - kx)$$

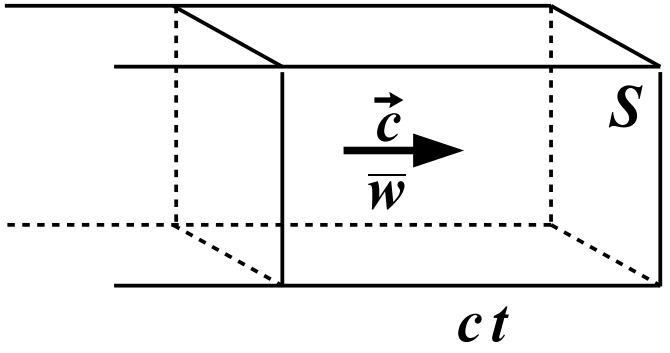
$$w_K = \frac{dW_K}{dV} = \frac{1}{2} \rho \omega^2 s_0^2 \sin^2(\omega t - kx)$$

$$\overline{w_K} = \frac{1}{4} \rho \omega^2 s_0^2$$

- povprečna gostota energije in energijskega toka

$$\overline{w} = 2\,\overline{w_K} = \frac{1}{2} \rho \omega^2 s_0^2$$

$$\overline{j} = \frac{\overline{W}}{S\,t} = \frac{S\,c\,t\,\overline{w}}{S\,t} = c\,\overline{w} = \frac{1}{2} c \rho \omega^2 s_0^2$$



Načelo superpozicije

- skupni odmik je vsota posameznih odmikov (linearno sredstvo)

$$s(x,t)=s_1(x,t)+s_2(x,t)$$

Interferenca

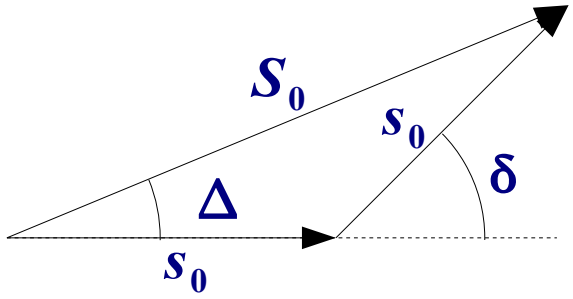
$$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

- dve valovanji z istima ω in k

$$\left.\begin{aligned} s_1(x,t)&=s_0\cos(\omega t-kx)\\ s_2(x,t)&=s_0\cos(\omega t-kx+\delta)\end{aligned}\right\} \quad s=s_1+s_2=S_0\cos(\omega t-kx+\Delta)$$

$$S_0=2s_0\cos\left(\frac{\delta}{2}\right) \qquad \Delta=\frac{\delta}{2}$$

- δ=0 [2Nπ]: konstruktivna interferenca
- δ=π [(2N+1)π)]: destruktivna interferenca



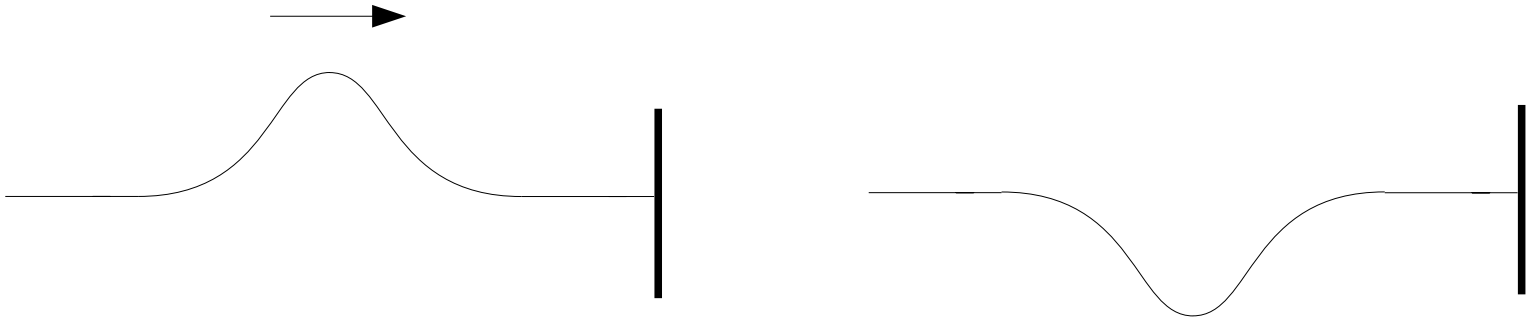
Stoječe valovanje

$$\left.\begin{aligned} s_1(x,t)&=s_0\cos(\omega t-kx)\\ s_2(x,t)&=s_0\cos(\omega t+kx)\end{aligned}\right\} \quad s=s_1+s_2=2s_0\cos(kx)\cos(\omega t)$$

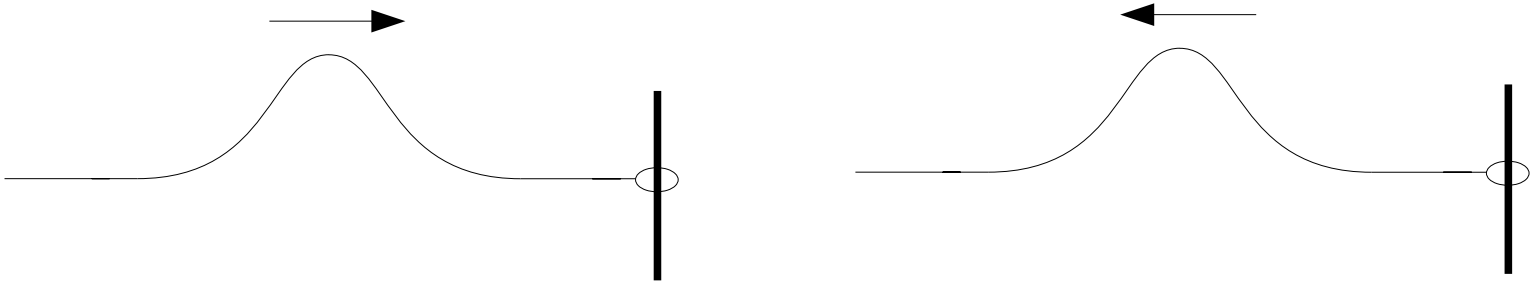
ni potujoči val !

Odboj valovanja

vpeto krajišče

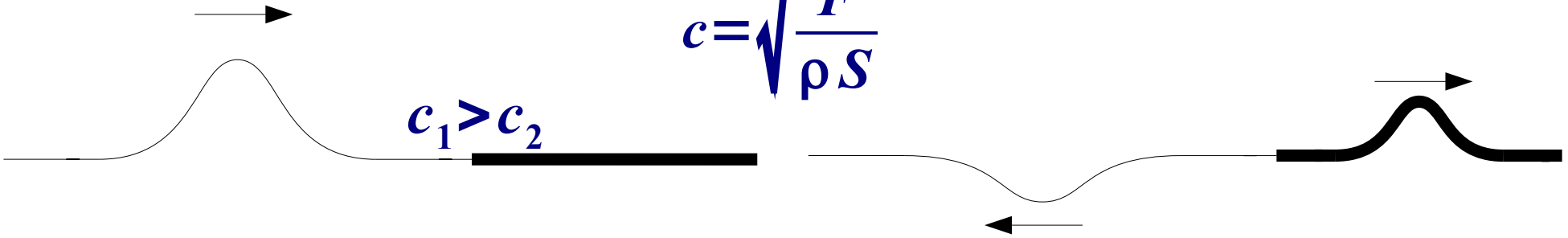


prosto krajišče

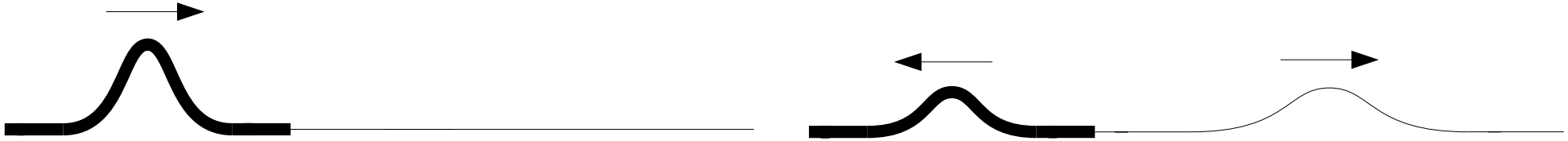


$$c=\sqrt{\frac{F}{\rho S}}$$

$c_1 > c_2$



$c_1 < c_2$



Lastno nihanje

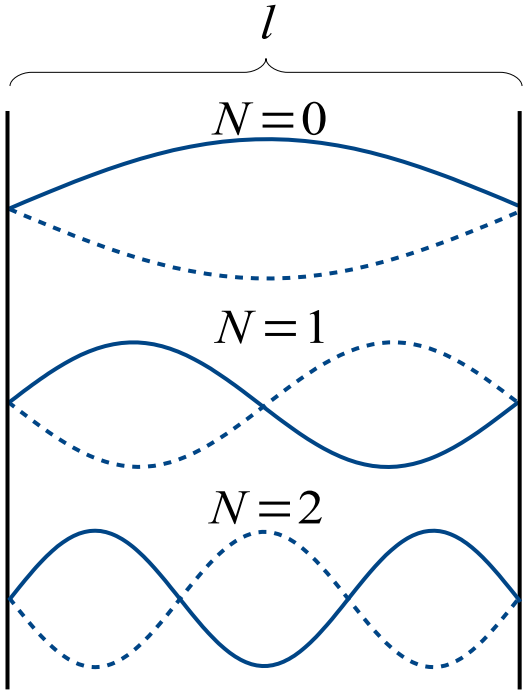
- odboj na prostem in vpetem krajišču
- stoječe valovanje:
  - obe krajišči vpeti
  - obe krajišči prosti
  - eno prosto in eno vpeto krajišče
- spekter lastnega nihanja

$$\nu_N = \frac{1}{2}(N+1)\frac{c}{l}$$

$$\nu_N = \frac{1}{2} N \frac{c}{l}$$

$$\nu_N = \frac{1}{4}(2N+1)\frac{c}{l}$$

N je število vozlov
l je dolžina strune
ali piščali



vpeti krajišči

$$\frac{1}{2}\lambda_0 = l$$

$$\frac{2}{2}\lambda_1 = l$$

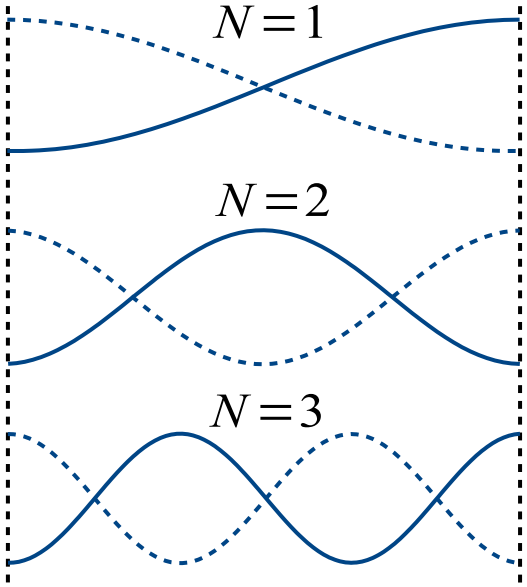
$$\frac{3}{2}\lambda_2 = l$$

$$\frac{N+1}{2}\lambda_N = l$$

$$c = \lambda_N \nu_N \Rightarrow \nu_N = \frac{c}{\lambda_N}$$

$$\nu_N = \frac{c}{\lambda_N} = \frac{N+1}{2}\frac{c}{l}$$

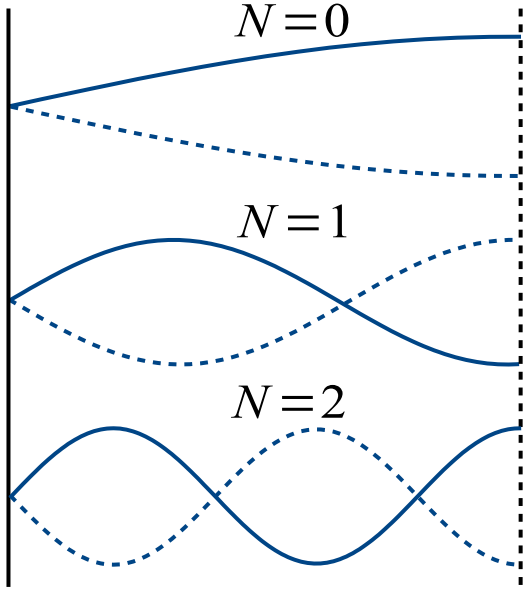
$$\nu_N = \frac{1}{2}(N+1)\frac{c}{l}$$



$$\frac{N}{2}\lambda_N = l$$

$$\nu_N = \frac{1}{2} N \frac{c}{l}$$

prosti krajišči

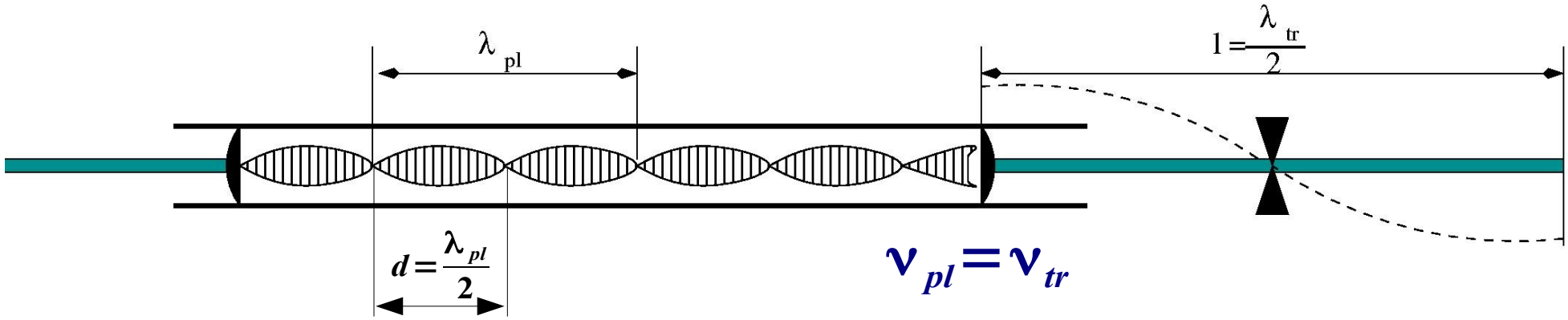


$$\frac{2N+1}{4}\lambda_N = l$$

$$\nu_N = \frac{1}{4}(2N+1)\frac{c}{l}$$

vpeto in prosto krajišče

- primer: Kundtova cev (vaja iz praktikuma)



$$\nu_{pl} = \nu_{tr}$$

$$\frac{c_{pl}}{\lambda_{pl}} = \frac{c_{tr}}{\lambda_{tr}} \Rightarrow c_{tr} = c_{pl}\frac{\lambda_{tr}}{\lambda_{pl}} = c_{pl}\frac{l}{d}$$

$$c = \lambda\nu \Rightarrow \nu = \frac{c}{\lambda}$$

Valovanje v dveh razsežnostih

$$s(\vec{r},t)=s_0(\vec{r})\cos(\omega t-\vec{k}\cdot\vec{r})$$

- ravni val

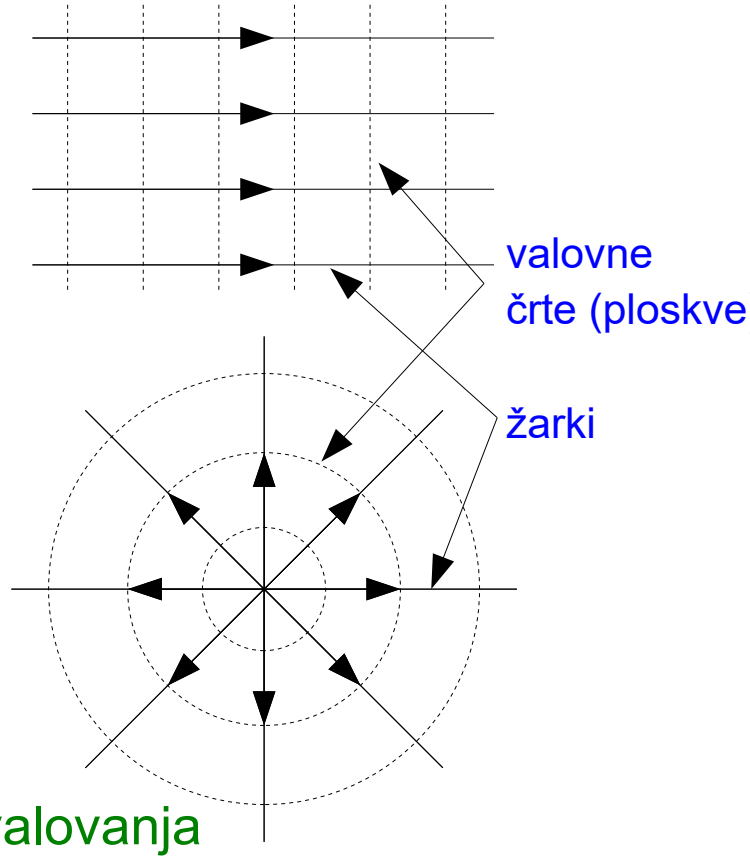
$$s(\vec{r},t)=s_0\cos(\omega t-kx)$$

- krožni val

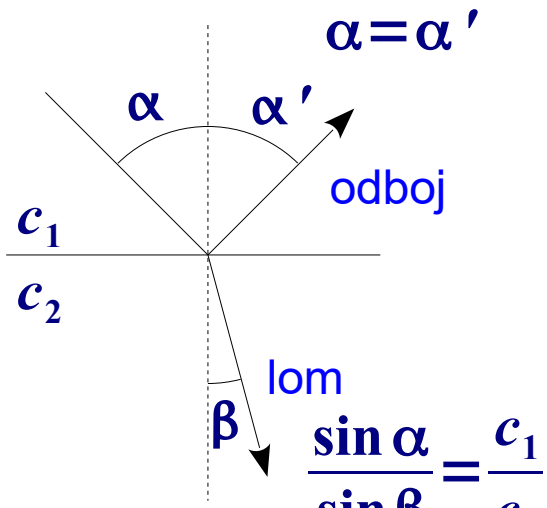
$$s(\vec{r},t)=s_0(r)\cos(\omega t-kr)$$

  - odvisnost gostote energijskega toka od oddaljenosti do izvira

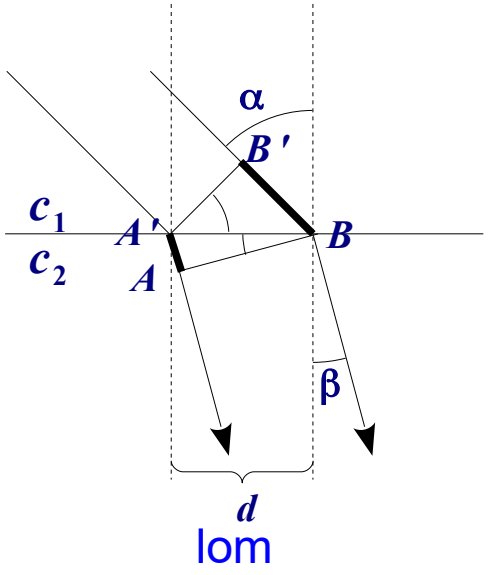
$$j\propto\frac{1}{r}\quad,\quad s_0\propto\sqrt{j}\propto\frac{1}{\sqrt{r}}$$



- odboj in lom valovanja – sprememba hitrosti širjenja valovanja



$$\frac{\sin\alpha}{\sin\beta}=\frac{c_1}{c_2}$$



$$\overline{B'B}=c_1t=\overline{A'B}\sin\alpha$$

$$\overline{A'A}=c_2t=\overline{A'B}\sin\beta$$

$$\frac{c_1}{c_2}=\frac{\sin\alpha}{\sin\beta}$$

• Fermatov princip – za potovanje med dvema točkama valovanje „izbere“ pot, za katero potrebuje najmanj časa

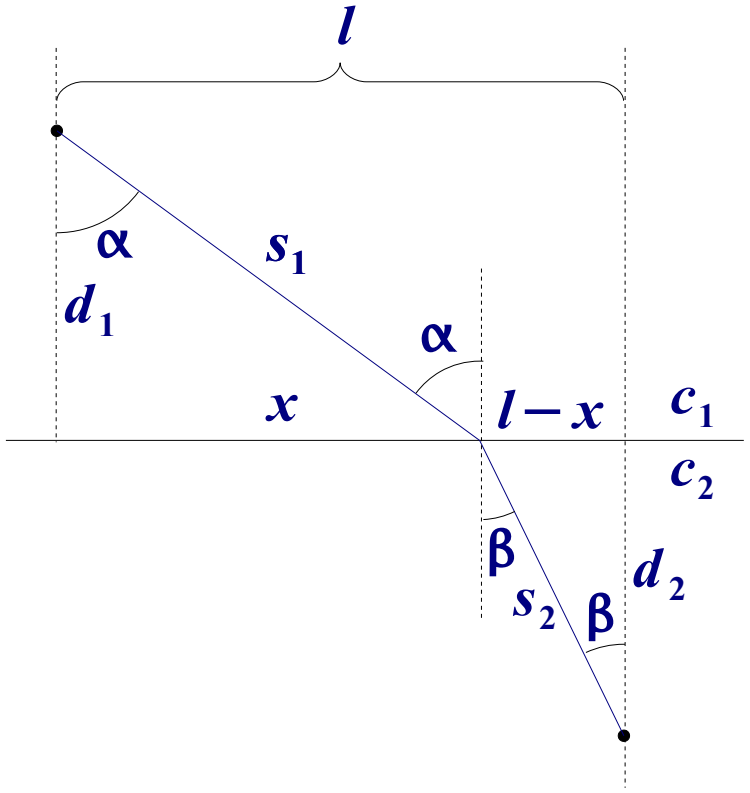
$$s_1 = \sqrt{d_1^2 + x^2}$$

$$s_2 = \sqrt{d_2^2 + (l-x)^2}$$

$$t = \frac{s_1}{c_1} + \frac{s_2}{c_2} = \frac{\sqrt{d_1^2 + x^2}}{c_1} + \frac{\sqrt{d_2^2 + (l-x)^2}}{c_2}$$

$$\frac{dt}{dx} = \frac{2x}{2c_1\sqrt{d_1^2 + x^2}} + \frac{-2(l-x)}{2c_2\sqrt{d_2^2 + (l-x)^2}} = 0$$

$$\frac{\sin\alpha}{c_1} - \frac{\sin\beta}{c_2} = 0 \Rightarrow \frac{\sin\alpha}{\sin\beta} = \frac{c_1}{c_2}$$

