D Mixing and CP Violation

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Part I

1. Introduction
2. Mixing phenomenology
3. Mixing measurements

Part II

1. CPV phenomenology
2. CPV measurements
3. Constraints on NP
4. Outlook

Belle Analysis School,
KEK, Feb 10 – 12, 2011

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University of Ljubljana
"Jožef Stefan" Institute
BELLE
Belle II

Few facts about statistics:

- Marko (Starič), main analyst on D^0 mixing discovery, never owed a boat
- Belle II analysis coordinator never gambles (ok, perhaps sometimes with predictions of accuracy to be achieved at Belle II)
- I'm not Damjan Golob,

and the priobability that Bruce gets 500 € from me is ε,

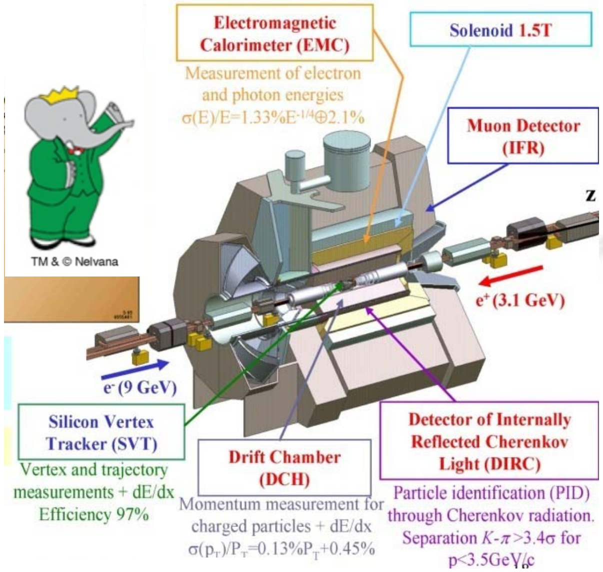
$$\lim_{N \to \infty} \varepsilon = 0$$

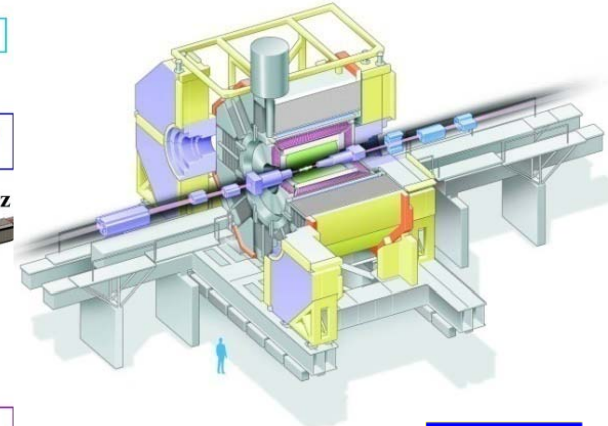
Introduction

Experiments

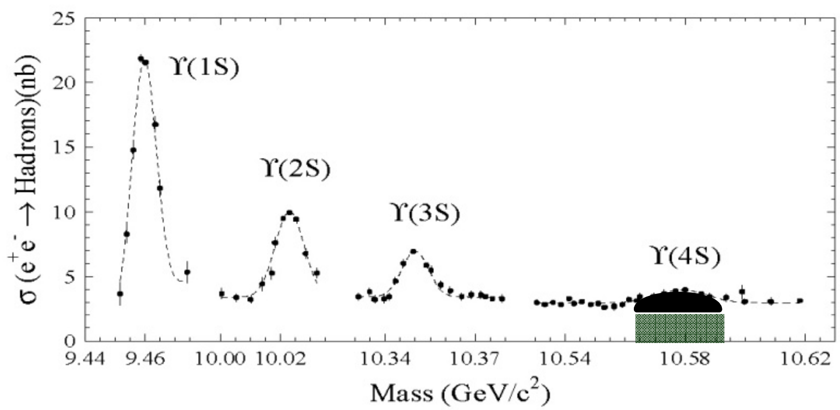
B-Factories

BaBar @ PEPII
SLAC



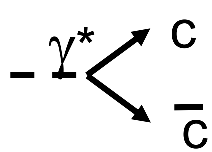


Belle @ KEKB
KEK



on resonance production
$e^+e^- \rightarrow \Upsilon(4S) \rightarrow B^0\bar{B}^0, B^+B^-$
$\sigma(B\bar{B}) \approx$ 1.1 nb (~10^9 $B\bar{B}$ pairs)

continuum production
$\sigma(c\bar{c}) \approx$ 1.3 nb (~1.3x10^9 $X_c\bar{Y}_c$ pairs)
$N_{rec}(D^{*+} \rightarrow D^0\pi^+ \rightarrow K^-\pi^+\pi^+) \approx 2.5\text{x}10^6$

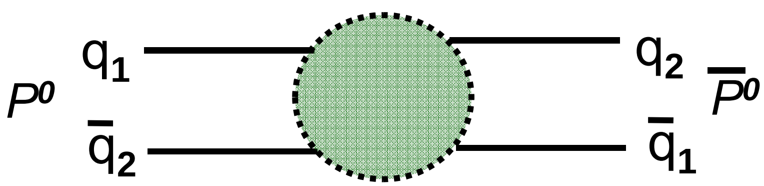


B-factory=
charm factory

Mixing phenomenology

Mixing of neutral mesons

Phenomena
in course of life neutral meson P^0 can transform into anti-meson $\overline{P}^0$



$P^0 = K^0, B_d^0, B_s^0$ and D^0

History

observation of K^0:
1950 (Caletch)
mixing in K^0:
1956 (Columbia) — 6 years
c quark mass
observation of B_d^0:
1983 (CESR)
mixing in B_d^0:
1987 (Desy) — 4 years
t quark mass
observation of B_s^0:
1992 (LEP)
mixing in B_s^0:
2006 (Fermilab) — 14 years
????
observation of D^0:
1976 (SLAC)
mixing in D^0:
2007 (KEK, SLAC) — 31 years
????
(evidence of)

Time evolution

D. Kirkby, Y. Nir, *CPV in Meson Decays*, in RPP

Schrödinger equation mixing affects the time evolution → oscillations

state initially produced as

$$\left|\psi(t=0)\right\rangle = a(0)\left|P^0\right\rangle + b(0)\left|\overline{P}^0\right\rangle$$

will evolve in time as

$$\left|\psi(t)\right\rangle = a(t)\left|P^0\right\rangle + b(t)\left|\overline{P}^0\right\rangle + \ldots$$

if interested in *a(t), b(t):*
effective Hamiltonian
H=M-(i/2)Γ (non-Hermitian)
and t-dependent Schrödinger eq.:

$$i\frac{\partial}{\partial t}\begin{bmatrix}\left|P^0(t)\right\rangle \\ \left|\overline{P}^0(t)\right\rangle\end{bmatrix} = \left(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma}\right)\begin{bmatrix}\left|P^0(t)\right\rangle \\ \left|\overline{P}^0(t)\right\rangle\end{bmatrix}$$

eigenstates:
(well defined $m_{1,2}$ and $\Gamma_{1,2}$)

$$\left|P_{1,2}\right\rangle = p\left|P^0\right\rangle \pm q\left|\overline{P}^0\right\rangle$$

Time evolution

Schrödinger equation

eigenvalues

diagonal elem.:

$P^0 \leftrightarrow P^0$

non-diagonal elem.:

$P^0 \leftrightarrow \overline{P}^0$

$$\begin{bmatrix} M - i\frac{\Gamma}{2} & M_{12} - i\frac{\Gamma_{12}}{2} \\ M_{12}^* - i\frac{\Gamma_{12}^*}{2} & M - i\frac{\Gamma}{2} \end{bmatrix} \begin{bmatrix} p \\ \pm q \end{bmatrix} = \lambda_{1,2} \begin{bmatrix} p \\ \pm q \end{bmatrix}$$

$$\lambda_{1,2} = M - i\frac{\Gamma}{2} \pm \frac{q}{p}\left[M_{12} - i\frac{\Gamma_{12}}{2}\right] \equiv m_{1,2} - i\frac{\Gamma_{1,2}}{2}, \quad \left(\frac{q}{p}\right)^2 = \frac{M_{12}{}^* - i\frac{\Gamma_{12}{}^*}{2}}{M_{12} - i\frac{\Gamma_{12}}{2}}$$

q/p: CPV;

if CPV neglected q/p=1

$P_{1,2}$ evolve in time according to $m_{1,2}$ and $\Gamma_{1,2}$:

$$\left|P_{1,2}(t)\right\rangle = e^{-i\lambda_{1,2} t}\left|P_{1,2}(t=0)\right\rangle$$

Time evolution

$$\left|P^0(t)\right\rangle = \frac{1}{2p}\left[\left|P_1(t)\right\rangle + \left|P_2(t)\right\rangle\right]$$

$$\left|\overline{P}^0(t)\right\rangle = \frac{1}{2q}\left[\left|P_1(t)\right\rangle - \left|P_2(t)\right\rangle\right]$$

Flavor states

state initially produced as pure P^0 or $\overline{P}^0$

$$\left|P^0(t)\right\rangle = \left[\left|P^0\right\rangle \cosh\left(\frac{ix+y}{2}\overline{\Gamma}t\right) - \frac{q}{p}\left|\overline{P}^0\right\rangle \sinh\left(\frac{ix+y}{2}\overline{\Gamma}t\right)\right] e^{-i\overline{m}t-\frac{\overline{\Gamma}}{2}t}$$

$$\left|\overline{P}^0(t)\right\rangle = \left[\left|\overline{P}^0\right\rangle \cosh\left(\frac{ix+y}{2}\overline{\Gamma}t\right) - \frac{p}{q}\left|P^0\right\rangle \sinh\left(\frac{ix+y}{2}\overline{\Gamma}t\right)\right] e^{-i\overline{m}t-\frac{\overline{\Gamma}}{2}t}$$

can at a later time t be $\overline{P}^0$ or P^0, depending on values of mixing parameters x, y:

$$x \equiv \frac{m_1 - m_2}{\overline{\Gamma}};\ y \equiv \frac{\Gamma_1 - \Gamma_2}{2\overline{\Gamma}};\ \overline{\Gamma} \equiv \frac{\Gamma_1 + \Gamma_2}{2};\ \overline{m} \equiv \frac{m_1 + m_2}{2}$$

Time evolution

M. Gronau et al., PLB508, 37 (2001)

Flavor states

coherent pair production from vector resonance

$e^+e^- \to V \to P^0 \overline{P}^0$

$V=\Upsilon(4S)$ B^0
$V=\Psi(3770)$ D^0
$V=\Phi$ K^0

$$\psi = \frac{1}{\sqrt{2}}\left[\left| P^0(\vec{p}_1)\right\rangle \left| \overline{P}^0(\vec{p}_2)\right\rangle \pm \left| \overline{P}^0(\vec{p}_1)\right\rangle \left| P^0(\vec{p}_2)\right\rangle \right]$$

initial state, C= ±1

$$\psi(t_1,t_2) = \frac{1}{\sqrt{2}} e^{-(\overline{m}-i\overline{\Gamma}/2)(t_1+t_2)} \left\{ \cos\left(\overline{\Gamma}\frac{x-iy}{2}(t_1 \pm t_2)\right)\left[\left| P^0(\vec{p}_1)\right\rangle \left| \overline{P}^0(\vec{p}_2)\right\rangle \pm \left| \overline{P}^0(\vec{p}_1)\right\rangle \left| P^0(\vec{p}_2)\right\rangle \right] \pm \right.$$
$$\left. \pm i \sin\left(\overline{\Gamma}\frac{x-iy}{2}(t_1 \pm t_2)\right)\left[\left| P^0(\vec{p}_1)\right\rangle \left| P^0(\vec{p}_2)\right\rangle - \left| \overline{P}^0(\vec{p}_1)\right\rangle \left| \overline{P}^0(\vec{p}_2)\right\rangle \right] \right\}$$

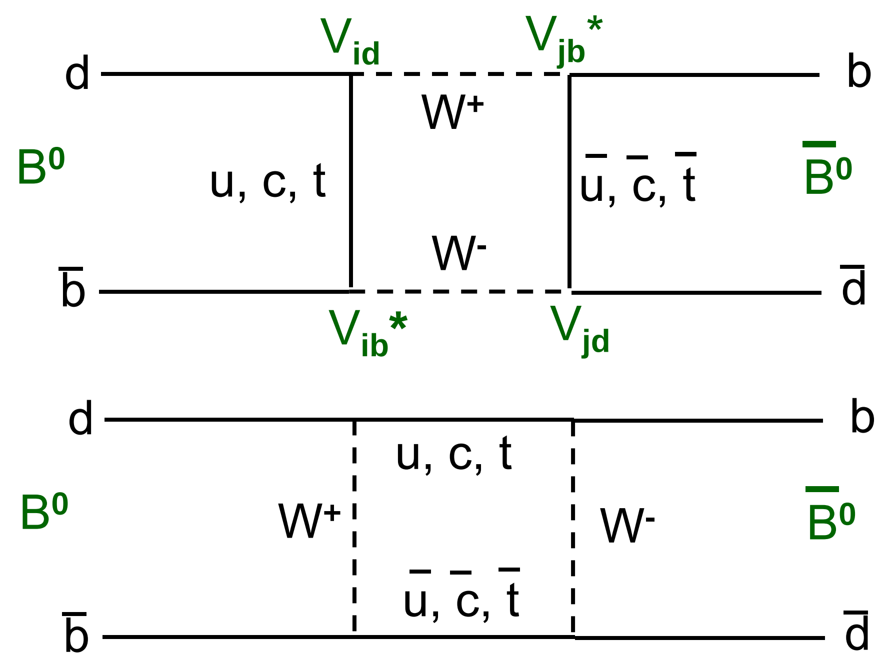
Mixing rate

Phenomenology

P⁰-$\overline{P}^0$ transition → box diagram at quark level

$$\left\langle \overline{B}^0 \middle| H_{wk} \middle| B^0 \right\rangle \propto \sum_{i,j=u,c,t} V_{ib}^* V_{id} V_{jd} V_{jb}^* \mathcal{F}(m_W^2, m_i^2, m_j^2)$$

P^0: any pseudo-scalar meson; specific example of B_d^0



if $m_i = m_j \Rightarrow$ due to CKM unitarity: no mixing

Mixing rate

Phenomenology
simplified treatment
based on dimension:

O. Nachtmann, Elem. Part. Phys., Springer-Verlag

$$\langle \overline{B}^0 | H_{wk} | B^0 \rangle \propto \sum_{i,j=u,c,t} V_{ib}^* V_{id} V_{jd} V_{jb}^* \mathcal{F}(m_W^2, m_i^2, m_j^2)$$

$$\mathcal{F}(m_W^2, m_i^2, m_j^2) \propto f_0 m_W^2 + f_1 m_i^2 + f_2 m_j^2 + f_3 m_i m_j + O(m_W^{-2})$$

for serious treatment see e.g.: A.J. Buras et al., Nucl.Phys.B245, 369 (1984)

CKM unitarity ⇒

$$\langle \overline{B}^0 | H_{wk} | B^0 \rangle \propto \sum_{i,j=u,c,t} V_{ib}^* V_{id} V_{jd} V_{jb}^* m_i m_j$$

Homework: contribution of which quark is dominant in the above expression?

