

```
Clear[k, δ, dn, N1, N2, θ1, D0, D1, D2, a1, a2, d];
```

```
Integrate[N1/a1, {x, 0, d}] + Integrate[(N1/a1 + N2/a2), {x, d, a1}] +
```

$$\frac{\text{Integrate}[N2/a2, \{x, a1, d+a2\}]}{a1} + \frac{(-a1 + a2 + d) N2}{a2} + \frac{(a1 - d) (a2 N1 + a1 N2)}{a1 a2}$$

```
Simplify[%]
```

$$N1 + N2$$

```
(Integrate[x*N1/a1, {x, 0, d}] + Integrate[x*(N1/a1 + N2/a2), {x, d, a1}]) +
```

$$\frac{\text{Integrate}[x*N2/a2, \{x, a1, d+a2\}]}{N1 + N2}$$

$$\frac{\frac{d^2 N1}{2 a1} + \frac{(-a1^2 + (a2+d)^2) N2}{2 a2} + \frac{(a1^2 - d^2) (a2 N1 + a1 N2)}{2 a1 a2}}{N1 + N2}$$

```
Simplify[%]
```

$$\frac{a1 N1 + (a2 + 2 d) N2}{2 (N1 + N2)}$$

```
(Integrate[x*x*N1/a1, {x, 0, d}] + Integrate[x*x*(N1/a1 + N2/a2), {x, d, a1}]) +
```

$$\frac{\text{Integrate}[x*x*N2/a2, \{x, a1, d+a2\}]}{N1 + N2}$$

$$\frac{\frac{d^3 N1}{3 a1} + \frac{(-a1^3 + (a2+d)^3) N2}{3 a2} + \frac{(a1^3 - d^3) (a2 N1 + a1 N2)}{3 a1 a2}}{N1 + N2}$$

```
Simplify[%]
```

$$\frac{a1^2 N1 + (a2^2 + 3 a2 d + 3 d^2) N2}{3 (N1 + N2)} - \left( \frac{a1 N1 + (a2 + 2 d) N2}{2 (N1 + N2)} \right)^2$$

$$- \frac{(a1 N1 + (a2 + 2 d) N2)^2}{4 (N1 + N2)^2} + \frac{a1^2 N1 + (a2^2 + 3 a2 d + 3 d^2) N2}{3 (N1 + N2)}$$

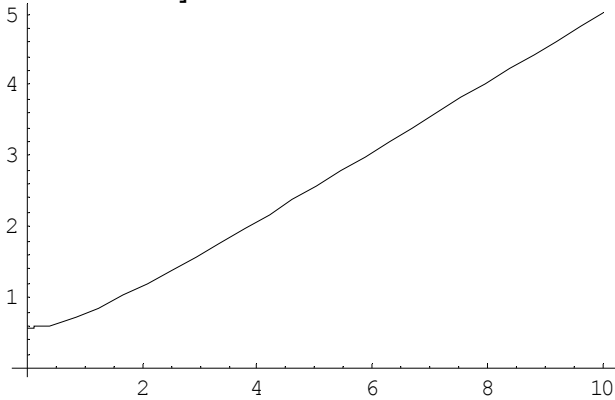
```
Simplify[%]
```

$$\frac{1}{12 (N1 + N2)^2} (-3 (a1 N1 + (a2 + 2 d) N2)^2 + 4 (N1 + N2) (a1^2 N1 + (a2^2 + 3 a2 d + 3 d^2) N2))$$

**N2 = 5; N1 = N2; a1 = 2; a2 = a1;**

**Plot[**

$$\text{Sqrt}\left[\frac{1}{12 (N1 + N2)^2} (-3 (a1 N1 + (a2 + 2 d) N2)^2 + 4 (N1 + N2) (a1^2 N1 + (a2^2 + 3 a2 d + 3 d^2) N2))\right], \{d, 0, 10\}$$