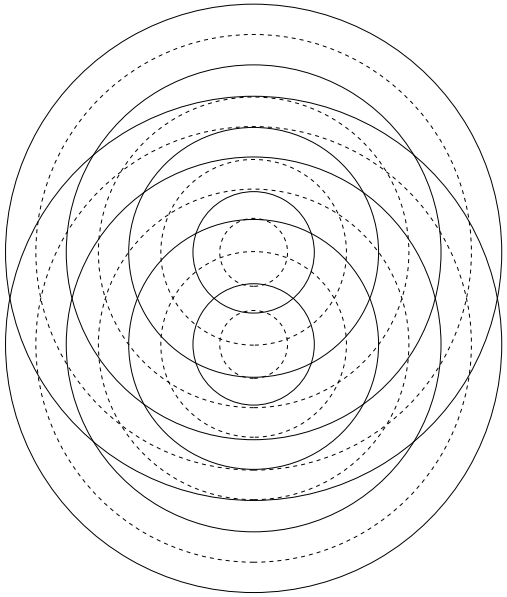


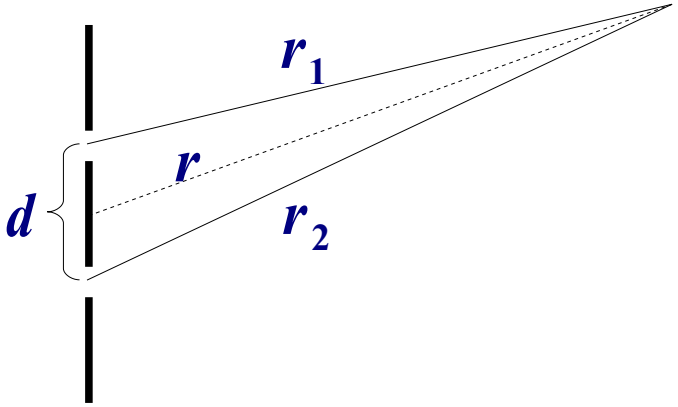
- interferenca dveh točkastih izvorov

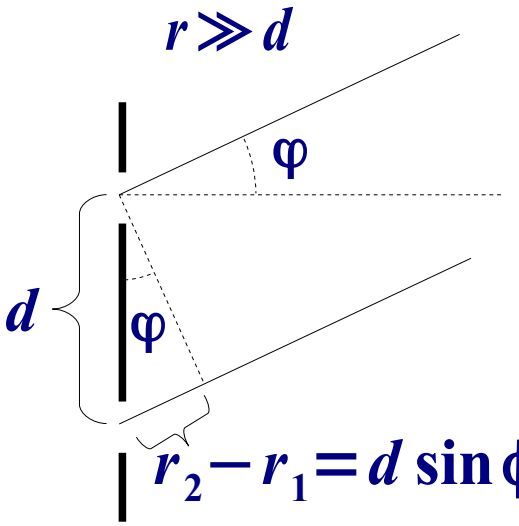
$$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$s(r_1,t)=s_0(r_1)\cos(\omega t-kr_1)$$

$$s(r_2,t)=s_0(r_2)\cos(\omega t-kr_2)$$







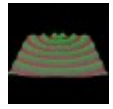
$$\delta=k\,d\sin\phi=\frac{2\pi\,d\sin\phi}{\lambda}$$

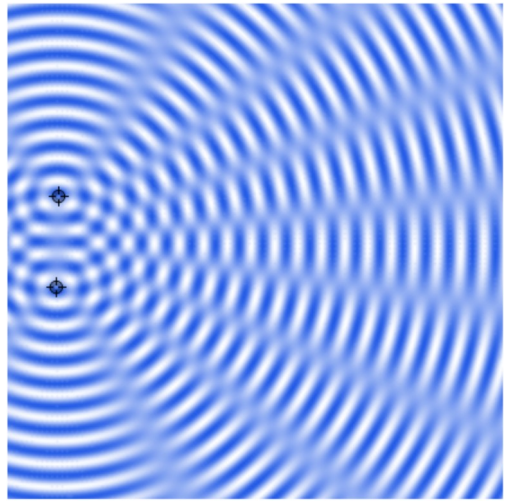
$r \gg d$

$$s(r,t)=s(r_1,t)+s(r_2,t)\approx s_0(r)\left(\cos(\omega t-kr_1)+\cos(\omega t-kr_2)\right)$$

$$s(r,t)\approx 2\,s_0(r)\cos\left(\omega t-k\frac{r_1+r_2}{2}\right)\cos\left(k\frac{r_2-r_1}{2}\right)$$

$$s(r,t)\approx 2\,s_0(r)\cos\left(k\frac{r_2-r_1}{2}\right)\cos(\omega t-kr)$$

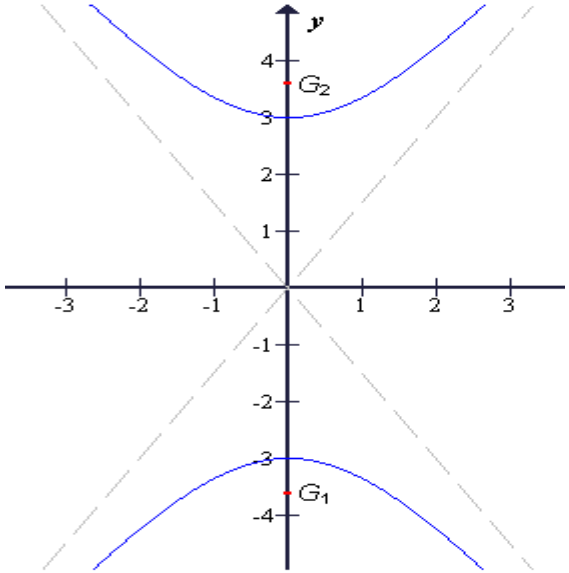




$$s(r,t) \approx \underbrace{2\,s_0(r)\cos\left(k\,\frac{r_2-r_1}{2}\right)}\;\underbrace{\cos(\omega t - kr)}$$

krajevno odvisna amplituda krožni val

$$s(r,t) \approx 2\,s_0(r)\cos\left(\frac{\pi d \sin\varphi}{\lambda}\right)\cos(\omega t - kr)$$



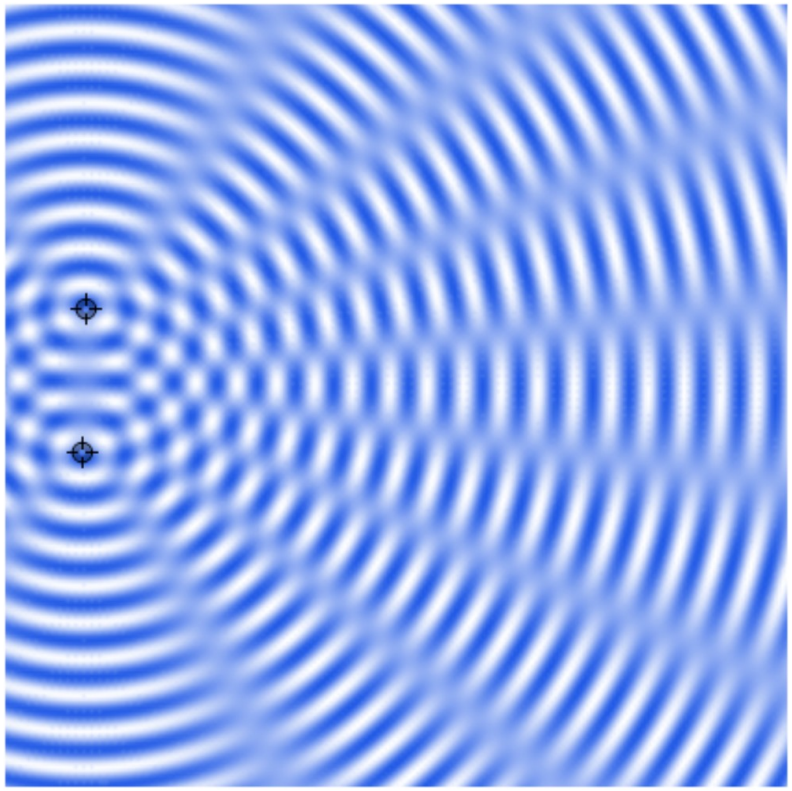
hiperbola

- ojačitve

$$\frac{\pi d \sin\varphi}{\lambda} = N\pi \;\Rightarrow\; d\sin\varphi = N\lambda$$

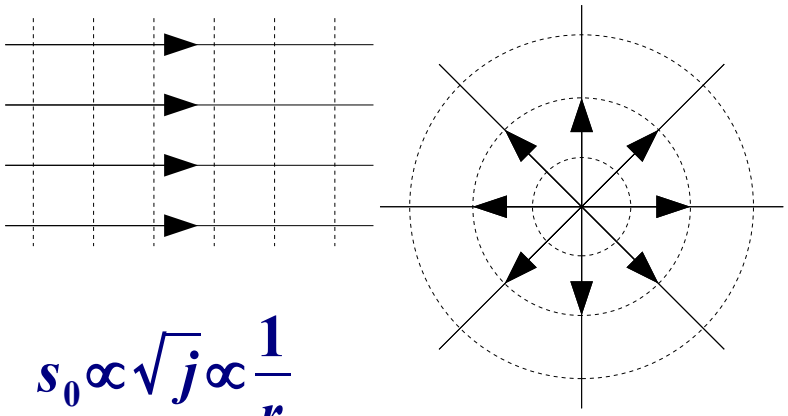
- oslabitve

$$\frac{\pi d \sin\varphi}{\lambda} = \frac{2N+1}{2}\pi \;\Rightarrow\; d\sin\varphi = \frac{2N+1}{2}\lambda$$



M

Valovanje v treh razsežnostih

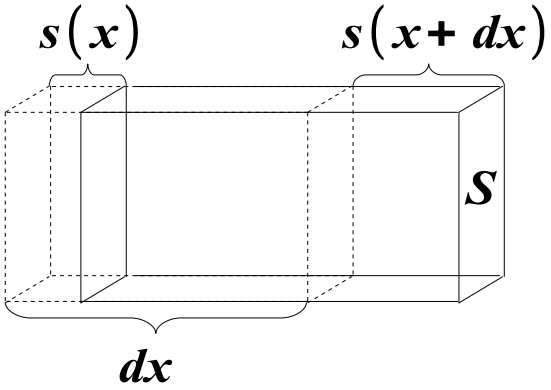


- ravni val $s(\vec{r},t)=s_0\cos(\omega t-kx)$
- krogelni val $s(\vec{r},t)=s_0(r)\cos(\omega t-kr)$
  - odvisnost gostote energijskega toka od oddaljenosti do izvira

$$j\propto\frac{1}{r^2}\ ,\quad s_0\propto\sqrt{j}\propto\frac{1}{r}$$

- akustika – zvočni valovi
  - zveza med amplitudo spremembe tlaka in odmikom

$$\frac{\Delta V}{V}=-\chi_s\Delta p$$



$$\frac{\cancel{S}\left(s(x+dx)-s(x)\right)}{\cancel{S}\,dx}=-\chi_s\Delta p$$

$$\cancel{S}\left(-p(x+dx)+p(x)\right)=\rho\cancel{S}\,dx\frac{\partial^2 s}{\partial t^2}$$

$$\frac{\partial s}{\partial x}=-\chi_s\Delta p$$

$$-\frac{\partial p}{\partial x}=\rho\frac{\partial^2 s}{\partial t^2}=\frac{1}{\chi_s}\frac{\partial^2 s}{\partial x^2}$$

$$\Delta p=-\frac{1}{\chi_s}\frac{\partial s}{\partial x}=-\frac{1}{\chi_s}k s_0\sin(\omega t-kx)$$

$$\frac{\partial^2 s}{\partial x^2}=\rho\chi_s\frac{\partial^2 s}{\partial t^2}$$

valovna enačba

$$\Delta p_0=\frac{1}{\chi_s}k s_0=c^2\rho k s_0=\rho\omega s_0 c$$

- jakost zvoka – povprečna gostota energijskega toka

$$\bar{j} = c\,\bar{w} = \frac{1}{2} c \rho \omega^2 s_0^2$$

$$jakost = -10 \log \frac{j}{j_0} \quad [decibel]$$

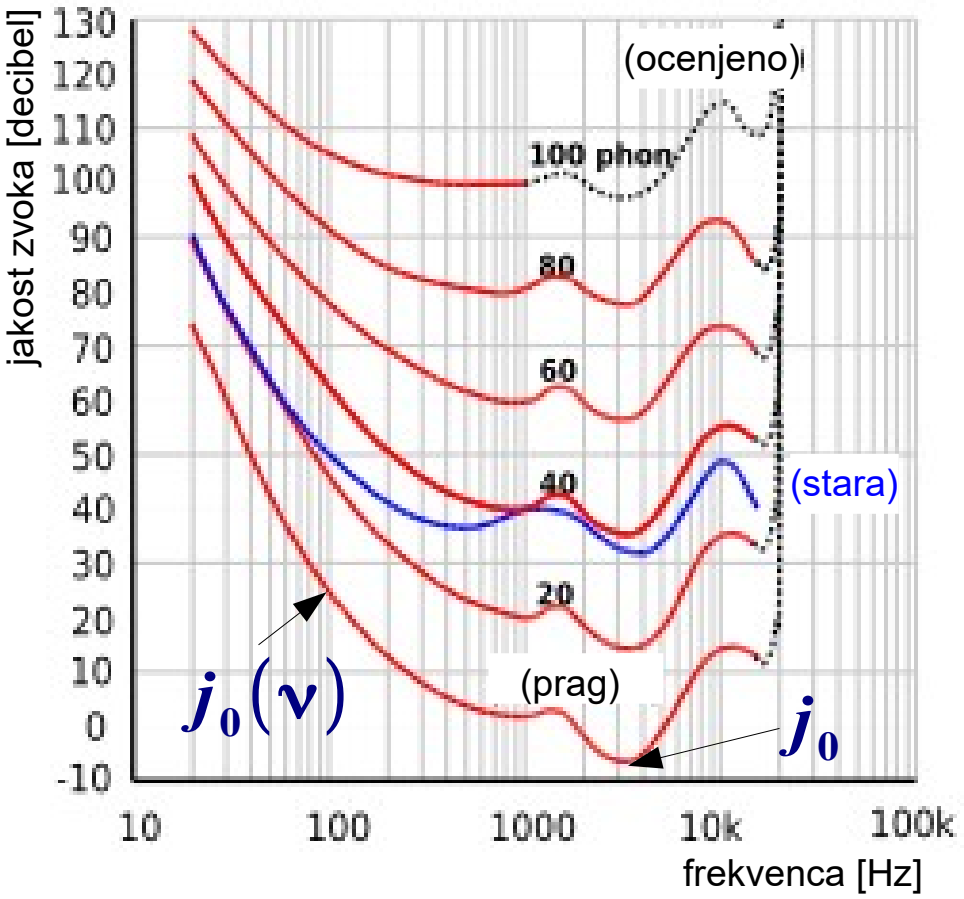
$$j_0 = 10^{-12}\, W/m^2$$

- glasnost – fiziološka količina

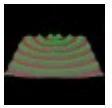
$$glasnost = -10 \log \frac{j}{j_0(\nu)} \quad [fon]$$

$$\Delta p_0 = c \rho \omega s_0 = \frac{2\bar{j}}{\omega s_0}$$

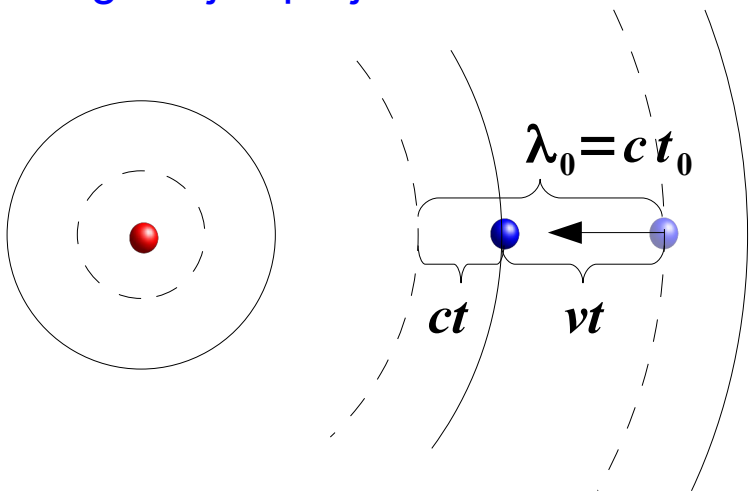
j [W/m^2]	s_0 [m]	Δp_0 [Pa]
10^{-12}	$1.1\ 10^{-11}$	$2.8\ 10^{-5}$
1	$1.1\ 10^{-5}$	28



- Dopplerjev pojav



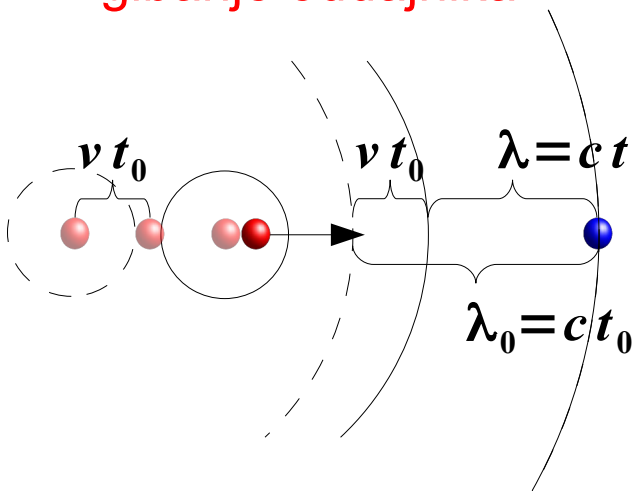
gibanje sprejemnika



$$\lambda_0 = c\,t + v\,t$$

$$\nu = \nu_0\left(1 \pm \frac{v}{c}\right)$$

gibanje oddajnika

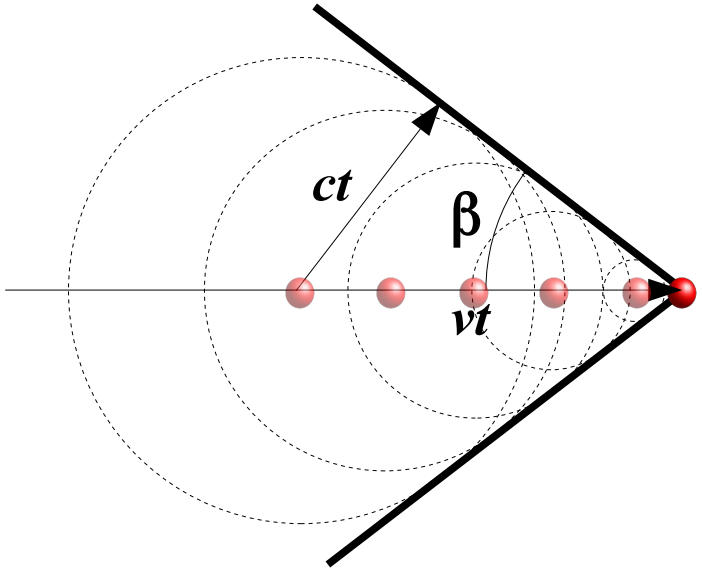


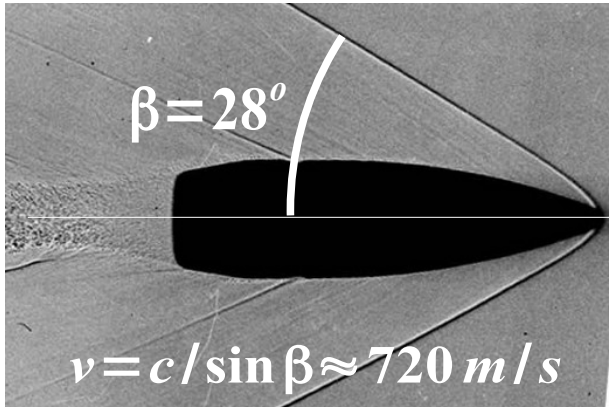
$$\lambda = \lambda_0 - v\,t_0$$

$$\nu = \nu_0 \frac{1}{\left(1 \mp \frac{v}{c}\right)}$$

- Machovo valovno čelo

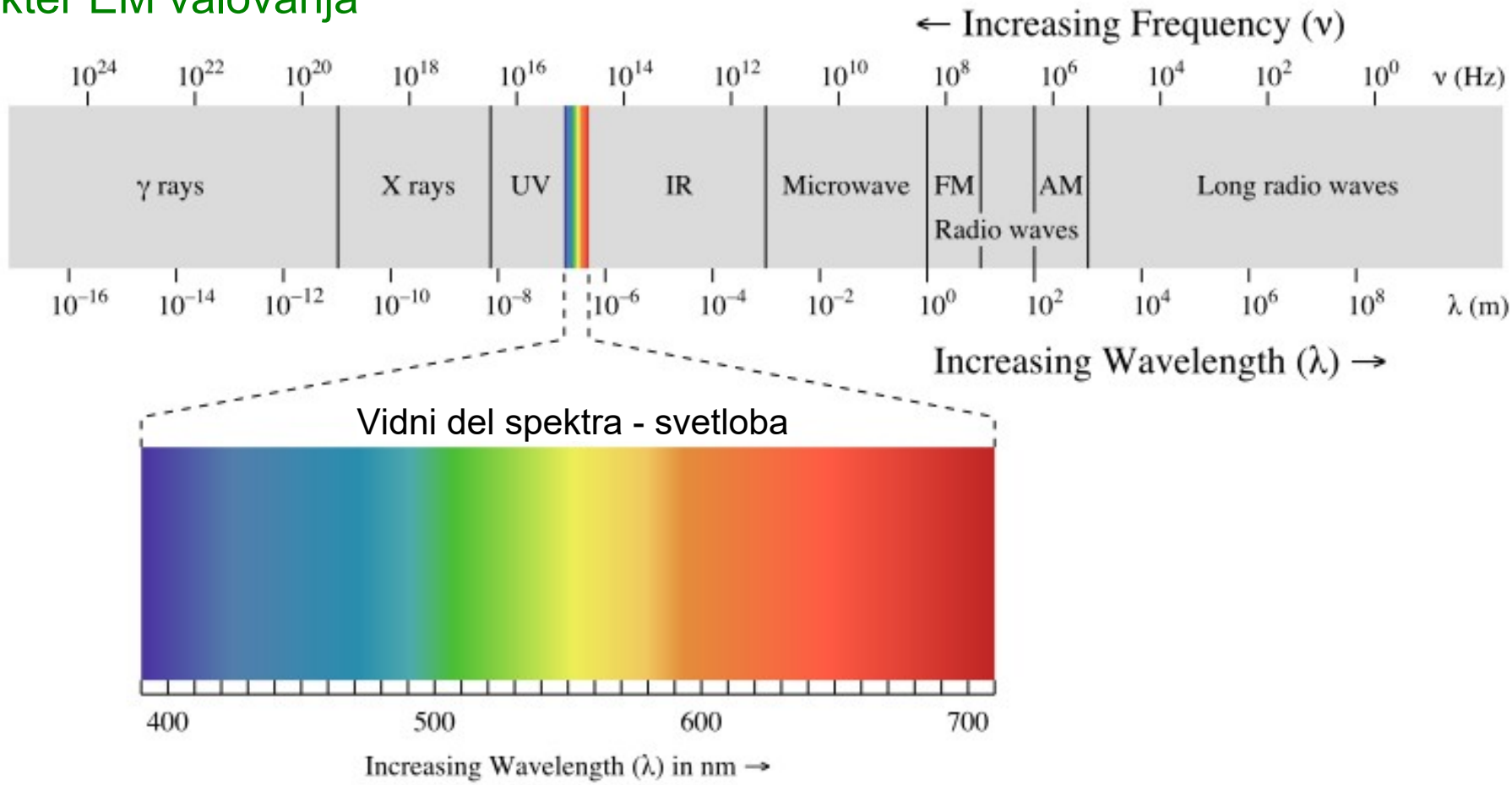
$$\sin\beta = \frac{c}{v}$$





Elektromagnetno valovanje

- spekter EM valovanja



- amplitudi E_0 in B_0
- hitrost širjenja EM valovanja c_0
- Lomni količnik n
- energija EM valovanja
- svetlobni tlak

Elektromagnetno valovanje – ravni val

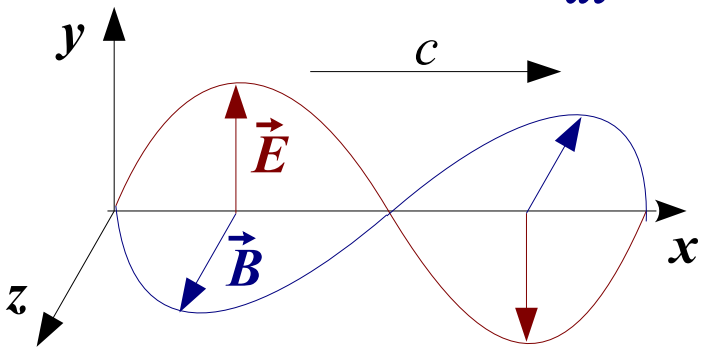
$$-\oint \vec{E}\cdot d\vec{r}=\int \frac{d\vec{B}}{dt}\cdot d\vec{S}$$