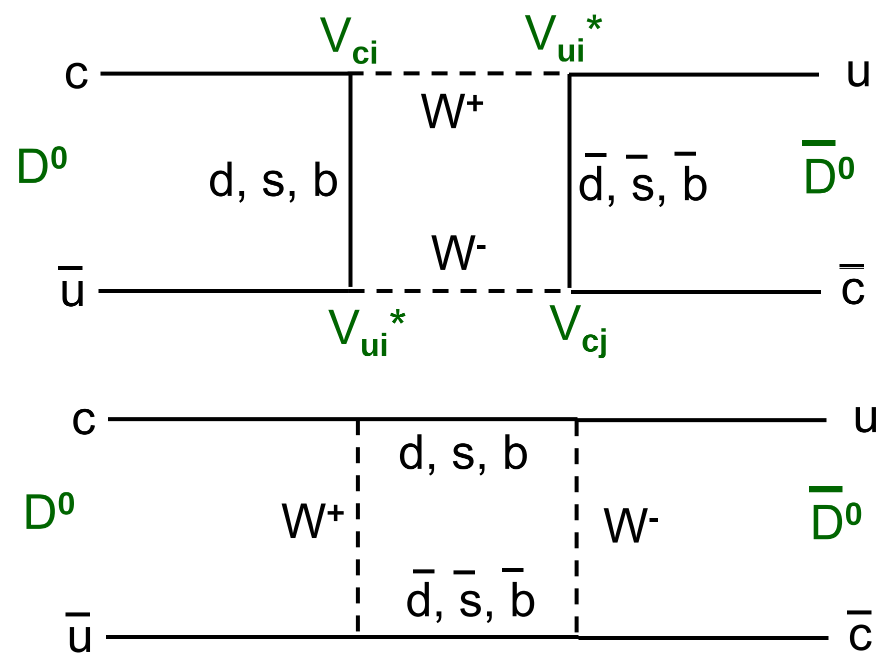
Mixing rate

Phenomenology

D^0 case

the only P^0 system with uplike q's

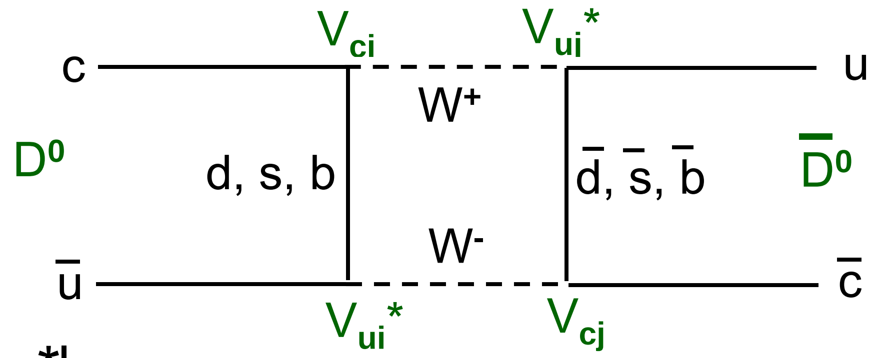
the system resisiting exp. observation for the longest time



$$\left\langle \overline{D}^0 \middle| H_{wk} \middle| D^0 \right\rangle \propto \sum_{i,j=d,s,b} V_{ui}^* V_{ci} V_{cj} V_{uj}^* m_i m_j$$

Mixing phenomenology

Mixing rate



Phenomenology

$|V_{cb}V_{ub}^*| << |V_{cs}V_{us}^*|, |V_{cd}V_{ud}^*|$ assuming unitarity in 2 generations $\Rightarrow$

$$\langle \overline{D}^0 | H_{wk} | D^0 \rangle \propto \underbrace{V_{us}^* V_{cs} V_{cd} V_{ud}^*}_{\text{DCS}} \underbrace{(m_s - m_d)^2}_{\text{SU(3) breaking}}$$

more involved (and correct) calculation:

A.F. Falk et al., PRD65, 054034 (2002)

G. Burdman, I. Shipsey, Ann.Rev.Nucl.Sci. 53, 431 (2003)

$$\langle \overline{D}^0 | H_w^{\Delta C=-2} | D^0 \rangle = \frac{G_F^2}{4\pi^2} \underbrace{V_{cs}^* V_{cd}^* V_{ud} V_{us}}_{\text{DCS}} \underbrace{\frac{(m_s^2 - m_d^2)^2}{m_c^2}}_{\text{SU(3) breaking}} \langle \overline{D}^0 | \bar{u}\gamma^\mu (1-\gamma_5) c \bar{u} \gamma_\mu (1-\gamma_5) c | D^0 \rangle$$

Mixing rate

Phenomenology

2nd order perturb. theory

see formula on p. I/7

$\Delta m - i\Delta\Gamma/2 = 2(q/p)[M - i\Gamma/2]_{12}$

$\Delta m,\ \Delta\Gamma = f(M_{12}, \Gamma_{12})$

$$(M - i\frac{\Gamma}{2})_{ij} = \frac{\langle D_i | H_{eff} | D_j \rangle}{2M_D} =$$

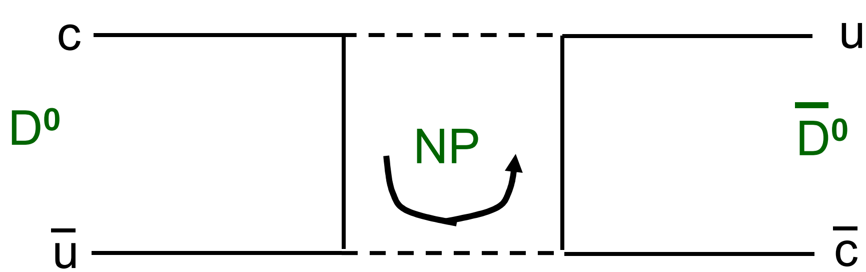
$$= M_D \delta_{ij} + \frac{1}{2M_D}\langle \overline{D}^0 | H_w^{\Delta C=-2} | D^0 \rangle +$$

$$+ \frac{1}{2M_D}\sum_n \frac{\langle \overline{D}^0 | H_w^{\Delta C=-1} | n \rangle \langle n | H_w^{\Delta C=-1} | D^0 \rangle}{M_D - E_n + i\varepsilon}$$

short distance $|x| \sim \mathcal{O}(10^{-5})$

common statement: mixing with large x sign of NP;

more appropriate: measurement of x yields complementary constraints on NP models (because of specific uplike *q* couplings)



Mixing rate

Phenomenology

2nd order perturb. theory

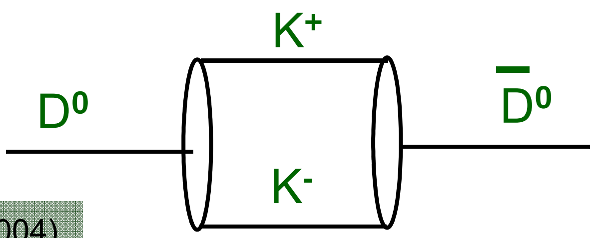
long distance

$$(M - i\frac{\Gamma}{2})_{ij} = \frac{\langle D_i | H_{eff} | D_j \rangle}{2M_D} =$$

$$= M_D \delta_{ij} + \frac{1}{2M_D} \langle \overline{D}^0 | H_w^{\Delta C=-2} | D^0 \rangle +$$

$$+ \frac{1}{2M_D} \sum_n \frac{\langle \overline{D}^0 | H_w^{\Delta C=-1} | n \rangle \langle n | H_w^{\Delta C=-1} | D^0 \rangle}{M_D - E_n + i\varepsilon}$$

difficult to calculate;
contributes to real and imaginary part ⇒
affects *x* and *y;*



two approaches:
OPE I.I. Bigi, N. Uraltsev, Nucl. Phys. B592, 92 (2001)
exclusive approach A.F. Falk et al., PRD69, 114021 (2004)
(principle can be easy understood, see p. II/26)

$$|x|, |y| \leq \mathcal{O}(10^{-2})$$

$$\frac{1}{M_D - E_n + i\varepsilon} = PV\left(\frac{1}{M_D - E_n}\right) + i\pi\delta(E_n - M_D)$$

Mixing of neutral mesons

Observables

(B-factories, hadron machines)

$$\left|D^0(t)\right\rangle = \left[\left|D^0\right\rangle \cosh\left(\frac{ix+y}{2}\overline{\Gamma} t\right) - \frac{q}{p}\left|\overline{D}^0\right\rangle \sinh\left(\frac{ix+y}{2}\overline{\Gamma} t\right)\right] e^{-i\overline{m}t - \frac{\overline{\Gamma}}{2}t}$$

$|x|, |y| << 1 \Rightarrow$

$$\frac{dN(D^0 \to f)}{dt} \propto e^{-\overline{\Gamma}t} \left| A_f + \frac{q}{p}\frac{ix+y}{2}\overline{A}_f \overline{\Gamma} t\right|^2$$

$$A_f = \left\langle f \middle| D^0 \right\rangle,\ \overline{A}_f = \left\langle f \middle| \overline{D}^0 \right\rangle$$

$$\frac{dN(\overline{D}^0 \to f)}{dt} \propto e^{-\overline{\Gamma}t} \left| \overline{A}_f + \frac{p}{q}\frac{ix+y}{2} A_f \overline{\Gamma} t\right|^2$$

Decay time distribution of experimentally accessible states D^0, $\overline{D}^0$ sensitive to mixing parameters x and y, depending on final state

Mixing of neutral mesons

Observables

(Charm-factories)

coherent production, $V(C=-1) \to D^0\overline{D}^0$

t-integrated rate

$$\Gamma(V \to D^0\overline{D}^0 \to f_1 f_2) = \frac{1}{2}|a_-|^2\left(\frac{1}{1-y^2}+\frac{1}{1+x^2}\right)+\frac{1}{2}|b_-|^2\left(\frac{1}{1-y^2}-\frac{1}{1+x^2}\right)$$

$$a_- = A_{f1}\overline{A}_{f2} - \overline{A}_{f1}A_{f2}; \quad b_- = \frac{p}{q}A_{f1}A_{f2} - \frac{q}{p}\overline{A}_{f1}\overline{A}_{f2}$$

Decay rate of experimentally accessible states D^0, $\overline{D}^0$ sensitive to mixing parameters x and y, depending on final state

Since they measure $$\frac{\Gamma(V \to D^0\overline{D}^0 \to f_1 X)}{\Gamma(V \to D^0\overline{D}^0 \to f_1 f_2)}$$

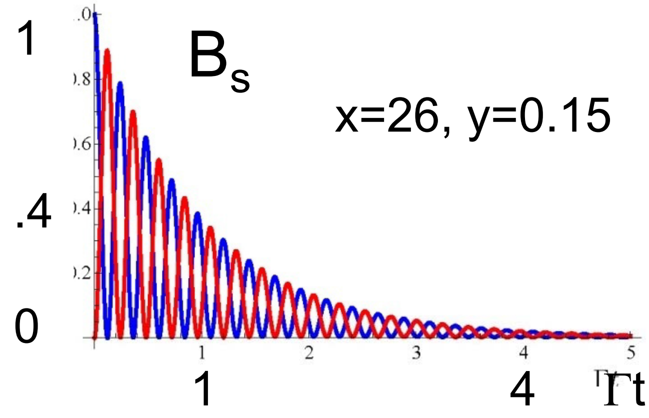
$\Rightarrow$ sensitive to y, x^2

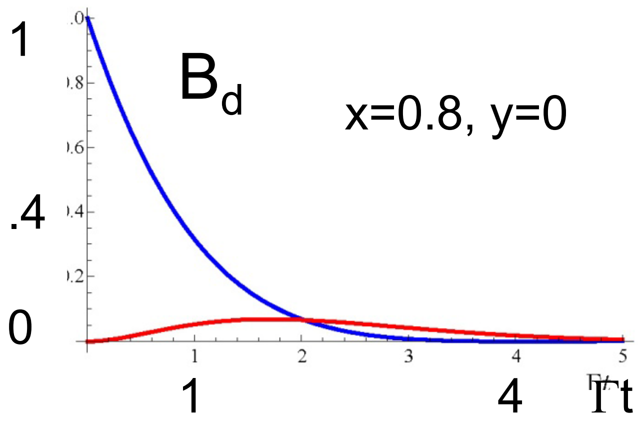
(see p. I/49-53)

Mixing of neutral mesons

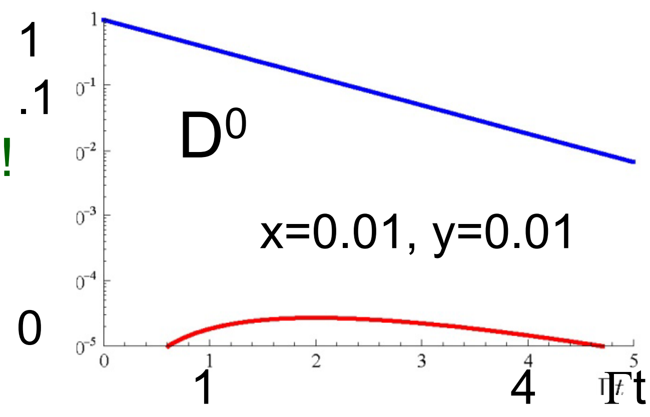
Observables

- blue: $P(P^0 \to P^0)$
- red: $P(P^0 \to \bar{P}^0)$





log scale!



