STRUCTURE OF LARGE AIR SHOWERS AT DEPTH 834 G CM^{-2} APPLICATIONS

John Linsley

Department of Physics and Astronomy, University of New Mexico
Albuquerque, New Mexico 87131, U.S.A.

It is shown how results presented in the two preceding papers can be applied to current problems in the study of cosmic rays. Applied to determination of the air shower size spectrum, they provide a means of improving the accuracy of existing results by eliminating the principal source of systematic error. Applied to the question of primary composition, they provide new information concerning the nature and magnitude of fluctuations.

1. Introduction. The following topics will be dealt with: size spectra above 10^{17}eV from Volcano Ranch, Yakutsk and Chacaltaya, dispersion of size with respect to field parameters such as Δ_{600}, the 'elongation rate (ER) theorem', application to η-fluctuations, application to α-fluctuations, comparison with other structure parameters, and future prospect. The terminology is the same as in the preceding two papers, which will be referred to as (I) and (II).

2. Size spectra above 10^{17}eV. VOLCANO RANCH. The new results supply information about inner structure that previously was lacking. Hence, the new size values that will be calculated for events registered in 1959-63 will be a good deal less uncertain than the old ones. However, the new result on outer structure disagrees with the old result at both ends of the size range 5.10^7 to 5.10^{10}. The effect on a size spectrum of revising the outer structure is complex, since it involves recalculating acceptance areas as well as individual size values. Furthermore, the differences between new and old values of $\langle\eta\rangle$ are outside the statistical errors. Hence, one or both of the measurements is affected by errors of a systematic nature. The old one is more likely to be at fault, since it implies a variation of $\langle\eta\rangle$ with size that is unacceptably rapid, according to a theoretical argument given below. Nevertheless, the question of systematic errors in both methods will have to be investigated before a final conclusion can be reached.

The new value of $\langle\alpha\rangle$ and the new formula for $\langle\eta\rangle$ give size values that are the same as before near 2.10^9. Near 5.10^7 they give values that are about 35% larger; near 10^{10}, about 25% smaller. By a crude method of estimation, the new values would give an average integral spectrum exponent equal to 1.9 for the interval $5.10^7 < N < 10^{10}$, as compared to 1.7 (upper limit 2.0) reported previously (Linsley 1963a).

The largest shower recorded at Volcano Ranch was fortunate in that the distance from its axis to the nearest detector was only $\sim$420m. Considering the lack of knowledge of inner structure at the time, and the experimental uncertainty in outer structure, its size was given as 5.10^{10}, "almost certainly within a factor of two" (Linsley 1963b). The recalculated size, using the average values of η and α given here, is $3.8.10^{10}$. Taking into account the instrumental correction factor 1.18 reported at the Denver Conference, the best value is now $4.5.10^{10}$.

YAKUTSK. In this case the classification parameter is particle density at 600m (except for small showers registered in the central portion of the array, which are treated separately). The interpolation formula is Equation 1 of (I), with α=1 and η determined empirically (Diminstein et al 1975). As I remarked in (I), there is very good agreement between that result and the one given here, when one compares showers that have travelled through the same thickness of atmosphere. Therefore, the present result for $\langle\eta\rangle$ does not suggest any change in the Yakutsk Δ_{600} spectrum. The present result for $\langle\alpha\rangle$ does, however, affect the conversion from Δ_{600} to average size (particle number). Size values derived on the assumption α=1 should be multiplied by a correction factor approximately equal to 1.45. The correction factor is almost independent of the characteristic distance (if $\gg R_o$), the density at that distance, or the zenith angle.

CHACALTAYA. Recent results (Aguirre et al 1977a) on the size spectrum at this very high elevation (550 g cm^{-2}) are inconsistent with earlier results at the same location (Bradt et al 1965). They are also inconsistent with many results obtained at lower elevations, combined with evidence concerning the rate of size attenuation. It seems clear that the discrepancy is closely related to a difference between the interpolation formula that is being used in this instance (Aguirre et al 1977b) and other representations of lateral structure. Since the formulas in question are empirical, any significant difference between them is to be blamed on systematic errors of some sort.

To compare my present result with the most recent formula given by the BASJE group I will use the notion of 'effective s' described in (I). The following formula is equivalent to Equation 1 of Aguirre et al (1977b):

$$s_{eff} = s + 2(1+x)/(1+2x)(1+1/C_2x^2) \qquad 0.1 < x < 3. \qquad (1)$$

where $s=.660-.105\log_{10}(N_e/10^7)+.125(\sec\theta-1)$ and $C_2=.100+.125(\sec\theta-1)$.