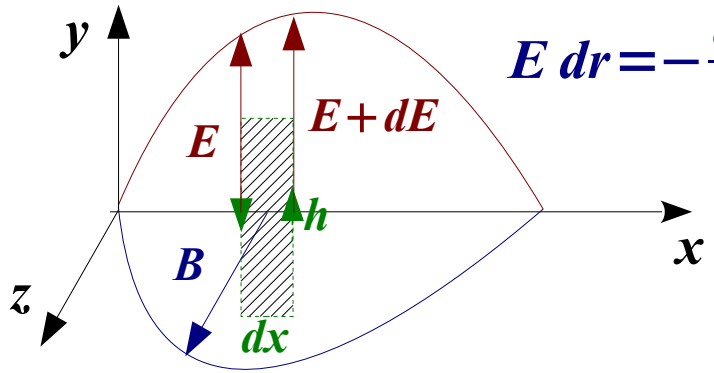
$$\frac{1}{\mu_0}\oint \vec{B}\cdot d\vec{r}=\epsilon_0\int \frac{d\vec{E}}{dt}\cdot d\vec{S}$$

Ravni val:

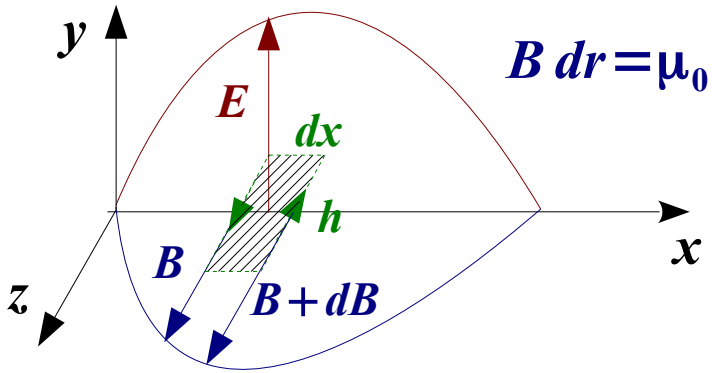
- v prostoru je samo polje, nabojev ni
- $\vec{E}, \vec{B} \perp \vec{k}$
- $\vec{E} \perp \vec{B}$
- $\vec{E}\times\vec{B} \parallel \vec{k}$
- **polji nihata sočasno**

$$E_y = E_0 \cos(\omega t - kx)$$
$$B_z = B_0 \cos(\omega t - kx)$$





$$E\,dr = -\frac{dB}{dt}\,dS$$



$$B\,dr = \mu_0\epsilon_0\frac{dE}{dt}\,dS$$

$$(E+dE)h - Eh = -\frac{dB}{dt}h\,dx \Rightarrow \frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

$$Bh - (B+dB)h = \mu_0\epsilon_0\frac{dE}{dt}h\,dx \Rightarrow -\frac{\partial B}{\partial x} = \mu_0\epsilon_0\frac{\partial E}{\partial t}$$

$$\frac{\partial E}{\partial x} = -E_0 k \sin(\omega t - kx)$$
$$\frac{\partial B}{\partial t} = -B_0 \omega \sin(\omega t - kx)$$
$$E_0 k = B_0 \omega$$
$$\boxed{E_0 = B_0 c_0}$$

$$\frac{\partial B}{\partial x} = -B_0 k \sin(\omega t - kx)$$
$$\frac{\partial E}{\partial t} = -E_0 \omega \sin(\omega t - kx)$$
$$B_0 k = \mu_0\epsilon_0 E_0 \omega$$
$$\boxed{c_0^2 = \frac{1}{\mu_0\epsilon_0}}$$

Energija EM valovanja

- EM valovanje nosi energijo v električnem in magnetnem polju

$$w_e = \frac{1}{2}\vec{E}\cdot\vec{D} = \frac{1}{2}\epsilon\epsilon_0 E_0^2 \cos^2(\omega t - kx)$$

$$w_m = \frac{1}{2}\vec{H}\cdot\vec{B} = \frac{1}{2}\frac{1}{\mu\mu_0} B_0^2 \cos^2(\omega t - kx)$$

$$w = w_e + w_m = \epsilon\epsilon_0 E_0^2 \cos^2(\omega t - kx) =$$
$$= \epsilon\epsilon_0 E_0 c B_0 \cos^2(\omega t - kx) =$$
$$= \frac{E_0 B_0}{c\mu\mu_0}\cos^2(\omega t - kx) =$$
$$= \frac{E_0 H_0}{c}\cos^2(\omega t - kx) = \frac{E H}{c}$$

$$w_{e,max} = \frac{1}{2}\epsilon\epsilon_0 E_0^2 = \frac{1}{2}\epsilon\epsilon_0 c^2 B_0^2 = \frac{1}{2}\frac{\cancel{\epsilon\epsilon_0}}{\cancel{\epsilon\epsilon_0}\mu\mu_0} B_0^2 = w_{m,max}$$

$$\bar{j} = c\,\bar{w} = c\cdot\frac{1}{2}\epsilon\epsilon_0 E_0^2$$

$$\boxed{\bar{j} = \frac{1}{2} c\,\epsilon\epsilon_0 E_0^2}$$

$$j = c\,w = E\,H$$

$$\vec{j} = \vec{E}\times\vec{H}$$

Poyntingov vector

Svetlobni tlak

- svetlobni snop ima gibalno količino in povzroča tlak na površino kamor vpada

gibalna količina

$$G = \frac{W}{c_0}$$

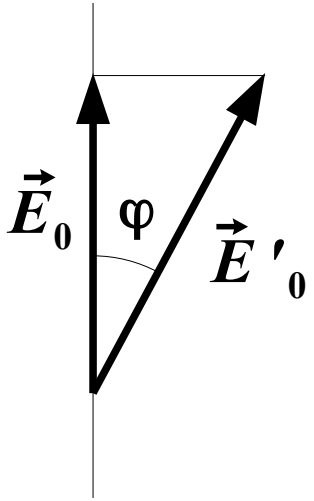
svetlobni tlak

absorbiran žarek $\bar{p} = \frac{\bar{j}}{c_0}$

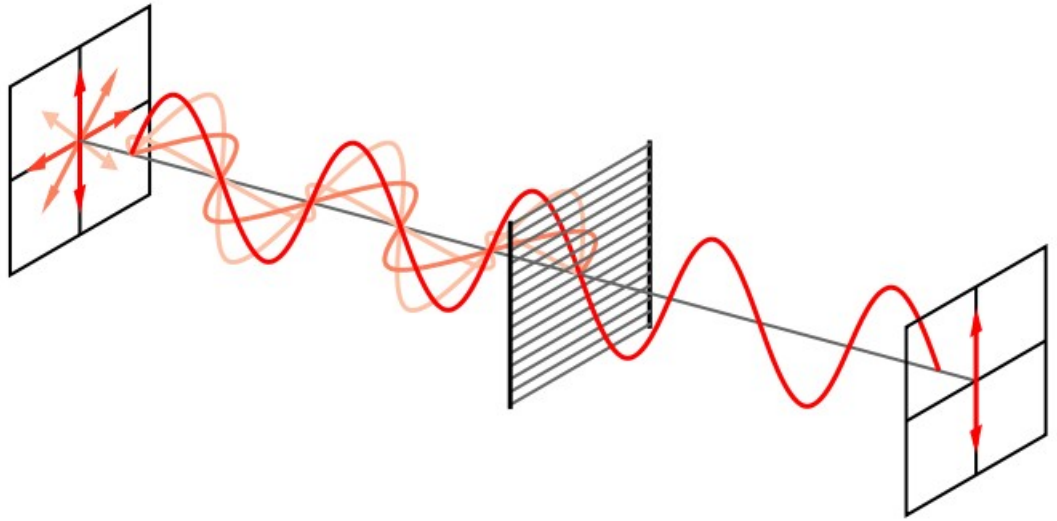
odbiti žarek $\bar{p} = \frac{2\bar{j}}{c_0}$

Polarizacija svetlobe

- smer polarizacije je smer E
- linearna polarizacija – smer E se ohranja
- nepolarizirana – smer E se naključno spreminja
- polarizator



$$E_0 = E'_0 \cos\varphi$$



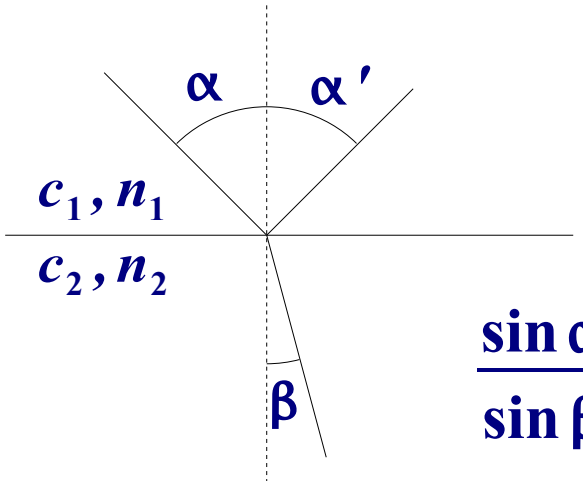
- gostota energijskega toka prepuščene svetlobe, ko je vpadna svetloba:
  - linearno polarizirana

$$\bar{j} = \frac{1}{2} c_0 \epsilon_0 E_0^2 = \frac{1}{2} c_0 \epsilon_0 E'^2_0 \cos^2\varphi = \bar{j}' \cos^2\varphi$$

  - ne polarizirana

$$\bar{j} = \frac{1}{2} c_0 \epsilon_0 \overline{E_0^2(t)} = \frac{1}{2} c_0 \epsilon_0 E'^2_0 \overline{\cos^2\varphi(t)} = \frac{1}{2} \bar{j}'$$

Odboj in lom

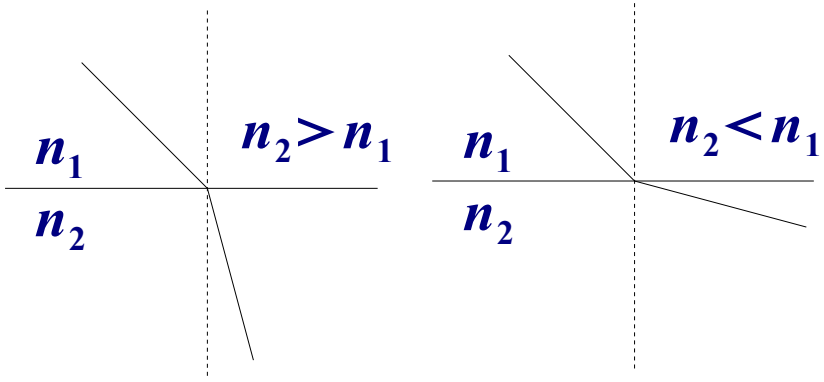


odboj

$$\alpha = \alpha'$$

lom

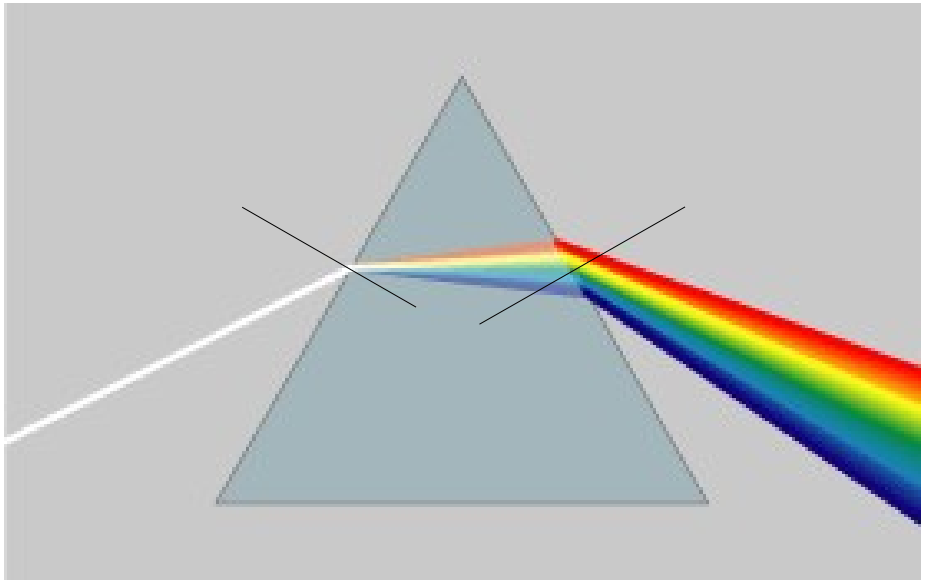
$$\frac{\sin\alpha}{\sin\beta} = \frac{c_1}{c_2} = \frac{n_2}{n_1}$$

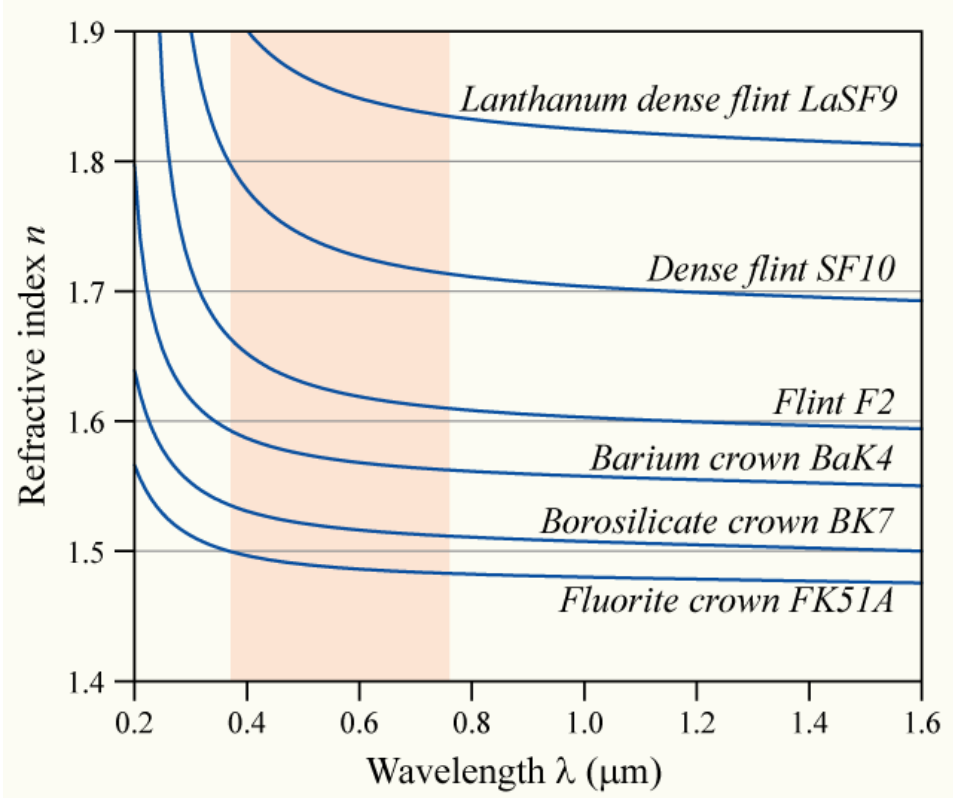


optično redkejše → gostejše

optično gostejše → redkejše

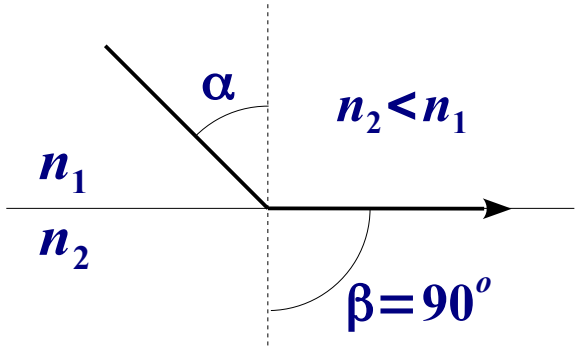
- disperzija – spektrometer na prizmo





• totalni odboj

$$\sin\alpha_T = \frac{n_2}{n_1}$$



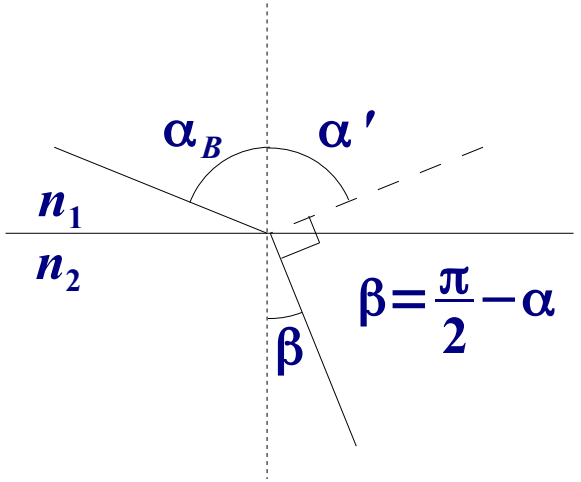
mejni primer

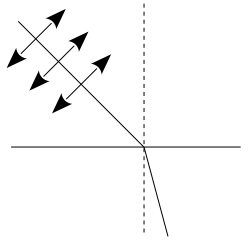
$$\frac{\sin\alpha}{\sin\beta} = \frac{c_1}{c_2} = \frac{n_2}{n_1}$$

• polarizacija pri odboju

Brewsterjev kot - α_B

$$\frac{\sin\alpha}{\sin\beta} = \frac{\sin\alpha}{\sin\left(\frac{\pi}{2}-\alpha\right)} = \frac{\sin\alpha}{\cos\alpha} = \tan\alpha_B = \frac{n_2}{n_1}$$





vzporedna polarizacija

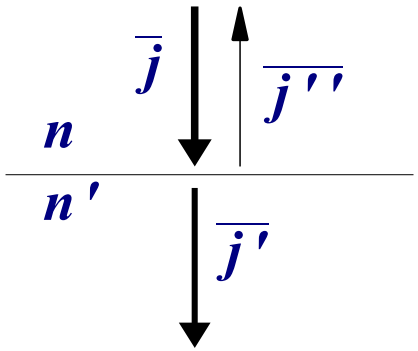
• gostota energijskega toka odbite in prepuščene svetlobe na meji med dvema sredstvoma (pravokotni vpad)

$$\frac{\bar{j}''}{\bar{j}} = \frac{(n-n')^2}{(n+n')^2}$$

odbita svetloba

$$\frac{\bar{j}'}{\bar{j}} = \frac{4nn'}{(n+n')^2}$$

prepuščena svetloba

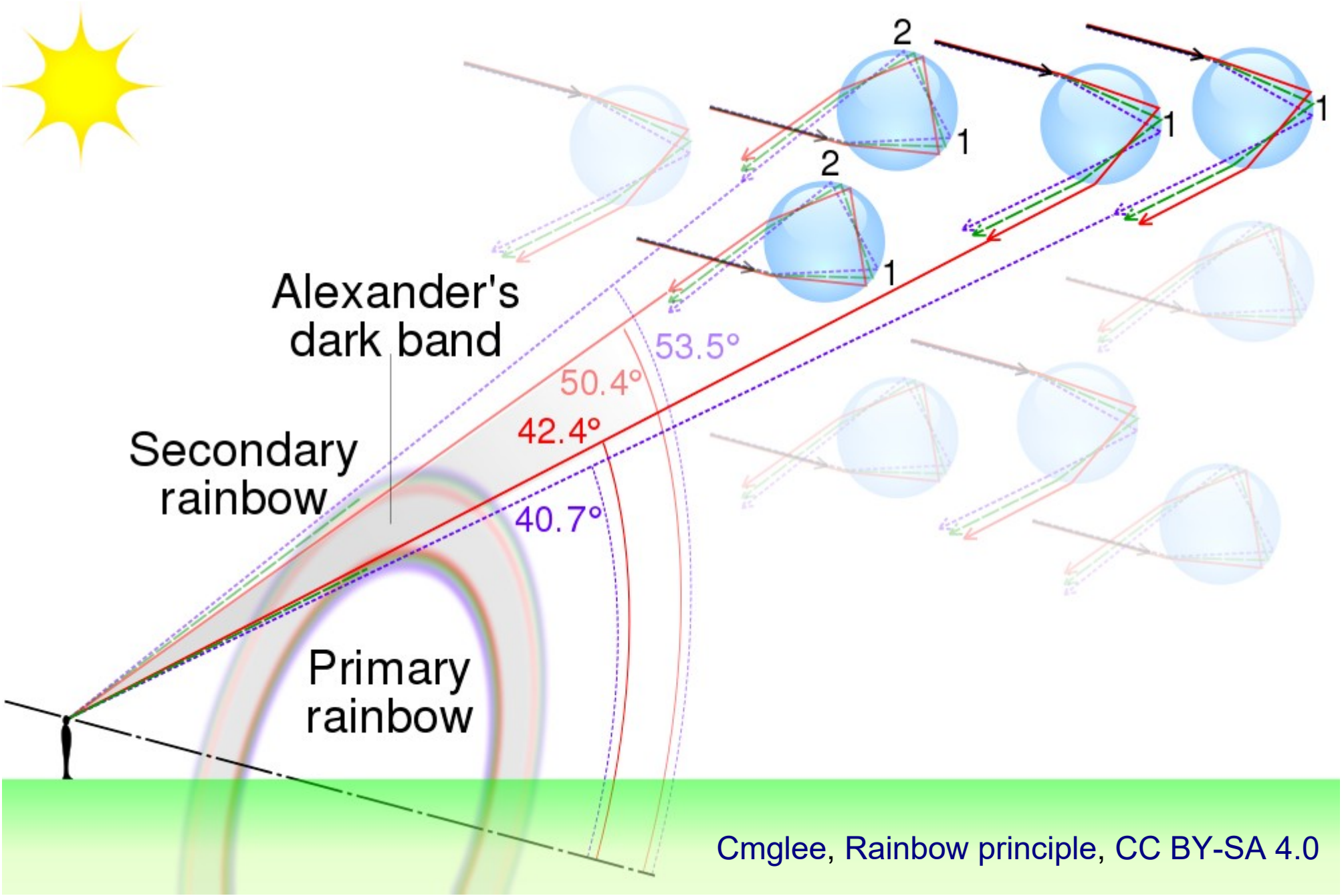


- fidografiji pri različnih smereh polarizatorja

navpična smer polarizatorja

vodoravna smer polarizatorja

- mavrica



Cmglee, Rainbow principle, CC BY-SA 4.0

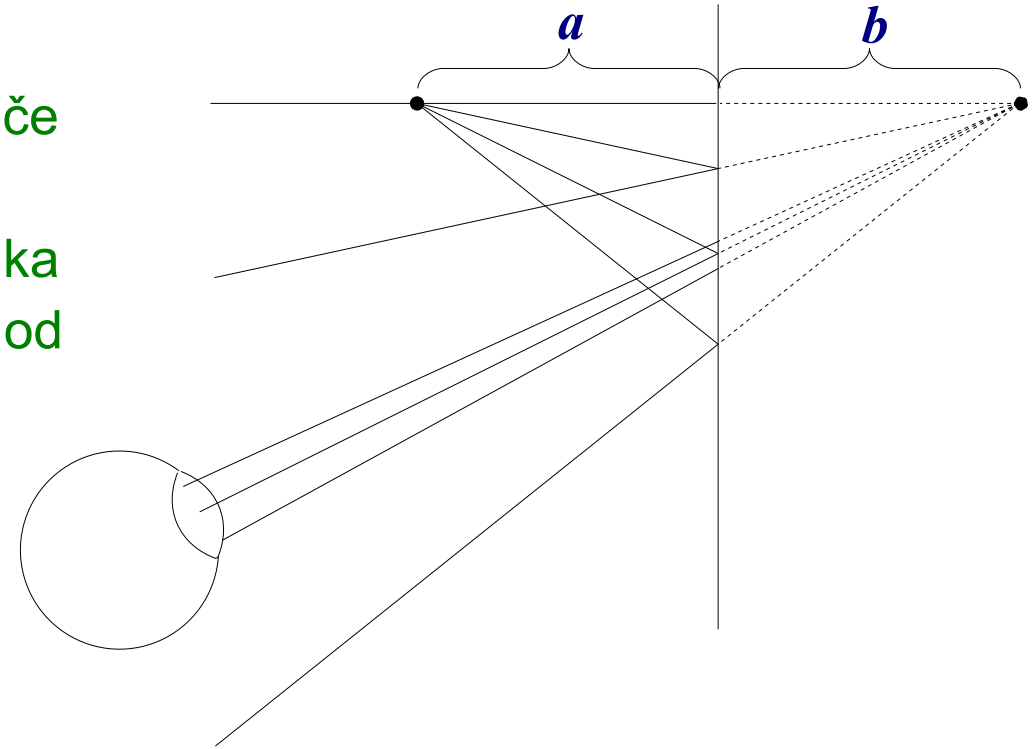
Geometrijska optika

- širjenje svetlobe v ravnih črtah - žarki
- predmeti so veliki v primerjavi z valovno dolžino
- preslikav: predmet – slika
  - prava (realna) slika – žarki se sekajo
  - navidezna (virtualna) – podaljški žarkov se sekajo