Mixing measurements

Experimental methods

$$\frac{1}{e^{-t}}\frac{d\Gamma(D^0 \to f)}{dt} = \left| A_f + \frac{q}{p}\frac{ix+y}{2} A_{\bar{f}}\, t \right|^2$$

(for easier notation: $\Gamma t \to t$)

Common exp. features

tagging

(B-factories, hadron machines)

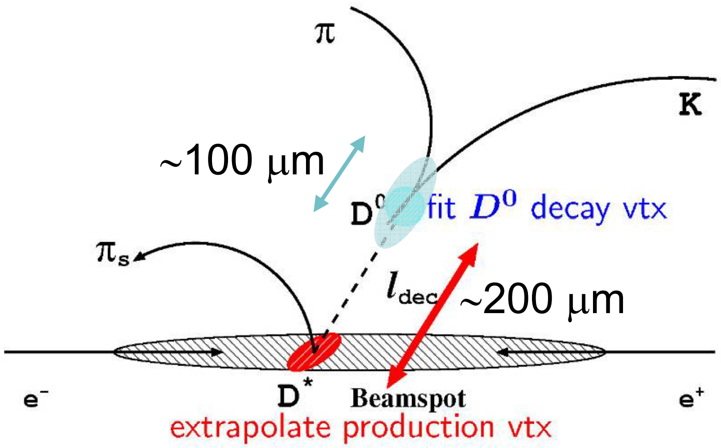
$D^{*+} \to D^0 \pi_s^+$

charge of $\pi_s \Rightarrow$ flavor of D^0;

$\Delta M = M(D^0\pi_s) - M(D^0)$

(or $q = \Delta M - m_\pi$) $\Rightarrow$

background reduction



decay time

(B-factories)

D^0 decay products vertex;

D^0 momentum & int. region;

$p^*(D^*) > 2.5$ GeV/c

eliminates D^0 from $b \to c$

Experimental methods

Decay modes

methods/precision/measured parameters depend on the decay mode

final states:	example, $D^0 \rightarrow$
semileptonic	$K^+ \ell \nu$
CP states	K^+K^-
WS hadronic 2-body states	$K^+\pi^-$
multi-body self conjugated states	$K_S\pi^-\pi^+$
and some decays which are a combination of those examples	$K^+\pi^-\pi^0$

(see p. I/46-48)

charm-factory:

(see p. I/49-53)

Mixing measurements

Semileptonic decays

Principle

$D^{*+} \rightarrow D^0 \; \pi_{slow}^+$

$D^0 \rightarrow K^{(*)-} e^+ \nu$ RS

$D^0 \rightarrow \overline{D}^0 \rightarrow K^{(*)+} e^- \overline{\nu}$ WS

t-integrated rates

$N_{WS}/N_{RS} = R_M = (x^2+y^2)/2$

Belle, PRD77, 112003 (2008), 492 fb^{-1}

Reconstruct ν:

$p_{miss} = p_{CMS} - p_{Ke\pi} - p_{rest}$

to improve resolution

$M(Ke\nu\pi_{slow}) \equiv M(D^{*+}),\; M^2(\nu) \equiv 0$

$f = K^- \ell^+ \nu$

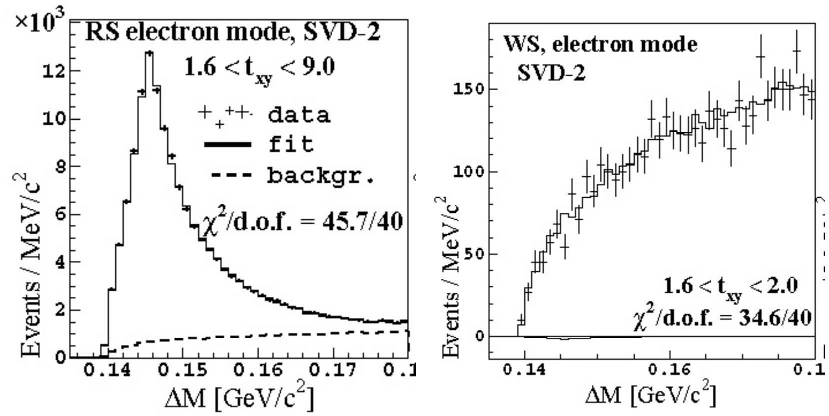
$$\overline{A}_f = A_{\bar{f}} = 0; \quad A_f = \overline{A}_{\bar{f}} \equiv A$$

RS $$\frac{\left|\left\langle f \middle| D^0(t) \right\rangle\right|^2}{e^{-t}} = |A|^2$$ derived from master formula on p. I/15

WS $$\frac{\left|\left\langle \bar{f} \middle| D^0(t) \right\rangle\right|^2}{e^{-t}} = |A|^2 \frac{x^2+y^2}{4} t^2$$

WS $$\frac{\left|\left\langle f \middle| \overline{D}^0(t) \right\rangle\right|^2}{e^{-t}} = |A|^2 \frac{x^2+y^2}{4} t^2$$

RS $$\frac{\left|\left\langle \bar{f} \middle| \overline{D}^0(t) \right\rangle\right|^2}{e^{-t}} = |A|^2$$



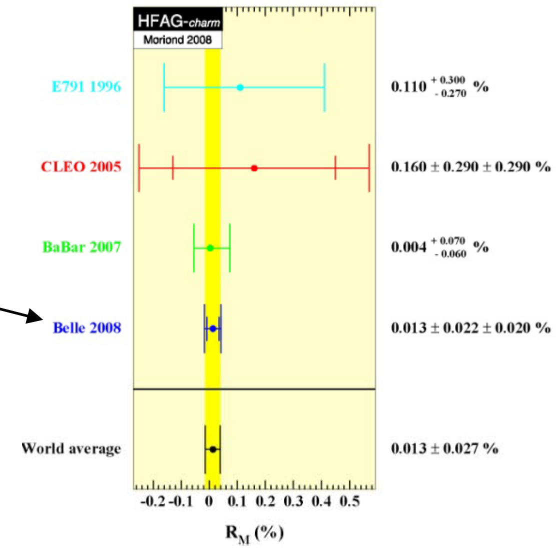
$N_{RS} \approx 330 \cdot 10^3$ $\quad$ $N_{RS} \approx 0$

Semileptonic decays

Results

$R_M=(1.3 \pm 2.2 \pm 2.0)\cdot 10^{-4}$
$R_M< 6.1\cdot 10^{-4}$ @ 90% C.L.

Belle, PRD77, 112003 (2008), 492 fb^{-1}



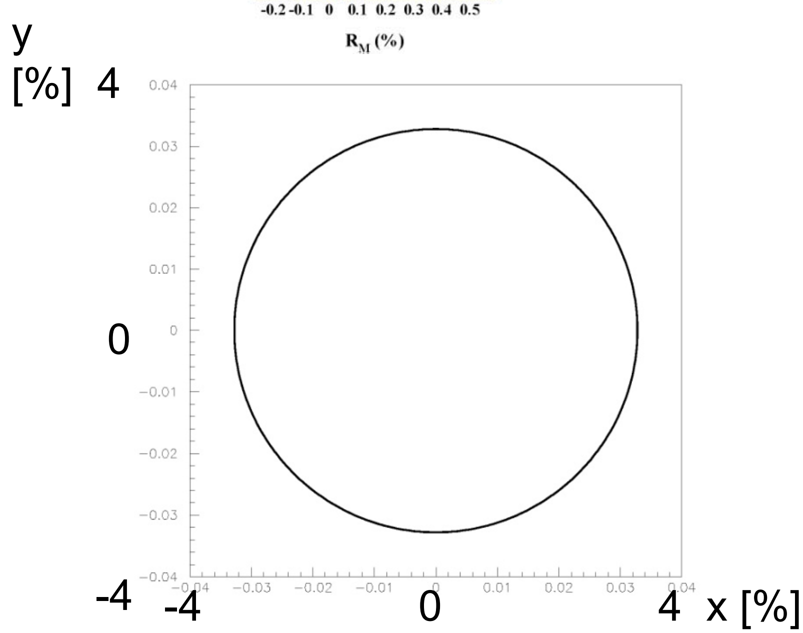
main syst.: WS bkg. Br's
WS bkg. ΔM shape

average of various measurements:
Heavy Flavor Averaging Group

HFAG, http://www.slac.stanford.edu/xorg/hfag/

$R_M=(1.3 \pm 2.7)\cdot 10^{-4}$

$R_M =(x^2+y^2)/2$



Decays to CP eigenstates

Principle

$D^0 \to K^+K^- / \pi^+\pi^-$

CP even final state;

if no CPV:

$CP|D_1\rangle = |D_1\rangle$

$|D_1\rangle$ is CP even state; only this component of $D^0/\bar{D}^0$ decays to $K^+K^- / \pi^+\pi^-$;

measuring lifetime in these decays $\Rightarrow \tau = 1/\Gamma_1$;

$D^0 \to K^- \pi^+$

$K^-\pi^+$: mixture of CP states $\Rightarrow$

$\tau = f(1/\Gamma_1, 1/\Gamma_2)$

$$f = \bar{f};\quad A_f = A_{\bar{f}};\quad \bar{A}_f = \bar{A}_{\bar{f}};\quad \left|\frac{A_f}{\bar{A}_{\bar{f}}}\right| = 1$$

$$\frac{\left|\left\langle f \middle| P^0(t)\right\rangle\right|^2}{e^{-t}} = \left|A_f\right|^2\left[1 - y\,t + \frac{x^2+y^2}{4}t^2\right]$$

$$\frac{\left|\left\langle f \middle| \bar{P}^0(t)\right\rangle\right|^2}{e^{-t}} = \left|A_f\right|^2\left[1 - y\,t + \frac{x^2+y^2}{4}t^2\right]$$

to linear order:

derived from master formula on p. I/15

$$\frac{\left|\left\langle f \middle| P^0(t)\right\rangle\right|^2}{e^{-t}} + \frac{\left|\left\langle f \middle| \bar{P}^0(t)\right\rangle\right|^2}{e^{-t}} = \left|A_f\right|^2\left[1 - y\,t\right]$$

$$\left|\left\langle f \middle| P^0(t)\right\rangle\right|^2 + \left|\left\langle f \middle| \bar{P}^0(t)\right\rangle\right|^2 \propto e^{-t}(1 - yt)$$

$$\approx e^{-t}e^{-yt} = e^{-(1+y)t}$$

when considering CPV expression is modified $\Rightarrow$ y in this mode called y_{CP}

Decays to CP eigenstates

Principle

$D^0 \rightarrow K^+K^- / \pi^+\pi^-$

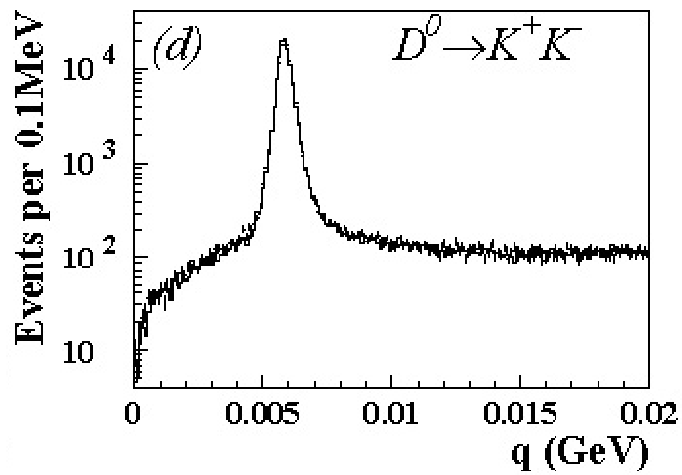
$$y_{CP} \equiv \frac{\tau(K^-\pi^+)}{\tau(K^-K^+)} - 1 \underset{no\ CPV}{=} y$$