**-Graphics-**

**Clear[N1, N2, a1, a2, d];**

**a1 = D1 \* Tan[θ1]; a2 = D2 \* Tan[θ1 - δ];**

**d = L \* Tan[θ1] - (L + D1) \* Tan[θ1 - δ]; D1 = k \* D0;**

**D2 = D0 - D1;**

$$\text{Sqrt}\left[\frac{1}{12 (N1 + N2)^2}$$

$$(-3 (a1 N1 + (a2 + 2 d) N2)^2 +$$

$$4 (N1 + N2) (a1^2 N1 + (a2^2 + 3 a2 d + 3 d^2) N2))]$$

$$\frac{1}{2\sqrt{3}} \left( \sqrt{\left( \frac{1}{(N1 + N2)^2} \right. \right.$$

$$(-3 (D0 k N1 \text{Tan}[\theta1] + N2 (- (D0 - D0 k) \text{Tan}[\delta - \theta1] + 2 ((20 + D0 k) \text{Tan}[\delta - \theta1] + 20 \text{Tan}[\theta1])))^2 +$$

$$4 (N1 + N2) (D0^2 k^2 N1 \text{Tan}[\theta1]^2 + N2$$

$$((D0 - D0 k)^2 \text{Tan}[\delta - \theta1]^2 - 3 (D0 - D0 k) \text{Tan}[\delta - \theta1]$$

$$((20 + D0 k) \text{Tan}[\delta - \theta1] + 20 \text{Tan}[\theta1]) +$$

$$3 ((20 + D0 k) \text{Tan}[\delta - \theta1] + 20 \text{Tan}[\theta1])^2)) \Bigg)$$

```

Clear[N1, N2, a1, a2, d, θ1, δ, D1, D2, D0, L, k, dn];
a1 = D1 * Tan[θ1]; a2 = D2 * Tan[θ1 - δ];
d = L * Tan[θ1] - (L + D1) * Tan[θ1 - δ]; D1 = k * D0;
D2 = D0 - D1; N1 = 50 * D1 * (Sin[θ1]) ^ 2 * Exp[-D1 / 2 / Lam1];
N2 = 50 * D2 * (Sin[θ1 - δ]) ^ 2 * Exp[-D2 / 2 / Lam2 - D1 / Lam1];
Lam1 = 3; Lam2 = 3;
Sqrt[
  1
  / (12 (N1 + N2) ^ 2
    (-3 (a1 N1 + (a2 + 2 d) N2) ^ 2 +
      4 (N1 + N2) (a1 ^ 2 N1 + (a2 ^ 2 + 3 a2 d + 3 d ^ 2) N2))]
  / (2 * Sqrt[3]
    (
      Sqrt[
        (
          -3 (
            50 D0 ^ 2 e ^ (-D0 k / 60) k ^ 2 Sin[θ1] ^ 2 Tan[θ1] +
            50 e ^ (-D0 k / 30 + 1 / 6 (-D0 + D0 k)) (D0 - D0 k)
              Sin[δ - θ1] ^ 2 (- (D0 - D0 k) Tan[δ - θ1] +
                2 ((D0 k + L) Tan[δ - θ1] + L Tan[θ1]))
            ) ^ 2
          +
          4 (
            50 e ^ (-D0 k / 30 + 1 / 6 (-D0 + D0 k)) (D0 - D0 k) Sin[δ - θ1] ^ 2 +
            50 D0 e ^ (-D0 k / 60) k Sin[θ1] ^ 2
          )
          (
            50 D0 ^ 3 e ^ (-D0 k / 60) k ^ 3 Sin[θ1] ^ 2 Tan[θ1] ^ 2 +
            50 e ^ (-D0 k / 30 + 1 / 6 (-D0 + D0 k)) (D0 - D0 k) Sin[δ - θ1] ^ 2
              ((D0 - D0 k) ^ 2 Tan[δ - θ1] ^ 2 - 3 (D0 - D0 k) Tan[
                δ - θ1] ((D0 k + L) Tan[δ - θ1] + L Tan[θ1]) +
                3 ((D0 k + L) Tan[δ - θ1] + L Tan[θ1]) ^ 2)
            )
          )
        )
      /
      (
        50 e ^ (-D0 k / 30 + 1 / 6 (-D0 + D0 k)) (D0 - D0 k) Sin[δ - θ1] ^ 2 +
        50 D0 e ^ (-D0 k / 60) k Sin[θ1] ^ 2
      )
    )
  )
]