Ravno zrcalo

- pri zrcalih je razdalja do slike pozitivna, če je slika na isti strani kot predmet
- pri ravnem zrcalu nastane navidezna slika na nasprotni strani in je enako oddaljena od zrcala kot predmet

$$b = -a$$



Krogelno zrcalo

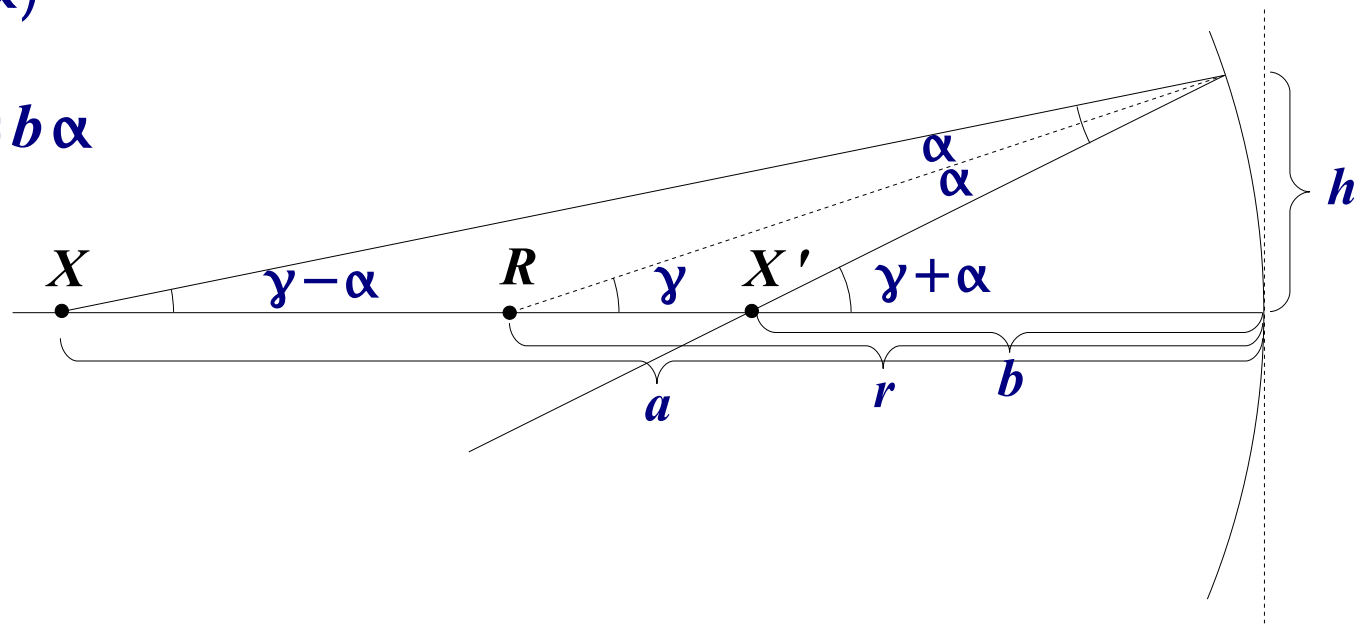
$$a(\gamma-\alpha)\approx r\gamma\approx b(\gamma+\alpha)$$

$$a\gamma-r\gamma=a\alpha \qquad r\gamma-b\gamma=b\alpha$$

$$\frac{a-r}{a}=\frac{\alpha}{\gamma}=\frac{r-b}{b}$$

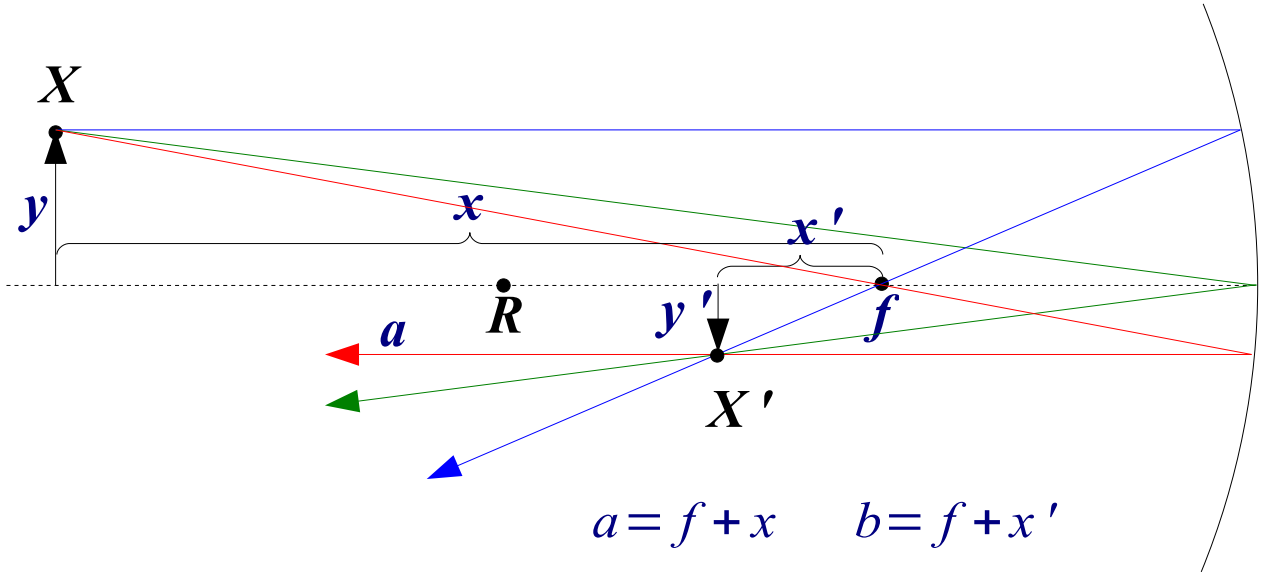
$$(a+b)r=2ab$$

$$\frac{1}{a}+\frac{1}{b}=\frac{1}{r/2}=\frac{1}{f}$$



$$\boxed{\frac{1}{a}+\frac{1}{b}=\frac{1}{f}} \qquad \boxed{f=\frac{1}{2}r}$$

$$\boxed{xx'=f^2}$$



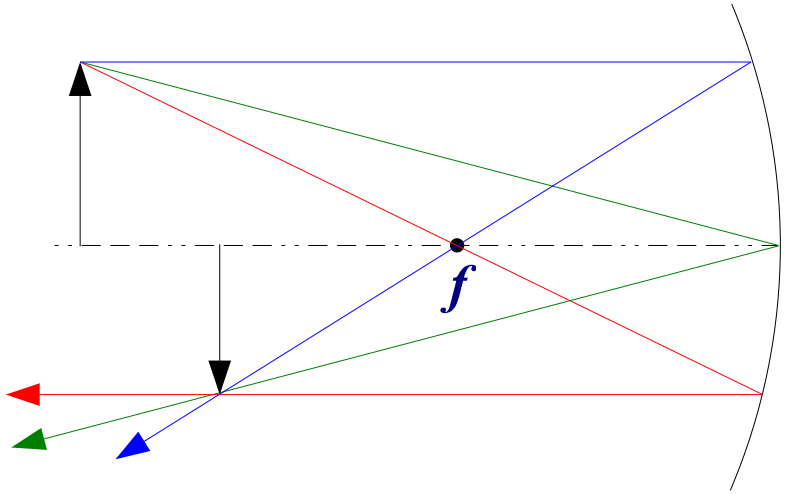
$$\frac{1}{f+x}+\frac{1}{f+x'}=\frac{1}{f}$$

$$a=f+x \qquad b=f+x'$$

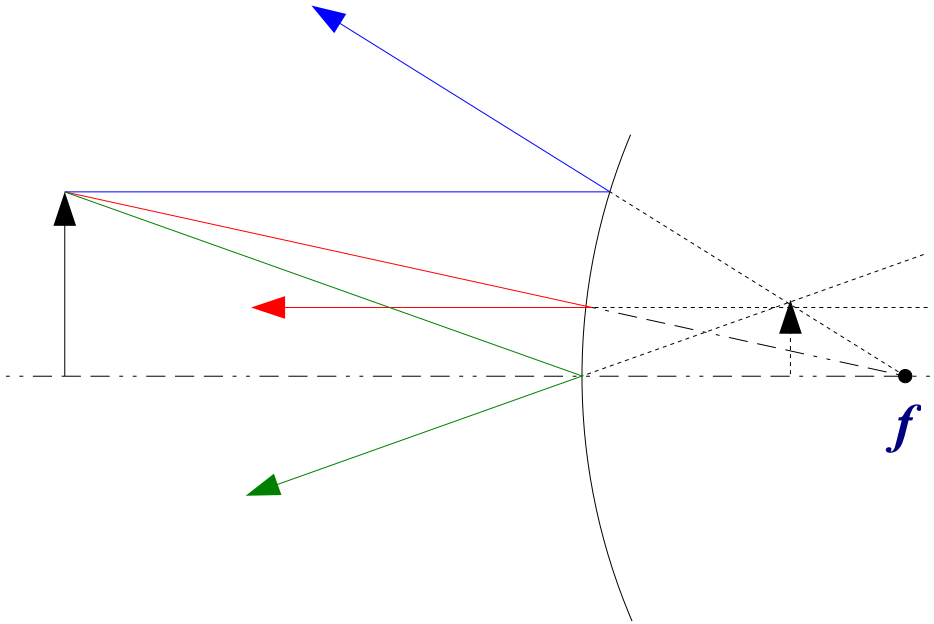
$$f(f+x')+f(f+x)=(f+x)(f+x')\Rightarrow f^2+\cancel{fx'}+\cancel{f^2}+\cancel{fx}=\cancel{f^2}+\cancel{fx}+\cancel{fx'}+xx'$$

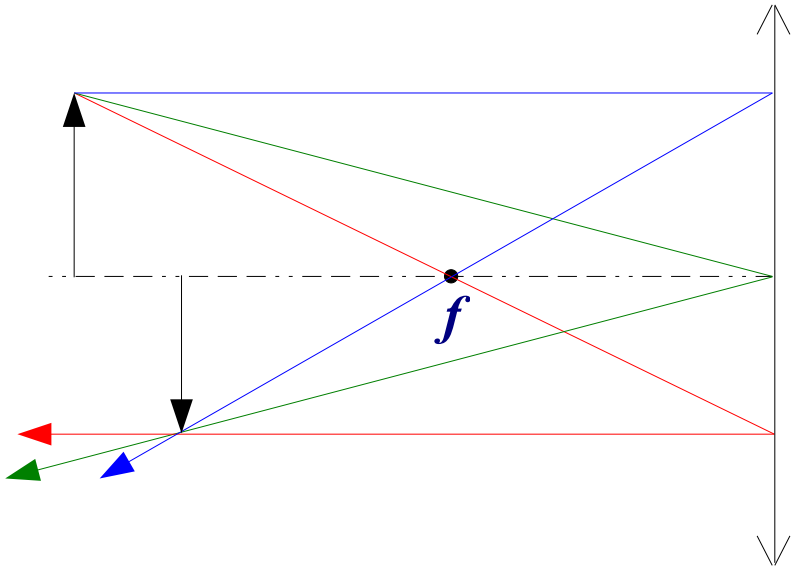
- načrtovanje žarkov

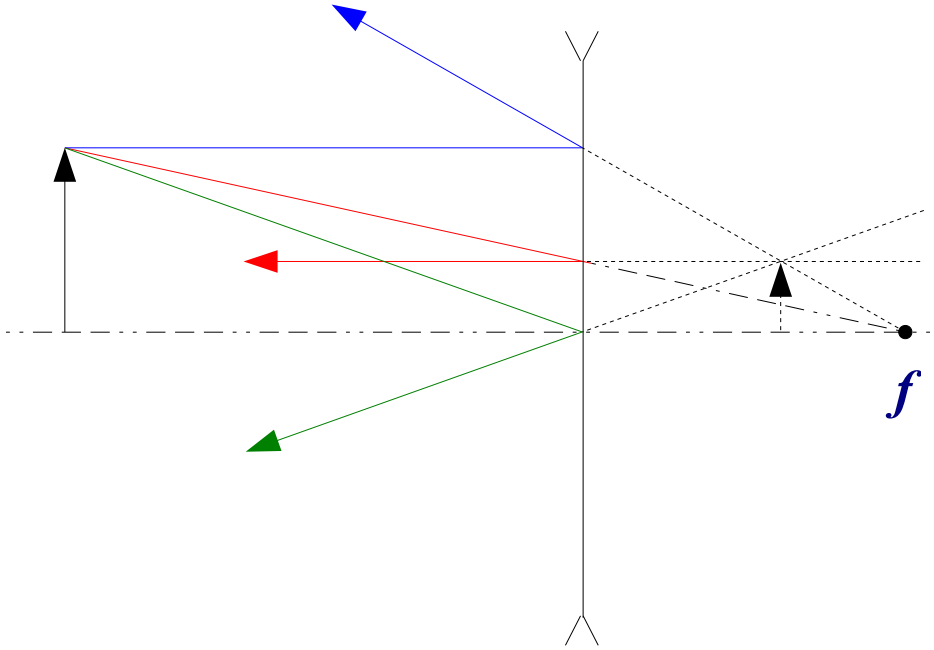
KONKAVNO ZRCALO



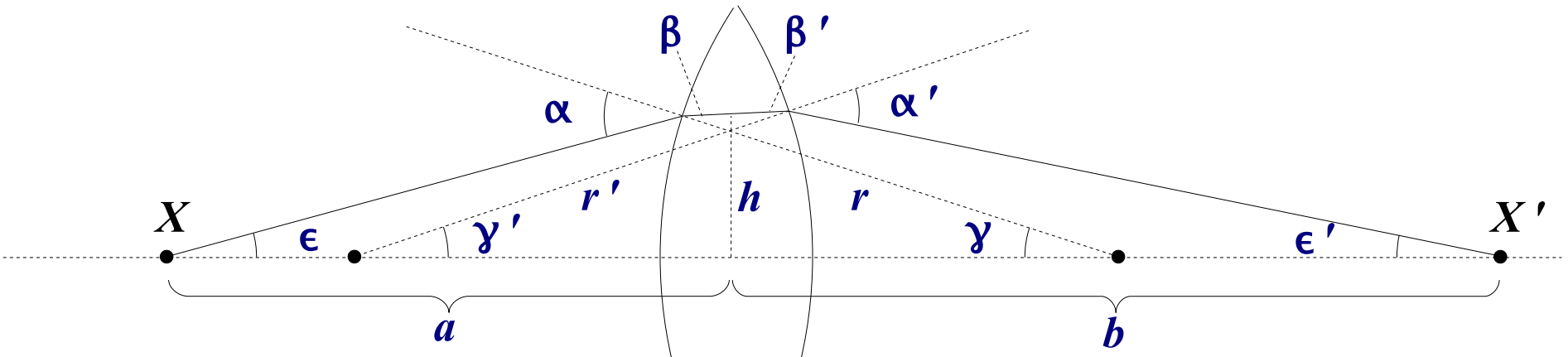
KONVEKSNO ZRCALO







Leče



$$\alpha = \epsilon + \gamma \qquad \alpha' = \epsilon' + \gamma'$$

$$\alpha \approx n\beta \qquad \alpha' \approx n\beta'$$

$$\beta + \beta' = \gamma + \gamma'$$

$$\gamma \approx \frac{h}{r} \; ; \; \epsilon \approx \frac{h}{a}$$

$$\gamma' \approx \frac{h}{r'} \; ; \; \epsilon' \approx \frac{h}{b}$$

$$n(\beta + \beta') = \epsilon + \epsilon' + \gamma + \gamma'$$

$$(n-1)(\gamma + \gamma') = \epsilon + \epsilon'$$

$$(n-1)\left(\frac{1}{r} + \frac{1}{r'}\right) = \frac{1}{a} + \frac{1}{b}$$

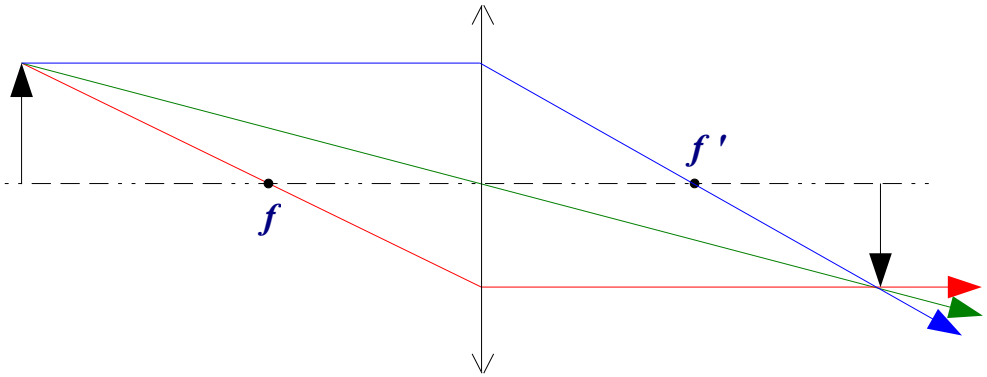
$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$$

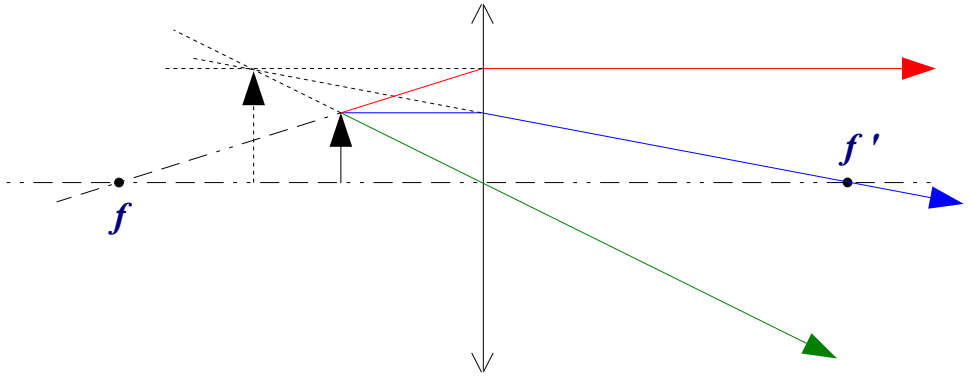
$$x\,x' = f^2$$

$$\frac{1}{f} = (n-1)\left(\frac{1}{r} + \frac{1}{r'}\right)$$

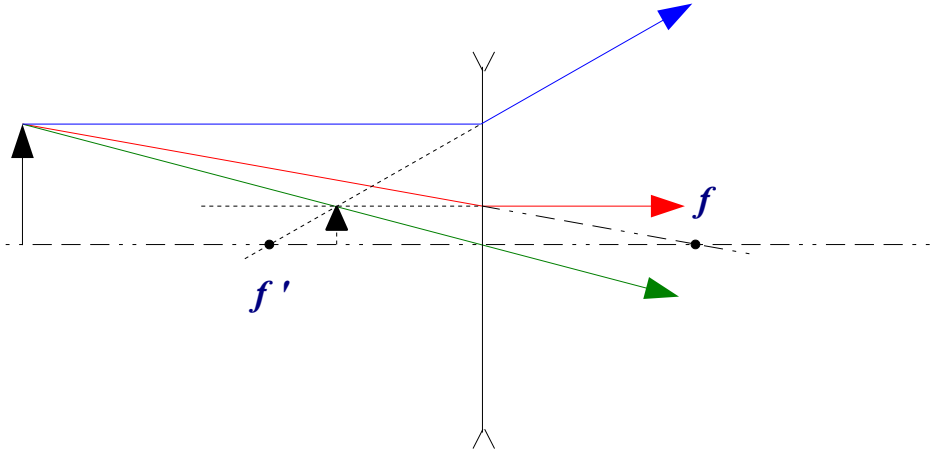
- načrtovanje žarkov

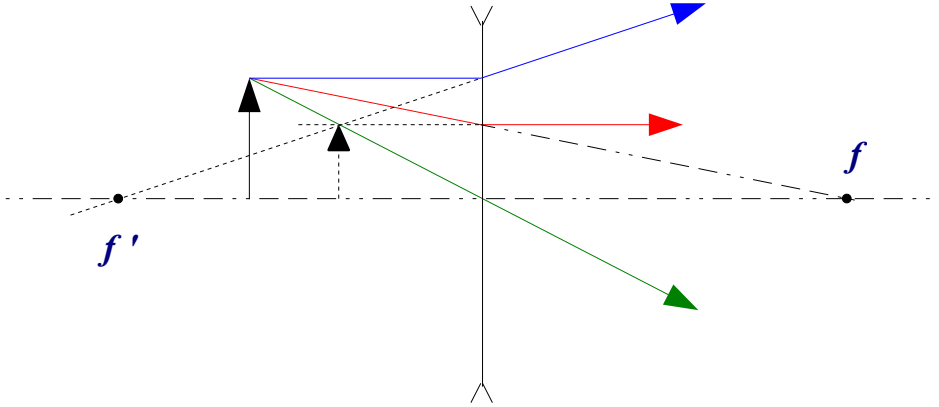
KONVEKSNA LEČA



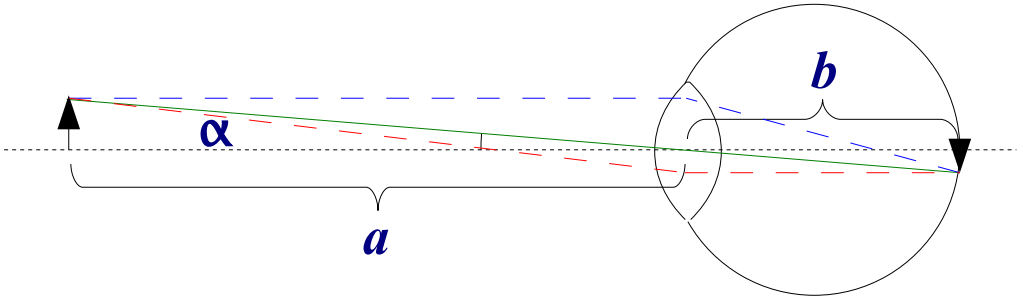


KONKAVNA LEČA





Optične naprave



Oko:

- goriščna razdalja se spreminja
- razdalja do slike je konstantna

$$b \approx 2\,cm$$

- velikost slike je odločena z zornim kotom

$$y' = b \tan \alpha$$

- najmanjši kot pri katerem še ločimo predmete je

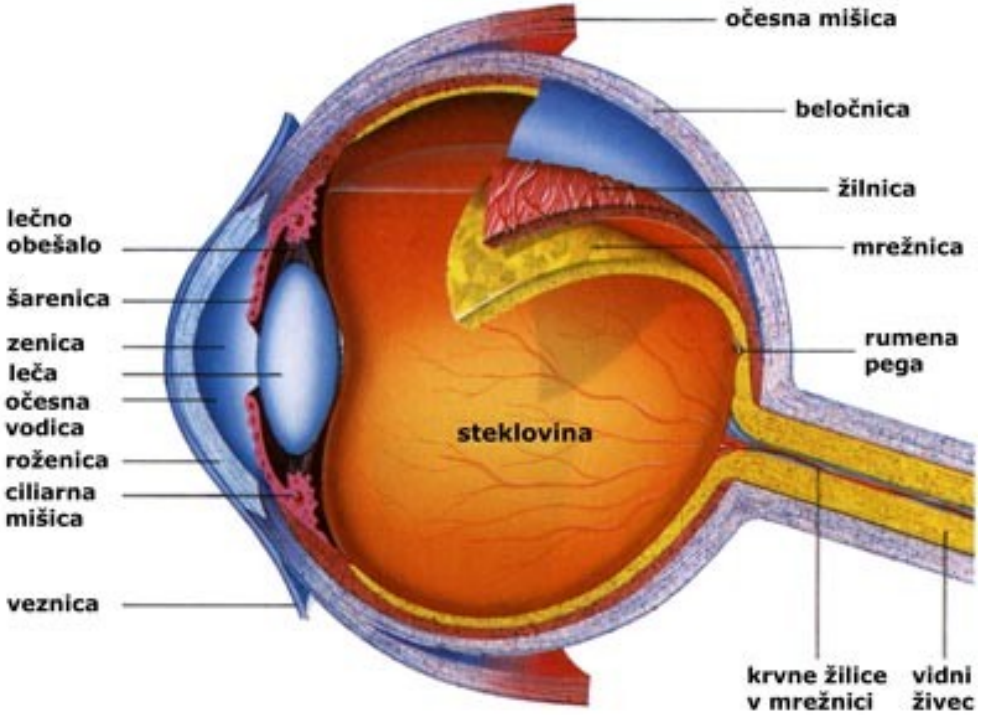
$$\alpha_0 \approx 1' \approx 3 \cdot 10^{-4}\,rad \qquad y'_0 \approx b\,\alpha_0 \approx 6\,\mu m$$

- najmanjša razdalja ko še vidimo ostro

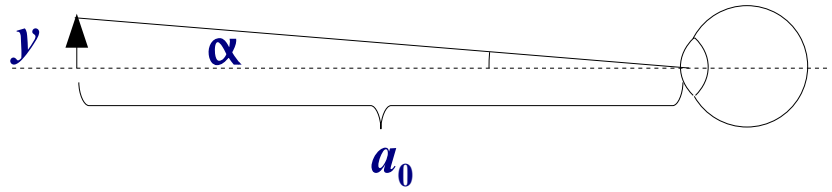
$$a_0 \approx 25\,cm \qquad y_0 \approx a_0\,\alpha_0 \approx 0{,}1\,mm$$

- občutljivost očesa (555 nm)

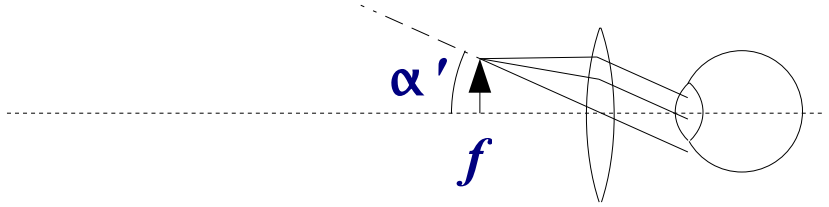
$$j_0 \approx 10^{-12}\,W/m^2$$



Lupa omogoča, da približamo predmet na manjšo razdaljo kot je a_0 saj ga preslika v neskončnost.