Results

$M(K^+K^-)$,
$q=M(K^+K^-\pi_s)-M(K^+K^-)-M(\pi)$,
σ_t,
selection optimized on MC

	K^+K^-	$K^-\pi^+$	$\pi^+\pi^-$
N_{sig}	111×10^3	1.22×10^6	49×10^3
P	98%	99%	92%

Belle, PRL 98, 211803 (2007), 540fb^{-1}



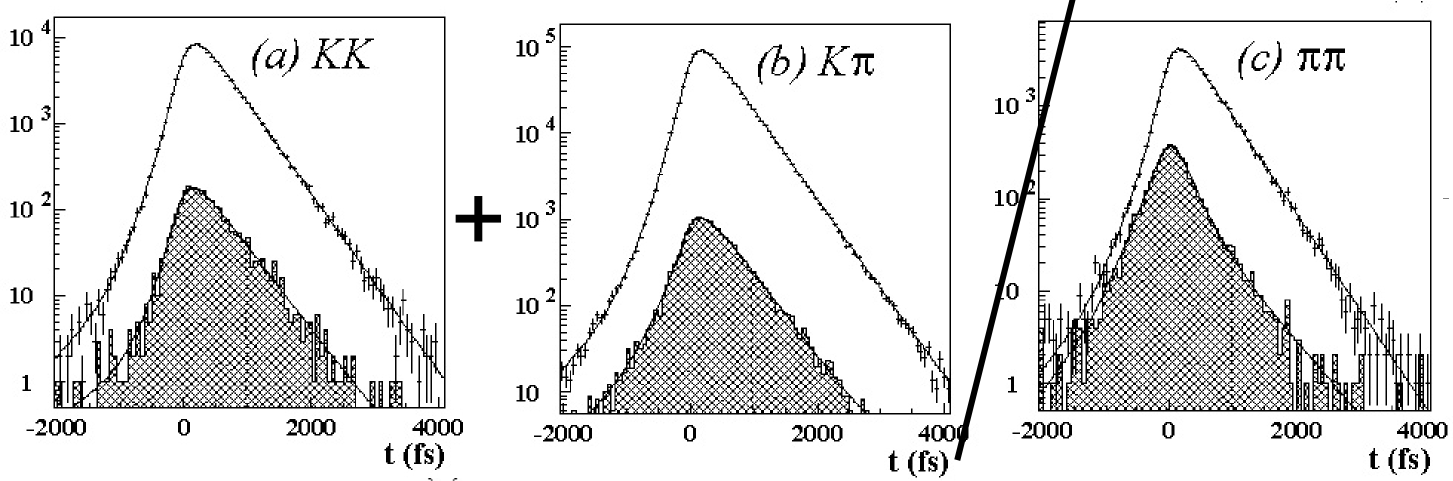
Belle, PRL 98, 211803 (2007), 540fb^{-1}

Decays to CP eigenstates

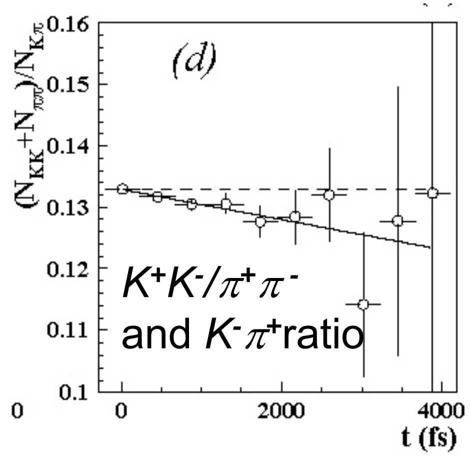
Results

$D^0 \to K^+K^- / \pi^+\pi^-$

simultaneous binned likelihood fit to decay-t, common free y_{CP}



χ^2/ndf= 1.084 (ndf=289)



$$y_{CP} = (1.31 \pm 0.32 \pm 0.25)\%$$

first evidence (one of ...) for D^0 mixing

dominant syst.: t acceptance linearity; small residual bias in τ;

Decays to CP eigenstates

Results

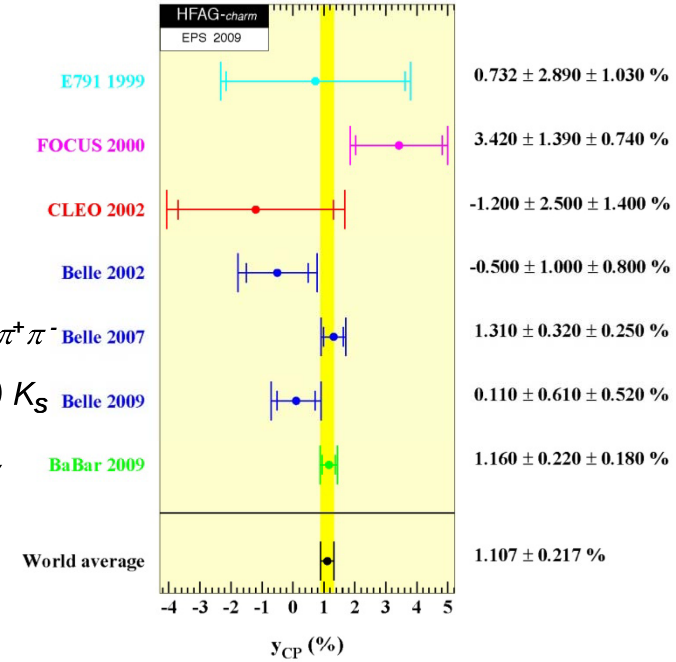
average y_{CP}

$$y_{CP} = (1.107 \pm 0.217)\%$$

HFAG, http://www.slac.stanford.edu/xorg/hfag/

$D^0 \to K^+K^- / \pi^+\pi^-$

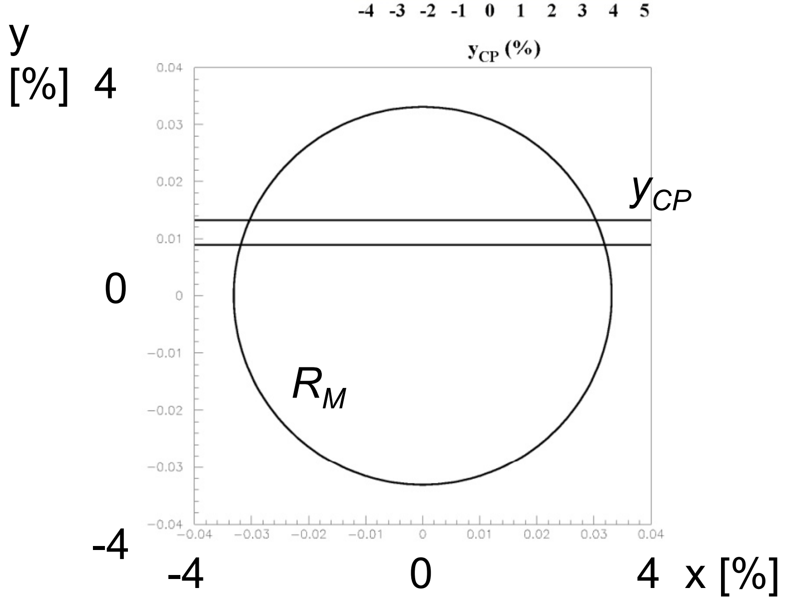
$D^0 \to (K^+K^-) K_S$



meas. can be performed with un-tagged (no $D^{*+} \to D^0\pi^+$) decays (larger stat., larger bkg.)

BaBar, PRD 80, 071103 (2009), 384fb^{-1}

$y_{CP} = y$



WS 2-body decays

Principle

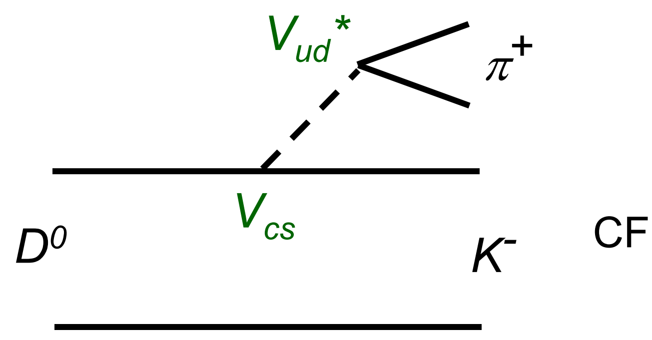
$D^{*+} \to D^0 \; \pi_{slow}^{\;+}$

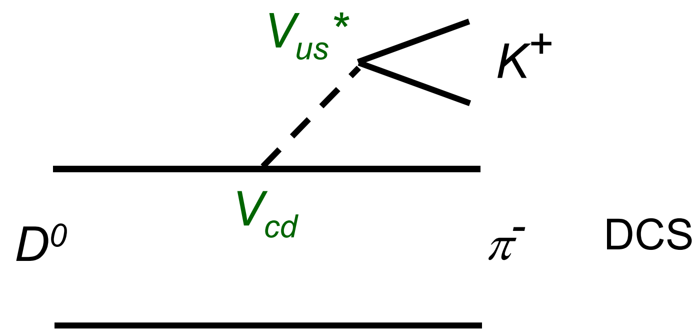
RS: $D^0 \to K^- \pi^+$

WS: $D^0 \to \overline{D}^0 \to K^+ \pi^-$

or

WS: $D^0 \to K^+ \pi^-$ (DCS)





interference between mixing and DCS for WS decays

$f=K^-\pi^+$

$$\frac{\overline{A}_f}{A_f} \equiv -\sqrt{R_D}\, e^{-i\delta}; \left|\frac{A_f}{\overline{A}_{\bar{f}}}\right| = 1$$

- sign due to relative sign of V_{us} and V_{cd}

$$\frac{\overline{A}_f}{A_f} \propto \frac{V_{cd}^* V_{us}}{V_{ud}^* V_{cs}} = \frac{-\lambda\,\lambda}{(1-\lambda^2)(1-\lambda^2)} = -\frac{\lambda^2}{(1-\lambda^2)^2}$$

WS 2-body decays

Principle

$D^{*+} \to D^0 \; \pi_{slow}^+$

RS: $D^0 \to K^- \pi^+$

WS: $D^0 \to \bar{D}^0 \to K^+ \pi^-$

t-dependence to separate DCS/mixed

δ: unknown strong phase DCS/CF;
not directly measurable at B-factories;
directly accesible at charm-factories

$$x' \equiv x\cos\delta + y\sin\delta; \quad y' \equiv y\cos\delta - x\sin\delta$$

$$y'' \equiv y\cos\delta + x\sin\delta$$

derived from master formula on p. I/15

$$\frac{\left|\langle f | P^0(t)\rangle\right|^2}{e^{-t}} = \left|A_f\right|^2 \left[1 - \sqrt{R_D}\, y'' t + R_D \frac{x^2+y^2}{4} t^2\right] \approx \left|A_f\right|^2$$

$$\frac{\left|\langle \bar{f} | \bar{P}^0(t)\rangle\right|^2}{e^{-t}} \approx \left|\bar{A}_{\bar{f}}\right|^2$$

$$\frac{\left|\langle \bar{f} | P^0(t)\rangle\right|^2}{e^{-t}} = \left|\bar{A}_{\bar{f}}\right|^2 \left[R_D + \sqrt{R_D}\, y' t + \frac{x^2+y^2}{4} t^2\right]$$

$$\frac{\left|\langle f | \bar{P}^0(t)\rangle\right|^2}{e^{-t}} = \left|A_f\right|^2 \left[R_D + \sqrt{R_D}\, y' t + \frac{x^2+y^2}{4} t^2\right]$$

$$\left|\langle K^+\pi^- | D^0(t)\rangle\right|^2 \propto \left[\underbrace{R_D}_{DCS} + \underbrace{\sqrt{R_D}\, y' t}_{interf.} + \underbrace{\frac{x'^2+y'^2}{4} t^2}_{mix}\right] e^{-t}$$

n.b.: $x'^2+y'^2 = x^2+y^2$

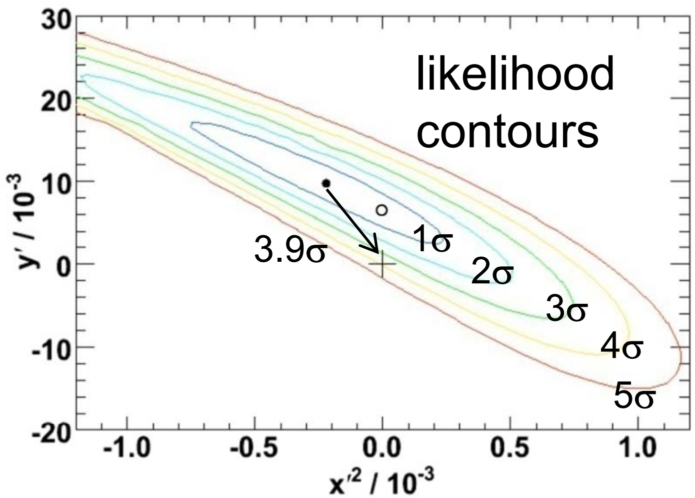
WS 2-body decays

Results

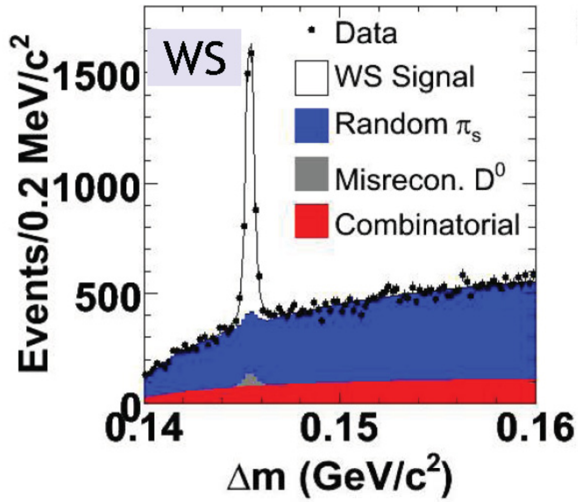
$D^0 \rightarrow K^+ \pi^-$

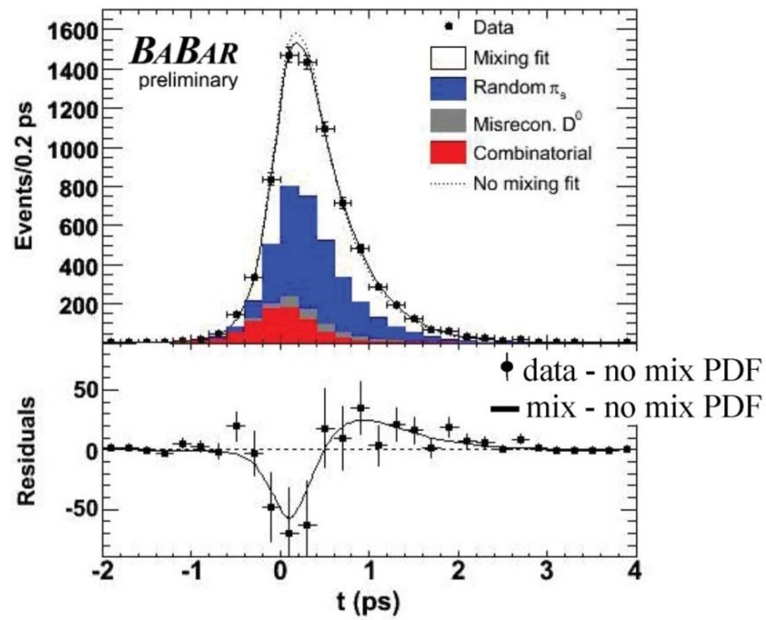
$R_D = (3.03 \pm 0.16 \pm 0.10) \cdot 10^{-3}$
$x'^2 = (-0.22 \pm 0.30 \pm 0.21) \cdot 10^{-3}$
$y' = (9.7 \pm 4.4 \pm 3.1) \cdot 10^{-3}$