(For simplicity I have omitted the second correction factor, after determining that it is unimportant in the present context.) The comparison is made in Table 1.

3. Size Dispersion. Using histograms of η and α, an assumption regarding their correlation, and the structure formula, one can compute the distribution of 'true' size for fixed particle density at a given radial distance. It is expected, as many authors have pointed out, that particle density at distances of order 5 to 10 Moliere units will fluctuate relatively little with respect to primary energy. Hence the result of such a computation is essentially the size distribution for fixed primary energy. I have obtained results of this kind in two interesting cases, assuming that η and α fluctuate independently. Both results are for $N\sim10^8$. One is for R=600m, depth

Table 1. Average effective s vs distance, for vertical showers at 3 elevations. Values in col. A are given by Eq. 1; those in col. B, by the present work, Eq. 8 of (I). In extrapolating the present result to the elevation of Chacaltaya, I used 95% confidence limits for coefficients a_1 and b_1 describing depth dependence. $N=10^8$ (5.10^7 for s.l.)

$x=R/R_o$	550 g cm^{-2} A	550 g cm^{-2} B	834 g cm^{-2} A	834 g cm^{-2} B	1020 g cm^{-2} A	1020 g cm^{-2} B
0.1	0.56	>0.72	0.62	0.84±.03	0.69	0.86±.03
0.3	0.57	>0.83	0.64	0.96 "	0.71	0.99 "
1.	0.68	>0.98	0.81	1.12 "	0.96	1.18 "
3.	1.10	>1.08	1.30	1.23 "	1.48	1.31 "

1020 g cm^{-2}; the other for R=530m, depth 834 g cm^{-2}. The size distributions prove to be approximately log-normal, with standard deviations corresponding to a factor 1.9 in the first case and 1.6 in the second. This amount of fluctuation is somewhat greater than has been predicted (for proton-initiated showers!) by the use of models (DeBeer et al 1968, Hillas et al 1971). The result underscores the importance of correctly evaluating selection effects in case of arrays that are 'closely packed', as discussed by Diminstein et al (1976).

4. The Elongation-Rate Theorem. The term 'elongation' denotes the increase in atmospheric depth of maximum development that results from increased primary energy, for showers of a given type. By extension, it refers also to the corresponding increase in depth at which showers attain a given 'age'. Here 'type' refers to the nature (mass number A) of the primary particle. 'Elongation rate' denotes the derivative $d(X_{max})/d(\ln E_o)$, where X_{max} is depth of maximum development of an average shower of energy E_o. The elongation-rate (ER) theorem states that the elongation rate is bounded above by X_o, and is very nearly equal to $(1-B)X_o$, where X_o is the characteristic length of cascade theory, equal by definition to 37.7 g cm^{-2} in air, and B is the exponent of E in the formula for pion multiplicity (more generally, the logarithmic derivative of multiplicity with respect to energy).

The ER theorem agrees satisfactorily with numerical results for all models that are in current use (Wdowczyk and Wolfendale 1973, Dixon et al 1973, Capdevielle et al 1975). It follows from the fact that in those models the locations (depths) of all energetically significant nuclear interactions are essentially energy independent. Therefore, the almost exclusive cause of elongation is the increase in energy of the neutral pion decay-photons, which elongates the individual electromagnetic cascades according to the well known formula of cascade theory. If pion multiplicity were energy independent, then the decay-photon energies would all be proportional to E_o and the elongation rate would be X_o. In case of energy dependent multiplicity, one requires an additional fact, that elongation of the shower as a whole is dominated by neutral pions produced in 'leading interactions' (interactions by the surviving nucleon or nucleons).