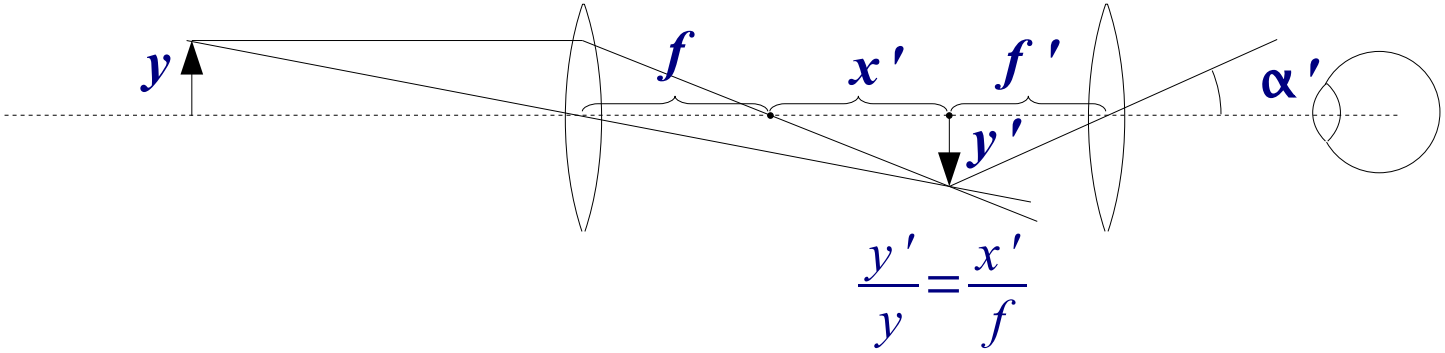


- povečava



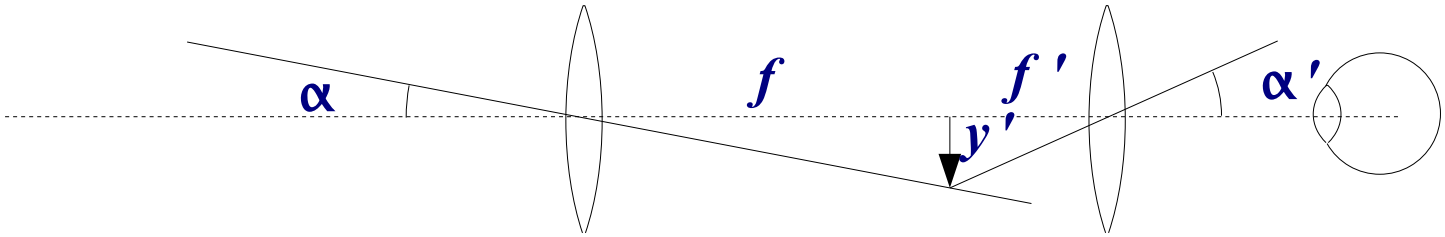
$$N = \frac{\tan\alpha'}{\tan\alpha} = \frac{y}{f}\frac{a_0}{y} = \frac{a_0}{f}$$

- mikroskop



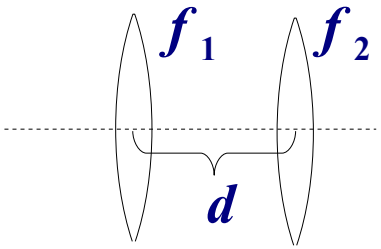
$$N = \frac{\tan\alpha'}{\tan\alpha} = \frac{y'}{f'}\frac{a_0}{y} = \frac{x'a_0}{f'f}$$

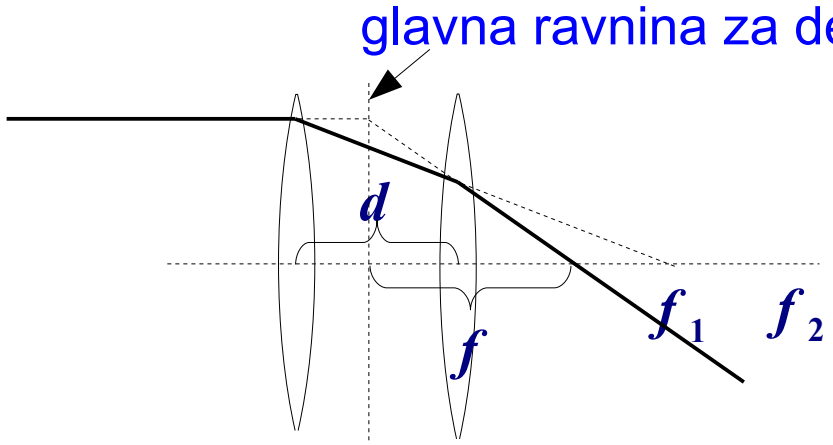
- teleskop (daljnogled s predmetom v ∞)



$$N = \frac{\tan\alpha'}{\tan\alpha} = \frac{y'}{f'}\frac{f}{y'} = \frac{f}{f'}$$

- lečje – sestav tankih leč





$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

Interferenca in uklon

$$j = c_0 \epsilon_0 E^2$$

Koherentna in nekoherentna valovanja

$$\bar{j} = \frac{1}{2} c_0 \epsilon_0 E_0^2$$

$$E_1 = E_{10} \cos(\omega t - kx) \qquad E_2 = E_{20} \cos(\omega t - kx + \delta)$$

$$j = c_0 \epsilon_0 (E_1 + E_2)^2 \propto E_1^2 + E_2^2 + 2 E_1 E_2$$

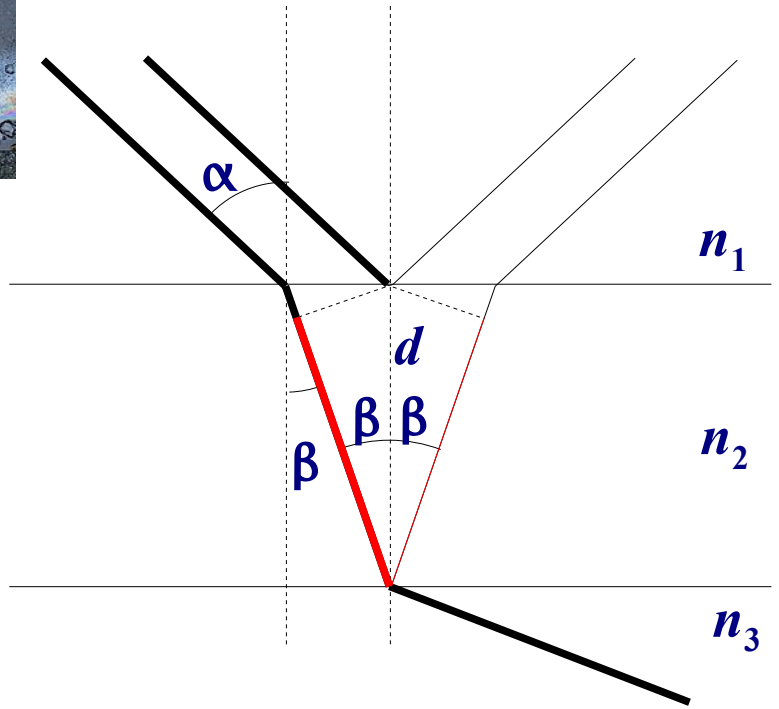
$$j \propto E_{10}^2 \cos^2(\omega t - kx) + E_{20}^2 \cos^2(\omega t - kx + \delta) +$$
$$+ 2 E_{10} E_{20} \cos(\omega t - kx) \cos(\omega t - kx + \delta)$$

$$\bar{j} \propto \frac{1}{2} E_{10}^2 + \frac{1}{2} E_{20}^2 + 2 E_{10} E_{20} \frac{1}{2} (\overline{\cos(2(\omega t - kx) + \delta)} + \overline{\cos(\delta)})$$

$$\bar{j} \propto \frac{1}{2} E_{10}^2 + \frac{1}{2} E_{20}^2 + E_{10} E_{20} \overline{\cos(\delta(t))}$$

- koherentni valovanji → interferenca
- nekoherentni valovanji → $\bar{j} = \bar{j}_1 + \bar{j}_2$

Odboj na tanki plasti

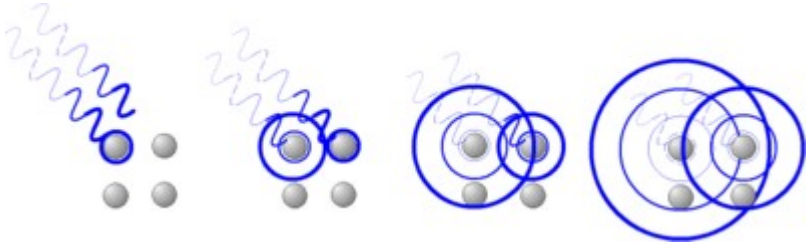


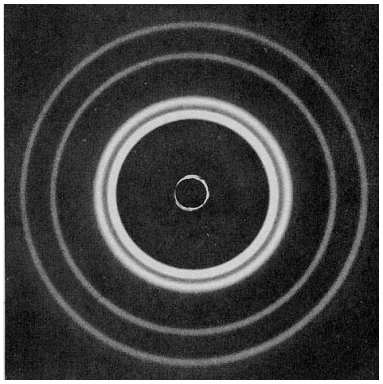
• odbita delna curka na tanki plasti sta koherentna in interferirata

$$\left.\begin{array}{l} n_1 > n_2 > n_3 \\ n_1 < n_2 < n_3 \end{array}\right\} \; 2 n_2 d \cos(\beta) = N \lambda_0$$

$$\left.\begin{array}{l} n_1 < n_2 > n_3 \\ n_1 > n_2 < n_3 \end{array}\right\} \; 2 n_2 d \cos(\beta) = \frac{2N+1}{2} \lambda_0$$

Braggov uklon





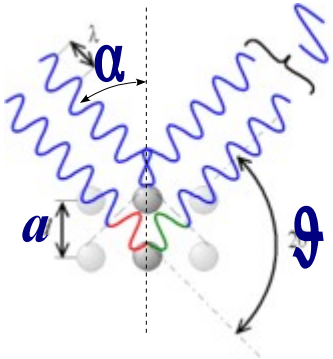
$$2 a \cos(\alpha) = N \lambda_0$$

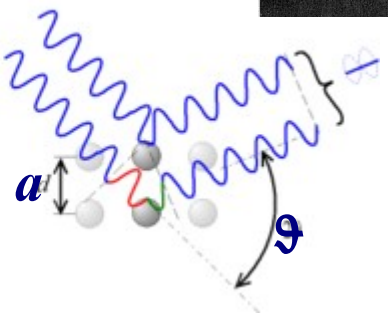
$$2 a \cos\left(\frac{\pi}{2} - \frac{\vartheta}{2}\right) = N \lambda_0$$

$$2 a \sin\left(\frac{\vartheta}{2}\right) = N \lambda_0$$

$$2\alpha + \vartheta = \pi$$

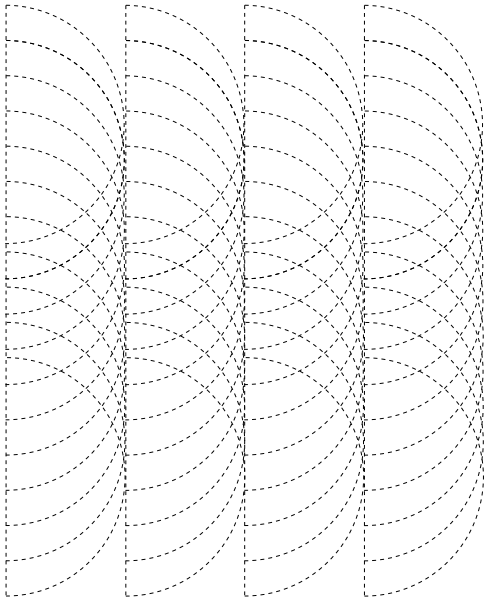
$$\alpha = \frac{\pi}{2} - \frac{\vartheta}{2}$$

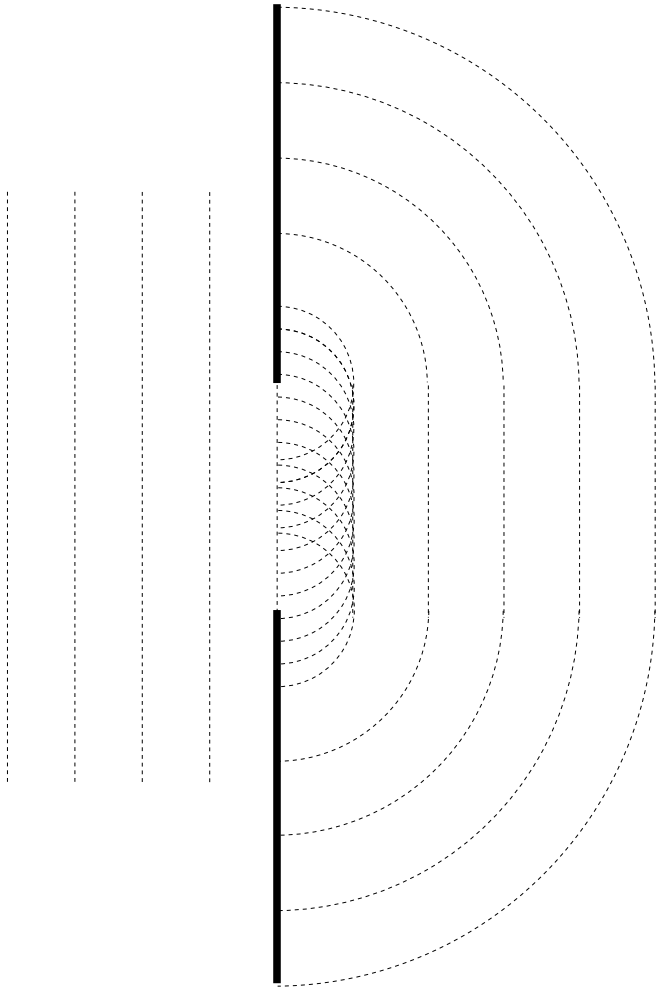




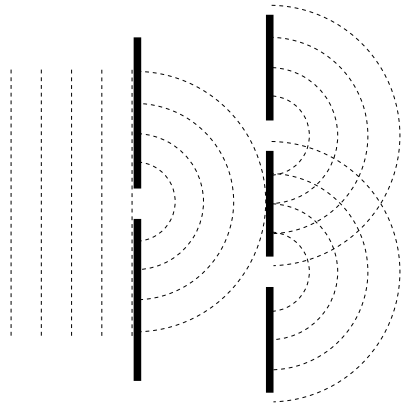
Uklon

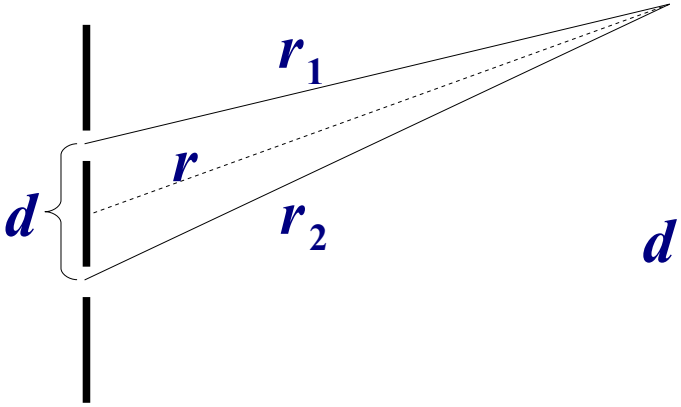
- uklon valovanja je širjenje valovanja v področje geometrijske sence.
- Huygensovo načelo: vsaka točka valovnega čela je izvir elementarnih krožnih valov (2D).

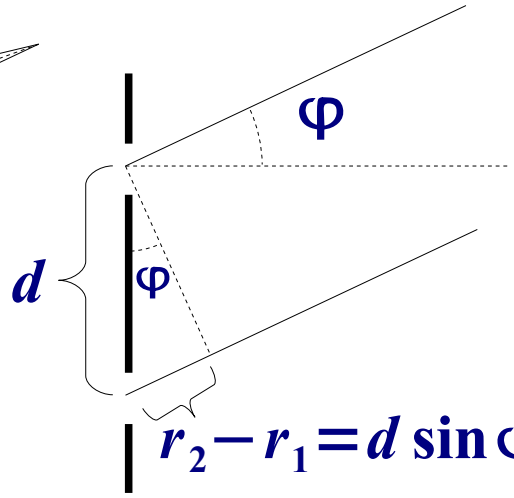




Youngov poskus







- delni valovanji iz obeh rež sta koherentni
- vsota delnih valovanj iz obeh rež

$$\delta = k\, d \sin\varphi = \frac{2\pi\, d \sin\varphi}{\lambda}$$

$$E_1(r_1, t) = E_{10}\cos(\omega t - k r_1)$$
$$E_2(r_2, t) = E_{10}\cos(\omega t - k r_2)$$

$$E = E_1 + E_2 = 2\,E_{10}\cos\left(k\,\frac{r_2 - r_1}{2}\right)\cos\left(\omega t - k\,\frac{r_2 + r_1}{2}\right)$$

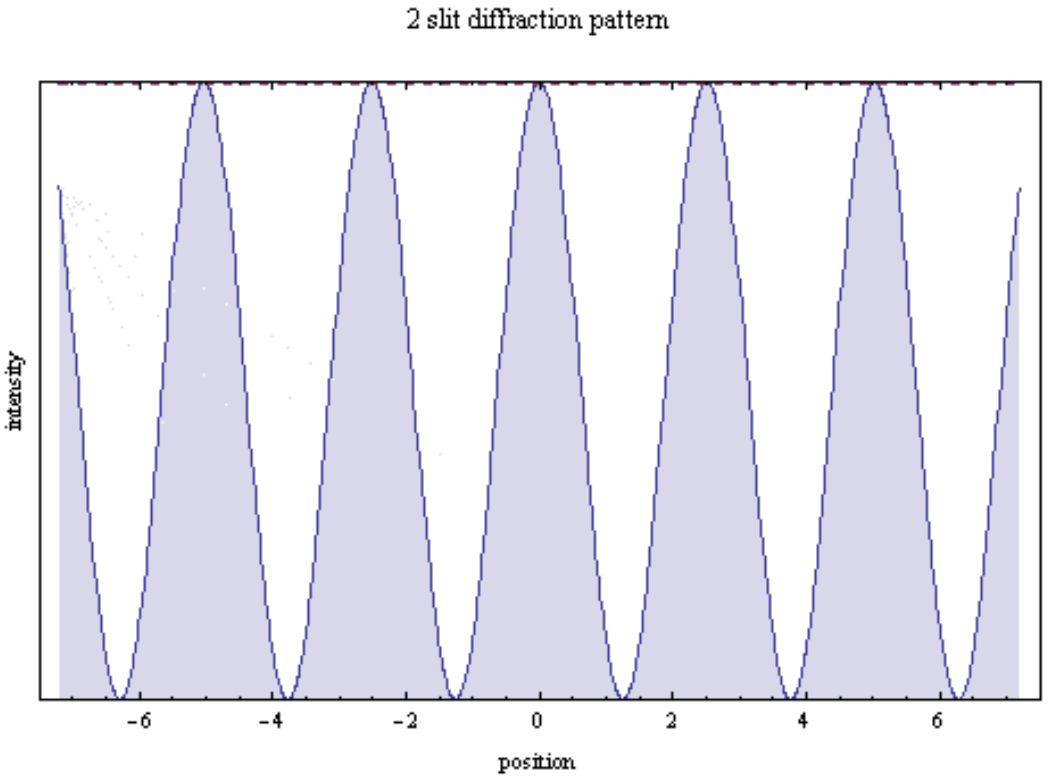
$$E = 2\,E_{10}\cos\left(\frac{1}{2}\,k\,d\sin\varphi\right)\cos(\omega t - k\,r)$$

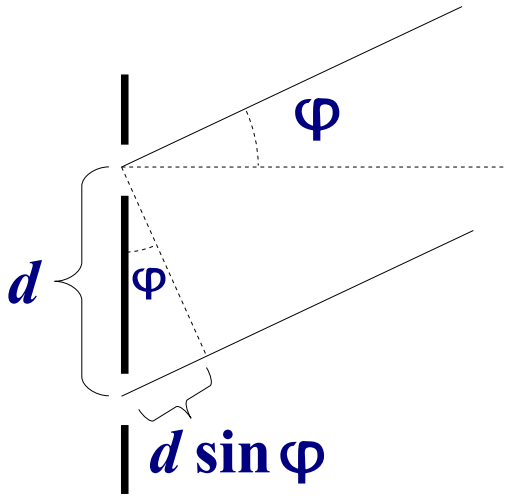
- interferenca na dveh ozkih režah

$$E_0 = 2E_{10}\cos\left(\frac{\delta}{2}\right) = 2E_{10}\cos\left(\frac{\pi d \sin\varphi}{\lambda}\right)$$