BaBar, PRL 98, 211802 (2007), 384fb^{-1}



$N_{WS} \approx 4000$;
(note: P worse than for K^+K^-)





first evidence (2nd of ....) for D^0 mixing

Mixing measurements

WS 2-body decays

Results

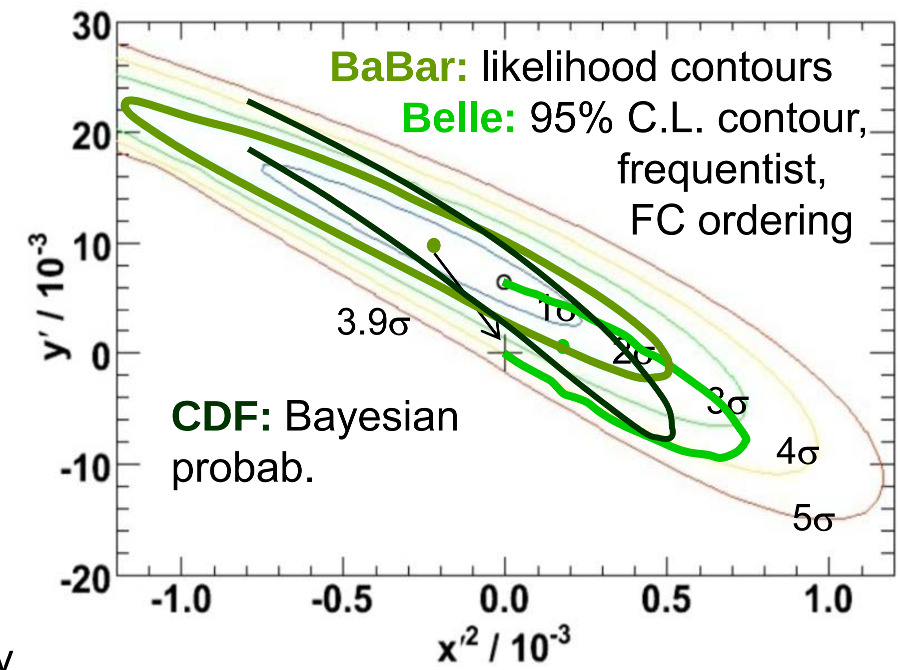
$D^0 \rightarrow K^+ \pi^-$

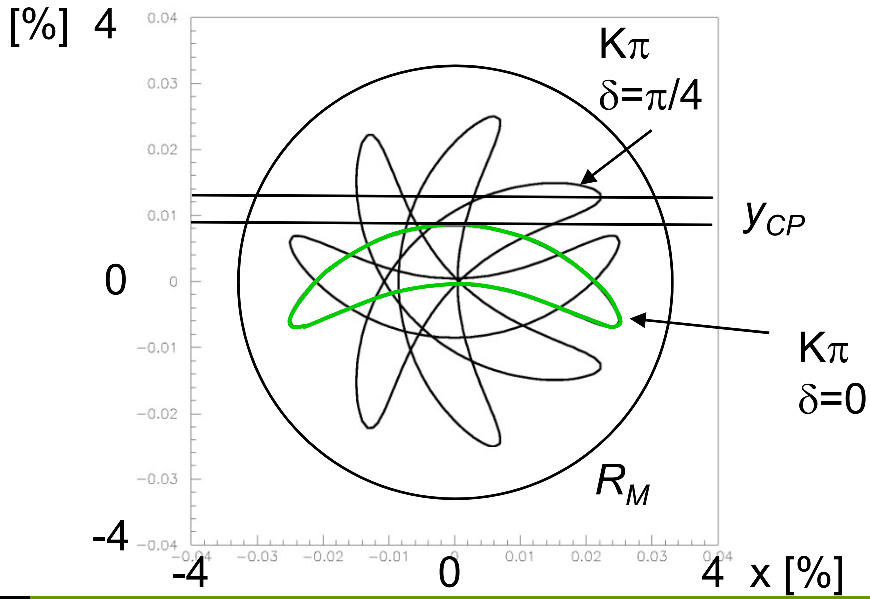
Belle, PRL 96, 151801 (2006), 400fb^{-1}

BaBar, PRL 98, 211802 (2007), 384fb^{-1}

CDF, PRL 100, 121802 (2008), 1.5fb^{-1}

$$x'^2 = (x \cos\delta + y \sin\delta)^2$$
$$y' = -x \sin\delta + y \cos\delta$$





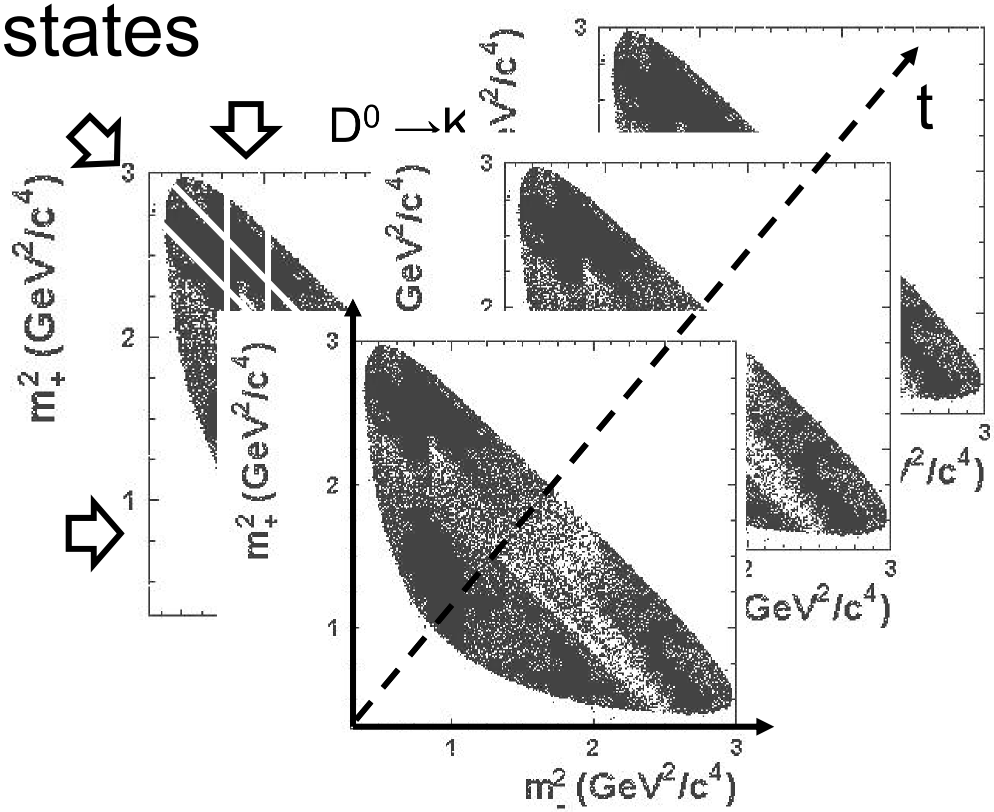
Multi-body self conjugated states

Principle

example $D^0 \to K_S \pi^+ \pi^-$
different types of interm. states;
CF: $D^0 \to K^{*-} \pi^+$
DCS: $D^0 \to K^{*+} \pi^-$
CP: $D^0 \to \rho^0 K_S$

if $f = \overline{f} \Rightarrow$ populate same Dalitz plot;
relative phases determined (unlike $D^0 \to K^+ \pi^-$);

specific regions of Dalitz plane →
specific admixture of interm. states →
specific t dependence $f(x, y)$;



by studying the decay time evolution of Dalitz plane → access directly x, y

"t-dependent Dalitz analyses"

Multi-body self conjugated states

Principle

example $D^0 \to K_S \pi^+ \pi^-$

t-dependent decay ampl.
depends on Dalitz variables
$m_\pm^2 = m^2(K_S\pi^\pm)$;
contains D^0 and $\overline{D}^0$ part
(due to mixing)
that propagate differently in time
$\lambda_{1,2}=f(x,y)$; see equations on p. I/6
(n.b.: $K^+\pi^-$: dependence on x'^2, y')

$$\mathcal{M}(m_-^2, m_+^2, t) \equiv \left\langle K_S \pi^+ \pi^- \middle| D^0(t) \right\rangle =$$

$$= \frac{1}{2} \underbrace{\mathcal{A}(m_-^2, m_+^2)}_{D^0 \to f} \left[e^{-i\lambda_1 t} + e^{-i\lambda_2 t} \right] +$$

$$+ \frac{1}{2} \underbrace{\overline{\mathcal{A}}(m_-^2, m_+^2)}_{\overline{D}^0 \to f} \left[e^{-i\lambda_1 t} - e^{-i\lambda_2 t} \right]$$

instantaneous amplitude:
sum of intermediate states

$$\mathcal{A}(m_-^2, m_+^2) =$$

$$= \sum a_r\, e^{i\Phi_r} B(m_-^2, m_+^2) + a_{NR}\, e^{i\Phi_{NR}}$$

$$\overline{\mathcal{A}}(m_-^2, m_+^2) =$$

$$= \sum a_r\, e^{i\Phi_r} B(m_+^2, m_-^2) + a_{NR}\, e^{i\Phi_{NR}}$$

Breit-Wigner

Multi-body self conjugated states

Belle, PRL 99, 131803 (2007), 540fb^{-1}

Results

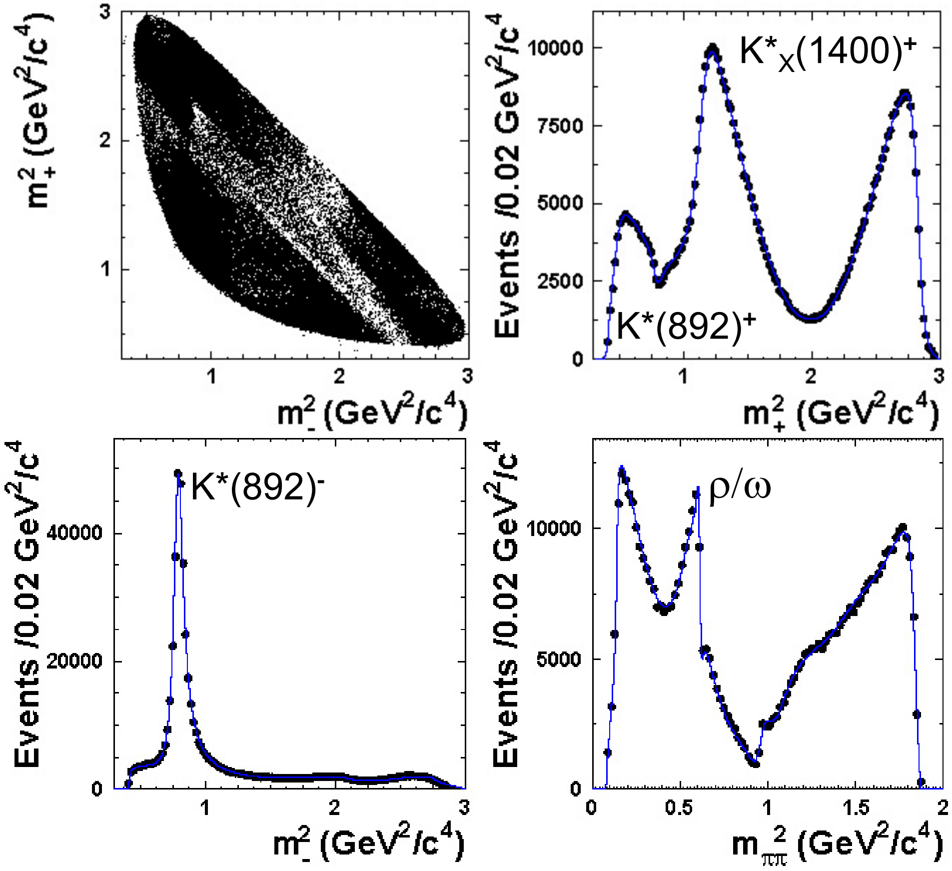
$D^0 \to K_S \pi^+ \pi^-$

$N_{sig} = 530 \cdot 10^3$
$P \approx 95\%$

3-dim fit (m_+^2, m_-^2, t);
complicated, ~40 free param.;
possibility of multiple solutions

usually fit to t-integrated Dalitz first to establish the appropriate model $(\mathcal{A}(m_+^2, m_-^2))$;

projection of fit in Dalitz plane

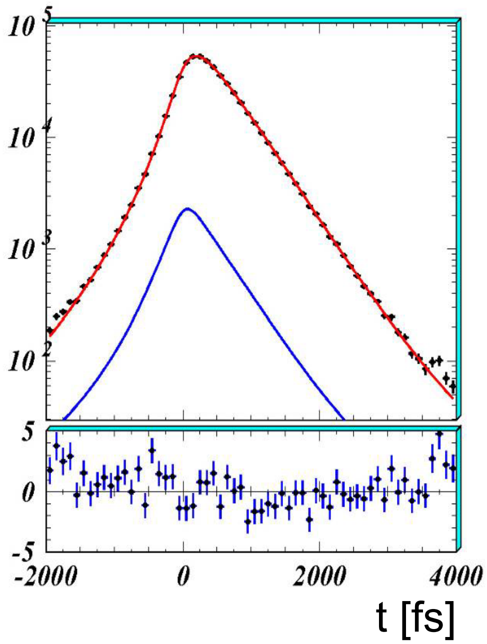


Multi-body self conjugated states

Belle, PRL 99, 131803 (2007), 540fb^{-1}

Results

$D^0 \rightarrow K_S \pi^+ \pi^-$



projection of fit in t distrib.