I find this theorem to be very useful, both for interpreting experimental results and for estimating properties of showers so large that they are consequently rare. The utility comes from a relation between depth dependence and energy dependence in which the elongation rate plays a key role. Let P represent the average value of some shower parameter. I will assume that P does not depend explicitly on primary energy, and that it belongs to one of the following types, or is intermediate between them. Type 1: depth dependence of the form $f(X/X_{max})$. Then

$$\left.\frac{\partial P}{\partial \ln E_o}\right|_X = -\frac{X}{X_{max}}(\text{elong. rate})\left.\frac{\partial P}{\partial X}\right|_{E_o} \qquad (2a)$$

In cascade theory, the age parameter 's' is of Type 1 and conforms to Equation 2a. Type 2: depth dependence of the form $f(X-X_{max})$. Then

$$\left.\frac{\partial P}{\partial \ln E_o}\right|_X = -(\text{elong. rate})\left.\frac{\partial P}{\partial X}\right|_{E_o} \qquad (2b)$$

Depth dependence similar to Type 2 is predicted, for $X > X_{max}$, by shower models in which nucleon-nucleus interactions are appreciably inelastic. Equation 2 (a or b) applies to showers of a single type, or to mixed primaries with constant or slowly varying composition.

5. Application to η-Fluctuations. To prepare for use of Equation 2, I first transform from the variable N to $E_o(N,X)$ with the following result:

$$\left.\frac{\partial\langle\eta\rangle}{\partial \ln E_o}\right|_X = b_2 \log_{10}e \left.\frac{\partial \ln N}{\partial \ln E_o}\right|_X, \quad \left.\frac{\partial\langle\eta\rangle}{\partial X}\right|_{E_o} = \frac{b_1}{X_1} + b_2 \log_{10}e \left.\frac{\partial \ln N}{\partial X}\right|_{E_o} \qquad (3)$$

where X_1=834 g cm^{-2} and b_1, b_2 are the coefficients in Equation 3 of (I). Using values 1.1 and $(-1/300)g^{-1}cm^2$ for $\partial \ln N/\partial \ln E_o|X$ and $\partial \ln N/\partial X|E_o$, respectively, I find that $\partial\langle\eta\rangle/\partial \ln E_o|X = 0.032\pm.014$, and $\partial\langle\eta\rangle/\partial X|E_o = -(8.6\pm1.0).10^{-4}g^{-1}cm^2$. Substituting those values in Equation 2a, I obtain elongation rate = $(X_{max}/X)(37\pm17)$g cm^{-2}, a result which is in reasonable accord with the ER theorem, although the error is too great to permit drawing a conclusion about the multiplicity index B. The same comment would apply if I had used Equation 2b.

In contrast to that result, the same analysis applied to the formula for $\langle\eta\rangle$ that I reported at the Jaipur Conference (Linsley 1963a, Eq. 2) gives the discordant result ER$\sim$100 g cm^{-2}. Of the two coefficients b_1 and b_2 it seems certain that b_2 is at fault. From an experimentalist's point of view that is not surprising, since measurements of energy (size) dependence are known to be more susceptible to methodological bias. The same difficulty occurs in case of the formula for s given by Aguirre et al (1977b), Eq. 1 above. That formula implies ER$\sim$200 g cm^{-2}.

If showers of a given type did not fluctuate, the probability distribution for η would consist of discrete lines belonging to the various A-values in the primary mass spectrum. Using the principle that superposition is valid for averages (Tomaszewski and Wdowczyk 1975), the separation of η-lines would be given by the following expression:

$$(\ln A_i - \ln A_j)\partial\langle\eta\rangle/\partial \ln E_o|X \qquad (4)$$

Even for the worst-case assumption that the primaries are an equal mixture of protons and Fe nuclei, the breadth due to line separation is only about 0.06, much less than the observed breadth of the η-distributions given in (II). Hence the observed breadth can be regarded as entirely due to 'intrinsic' fluctuations arising from stochastic variations in the location and character of the leading interactions.

There is a simple well-known fluctuation model in which the only random variable is the depth of the initial interaction. According to that model, η-deviations would be equal to (b_1/X_1) times the corresponding deviations in $X_{initial}$. The breadth of the η-distribution would be equal to the same factor times the primary mean free path, and would not depend on depth (over the range described by b_1). Clearly that is not the case. The value of $\sigma_\eta X_1/b_1$ is much too large (225 g cm^{-2} at $X \sim X_1$), and it increases with increasing depth, as shown by Figure 2 of (II). This indicates that in real showers the fluctuations in outer structure result from numerous contributions, some of which involve nuclear interactions that occur deep in the atmosphere (as deep, say, as X_{max}).