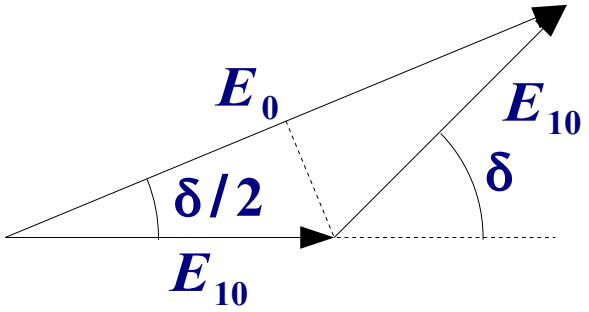
$$\bar{j} = \frac{1}{2} c_0 \epsilon_0 E_0^2$$

$$\bar{j} \propto \cos^2\left(\frac{\delta}{2}\right) = \cos^2\left(\frac{\pi d \sin\varphi}{\lambda}\right)$$





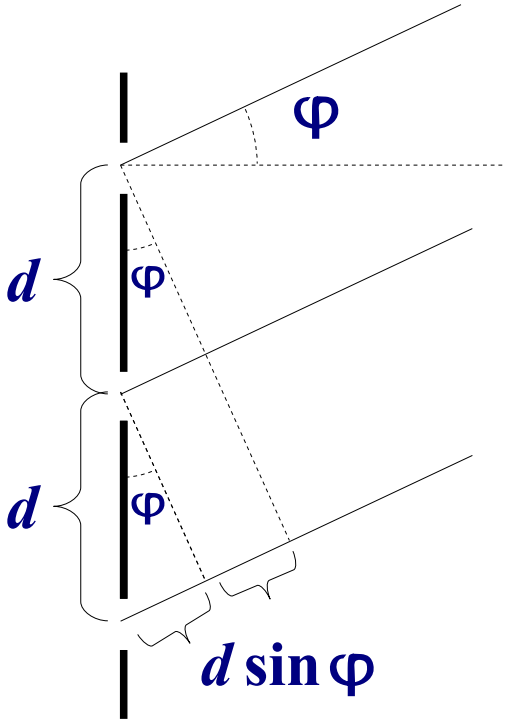
$$\delta = k\, d \sin\varphi = \frac{2\pi d \sin\varphi}{\lambda}$$

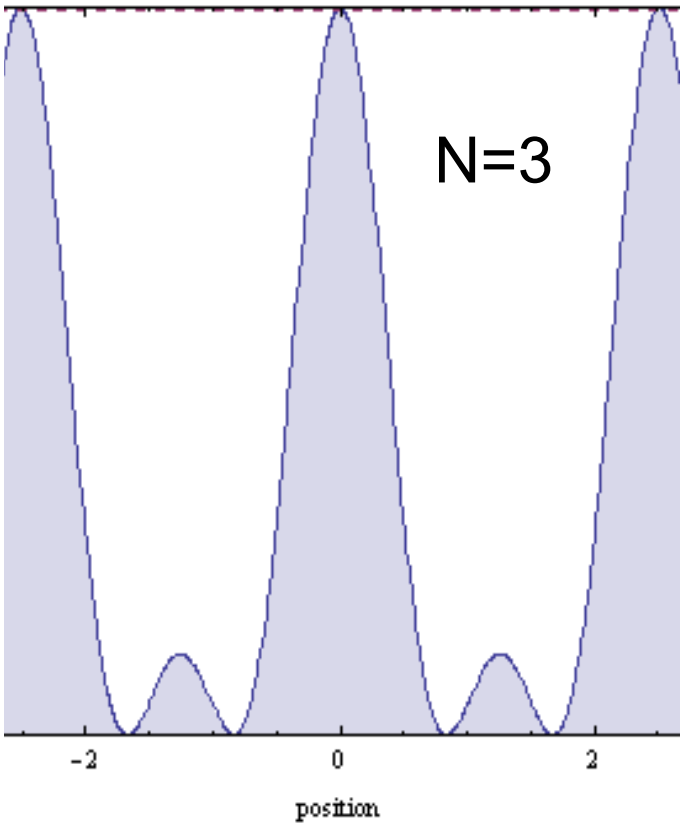


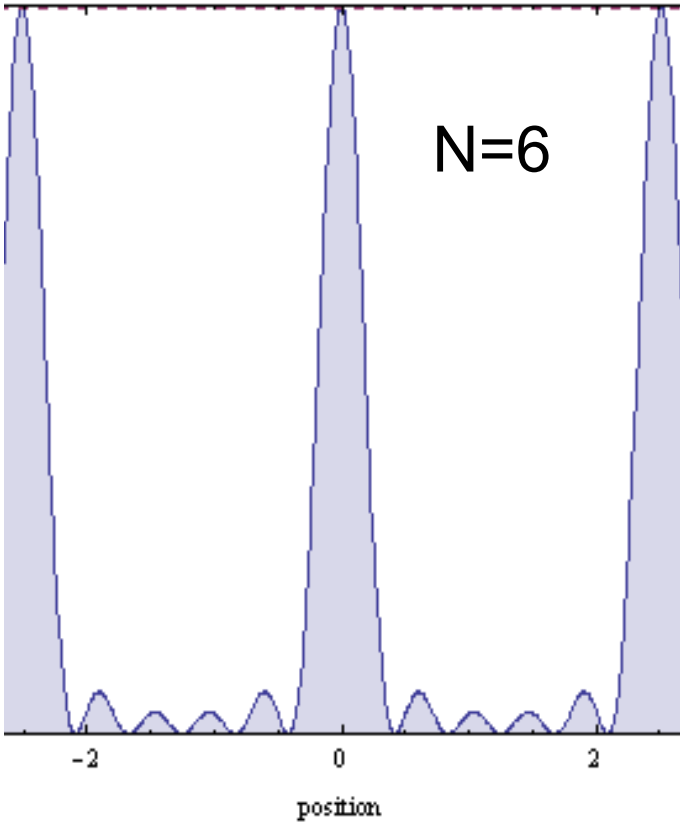
Uklonska mrežica – N ozkih rež

$$\left.\begin{aligned}\frac{E_0}{2} &= r\sin\left(\frac{N\delta}{2}\right)\\ \frac{E_{10}}{2} &= r\sin\left(\frac{\delta}{2}\right)\end{aligned}\right\} \quad E_0 = E_{10}\frac{\sin\left(\frac{N\delta}{2}\right)}{\sin\left(\frac{\delta}{2}\right)}$$

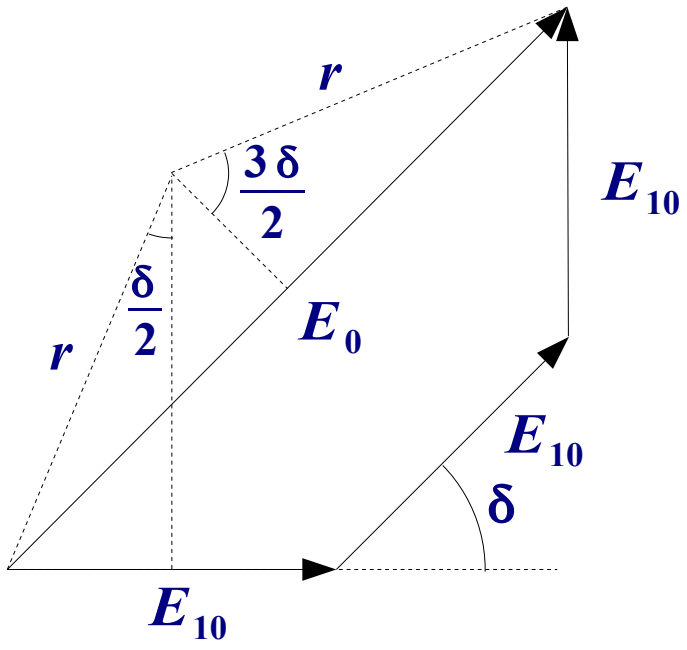
$$\bar{j} \propto \frac{\sin^2(N\delta/2)}{\sin^2(\delta/2)} = \frac{\sin^2(N\pi d\sin\varphi/\lambda)}{\sin^2(\pi d\sin\varphi/\lambda)}$$







$$\delta = k\,d\sin\varphi = \frac{2\pi\,d\sin\varphi}{\lambda}$$



- glavni maksimum reda n

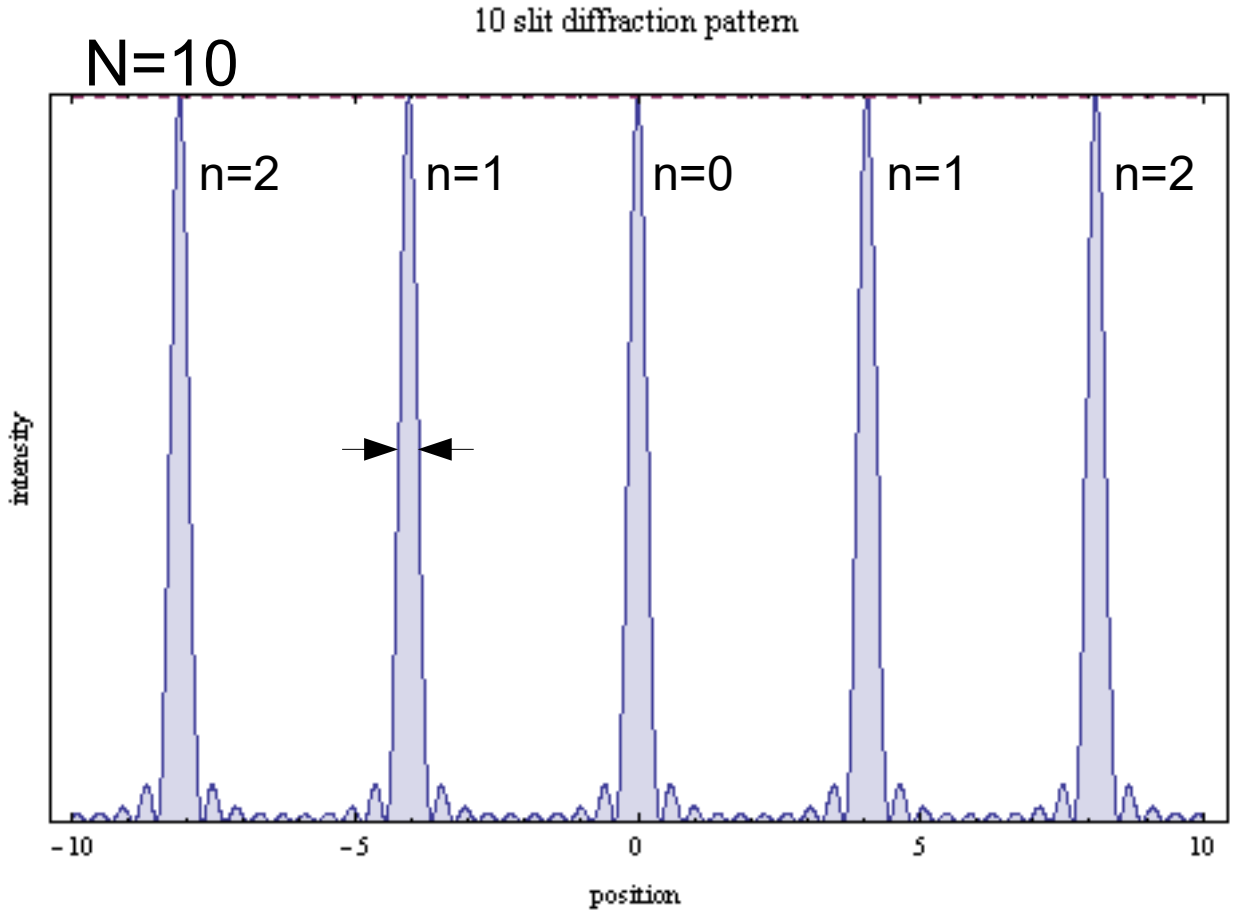
$$\frac{\pi d \sin\varphi}{\lambda} = n\pi \Rightarrow \sin\varphi = \frac{n\lambda}{d}$$

- širina na polovični višini

$$\Delta\varphi \approx \frac{\lambda}{N d \cos\varphi}$$

- ničle

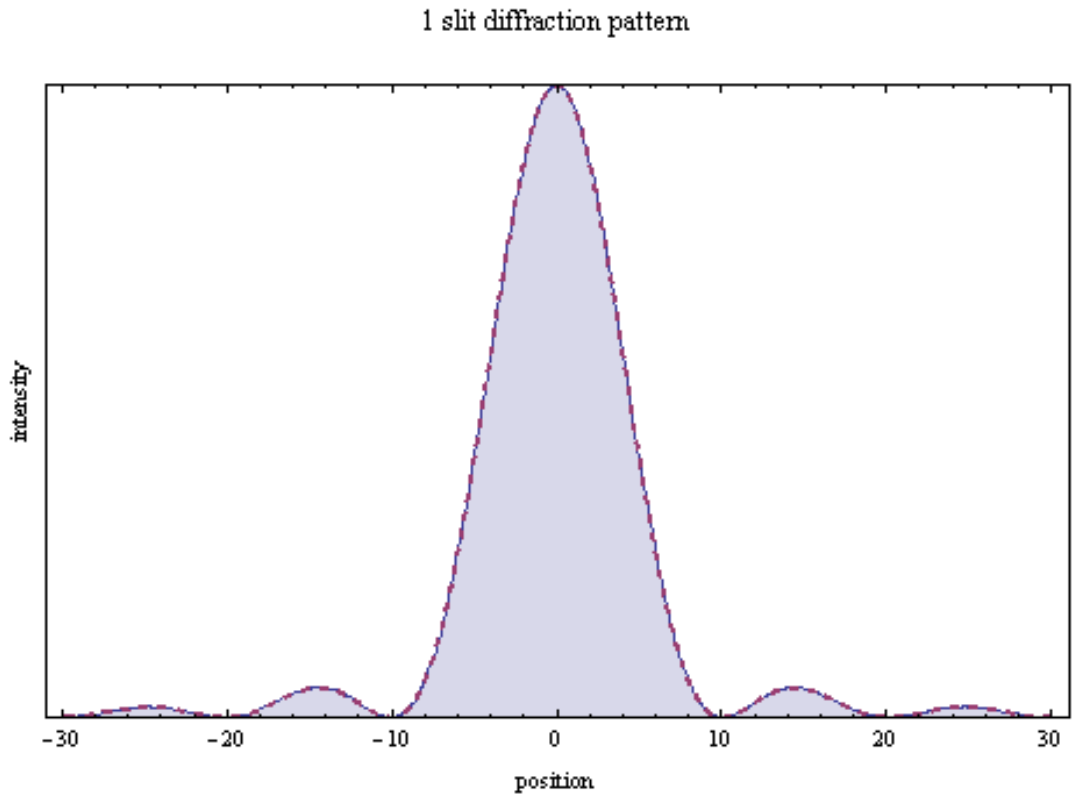
$$\frac{N\pi d \sin\varphi}{\lambda} = m\pi; \quad m = 1,2 .. N-1$$

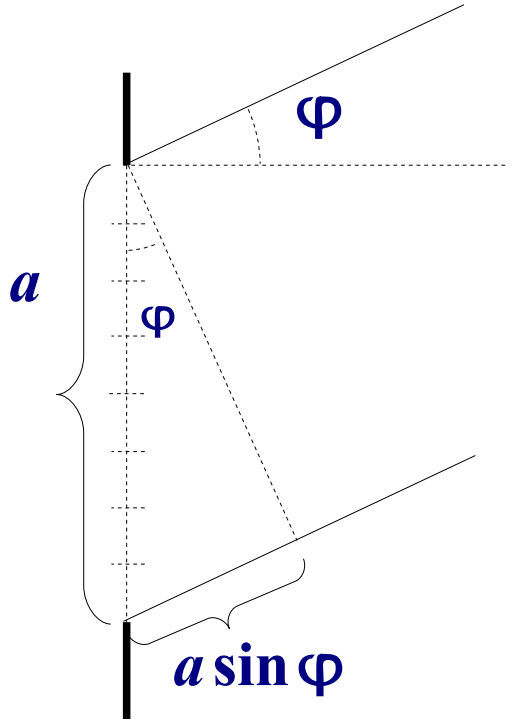


Uklon na široki reži

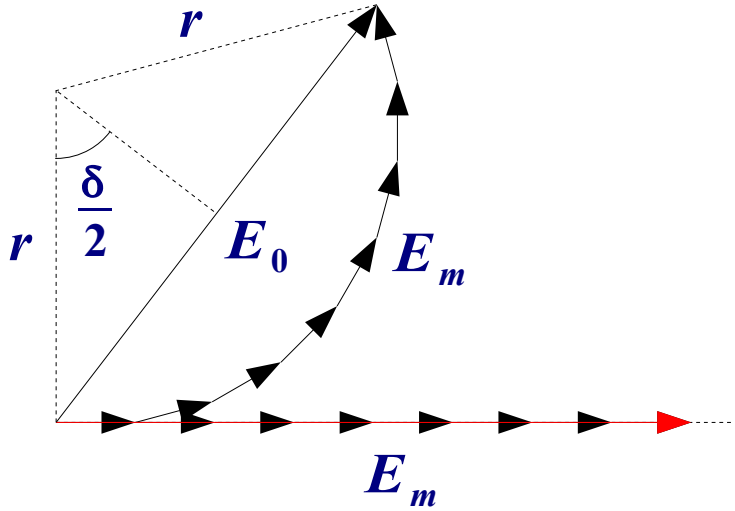
$$\left.\begin{aligned} \frac{E_0}{2} &= r\sin\left(\frac{\delta}{2}\right) \\ E_m &= r\,\delta \end{aligned}\right\} \quad E_0 = E_m \frac{\sin\left(\frac{\delta}{2}\right)}{\frac{\delta}{2}}$$

$$\bar{j} \propto \frac{\sin^2(\delta/2)}{(\delta/2)^2} = \frac{\sin^2(\pi\, a \sin\varphi/\lambda)}{(\pi\, a \sin\varphi/\lambda)^2}$$





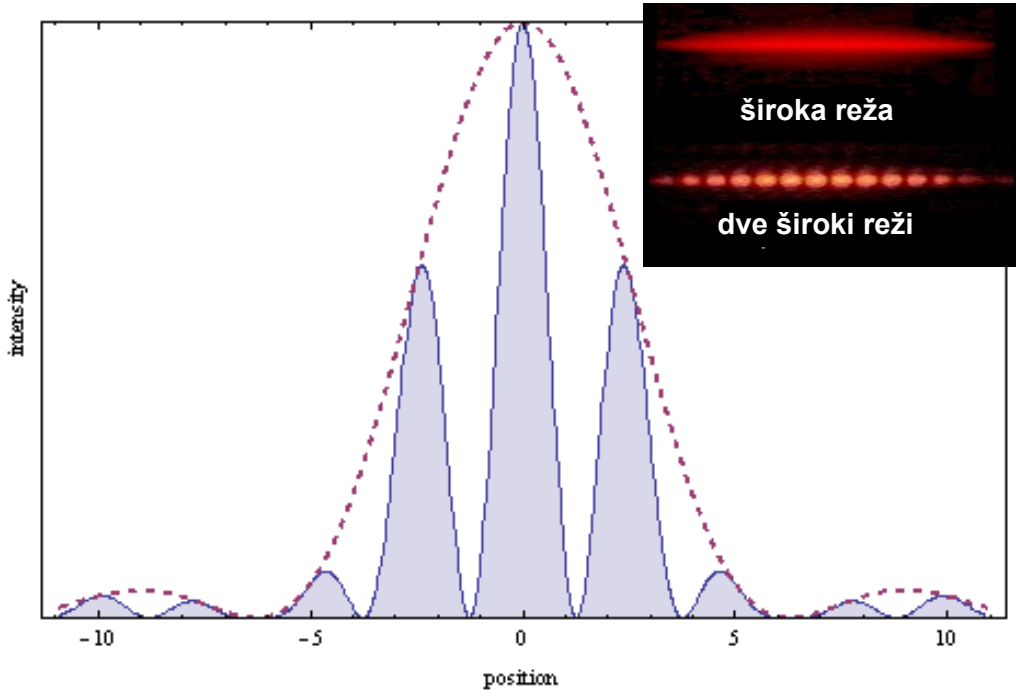
$$\delta = k\, a \sin\varphi = \frac{2\pi\, a \sin\varphi}{\lambda}$$

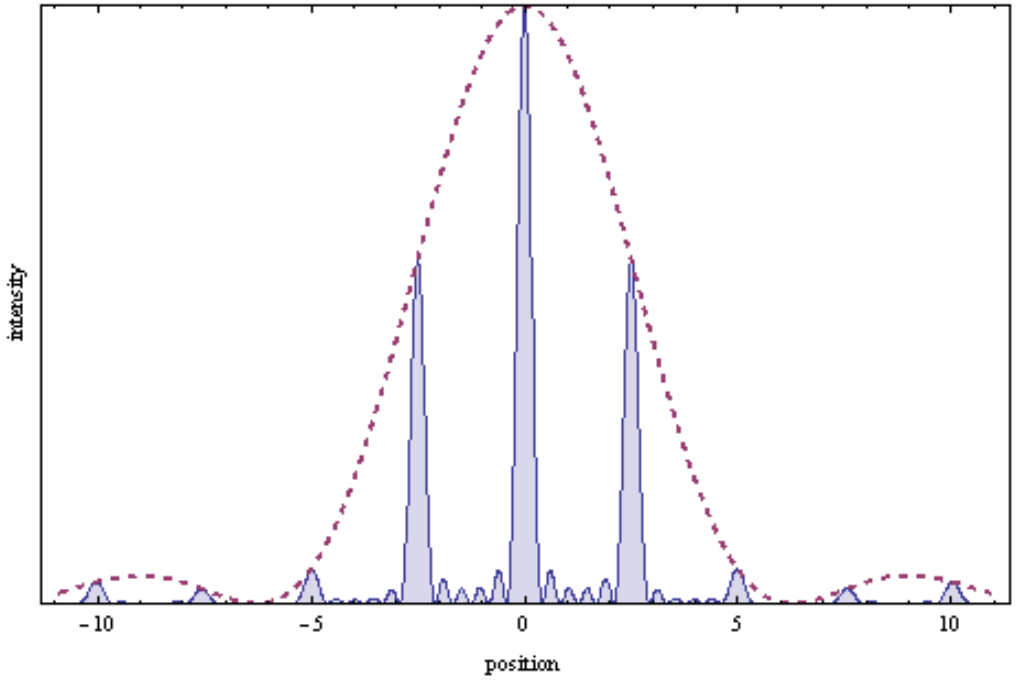


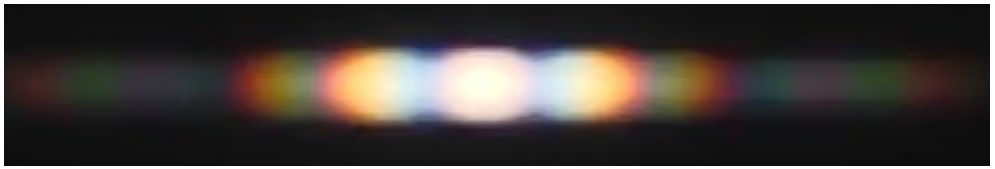
Uklon na N širokih režah

- interferenca na širokih režah je kombinacija režah uklonskih slik ene široke reže in uklonske mrežice

$$\bar{j} \propto \frac{\sin^2(N\pi d \sin\varphi/\lambda)}{\sin^2(\pi d \sin\varphi/\lambda)} \cdot \frac{\sin^2(\pi a \sin\varphi/\lambda)}{(\pi a \sin\varphi/\lambda)^2}$$