$$x = (0.80 \pm 0.29 \pm {}^{0.13}_{0.16})\%$$

$$y = (0.33 \pm 0.24 \pm {}^{0.10}_{0.14})\%$$

dominant syst.: model dependency (param. of resonances); Dalitz model for bkg.;

Results

other analogous modes: $D^0 \rightarrow K_S K^+ K^-$
$\pi^0 \pi^+ \pi^-$

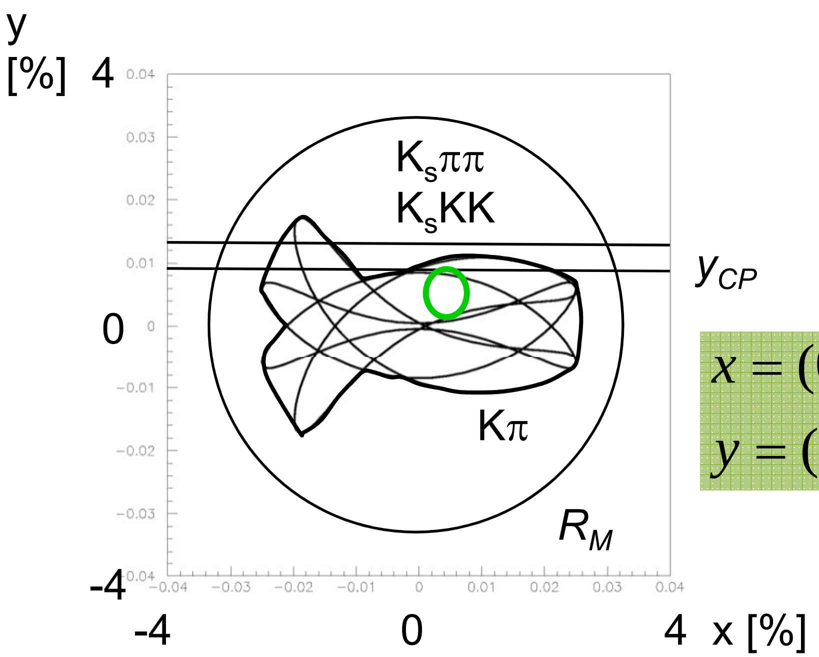
sensitivity to x, y depends on relative phases of interm. states (interference); difficult to predict

Multi-body self conjugated states

Results

$D^0 \rightarrow K_S \pi^+ \pi^- / K_S K^+ K^-$

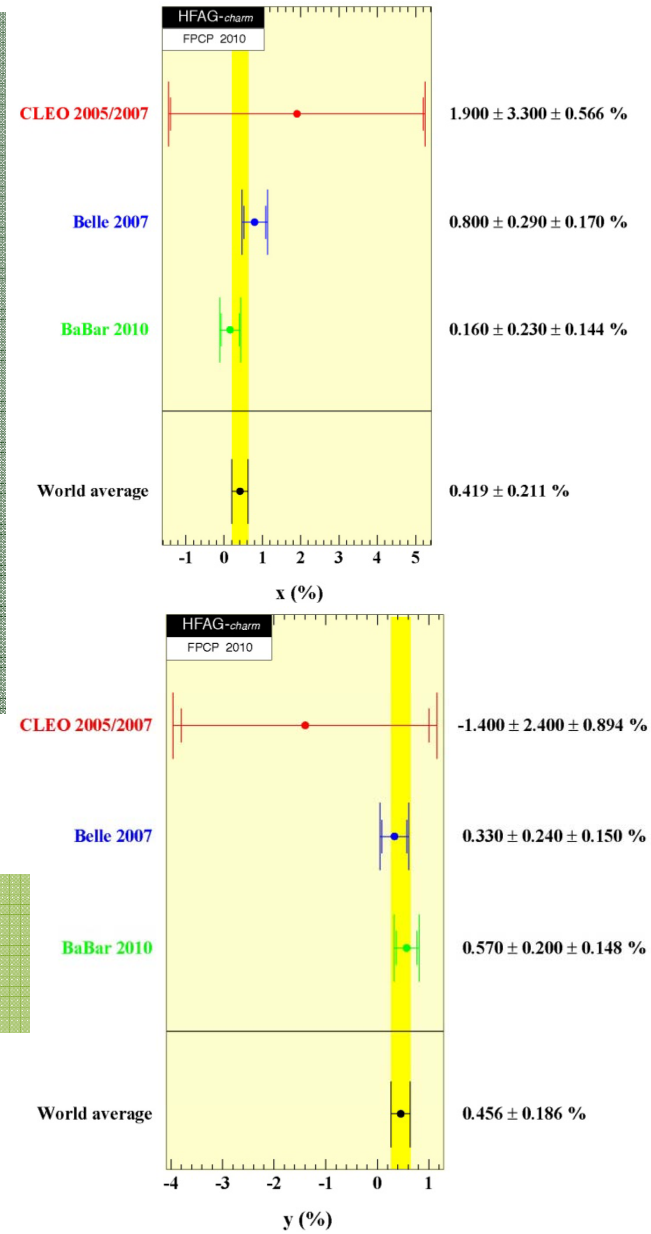
t-dependent Dalitz analyses:
most precise determination of
mixing parameters



$x = (0.42 \pm 0.21)\%$

$y = (0.46 \pm 0.19)\%$

HFAG, http://www.slac.stanford.edu/xorg/hfag/



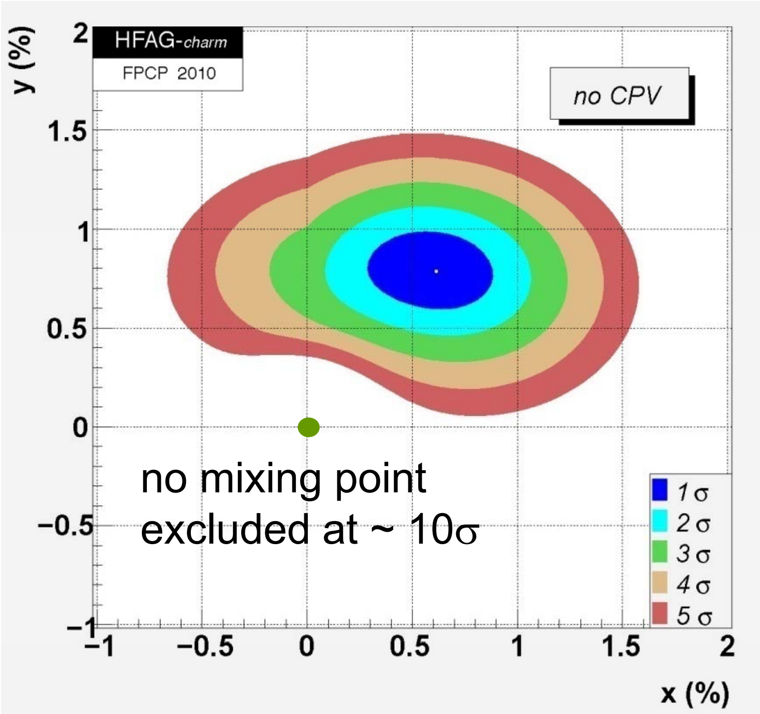
Averages

Results

HFAG, http://www.slac.stanford.edu/xorg/hfag/

χ^2 fit including correlations among measured quantities

Parameter	No CPV
x (%)	$0.61^{+0.19}_{-0.20}$
y (%)	0.79 ± 0.13
δ (°)	$26.6^{+11.2}_{-12.1}$
R_D (%)	$0.3317^{+0.0080}_{-0.0081}$
A_D (%)	–
$\|q/p\|$	–
ϕ (°)	–
$\delta_{K\pi\pi}$ (°)	$21.6^{+22.1}_{-23.2}$



n.b.: $x(D^0) \approx 0.01$; $x(K^0) \approx 1$; $x(B_d) \approx 0.8$; $x(B_s) \approx 25$;

$(x,y) \neq (0,0)$: 10 σ;

$x \propto m_1 - m_2$, $y \propto \Gamma_1 - \Gamma_2$; D_1: CP=+1;

x, y >0 $\Rightarrow$ CP even state heavier and shorter lived;

(unlike K^0 system)

Additional material

Introduction

Flavor physics

You always admire what you really don't understand.

B. Pascal (1623 - 1662)

Questions (to SM)

Why are we humans and not anti-humans?

Sakharov, CP violation;
CPV in SM small

Why are some large and some small?

Hierarchy, three generations

Why am I massive?

Origin of EW symmetry breaking;
beyond SM theories may explain,
but at what scale?
Precission needed

Charm physics

Charm is... a way of getting the answer yes without having asked any clear question.

A. Camus (1913 - 1960)

Dual role

- experimental tests of theor. predictions (most notably of (L)QCD); improve precision of CKM measurements (B physics);

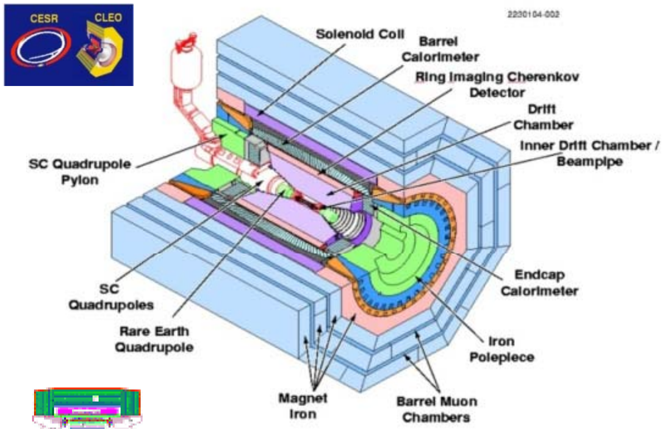
example: leptonic decays of D mesons → decay constants, tests of LQCD;

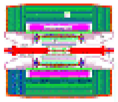
- standalone field of SM tests and searches for new phenomena (SM and/or NP);

example: mixing and CPV in D^0 system

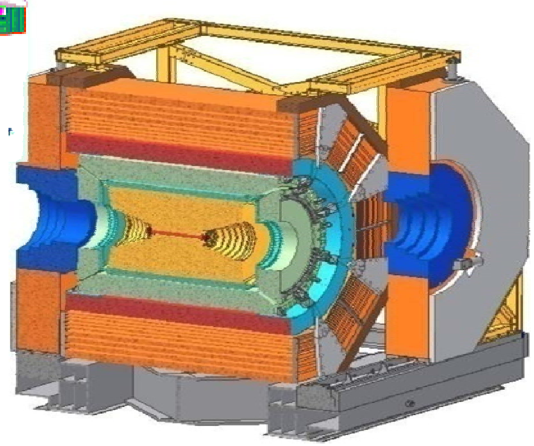
Experiments

Charm-Factories
Cleo-c @ CESR
Cornell





BESIII @ BEPC-II
IHEP



$e^+e^- \to \psi(3770) \to D^0\bar{D}^0, D^+D^-$

Cleo-c:
~800 pb^{-1} of data
available at $\psi(3770)$; 2.8×10^6 $D^0\bar{D}^0$

$N_{rec}(D^0 \to K^- \pi^+) \approx 150 \times 10^3$ (single tag)

BES-III:
~900 pb^{-1} of data (?)
available at $\psi(3770)$;

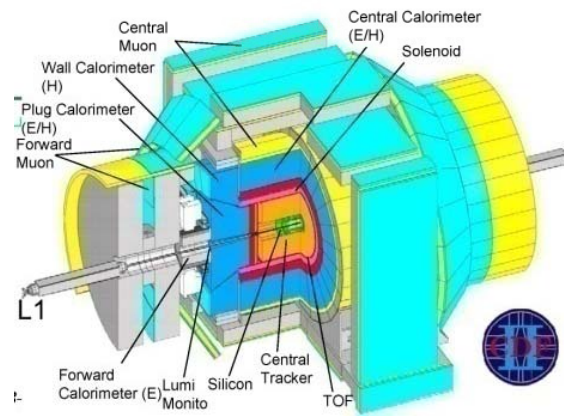
$D\bar{D}$ in coherent (C = -1) state

Experiments

$p\overset{(-)}{p}$ Colliders

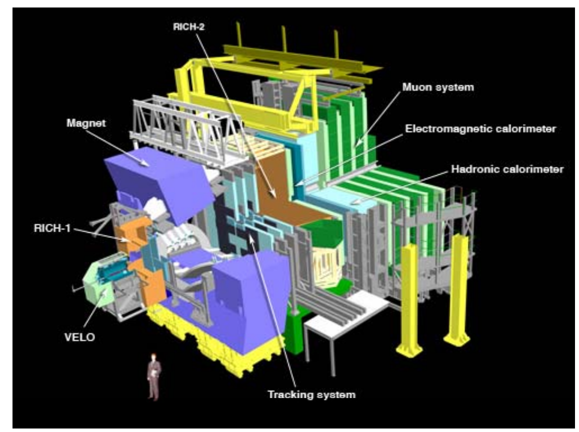
D0, CDF @ Tevatron
Fermilab

$\sim$ 6 fb^{-1} available
$N_{rec}(D^{*+} \to D^0 \pi^+ \to K^- \pi^+ \pi^+) \approx 7\times10^6$



LHCb @ LHC
CERN

For 2 fb^{-1} (currently 1 pb^{-1})
$N_{rec}(D^{*+} \to D^0 \pi^+ \to K^- \pi^+ \pi^+) \approx 15\times10^6$



diverse exp. conditions to study charm physics

We all live with the objective of being happy; our lives are all different and yet the same.