6. Application to α-Fluctuations. Even stronger evidence that fluctuations are not confined to the early stages of development is afforded by the result for α. According to the simple model described above, a parameter like α, whose average value has hardly any depth dependence, would show very little fluctuation, contrary to what is observed. The actual behavior of α, and the different behavior of α and η, can be understood if inner structure is strongly affected by leading interactions in the $\sim$100 g cm^{-2}-thick layer of atmosphere just above ground level, whereas outer structure is determined by the accumulated effect of the leading interactions above that layer. It appears, however, that in order to account for α-fluctuations as large as those observed, the number of leading nucleons that reach ground level must be small, of order unity.

7. Comparison with Other Parameters. Results on the parameter $t_{\frac{1}{2}}$ obtained at Leeds by the study of pulse profiles have been analyzed along similar lines (Barrett et al 1977). The average value varies with depth and energy in a manner consistent with the ER theorem (provided that the energy coefficient error estimate is not insisted upon). However, the 'characteristic length' analogous to $\sigma_\eta X_1/b_1$ above has a much smaller value, $\sim$80 g cm^{-2}.

Another parameter studied at Leeds, called B(100), resembles η in terms of the operations used to measure it (Lapikens et al 1977). However, the characteristic length derived from fluctuation and depth dependence is also a good deal smaller ($\sim$100 g cm^{-2}). Of course, this 'disagreement' casts no doubt whatever on any of the measurements. It simply indicates that each of the parameters has something different to tell us.

All of these parameters, and some others I have not mentioned, presumably share the property of having no explicit energy dependence. In such cases the line separations describing mixed composition can be estimated using Equation 4 above. In case of $t_{\frac{1}{2}}$, the breadth due to line separation estimated thus would be approximately equal to the observed breadth, for an equal mixture of protons and Fe nuclei (and hence the resultant breadth would be about 1.4 times the observed breadth).

Another important class of shower parameters have explicit energy dependence, namely those that involve muon size or muon density. For such parameters, the total energy coefficient equals the term given by Equation 2 plus a term which typically is as great or greater than the first in magnitude. Thus, in cases where the intrinsic line breadth is small, the H-Fe line separation may be several times the intrinsic breadth, and it should be feasible to detect a rather small proportion of Fe nuclei ($\sim$10%) in a primary beam consisting mainly of protons, or vice-versa (Linsley and Scarsi 1962).

8. Future Prospect. Evidence seems to be building that a large proportion (90% ?) of primary cosmic rays above 10^{17}eV are protons. However, the need is not for such guesses but for detailed theoretical work that will unify many new results by many authors. Models should be used which correctly predict the average characteristics of showers, to the full extent they are known. In the calculation of fluctuations the recommendations by Tomaszewski and Wdowczyk (1975) should be followed for showers initiated by heavy nuclei. Fluctuation calculations should not be limited to a single parameter at a single level, but should explore the primary mass effect over the entire range covered by experiments, especially with regard to depth dependence. The principle that the early stages of shower development are the most sensitive to differences in primary mass is questionable, in light of the results by Waddington and Freier (1973), which gave quantitative expression to the concern voiced earlier by Peters (1963). It should be re-examined. It may be found that the most favorable region for making the desired discrimination is just below the level where fragmentation has been essentially completed, so the leading nucleons have become independent but have not lost all memory of their initially common energy. Favorable experiments would be those (at whatever level they might be carried out) that are able to sense conditions in the region so described.

9. Acknowledgement. This work was supported by the U.S. National Science Foundation.

10. References.

Aguirre, C., G.R. Mejia, H. Yoshii, T. Kaneko, P.K. MacKeown, F. Kakimoto, Y. Mizumoto, K. Suga, M. Nagano, K. Kamata, K. Murakami, K. Nishi and Y. Toyoda (1977a) paper EA-59, this Conference.

Aguirre, C., A. Trepp, H. Yoshii, P.K. MacKeown, T. Kaneko, F. Kakimoto, Y. Mizumoto, K. Suga, M. Nagano, K. Kamata, K. Murakami, K. Nishi and Y. Toyoda (1977b) paper EA-57, this Conference.

Aseikin, V.S., V.P. Bobova, A.G. Dubovij, N.V. Kabanova, I.N. Kirov, N.M. Nesterova, S.I. Nikolski, N.M. Nikolskaja, V.A. Romakhin, I.N. Stamenov, E.I. Tukish and V.C. Janminchev (1975) Proc. 14th ICCR (Munich) 8, 2807-12.

Barrett, M.L., R. Walker, A.A. Watson and P. Wild (1977) paper EA-48, this Conference.