```

```

Clear[N1, N2, a1, a2, d, θ1, δ, D1, D2, D0, L, k, dn];
a1 = D1 * Tan[θ1]; a2 = D2 * Tan[θ1 - δ];
d = L * Tan[θ1] - (L + D1) * Tan[θ1 - δ]; D1 = k * D0;
D2 = D0 - D1; N1 = 50 * D1 * (Sin[θ1]) ^ 2 * Exp[-D1 / 2 / Lam1];
N2 = 50 * D2 * (Sin[θ1 - δ]) ^ 2 * Exp[-D2 / 2 / Lam2 - D1 / Lam1];
Lam1 = 3; Lam2 = 3;
Sqrt[
  (
    1
    / (12 (N1 + N2) ^ 2
      (-3 (a1 N1 + (a2 + 2 d) N2) ^ 2 +
        4 (N1 + N2) (a1 ^ 2 N1 + (a2 ^ 2 + 3 a2 d + 3 d ^ 2) N2)) +
    pad ^ 2 / 12) / (N1 + N2) ]

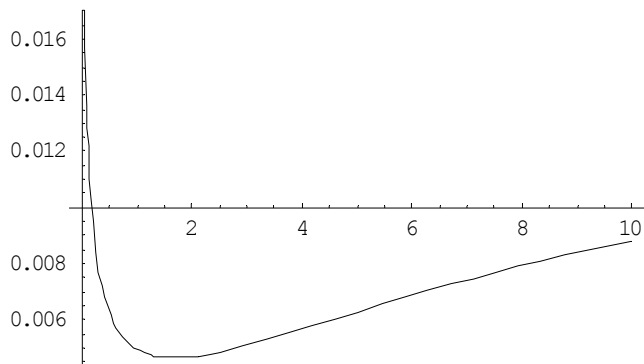
```

$$\begin{aligned}
& \sqrt{\left( \left( 0.016875 + \left( -3 \left( 50 D0^2 e^{-\frac{D0k}{6}} k^2 \sin[\theta 1]^2 \tan[\theta 1] + \right. \right. \right. \right. \\
& \quad 50 e^{-\frac{D0k}{3} + \frac{1}{6} (-D0+D0k)} (D0 - D0 k) \\
& \quad \sin[\delta - \theta 1]^2 (- (D0 - D0 k) \tan[\delta - \theta 1] + \\
& \quad \quad \left. \left. 2 ((D0 k + L) \tan[\delta - \theta 1] + L \tan[\theta 1]) \right) \right)^2 + \right. \\
& \quad \left. 4 \left( 50 e^{-\frac{D0k}{3} + \frac{1}{6} (-D0+D0k)} (D0 - D0 k) \sin[\delta - \theta 1]^2 + \right. \right. \\
& \quad \quad \left. \left. 50 D0 e^{-\frac{D0k}{6}} k \sin[\theta 1]^2 \right) \right. \\
& \quad \left( 50 D0^3 e^{-\frac{D0k}{6}} k^3 \sin[\theta 1]^2 \tan[\theta 1]^2 + \right. \\
& \quad 50 e^{-\frac{D0k}{3} + \frac{1}{6} (-D0+D0k)} (D0 - D0 k) \sin[\delta - \theta 1]^2 \\
& \quad \left( (D0 - D0 k)^2 \tan[\delta - \theta 1]^2 - 3 (D0 - D0 k) \tan[ \right. \\
& \quad \quad \delta - \theta 1] ((D0 k + L) \tan[\delta - \theta 1] + L \tan[\theta 1]) + \\
& \quad \quad \left. \left. 3 ((D0 k + L) \tan[\delta - \theta 1] + L \tan[\theta 1])^2 \right) \right) \left. \right) / \\
& \quad \left( 12 \left( 50 e^{-\frac{D0k}{3} + \frac{1}{6} (-D0+D0k)} (D0 - D0 k) \sin[\delta - \theta 1]^2 + \right. \right. \\
& \quad \quad \left. \left. 50 D0 e^{-\frac{D0k}{6}} k \sin[\theta 1]^2 \right)^2 \right) \left. \right) / \\
& \quad \left( 50 e^{-\frac{D0k}{3} + \frac{1}{6} (-D0+D0k)} (D0 - D0 k) \sin[\delta - \theta 1]^2 + \right. \\
& \quad 50 \\
& \quad D0 \\
& \quad e^{-\frac{D0k}{6}} \\
& \quad k \\
& \quad \left. \left. \left. \sin[\theta 1]^2 \right) \right) \right)
\end{aligned}$$

```

Clear[N1, N2, a1, a2, d, θ1, δ, D1, D2, D0, L, k, dn, pad];
a1 = D1 * Tan[θ1]; a2 = D2 * Tan[θ1 - δ];
d = L * Tan[θ1] - (L + D1) * Tan[θ1 - δ]; D1 = k * D0;
D2 = D0 - D1; N1 = 50 * D1 * (Sin[θ1]) ^ 2 * Exp[-D1 / 2 / Lam1];
N2 = 50 * D2 * (Sin[θ1 - δ]) ^ 2 * Exp[-D2 / 2 / Lam2 - D1 / Lam1];
Lam1 = 3; Lam2 = 3; L = 20 - D0; k = 0.5; δ = 0; θ1 = 0.3;
L = 20; pad = 0.6;
Plot[
  Sqrt[
    (
      1
      / (12 (N1 + N2) ^ 2
        (-3 (a1 N1 + (a2 + 2 d) N2) ^ 2 +
          4 (N1 + N2) (a1 ^ 2 N1 + (a2 ^ 2 + 3 a2 d + 3 d ^ 2) N2)) +
      pad ^ 2 / 12) / (N1 + N2) ] / (L + D0 / 2), {D0, 0, 10}]