Anne Frank (1929 -1945)

huge statistics in more requiring exp. environment

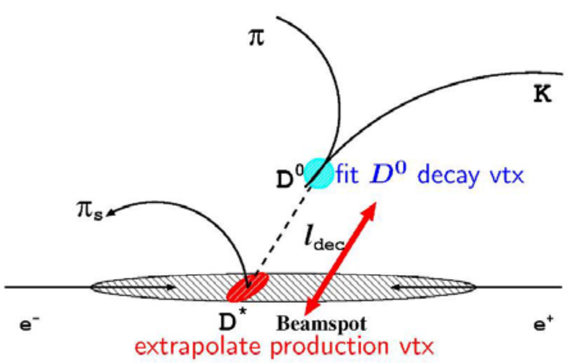
Experimental methods

B-factories

decay time

D^0 decay products vertex;
D^0 momentum & int. region;

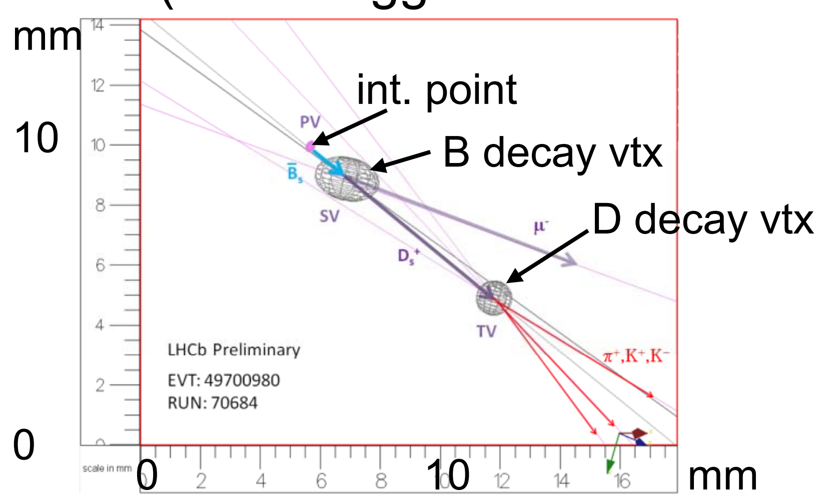
$p^*(D^*) > 2.5$ GeV/c
eliminates D^0 from $b \to c$



hadron machines

Tevatron: transverse decay length
LHCb: decay length between B ($B \to D^*X$) and D^0 vtx

Tevatron: impact param. distribution
LHCb: using D^0 from B
(better trigger ε and vtx resol.)



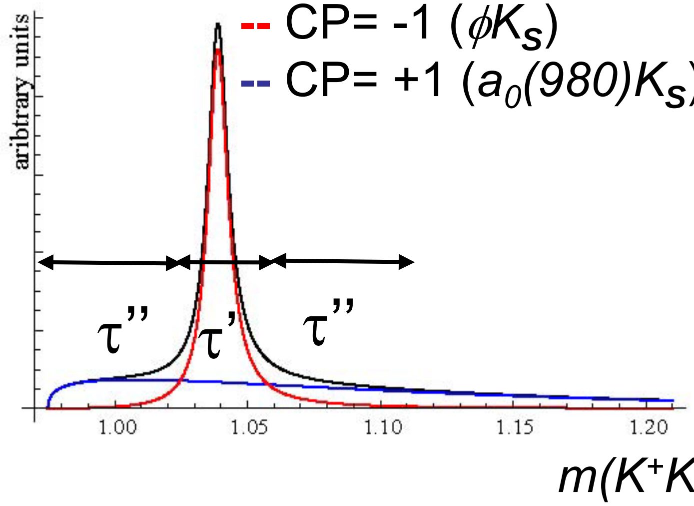
Decays to CP eigenstates

Principle

$D^0 \to (K^+K^-)\,K_S$
$(D^0 \to \phi\, K_S,\ a_0(980)\, K_S, ...)$
mixture of CP= ±1 states

$\tau(\phi K_S) = 1/\Gamma_2 > 1/\Gamma_1 = \tau(K^+K^-)$

$$\tau' = f_{CP=+1}\frac{\tau}{1+y_{CP}} + (1-f_{CP=+1})\frac{\tau}{1-y_{CP}}$$



$D^0 \to (K^+K^-)\,K_S$ is topologically different than $D^0 \to K^-\pi^+$;

small biases in the τ measurement would not cancel in the ratio $\tau(K^-\pi^+)/\tau(K^+K^-K_S)$

measure τ for $K^+K^-K_S$ only in different $m(K^+K^-)$ regions

$$\Delta\tau = \frac{\tau' - \tau''}{\tau' + \tau''} \approx y_{CP}\,(f'_{CP=+1} - f''_{CP=+1})$$

Decays to CP eigenstates

Results

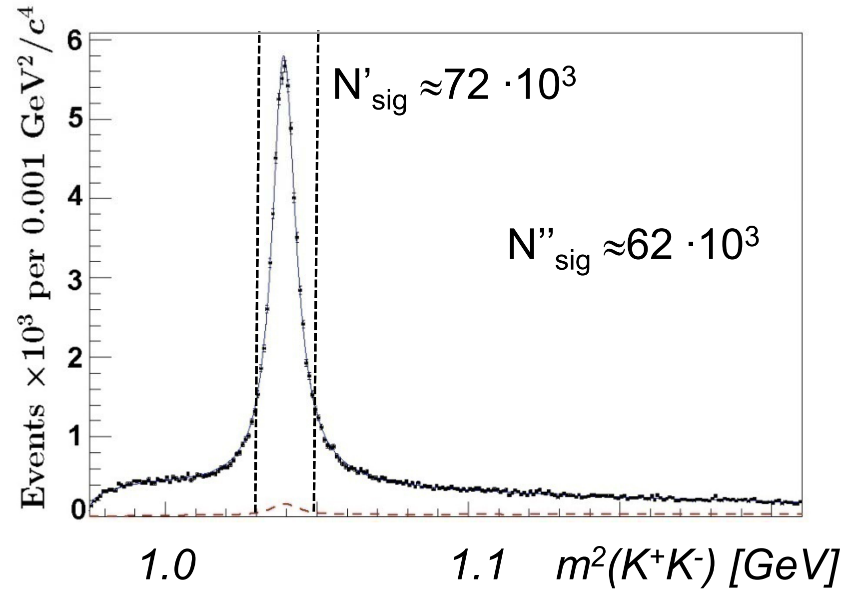
$D^0 \to (K^+K^-)\,K_S$

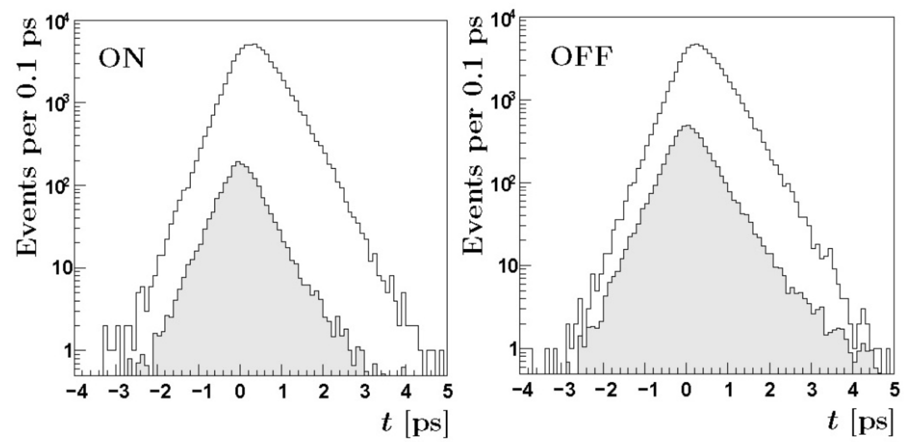
$(D^0 \to \phi K_S,\ a_0(980)\,K_S, \ldots)$

$$y_{CP} = (0.11 \pm 0.61 \pm 0.52)\%$$

Belle, PRD 80, 052006 (2009), 673fb^{-1}

main syst.: residual biases in τ





Mixing measurements

WS 2-body decays

Results

$D^0 \rightarrow K^+ \pi^-$

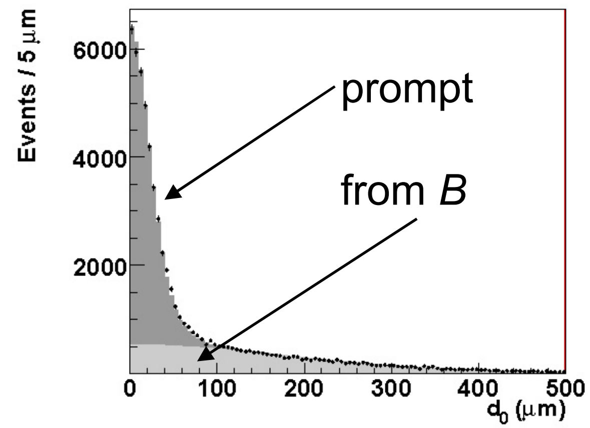
CDF: divide data into 20 *t* bins;

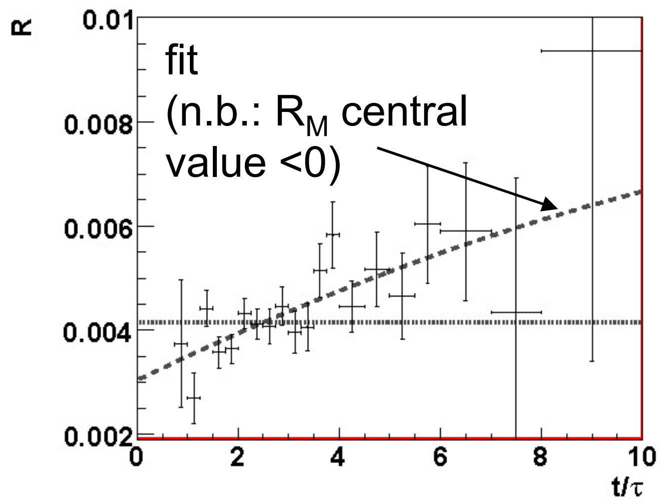
in each bin determine yield of prompt (not from *B*) RS and WS events, based on imp. parameter distr.;

plot WS/RS ratio in bins of *t*;

fit the distribution;

CDF, PRL 100, 121802 (2008), 1.5fb^{-1}





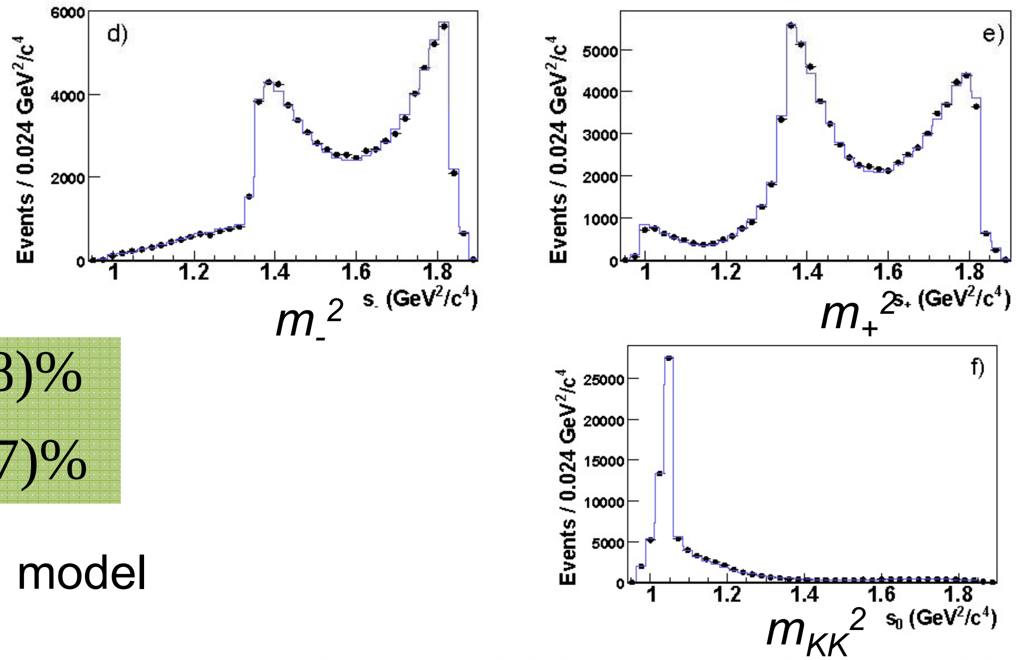
$$\frac{\left|\left\langle K^+\pi^- \middle| D^0(t)\right\rangle\right|^2}{\left|\left\langle K^-\pi^+ \middle| D^0(t)\right\rangle\right|^2} = R_D + \sqrt{R_D}\, y' t + \frac{x'^2 + y'^2}{4} t^2$$

Multi-body self conjugated states

BaBar, arXiv:1004.5053, 470 fb^{-1}

Results

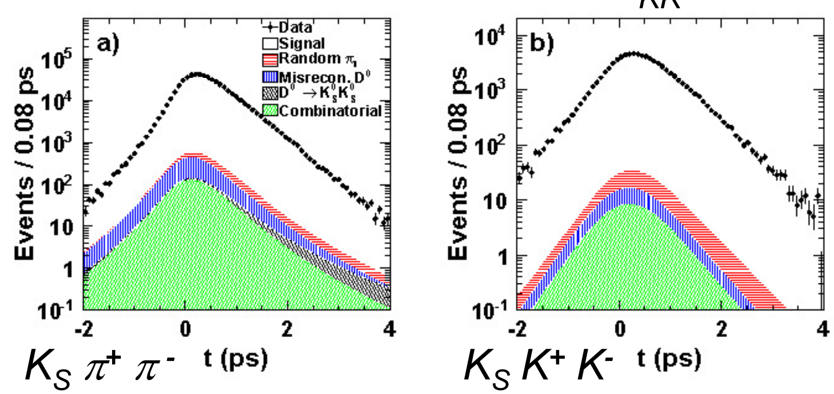
simultaneous:
$D^0 \rightarrow K_S \pi^+ \pi^- / K_S K^+ K^-$
profit from resonances that are present in both final states, e.g. $a_0(980)$



$$x = (0.16 \pm 0.23 \pm 0.12 \pm 0.08)\%$$

$$y = (0.57 \pm 0.20 \pm 0.13 \pm 0.07)\%$$

first error stat., second syst., third model



Multi-body flavor specific states

Principle

$D^0 \to K^+ \pi^- \pi^0$

properties: mixture of 2-body WS ($K\pi$) and t-dependent Dalitz ($K_S\pi\pi$);