Fotometrija

Svetlobni tok

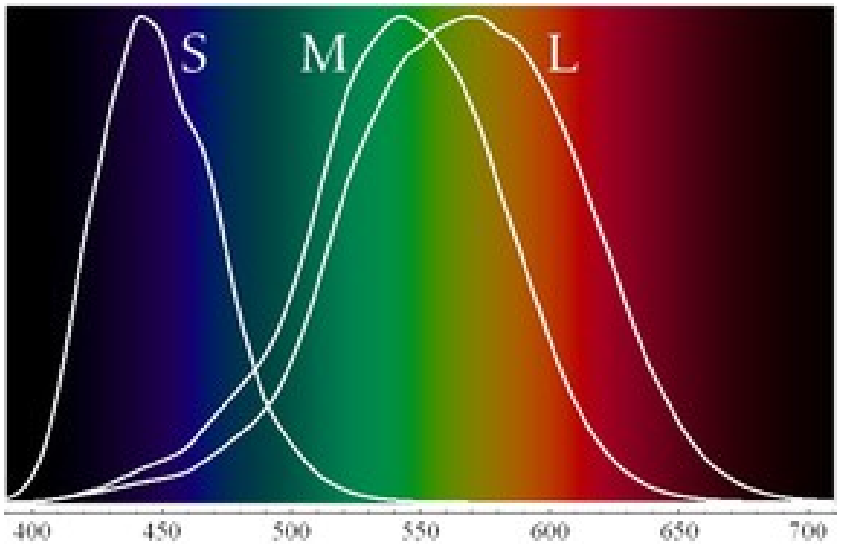
- svetlobni tok je moč, ki jo prinaša svetloba

$$P = j\,S \quad [W]$$

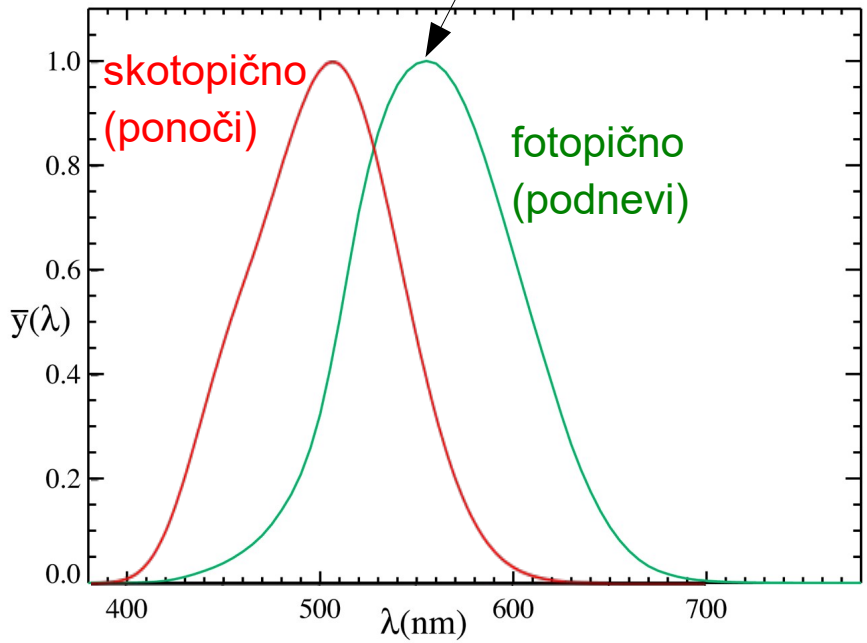
- merimo ga s kalorimetri, ki merijo prejeto energijo (bolometer)
- fiziološko merilo upošteva relativno občutljivost očesa

V(λ) – enota je lumen

$$1\,\mathrm{W} = V(\lambda)\cdot 683\ lm$$



relativna občutljivost čepkov



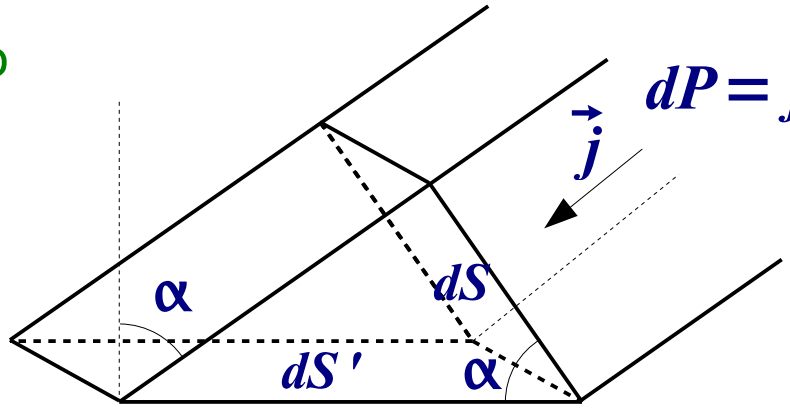
relativna svetlobna občutljivost očesa

Osvetljenost

- gostota svetlobnega toka na osvetljeno površino

$$j' = \frac{dP}{dS'} \quad \left[\frac{lm}{m^2} = lux = lx\right]$$

$$j' = \frac{dP}{dS'} = j\frac{dS}{dS'} = j\cos\alpha$$

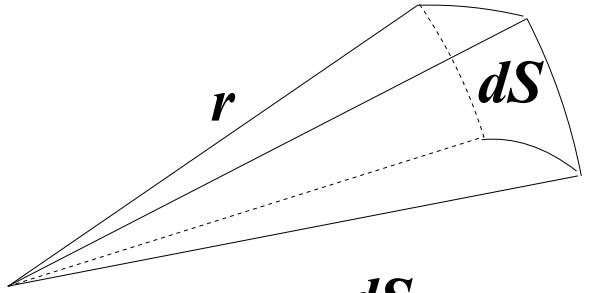


$$dS = dS'\cos\alpha$$

Svetilnost

- gostota svetlobnega toka izsevanega v prostorski kot

$$I = \frac{dP}{d\Omega} \quad \left[\frac{W}{sr}; \frac{lm}{sr} = cd\right]$$



$$d\Omega = \frac{dS}{r^2} \quad [sr]$$

- točkast izotropen izvor $I = \frac{P}{4\pi}$

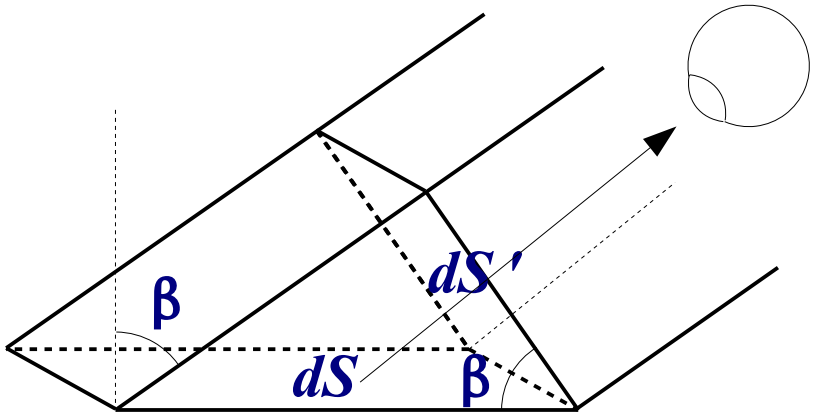
$$j(r) = \frac{dP}{dS} = \frac{dP}{r^2\, d\Omega} = \frac{1}{r^2} I\,; \quad I = r^2\, j(r)$$

Svetlost

- svetilnost na enoto efektivne površine

$$B = \frac{dI}{dS'} \quad \left[\frac{W}{sr\,m^2}; \frac{cd}{m^2}\right]$$

$$B(\beta) = \frac{dI(\beta)}{dS'}$$



$$dS' = dS\cos\beta$$

- Lambertov zakon – hrapava ravna svetila so enako svetla, če jih pogledamo iz katere koli smeri

$$B(\beta) = B(0) \Rightarrow \frac{I(\beta)}{S\cos\beta} = \frac{I(0)}{S}$$

$$I(\beta) = I(0)\cos\beta$$

Sevanje črnega telesa

- Stefan - Boltzmannov zakon

$$j^* = \frac{P}{S^*} = \sigma T^4$$

$$\sigma = 5.67 \cdot 10^{-8} \frac{W}{m^2 K^4}$$

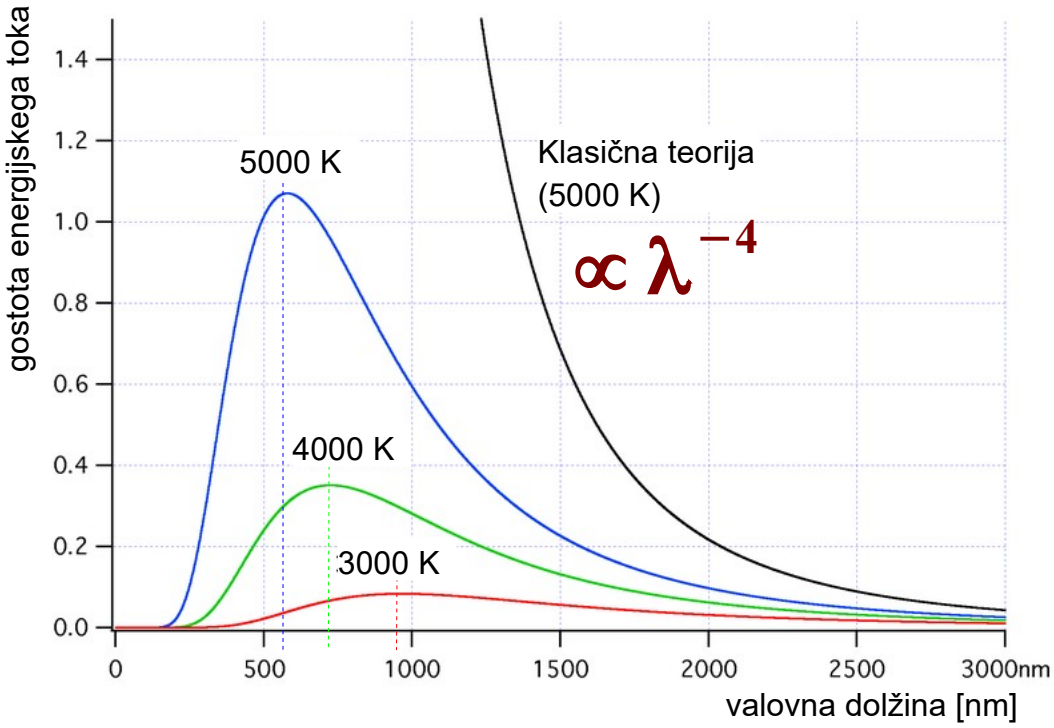
Stefanova konstanta

- Wienov zakon

$$\lambda_0 T = k_w = 2.90 \cdot 10^{-3}\, mK$$

- Planckov zakon

$$\frac{dj^*}{d\lambda} = \frac{2\pi h c^2}{\lambda^5} \frac{1}{e^{(hc/\lambda k T)} - 1}$$



- če telo ni črno dodamo izsevnost

$$j^* = e\,\sigma\,T^4 \qquad e = \frac{j^*}{j_0^*}$$

Ag	0.02
Al	0.08
Cu	0.15
medenina	0.60
saje	0.95

- zveza med izsevnostjo in odbojnostjo – termodinamično ravnovesje

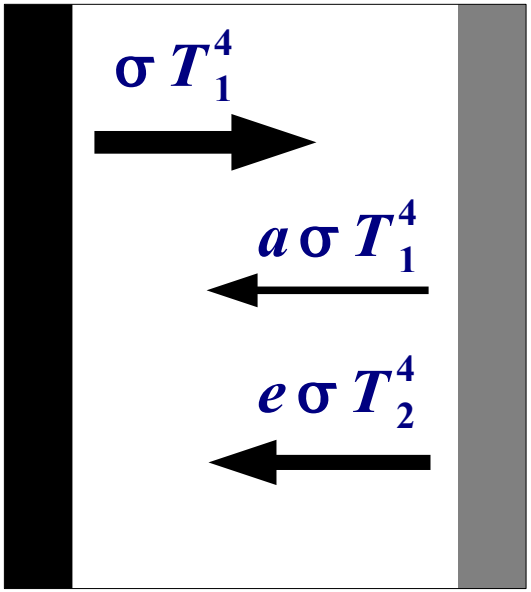
$$e = 1 - a$$

$$j = \sigma\,T_1^4 - a\,\sigma\,T_1^4 - e\,\sigma\,T_2^4$$

$$T_1 = T_2 \;\Rightarrow\; j = 0$$

$$\sigma\,T^4 - a\,\sigma\,T^4 - e\,\sigma\,T^4 = 0$$

$$1 - a - e = 0$$



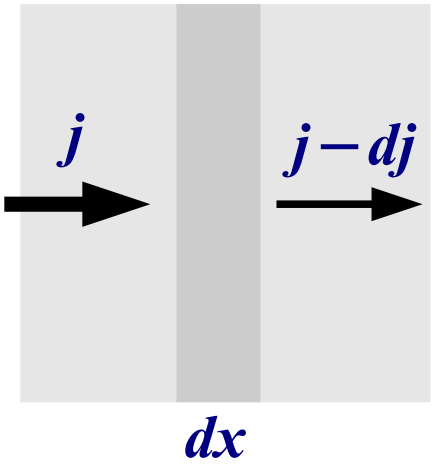
Absorpcija

- prepuščena gostota energijskega toka

$$dj = -\mu\, j\, dx \;\Rightarrow\; \frac{dj}{j} = -\mu\, dx$$

$$\int_{j_0}^{j} \frac{dj}{j} = -\mu \int_{0}^{x} dx \;\Rightarrow\; \ln\frac{j}{j_0} = -\mu\, x$$

$$j = j_0 e^{-\mu x}$$

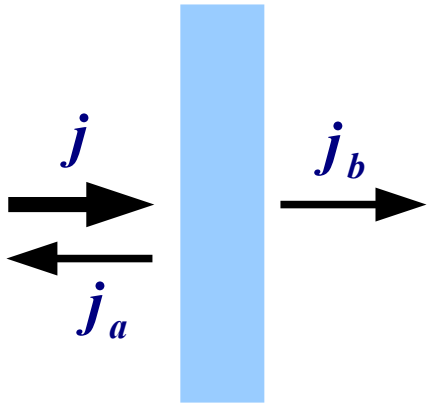


- absorpcijski koeficient $\mu\,[m^{-1}]$

Odbojnost in prepustnost

- odbojnost (albedo) $a = \frac{j_a}{j}$

- prepustnost $b = \frac{j_b}{j}$



Standardni model
OSNOVNIH DELCEV IN INTERAKCIJ

FERMIONS
matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2

Flavor	Mass GeV/c^2	Electric charge
ν_e electron neutrino	<1×10^{-8}	0
e electron	0.000511	–1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	–1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	–1

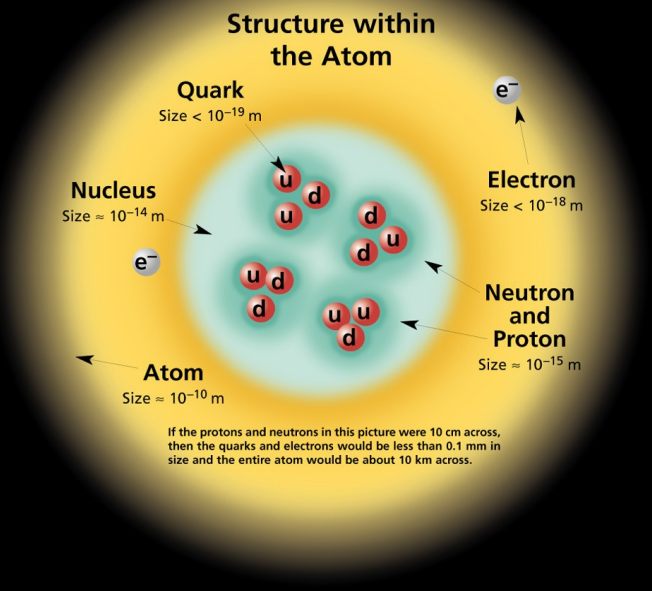
Quarks spin = 1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.003	2/3
d down	0.006	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	175	2/3
b bottom	4.3	–1/3

Spin is the intrinsic angular momentum of particles. Spin is given in units of $\hbar$, which is the quantum unit of angular momentum, where $\hbar = h/2\pi = 6.58\times10^{-25}$ GeV s = 1.05×10^{-34} J s.

Electric charges are given in units of the proton's charge. In SI units the electric charge of the proton is 1.60×10^{-19} coulombs.

The **energy** unit of particle physics is the electronvolt (eV), the energy gained by one electron in crossing a potential difference of one volt. **Masses** are given in GeV/c^2 (remember $E = mc^2$), where 1 GeV = 10^9 eV = 1.60×10^{-10} joule. The mass of the proton is 0.938 GeV/c^2 = 1.67×10^{-27} kg.



BOSONS
force carriers
spin = 0, 1, 2, ...

Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.4	–1
W^+	80.4	+1
Z^0	91.187	0

Strong (color) spin = 1

Name	Mass GeV/c^2	Electric charge
g gluon	0	0

Color Charge
Each quark carries one of three types of "strong charge," also called "color charge." These charges have nothing to do with the colors of visible light. There are eight possible types of color charge for gluons. Just as electrically-charged particles interact by exchanging photons, in strong interactions color-charged particles interact by exchanging gluons. Leptons, photons, and ***W*** and ***Z*** bosons have no strong interactions and hence no color charge.

Quarks Confined in Mesons and Baryons
One cannot isolate quarks and gluons; they are confined in color-neutral particles called **hadrons**. This confinement (binding) results from multiple exchanges of gluons among the color-charged constituents. As color-charged particles (quarks and gluons) move apart, the energy in the color-force field between them increases. This energy eventually is converted into additional quark-antiquark pairs (see figure below). The quarks and antiquarks then combine into hadrons; these are the particles seen to emerge. Two types of hadrons have been observed in nature: **mesons** $q\bar{q}$ and **baryons** qqq.

Residual Strong Interaction
The strong binding of color-neutral protons and neutrons to form nuclei is due to residual strong interactions between their color-charged constituents. It is similar to the residual electrical interaction that binds electrically neutral atoms to form molecules. It can also be viewed as the exchange of mesons between the hadrons.

PROPERTIES OF THE INTERACTIONS

Property \ Interaction	Gravitational	Weak (Electroweak)	Electromagnetic (Electroweak)	Strong: Fundamental	Strong: Residual
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge	See Residual Strong Interaction Note
Particles experiencing:	All	Quarks, Leptons	Electrically charged	Quarks, Gluons	Hadrons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons	Mesons
Strength relative to electromag for two u quarks at: 10^{-18} m	10^{-41}	0.8	1	25	Not applicable to quarks
Strength relative to electromag for two u quarks at: 3×10^{-17} m	10^{-41}	10^{-4}	1	60	
for two protons in nucleus	10^{-36}	10^{-7}	1	Not applicable to hadrons	20

Baryons qqq and Antibaryons $\bar{q}\bar{q}\bar{q}$
Baryons are fermionic hadrons.
There are about 120 types of baryons.

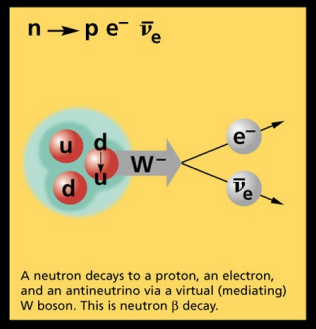
Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
p	proton	uud	1	0.938	1/2
$\bar{p}$	anti-proton	$\bar{u}\bar{u}\bar{d}$	–1	0.938	1/2
n	neutron	udd	0	0.940	1/2
Λ	lambda	uds	0	1.116	1/2
Ω^-	omega	sss	–1	1.672	3/2

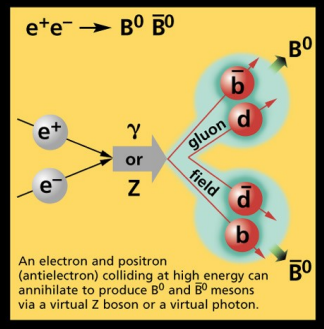
Mesons $q\bar{q}$
Mesons are bosonic hadrons.
There are about 140 types of mesons.

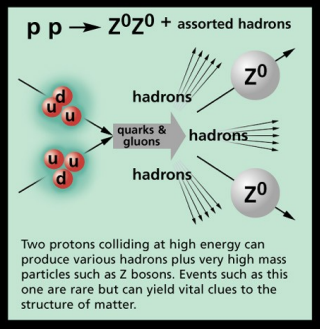
Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
π^+	pion	$u\bar{d}$	+1	0.140	0
K^-	kaon	$s\bar{u}$	–1	0.494	0
ρ^+	rho	$u\bar{d}$	+1	0.770	1
B^0	B-zero	$d\bar{b}$	0	5.279	0
η_c	eta-c	$c\bar{c}$	0	2 .980	0

Matter and Antimatter
For every particle type there is a corresponding antiparticle type, denoted by a bar over the particle symbol (unless + or – charge is shown). Particle and antiparticle have identical mass and spin but opposite charges. Some electrically neutral bosons (e.g., Z^0, γ, and $\eta_c = c\bar{c}$, but not $K^0 = d\bar{s}$) are their own antiparticles.

Figures
These diagrams are an artist's conception of physical processes. They are ***not*** exact and have ***no*** meaningful scale. Green shaded areas represent the cloud of gluons or the gluon field, and red lines the quark paths.







The Particle Adventure
Visit the award-winning web feature *The Particle Adventure* at
http://ParticleAdventure.org

This chart has been made possible by the generous support of:
U.S. Department of Energy
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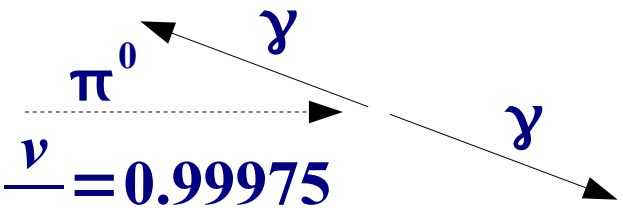
Lawrence Berkeley National Laboratory
Stanford Linear Accelerator Center
American Physical Society, Division of Particles and Fields
BURLE INDUSTRIES, INC.