```

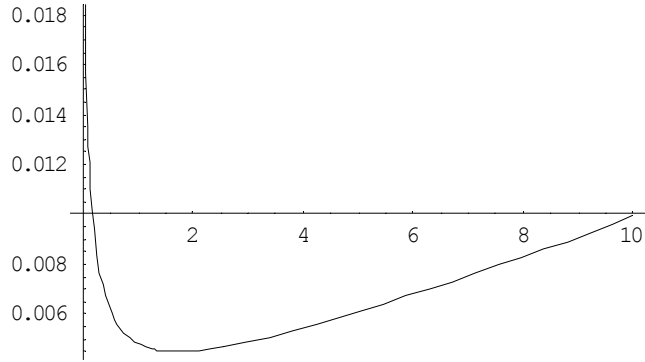


-Graphics-

```

Clear[N1, N2, a1, a2, d, θ1, δ, D1, D2, D0, L, k, dn, pad];
a1 = D1 * Tan[θ1]; a2 = D2 * Tan[θ1 - δ];
d = L * Tan[θ1] - (L + D1) * Tan[θ1 - δ]; D1 = k * D0;
D2 = D0 - D1; N1 = 50 * D1 * (Sin[θ1]) ^ 2 * Exp[-D1 / 2 / Lam1];
N2 = 50 * D2 * (Sin[θ1 - δ]) ^ 2 * Exp[-D2 / 2 / Lam2 - D1 / Lam1];
Lam1 = 30; Lam2 = 30; L = 20 - D0; k = 0.5; δ = 0; θ1 = 0.3;
pad = 0.6;
Plot[
  Sqrt[
    (
      1
      / (12 (N1 + N2) ^ 2
        (-3 (a1 N1 + (a2 + 2 d) N2) ^ 2 +
          4 (N1 + N2) (a1 ^ 2 N1 + (a2 ^ 2 + 3 a2 d + 3 d ^ 2) N2)) +
      pad ^ 2 / 12) / (N1 + N2) ] / (L + D0 / 2), {D0, 0, 10}]