WS: interference mixing/DCS; t-dependence similar as for $K\pi$;

WS and RS Dalitz distribution; in each relative phases determined; one unknown relative phase between chosen point in RS and WS Dalitz plane;

$f=K^- \pi^+ \pi^0$

$$\left|\left\langle K^+\pi^-\pi^0 \middle| D^0(t)\right\rangle\right|^2 \propto [\underbrace{|A_{\bar f}|^2}_{DCS} +$$

$$+\underbrace{|A_{\bar f}||\bar A_{\bar f}|(y'_{K\pi\pi}\cos\delta_f - x'_{K\pi\pi}\sin\delta_f)t}_{interf.} +$$

$$+\underbrace{|\bar A_{\bar f}|^2 \frac{x'^2_{K\pi\pi} + y'^2_{K\pi\pi}}{4} t^2}_{mix}]e^{-t}$$

$$x'_{K\pi\pi} = x\cos\delta_{K\pi\pi} + y\sin\delta_{K\pi\pi}$$
$$y'_{K\pi\pi} = y\cos\delta_{K\pi\pi} - x\sin\delta_{K\pi\pi}$$

$A_{\bar f}, \bar A_{\bar f}$ and $\delta_f = \bar\delta - \delta$ functions of $m_{K\pi}^2$, $m_{\pi\pi}^2$

$\delta_{K\pi\pi}$: unknown strong phase DCS/CF; not directly measurable at B-factories; directly accesible at charm-factories

Multi-body flavor specific states

Principle

$D^0 \rightarrow K^+ \pi^- \pi^0$

$A_{\bar{f}}, \bar{A}_{\bar{f}}, \delta_f$ determined from RS t-integrated Dalitz distrib.;

mixing parameters from WS t-dependent Dalit distrib.

$$\left|\langle K^+\pi^-\pi^0 | D^0(t)\rangle\right|^2 \propto [\underbrace{|A_{\bar{f}}|^2}_{DCS} + \underbrace{+|A_{\bar{f}}||\bar{A}_{\bar{f}}|(y'_{K\pi\pi}\cos\delta_f - x'_{K\pi\pi}\sin\delta_f)t}_{interf.} + \underbrace{+|\bar{A}_{\bar{f}}|^2 \frac{x'^2_{K\pi\pi} + y'^2_{K\pi\pi}}{4} t^2}_{mix}]e^{-t}$$

Results

$x'_{K\pi\pi} = (2.61\ ^{+0.57}_{-0.68} \pm 0.39)\%$
$y'_{K\pi\pi} = (-0.06\ ^{+0.55}_{-0.64} \pm 0.34)\%$

BaBar, PRL 103, 211801 (2009), 384 fb^{-1}

Resonance	a_j^{DCS}	δ_j^{DCS} (degrees)	f_j (%)
$\rho(770)$	1 (fixed)	0 (fixed)	39.8 ± 6.5
$K_2^{*0}(1430)$	0.088 ± 0.017	-17.2 ± 12.9	2.0 ± 0.7
$K_0^{*+}(1430)$	6.78 ± 1.00	69.1 ± 10.9	13.1 ± 3.3
$K^{*+}(892)$	0.899 ± 0.005	-171.0 ± 5.9	35.6 ± 5.5
$K_0^{*0}(1430)$	1.65 ± 0.59	-44.4 ± 18.5	2.8 ± 1.5
$K^{*0}(892)$	0.398 ± 0.038	24.1 ± 9.8	6.5 ± 1.4
$\rho(1700)$	5.4 ± 1.6	157.4 ± 20.3	2.0 ± 1.1

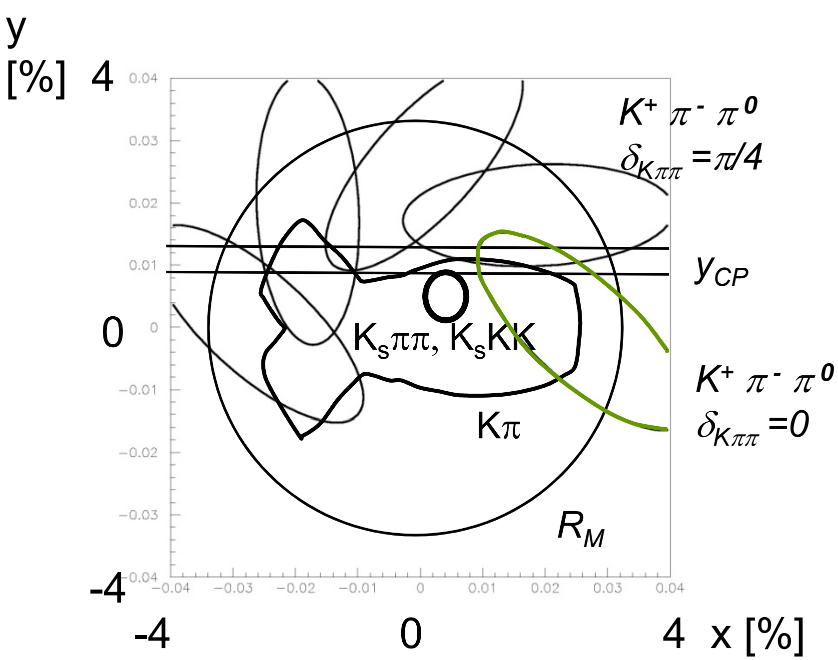
results of Dalitz fit for WS decays

Multi-body flavor specific states

Results

$D^0 \rightarrow K^+ \pi^- \pi^0$

$x'_{K\pi\pi} = x \cos\delta_{K\pi\pi} + y \sin\delta_{K\pi\pi}$
$y'_{K\pi\pi} = -x \sin\delta_{K\pi\pi} + y \cos\delta_{K\pi\pi}$



Charm-factories

derived from master formula on p. I/15

$$\Gamma(V \to D^0\overline{D}^0 \to f_1 f_2) =$$

$$= \frac{1}{2}|a_-|^2\left(\frac{1}{1-y^2}+\frac{1}{1+x^2}\right)+\frac{1}{2}|b_-|^2\left(\frac{1}{1-y^2}-\frac{1}{1+x^2}\right)$$

Principle

coherence of $D^0\overline{D}^0$ pair affects t-integrated rates;
example: $f_1 = K^-\pi^+$, $f_2 = e^-X$

$$A_{f1} \equiv A,\ \overline{A}_{f2} \equiv A_e;\quad \overline{A}_{f1} = -\sqrt{R_D}e^{-i\delta}A,\ A_{f2} = 0$$

$$a_- = AA_e;\quad b_- = \frac{q}{p}\sqrt{R_D}e^{-i\delta}AA_e \approx \sqrt{R_D}e^{-i\delta}AA_e$$

$$\Gamma(V \to D^0\overline{D}^0 \to K^-\pi^+, e^-X) =$$

$$= \frac{1}{2}|AA_e|^2\left\{2+x^2(1+R_D)-y^2(1-R_D)\right\}$$

for $D^0 \to f_1$ and $\overline{D}^0 \to f_2$ ("double tagged", DT events);
sensititiviy to x, y is in 2nd order only

$$\Gamma(V \to D^0\overline{D}^0 \to K^-\pi^+, e^-X) = |AA_e|^2\left\{1+\frac{x^2-y^2}{2}\right\}$$

Mixing measurements

Charm-factories

Principle

one can also reconstruct only single final state, e.g. $K^-\pi^+$ ("single tagged", ST events);

each event contains D^0 and $\overline{D}^0$, inclusive singe tag rate equals the rate of non-coherent decays;

sensitivity of ST events to $\sqrt{(R_D)}\, y \cos\delta$ is in 1st order;

DT/ST ratio
(ST provides sensitivity to mixing parameters, DT normalization)

f= $K^-\pi^+$

derived from master formula on p. I/15

$$\frac{1}{2e^{-t}}\left(\left|\langle f|P^0(t)\rangle\right|^2+\left|\langle f|\overline{P}^0(t)\rangle\right|^2\right)\approx$$

$$\approx\frac{1}{2}|A|^2\left[1+\sqrt{R_D}\,y''t\right]+\frac{1}{2}|A|^2\left[R_D+\sqrt{R_D}\,y't\right]$$

$$\Gamma(V\to D^0\overline{D}^0\to K^-\pi^+X)=$$

$$=\frac{1}{2}\int\left[\left|\langle f|P^0(t)\rangle\right|^2+\left|\langle f|\overline{P}^0(t)\rangle\right|^2\right]dt\approx$$

$$=|A|^2\left[1+R_D+2\sqrt{R_D}\,y\cos\delta\right]$$

f= e^-X

$$\frac{1}{2e^{-t}}\left(\left|\langle f|P^0(t)\rangle\right|^2+\left|\langle f|\overline{P}^0(t)\rangle\right|^2\right)=|A_e|^2$$

$$\frac{\Gamma(K^-\pi^+,e^-X)}{\Gamma(K^-\pi^+)\Gamma(e^-X)}\approx1-R_D-2\sqrt{R_D}\,y\cos\delta$$

Charm-factories

Principle

various decay modes,
effective rates;
$S_\pm$: CP= ±1 eigenstate
e^- : semileptonic state
r : $\sqrt{R_D}$

Cleo, PRD 78, 012001 (2008), 281pb^{-1}

Mode	Correlated
$K^-\pi^+$	$1+R_{\rm WS}$
S_+	2
S_-	2
$K^-\pi^+, K^-\pi^+$	$R_{\rm M}$
$K^-\pi^+, K^+\pi^-$	$(1+R_{\rm WS})^2 - 4r\cos\delta(r\cos\delta+y)$
$K^-\pi^+, S_+$	$1+R_{\rm WS}+2r\cos\delta+y$
$K^-\pi^+, S_-$	$1+R_{\rm WS}-2r\cos\delta-y$
$K^-\pi^+, e^-$	$1-ry\cos\delta-rx\sin\delta$
S_+, S_+	0
S_-, S_-	0
S_+, S_-	4
S_+, e^-	$1+y$
S_-, e^-	$1-y$

ST; $\dfrac{\Gamma(f)}{\Gamma^{uncorr}(f)}$

DT; $\dfrac{\Gamma(f_1,f_2)}{\Gamma^{uncorr}(f_1)\,\Gamma^{uncorr}(f_2)}$

Charm-factories

Principle

carefull when reading the table:

$\rightarrow \dfrac{\Gamma(K^-\pi^+)}{\Gamma^{uncorr}(K^-\pi^+)}$ *1+R_WS* $1+R_{WS}$

$\rightarrow \dfrac{\Gamma(\Gamma(K^-\pi^+, e^-X)}{\Gamma^{uncorr}(K^-\pi^+)\,\Gamma^{uncorr}(e^-X)}$ $1-\sqrt{R_D}\, x\sin\delta - \sqrt{R_D}\, y\cos\delta$

ST rates

$$\Gamma(V \to D^0\overline{D}^0 \to K^-\pi^+ X) = $$
$$= \frac{1}{2}\left(\Gamma(D^0 \to K^-\pi^+) + \Gamma(\overline{D}^0 \to K^-\pi^+)\right)$$

uncorrelated

$$\Gamma^{uncorr}(D^0 \to K^-\pi^+) = $$
$$= \frac{1}{2}\left(\Gamma(D^0 \to K^-\pi^+) + \Gamma(\overline{D}^0 \to K^+\pi^-)\right)$$

$$\rightarrow \Gamma(V \to D^0\overline{D}^0 \to K^-\pi^+ X) = (1+R_{WS})\Gamma^{uncorr}(D^0 \to K^-\pi^+) =$$
$$= (1+R_{WS})\frac{1}{2}\left(\Gamma(D^0 \to K^-\pi^+) + \Gamma(\overline{D}^0 \to K^+\pi^-)\right) =$$
$$= \underbrace{(1 + R_D + \sqrt{R_D}\,y' + R_M)}_{1+R_{WS}}|A|^2(1+\sqrt{R_D}\,y'') \approx |A|^2\left[1 + R_D + 2\sqrt{R_D}\,y\cos\delta\right]$$

$$\rightarrow \frac{\Gamma(V \to D^0\overline{D}^0 \to K^-\pi^+, e^-X)}{\Gamma^{uncorr}(D^0 \to K^-\pi^+)\Gamma^{uncorr}(D^0 \to e^-X)} = \frac{|AA_e|^2\left(1+(x^2-y^2)/2\right)}{|A|^2\left(1+\sqrt{R_D}\,y''\right)|A_e|^2} \approx$$
$$\approx 1 - \sqrt{R_D}\,y'' = 1 - \sqrt{R_D}\,y\cos\delta - \sqrt{R_D}\,x\sin\delta$$

derived from equations on p. I/49, 49

Mixing measurements

Cleo, PRD 78, 012001 (2008), 281pb^{-1}

Charm-factories

Results

examples of DT, $\overline{M} = (M_{f1}+M_{f2})/2$

$N_{ST}(K^-\pi^+ + K^+\pi^-) \sim 51\cdot 10^3$
$N_{DT}(K^-\pi^+, K^+\pi^-) \sim 600$

fit to several measured
ST and DT rates

$y=(-5.207 \pm 5.571 \pm 2.737)\%$
$\sqrt{(R_D)}\cos\delta = (8.878 \pm 3.369 \pm 1.579)\%$

using WA value of R_D:
$\cos\delta = (1.54 \pm 0.65)$;
naively: $\delta \in [0^o, 27^o]$ & $[153^o, 180^o]$