Posebna teorija relativnosti

- Einsteinova postulata:
  - fizikalni zakoni so enaki v vseh inercialnih opazovalnih sistemih
  - hitrost svetlobe v vakuumu ima enako vrednost v vseh inercialnih opazovalnih sistemih (v vseh smereh)

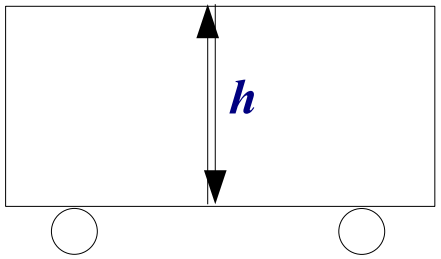
$$\pi^0 \rightarrow \gamma + \gamma$$



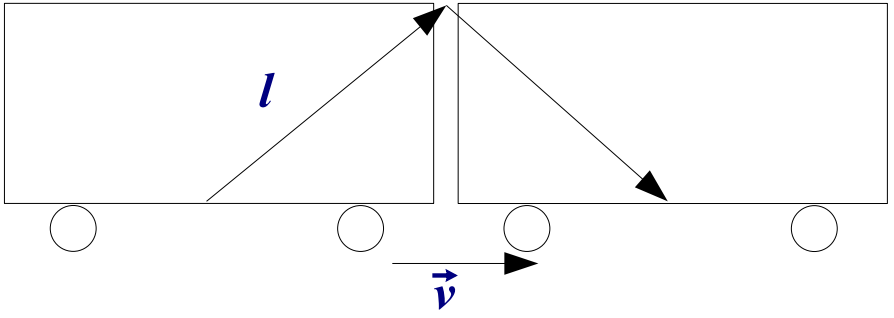
$$\beta = \frac{v}{c_0} = 0.99975$$

- podaljšanje časa

$$\Delta t_0 = \frac{2h}{c_0} \qquad \Delta t = \frac{2l}{c_0} = \frac{2\sqrt{\left(\frac{1}{2}c_0 \Delta t_0\right)^2 + \left(\frac{1}{2}v\Delta t\right)^2}}{c_0}$$



$$c_0^2 \Delta t^2 = c_0^2 \Delta t_0^2 + v^2 \Delta t^2$$



$$\Delta t = \gamma \Delta t_0 \qquad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c_0^2}}} = \frac{1}{\sqrt{1-\beta^2}}$$

- skrčenje dolžin

$$\Delta x = \frac{\Delta x_0}{\gamma}$$

- Lorentzove transformacije

$$x' = \gamma (x - v t) \qquad x' = \gamma (x - \beta c_0 t) \qquad x = \gamma (x' + \beta c_0 t)$$

$$y' = y \qquad y' = y$$

$$z' = z \qquad z' = z$$

$$t' = \gamma \left(t - \frac{v x}{c_0^2}\right) \qquad c_0 t' = \gamma (c_0 t - \beta x) \qquad c_0 t = \gamma (c_0 t' + \beta x)$$

- gibalna količina in polna energija

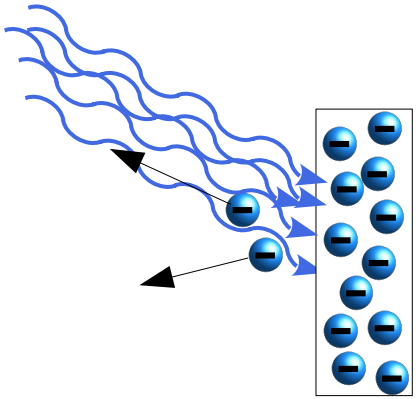
$$p = \gamma m v$$

$$E = \gamma m c_0^2$$

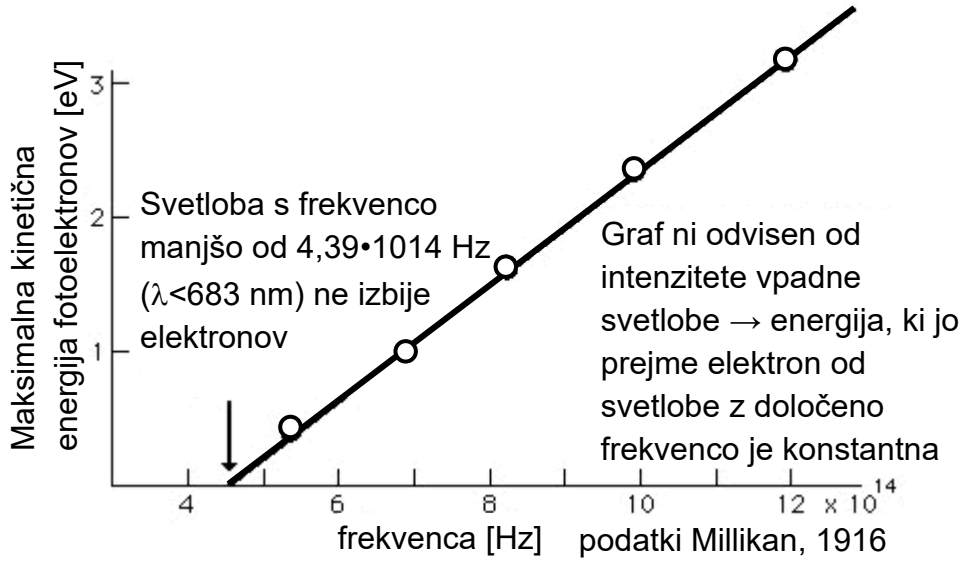
$$E^2 = p^2 c_0^2 + m^2 c_0^4$$

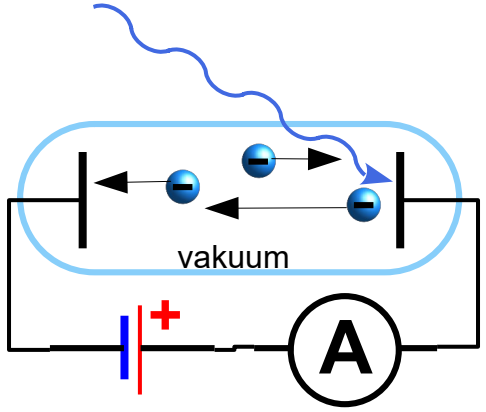
Fotoefekt

- fotoefekt – svetloba odda energijo elektronu v paketu - kvantu



$$W_{k,max} = h\nu - W_0$$



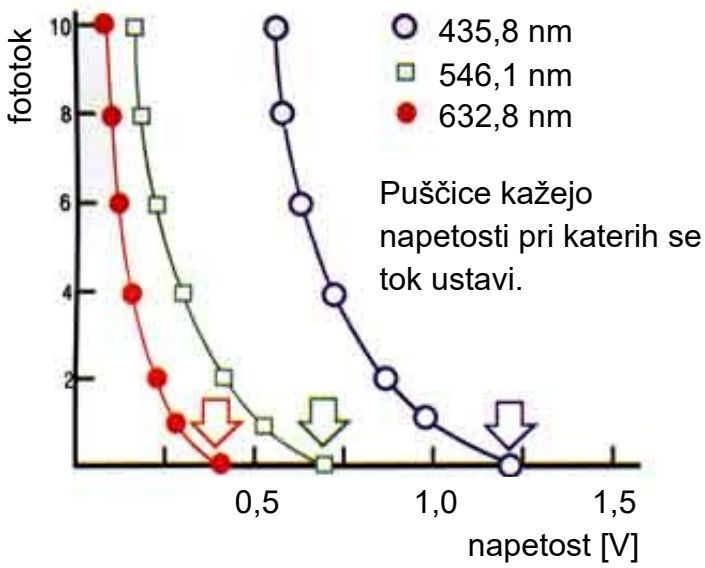


- energija in gibalna količina fotona

$$E = h\nu \qquad p = \frac{E}{c_0} = \frac{h}{\lambda}$$

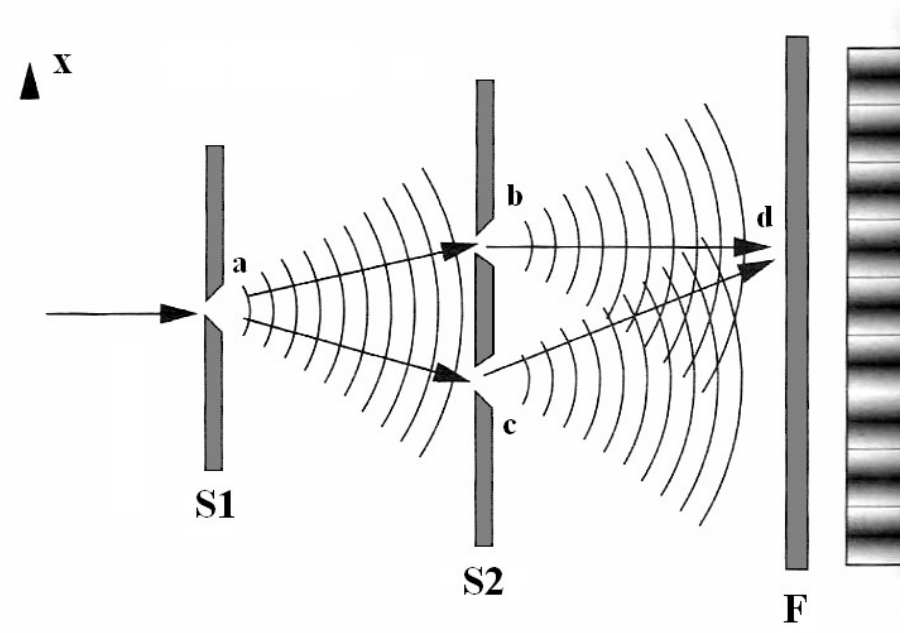
$$h = 6.63 \cdot 10^{-34}\ J\,s$$

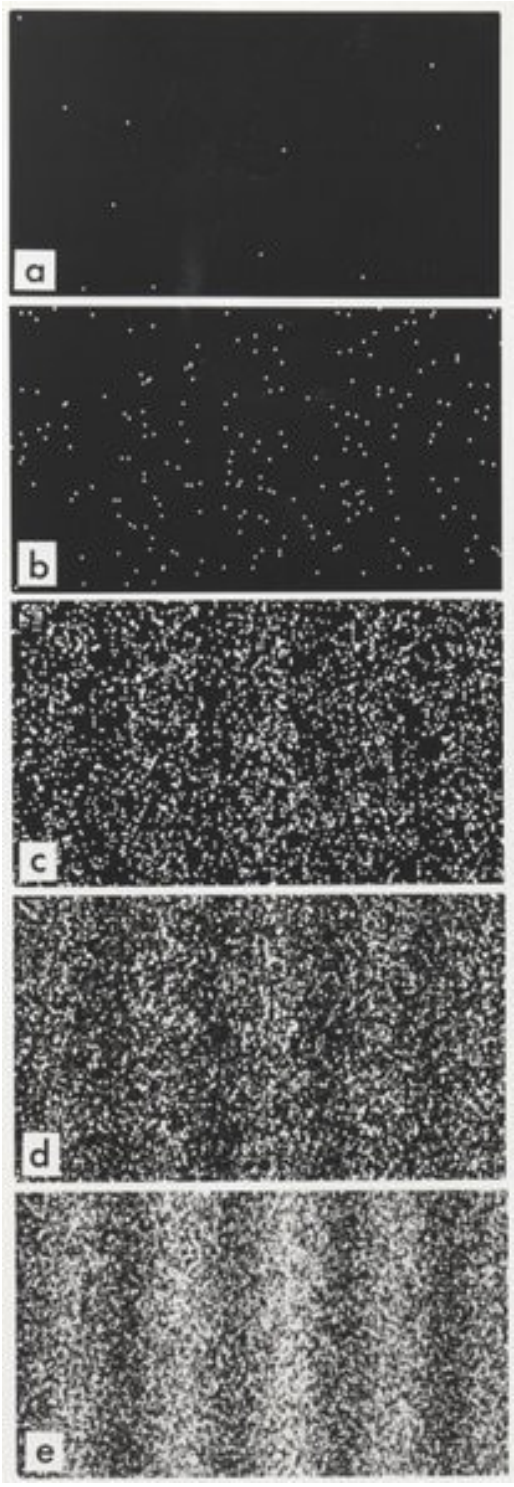
Planckova konstanta



Youngov poskus s posameznimi fotoni

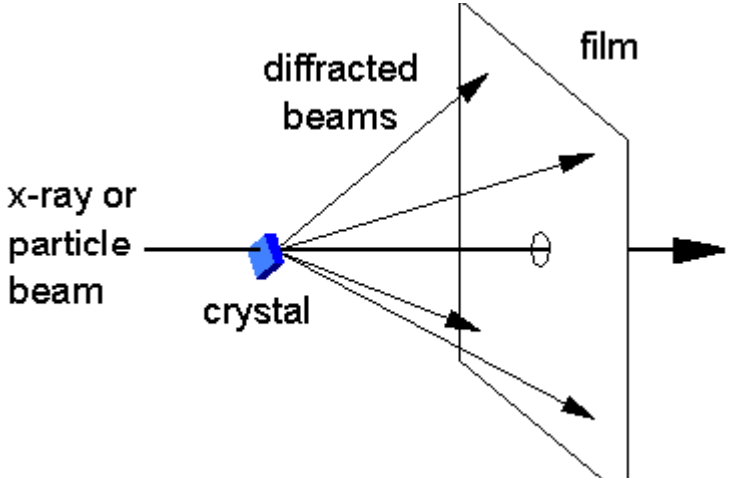
- tudi pri zelo šibki osvetlitvi dveh rež, ko na zaslonu zaznavamo posamezne svetlobne kvante, nastane interferenčna slika

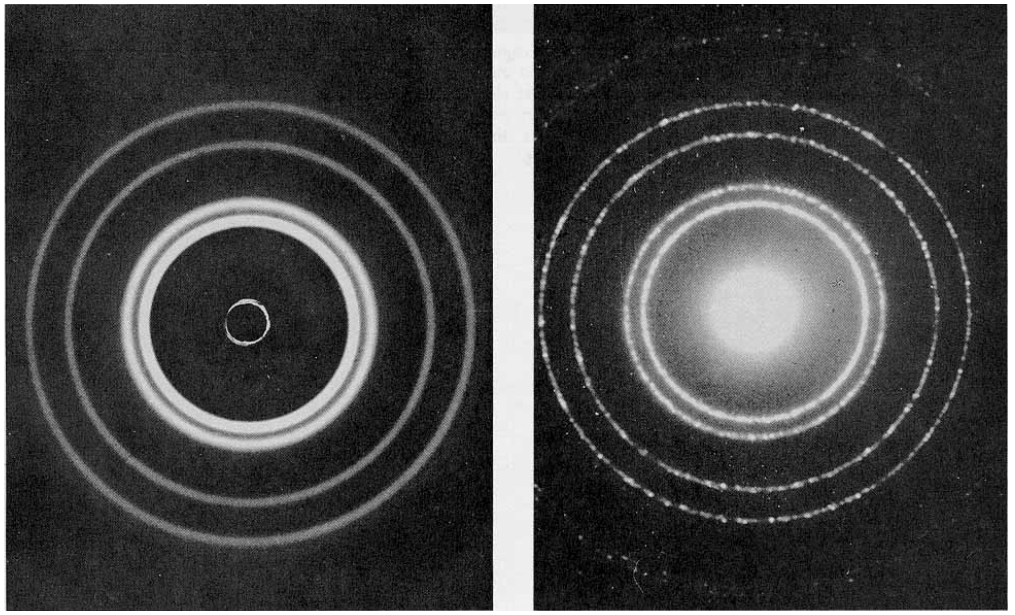




- masni valovi – de Broglieva valovna dolžina
- tudi delci kažejo lastnosti valovanja
- curek elektronov na kristalu ustvari enako interferenčno sliko kot žarki X

$$\lambda = \frac{h}{p}$$

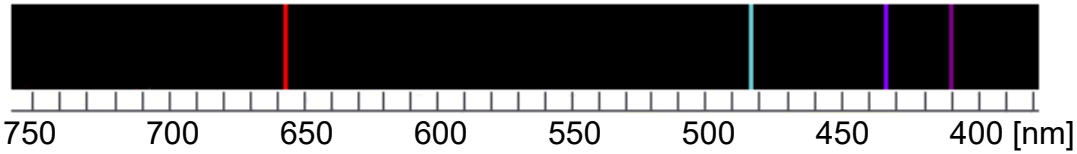




- Heisenbergovo načelo nedoločenosti

$$\Delta p \cdot \Delta x \geqslant \frac{h}{2\pi}$$

Atom – Bohrov model



- Bohrov model vodikovega atoma:
  - elektron kroži

$$F_e = ma_r \Rightarrow \frac{e_0^2}{4\pi\epsilon_0 r^2} = m_e \omega^2 r$$

  - vrtilna količina je kvantizirana

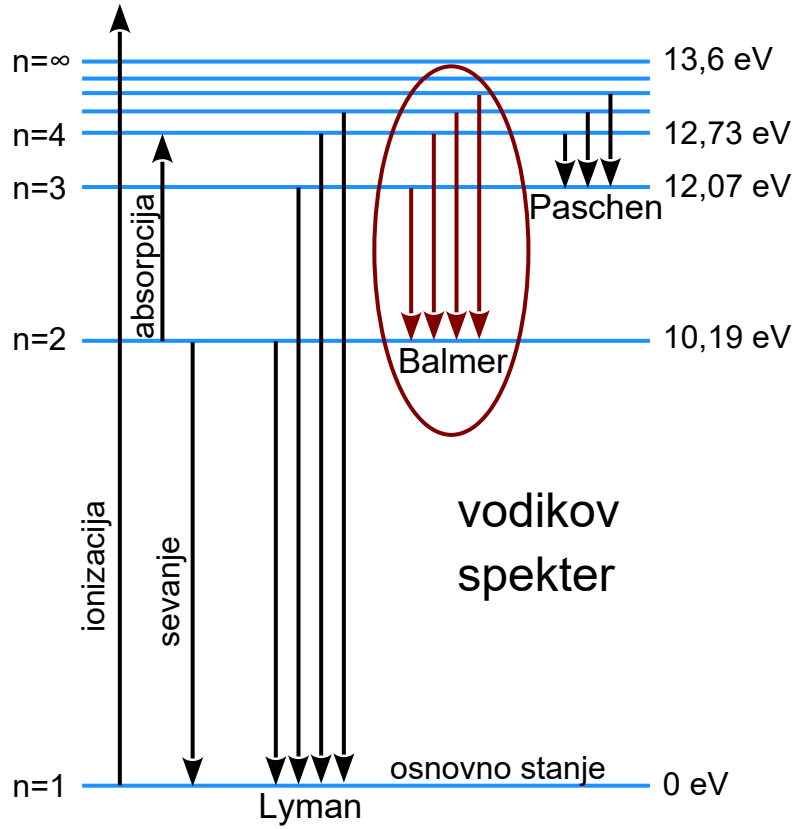
$$\Gamma = n\frac{h}{2\pi} = n\hbar$$

$$r_n = \frac{n^2\hbar^2 4\pi\epsilon_0}{m_e e_0^2} = n^2 r_B$$

  - energija elektrona

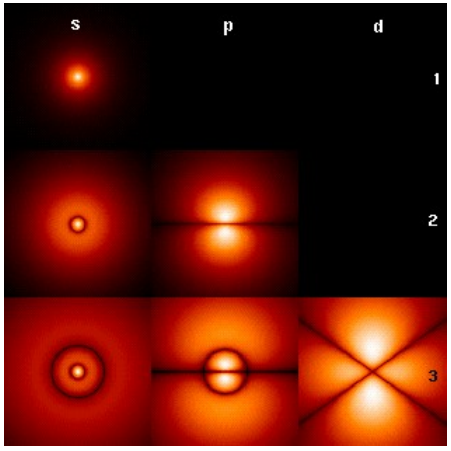
$$E_n = \frac{\Gamma_n^2}{2m_e r_n^2} - \frac{e_0^2}{4\pi\epsilon_0 r_n} = \frac{-\hbar^2}{2m_e r_B^2 n^2}$$

$$h\nu = \frac{hc}{\lambda} = E_n - E_{n'} = \frac{\hbar^2}{2m_e r_B^2}\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)$$

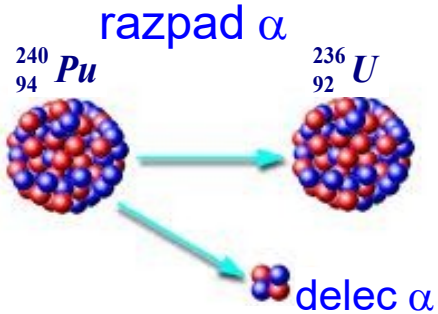


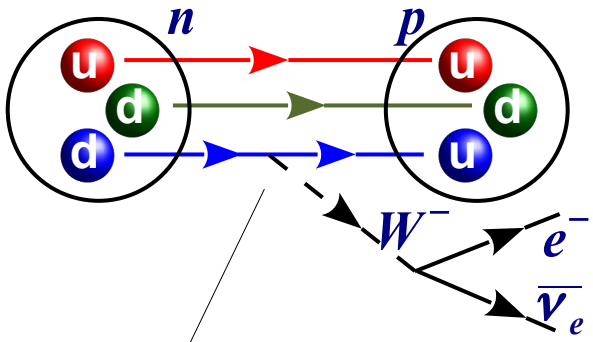
$$\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

Balmerjeva serija



Jedro



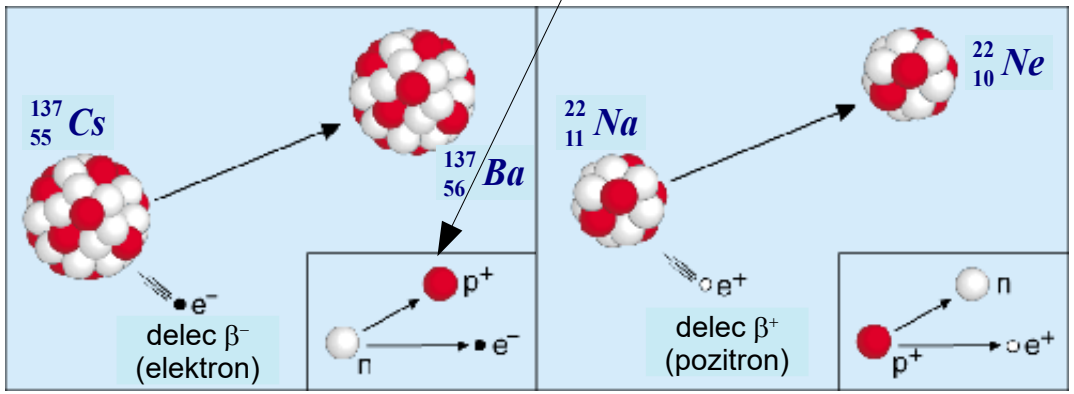


- Jedrske reakcije:
  - Razpad α
  - Razpad β

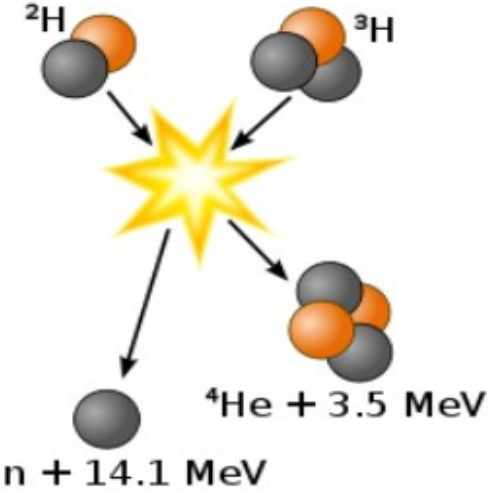
$$^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^- + \overline{\nu}_e$$ (razpad β^-)

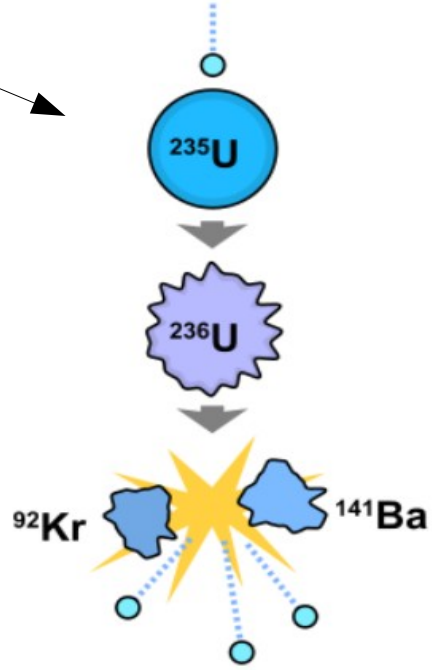
$$^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + e^+ + \nu_e$$ (razpad β^+)

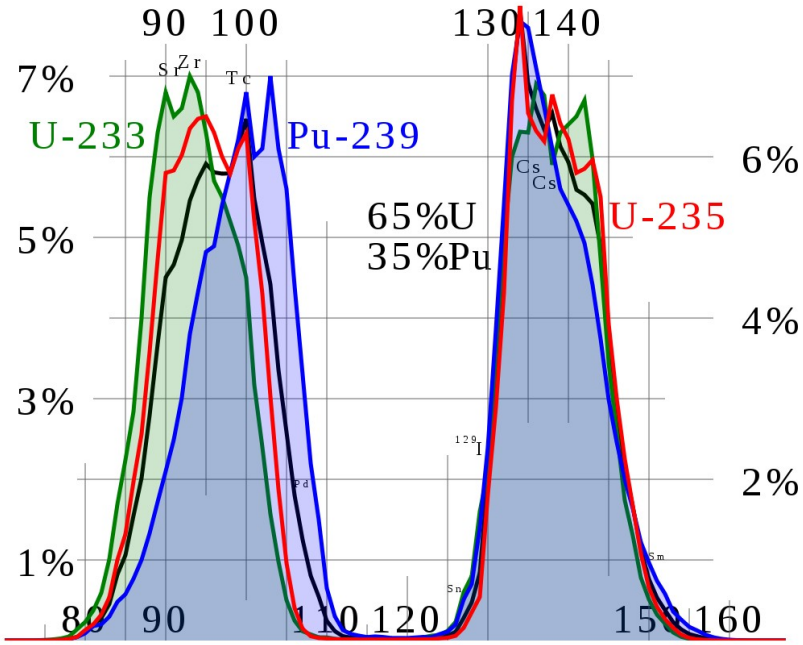
$$e^- + {}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + \nu_e$$ (zajetje elektrona)



- Cepitev jeder – fisija
  - jedrski reaktor
- Zlivanje jeder - fuzija







Porazdelitev mas produktov po cepitvi jeder nekaterih izotopov.