```



-Graphics-

```
Clear[N1, N2, a1, a2, d, θ1, δ, D1, D2, D0, L, k, dn, pad];
```

```
a1 = D1 * Tan[θ1]; a2 = D2 * Tan[θ1 - δ];
```

```
d = L * Tan[θ1] - (L + D1) * Tan[θ1 - δ]; D1 = k * D0;
```

```
D2 = D0 - D1; N1 = 50 * D1 * (Sin[θ1]) ^ 2 * Exp[-D1 / 2 / Lam1];
```

```
N2 = 50 * D2 * (Sin[θ1 - δ]) ^ 2 * Exp[-D2 / 2 / Lam2 - D1 / Lam1];
```

```
Lam1 = 30; Lam2 = 30; L = 20 - D0; k = 0.5; θ1 = 0.3;
```

```
pad = 0.6; D0 = 4;
```

```
Plot[
```

```
  Sqrt[
```

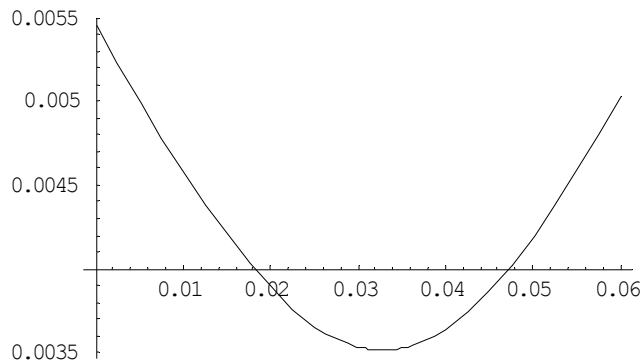
$$\left( \frac{1}{12 (N1 + N2)^2} \right.$$

$$\left. (-3 (a1 N1 + (a2 + 2 d) N2)^2 + \right.$$

$$\left. 4 (N1 + N2) (a1^2 N1 + (a2^2 + 3 a2 d + 3 d^2) N2) \right) +$$

$$\left. \text{pad}^2 / 12 \right) / (N1 + N2) / (L + D0 / 2),$$

```
{δ, 0, 0.06}]
```

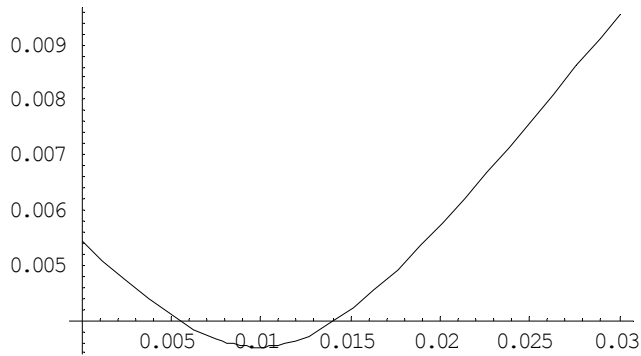


-Graphics-

```

Clear[N1, N2, a1, a2, d, θ1, δ, D1, D2, D0, L, k, dn, pad];
δ = dn / θ1; a1 = D1 * Tan[θ1]; a2 = D2 * Tan[θ1 - δ];
d = L * Tan[θ1] - (L + D1) * Tan[θ1 - δ]; D1 = k * D0;
D2 = D0 - D1; N1 = 50 * D1 * (Sin[θ1]) ^ 2 * Exp[-D1 / 2 / Lam1];
N2 = 50 * D2 * (Sin[θ1 - δ]) ^ 2 * Exp[-D2 / 2 / Lam2 - D1 / Lam1];
Lam1 = 30; Lam2 = 30; L = 20 - D0; k = 0.5; θ1 = 0.3;
pad = 0.6; D0 = 4;
Plot[
  Sqrt[
    (
      1
      / (12 (N1 + N2)^2
        (-3 (a1 N1 + (a2 + 2 d) N2)^2 +
          4 (N1 + N2) (a1^2 N1 + (a2^2 + 3 a2 d + 3 d^2) N2)) +
      pad^2 / 12) / (N1 + N2) ] / (L + D0 / 2),
    {dn, 0, 0.03}]