Betev, B., T. Stanev, Ch. Vankóv, N.M. Nesterova and V.A. Romachin (1975) Proc. 14th ICCR (Munich) 8, 2989-94.

References (continued)

Bradt, H., G. Clark, M. LaPointe, V. Domingo, I. Escobar, K. Kamata, K. Murakami, K. Suga and Y. Toyoda (1965) Proc. 9th ICCR (London) 2, 715-7.

Capdevielle, J-N., J. Procureur and M-F. Bourdeau (1975) Proc. 14th ICCR (Munich) 2930-5.

De Beer, J.V., B. Holyoak, H. Oda, J. Wdowczyk and A.W. Wolfendale (1968) J. Phys. A 1, 72-81.

Diminstein, O.S., T.A. Egorov, N.N. Efimov, N.N. Efremov, A.V. Glushkov, L.I. Kaganov, M.I. Pravdin and G.B. Khristiansen (1975) Proc. 14th ICCR (Munich) 12, 4334-8.

Diminstein, O.S., N.N. Efimov, A.V. Glushkov, L.I. Kaganov and M.I. Pravdin (1976) "Measurement Problems of the Primary Energy Spectrum of Superhigh Energy Particles", presented at the European Symposium, Leeds.

Dixon, H.E., J.C. Earnshaw, J.R. Hook, G. J. Smith and K.E. Turver (1973) Proc. 13th ICCR (Denver) 2473-88.

Fukui, S., H. Hasegawa, T. Matano, I. Miura, M. Oda, K. Suga, G. Tanahashi and Y. Tanaka (1960) Progr. Theor. Phys. (Kyoto) 16, Suppl. 16, 1-53.

Greisen, K. (1956) Progress in Cosmic Ray Physics (J.G. Wilson, Ed., North-Holland Publ. Co., Amsterdam) Vol. 3, 1-141.

Greisen, K. (1960) Ann. Rev. Nucl. Sci. 10, 63-108.

Hillas, A.M., D.J. Marsden, J.D. Hollows and H.W. Hunter (1971) Proc. 12th ICCR (Hobart) 1001-6.

Kamata, K. and J. Nishimura (1958) Progr. Theor. Phys. (Kyoto) Suppl. 6, 93-100.

Kawaguchi, S., K. Suga and H. Sakuyama (1973) Proc. 13th ICCR (Denver) 4, 2562-7.

Kawaguchi, S., K. Suga and H. Sakuyama (1975) Proc. 14th ICCR (Munich) 8, 2826-30.

Khristiansen, G.B., O.V. Vedeneev, G.V. Kulikov, V.I. Nazarov and V.I. Solovjeva (1971) Proc. 12th ICCR (Hobart) 6, 2097-108.

Lapikens, J., H.M. Norwood, R.J.O. Reid, S. Ridgway and A.A. Watson (1977) paper EA-47, this Conference.

Linsley, J. and L. Scarsi (1962) Phys. Rev. Lett. 9, 123-5.

Linsley, J., L. Scarsi and B. Rossi (1962) J. Phys. Soc. Japan 17, Suppl. A-III, 91-102.

Linsley, J. (1963a) Proc. 8th ICCR (Jaipur) 4, 77-99.

Linsley, J. (1963b) Phys. Rev. Lett. 10, 146-8.

Linsley, J. (1973a) Proc. 13th ICCR (Denver) 5, 3212-9; (1973b) ibid. 4, 2742-7.

Nikolsky, S.I. (1962) Proc. 5th Interamerican Seminar on Cosmic Rays (La Paz, Bolivia) 2, XLVIII.

Peters, B. (1963) comment in Proc. 8th ICCR (Jaipur) 4, p.124.

References (continued)

Tomaszewski, A. and J. Wdowczyk (1975) Proc. 14th ICCR (Munich) 8, 2899-903.

Vernov, S.N., G.B. Khristiansen, A.T. Abrosimov, V.B. Atrashkevich, I.F. Beliaeva, O.V. Vedeneev, V.A. Dmitriev, G.V. Kulikov, J.A. Nechin, V.I. Solovieva, K.I. Soloviev, J.A. Fomin and B.A. Khrenov (1963) Proc. 8th ICCR (Jaipur) 4, 173-8.

Waddington, C.J. and P.S. Freier (1973) Proc. 13th ICCR (Denver) 4, 2449-54.

Wdowczyk, J., and A.W. Wolfendale (1973) Proc. 13th ICCR (Denver) 3, 2336-47.