```



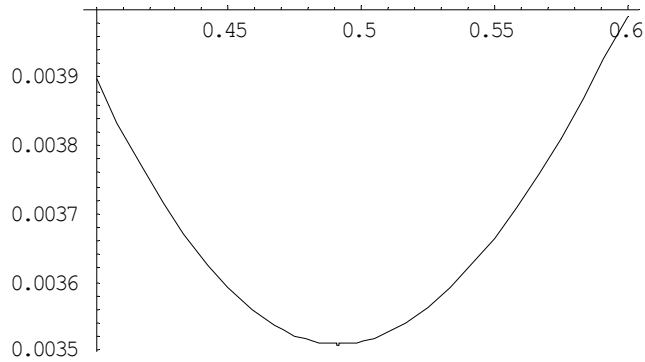
-Graphics-



```

Clear[N1, N2, a1, a2, d, θ1, δ, D1, D2, D0, L, k, dn, pad];
δ = dn / θ1; a1 = D1 * Tan[θ1]; a2 = D2 * Tan[θ1 - δ];
d = L * Tan[θ1] - (L + D1) * Tan[θ1 - δ]; D1 = k * D0;
D2 = D0 - D1; N1 = 50 * D1 * (Sin[θ1]) ^ 2 * Exp[-D1 / 2 / Lam1];
N2 = 50 * D2 * (Sin[θ1 - δ]) ^ 2 * Exp[-D2 / 2 / Lam2 - D1 / Lam1];
Lam1 = 30; Lam2 = 30; L = 20 - D0; θ1 = 0.3; pad = 0.6;
D0 = 4; dn = 0.01;
Plot[
  Sqrt[
    (
      1
      / (12 (N1 + N2) ^ 2
        (-3 (a1 N1 + (a2 + 2 d) N2) ^ 2 +
          4 (N1 + N2) (a1 ^ 2 N1 + (a2 ^ 2 + 3 a2 d + 3 d ^ 2) N2)) +
      pad ^ 2 / 12) / (N1 + N2) ] / (L + D0 / 2),
    {k, 0.4, 0.6}]

```

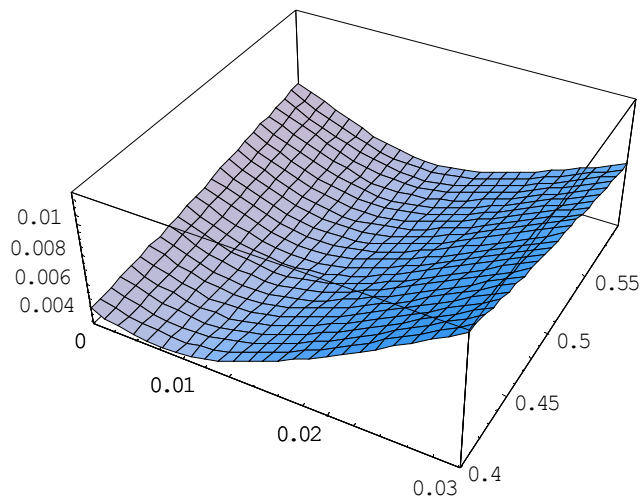


-Graphics-

```

Clear[N1, N2, a1, a2, d, θ1, δ, D1, D2, D0, L, k, dn, pad];
δ = dn / θ1; a1 = D1 * Tan[θ1]; a2 = D2 * Tan[θ1 - δ];
d = L * Tan[θ1] - (L + D1) * Tan[θ1 - δ]; D1 = k * D0;
D2 = D0 - D1; N1 = 50 * D1 * (Sin[θ1]) ^ 2 * Exp[-D1 / 2 / Lam1];
N2 = 50 * D2 * (Sin[θ1 - δ]) ^ 2 * Exp[-D2 / 2 / Lam2 - D1 / Lam1];
Lam1 = 30; Lam2 = 30; L = 20 - D0; θ1 = 0.3; pad = 0.6;
D0 = 4;
Plot3D[
  Sqrt[
    (
      1
      / (12 (N1 + N2)^2
        (-3 (a1 N1 + (a2 + 2 d) N2)^2 +
          4 (N1 + N2) (a1^2 N1 + (a2^2 + 3 a2 d + 3 d^2) N2)) +
      pad^2 / 12) / (N1 + N2) ] / (L + D0 / 2) ,
    {dn, 0, 0.03}, {k, 0.4, 0.6}]

```

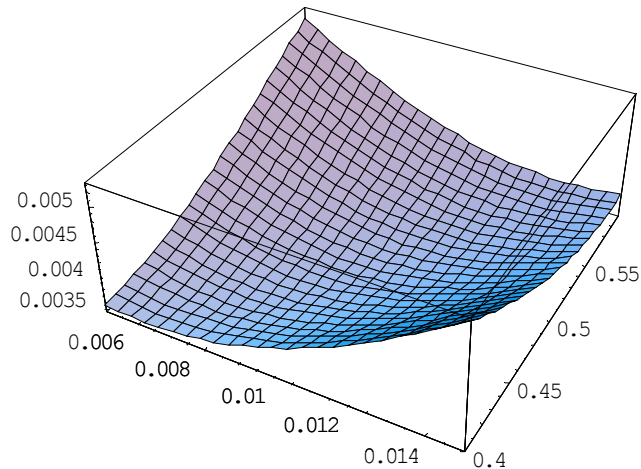


-SurfaceGraphics-

```

Clear[N1, N2, a1, a2, d, θ1, δ, D1, D2, D0, L, k, dn, pad];
δ = dn/θ1; a1 = D1 * Tan[θ1]; a2 = D2 * Tan[θ1 - δ];
d = L * Tan[θ1] - (L + D1) * Tan[θ1 - δ]; D1 = k * D0;
D2 = D0 - D1; N1 = 50 * D1 * (Sin[θ1]) ^ 2 * Exp[-D1 / 2 / Lam1];
N2 = 50 * D2 * (Sin[θ1 - δ]) ^ 2 * Exp[-D2 / 2 / Lam2 - D1 / Lam1];
Lam1 = 30; Lam2 = 30; L = 20 - D0; θ1 = 0.3; pad = 0.6;
D0 = 4;
FindMinimum[
  Sqrt[
    (
      1
      / (12 (N1 + N2)^2
        (-3 (a1 N1 + (a2 + 2 d) N2)^2 +
          4 (N1 + N2) (a1^2 N1 + (a2^2 + 3 a2 d + 3 d^2) N2)) +
      pad^2 / 12) / (N1 + N2) ] / (L + D0 / 2), {dn, 0.01},
    {k, 0.5}]
{0.00348694, {dn→0.00823044, k→0.455113}}
Clear[N1, N2, a1, a2, d, θ1, δ, D1, D2, D0, L, k, dn, pad];
δ = dn/θ1; a1 = D1 * Tan[θ1]; a2 = D2 * Tan[θ1 - δ];
d = L * Tan[θ1] - (L + D1) * Tan[θ1 - δ]; D1 = k * D0;
D2 = D0 - D1; N1 = 50 * D1 * (Sin[θ1]) ^ 2 * Exp[-D1 / 2 / Lam1];
N2 = 50 * D2 * (Sin[θ1 - δ]) ^ 2 * Exp[-D2 / 2 / Lam2 - D1 / Lam1];
Lam1 = 30; Lam2 = 30; L = 20 - D0; θ1 = 0.3; pad = 0.6;
D0 = 4;
Plot3D[
  Sqrt[
    (
      1
      / (12 (N1 + N2)^2
        (-3 (a1 N1 + (a2 + 2 d) N2)^2 +
          4 (N1 + N2) (a1^2 N1 + (a2^2 + 3 a2 d + 3 d^2) N2)) +
      pad^2 / 12) / (N1 + N2) ] / (L + D0 / 2),
    {dn, 0.005, 0.015}, {k, 0.4, 0.6}]

```



**-SurfaceGraphics-**

```
Clear[N1, N2, a1, a2, d, θ1, δ, D1, D2, D0, L, k, dn, pad];
```

```
δ = dn/θ1; a1 = D1 * Tan[θ1]; a2 = D2 * Tan[θ1 - δ];
```

```
d = L * Tan[θ1] - (L + D1) * Tan[θ1 - δ]; D1 = k * D0;
```

```
D2 = D0 - D1; N1 = 50 * D1 * (Sin[θ1]) ^ 2 * Exp[-D1 / 2 / Lam1];
```

```
N2 = 50 * D2 * (Sin[θ1 - δ]) ^ 2 * Exp[-D2 / 2 / Lam2 - D1 / Lam1];
```

```
Lam1 = 30; Lam2 = 30; L = 20 - D0; θ1 = 0.3; pad = 0.6;
```

```
D0 = 4; dn = 0.0082;
```

```
Plot[
```

```
  Sqrt[
```

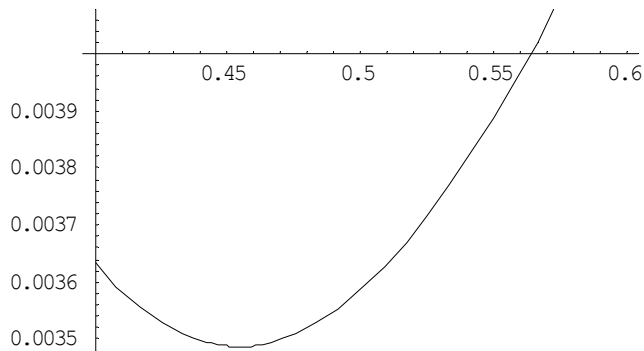
$$\left( \frac{1}{12 (N1 + N2)^2} \right.$$

$$\left( -3 (a1 N1 + (a2 + 2 d) N2)^2 + \right.$$

$$\left. 4 (N1 + N2) (a1^2 N1 + (a2^2 + 3 a2 d + 3 d^2) N2) \right) +$$

$$\left. pad^2 / 12 \right) / (N1 + N2) / (L + D0 / 2),$$

```
{k, 0.4, 0.6}]
```

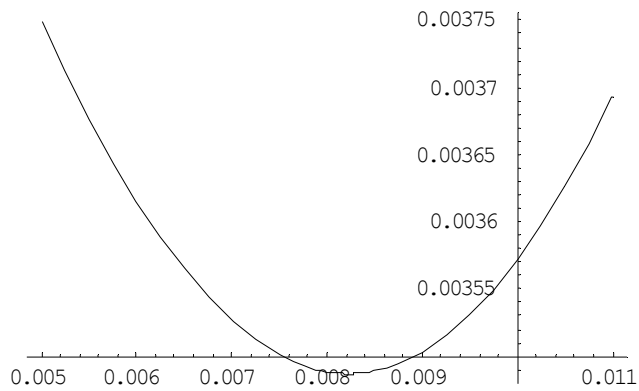


**-Graphics-**

```

Clear[N1, N2, a1, a2, d, θ1, δ, D1, D2, D0, L, k, dn, pad];
δ = dn / θ1; a1 = D1 * Tan[θ1]; a2 = D2 * Tan[θ1 - δ];
d = L * Tan[θ1] - (L + D1) * Tan[θ1 - δ]; D1 = k * D0;
D2 = D0 - D1; N1 = 50 * D1 * (Sin[θ1]) ^ 2 * Exp[-D1 / 2 / Lam1];
N2 = 50 * D2 * (Sin[θ1 - δ]) ^ 2 * Exp[-D2 / 2 / Lam2 - D1 / Lam1];
Lam1 = 30; Lam2 = 30; L = 20 - D0; θ1 = 0.3; pad = 0.6;
D0 = 4; k = 0.455;
Plot[
  Sqrt[
    (
      1
      / (12 (N1 + N2)^2
        (-3 (a1 N1 + (a2 + 2 d) N2)^2 +
          4 (N1 + N2) (a1^2 N1 + (a2^2 + 3 a2 d + 3 d^2) N2)) +
      pad^2 / 12) / (N1 + N2) ] / (L + D0 / 2),
    {dn, 0.005, 0.011}]

```



-Graphics-