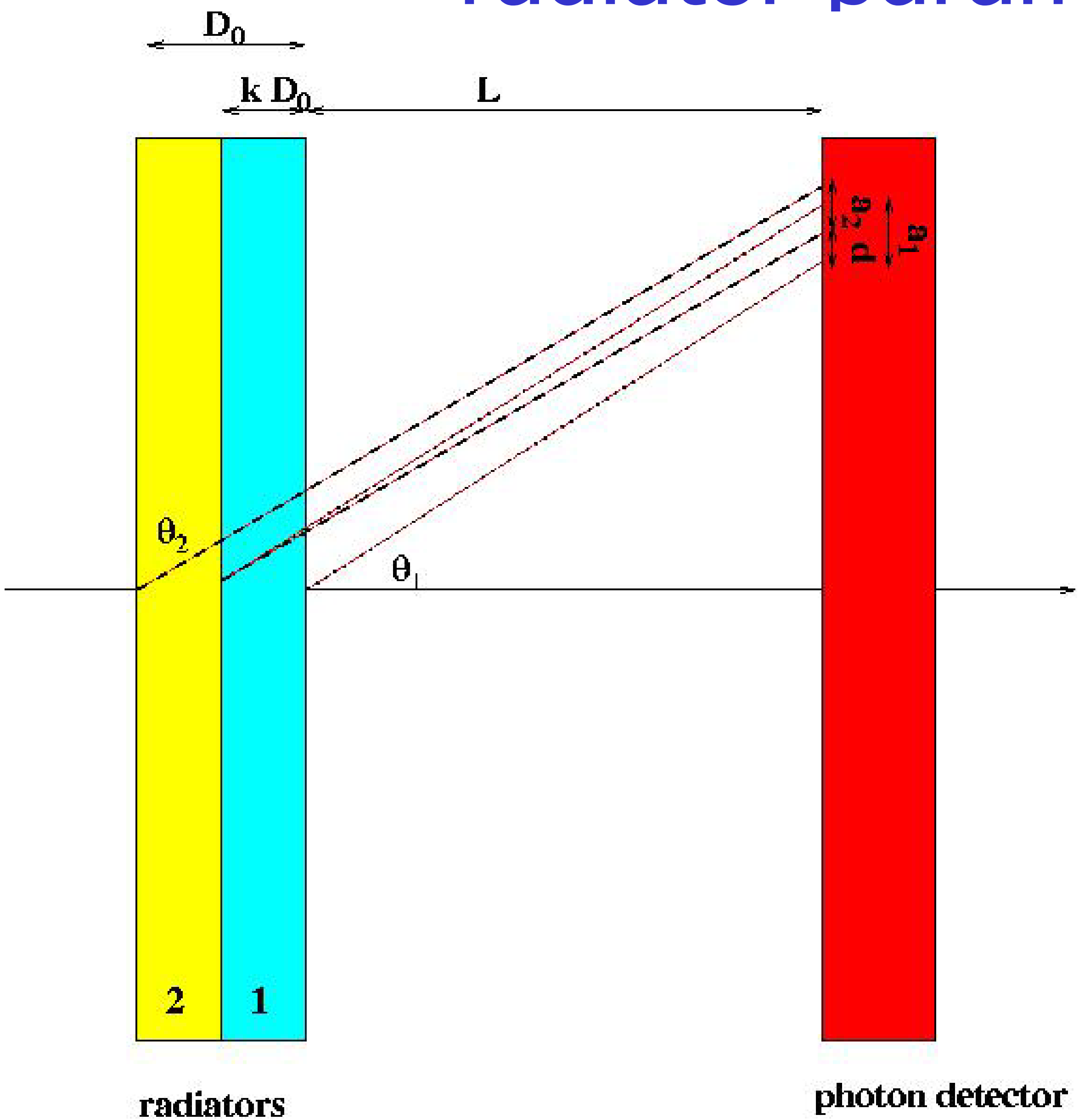
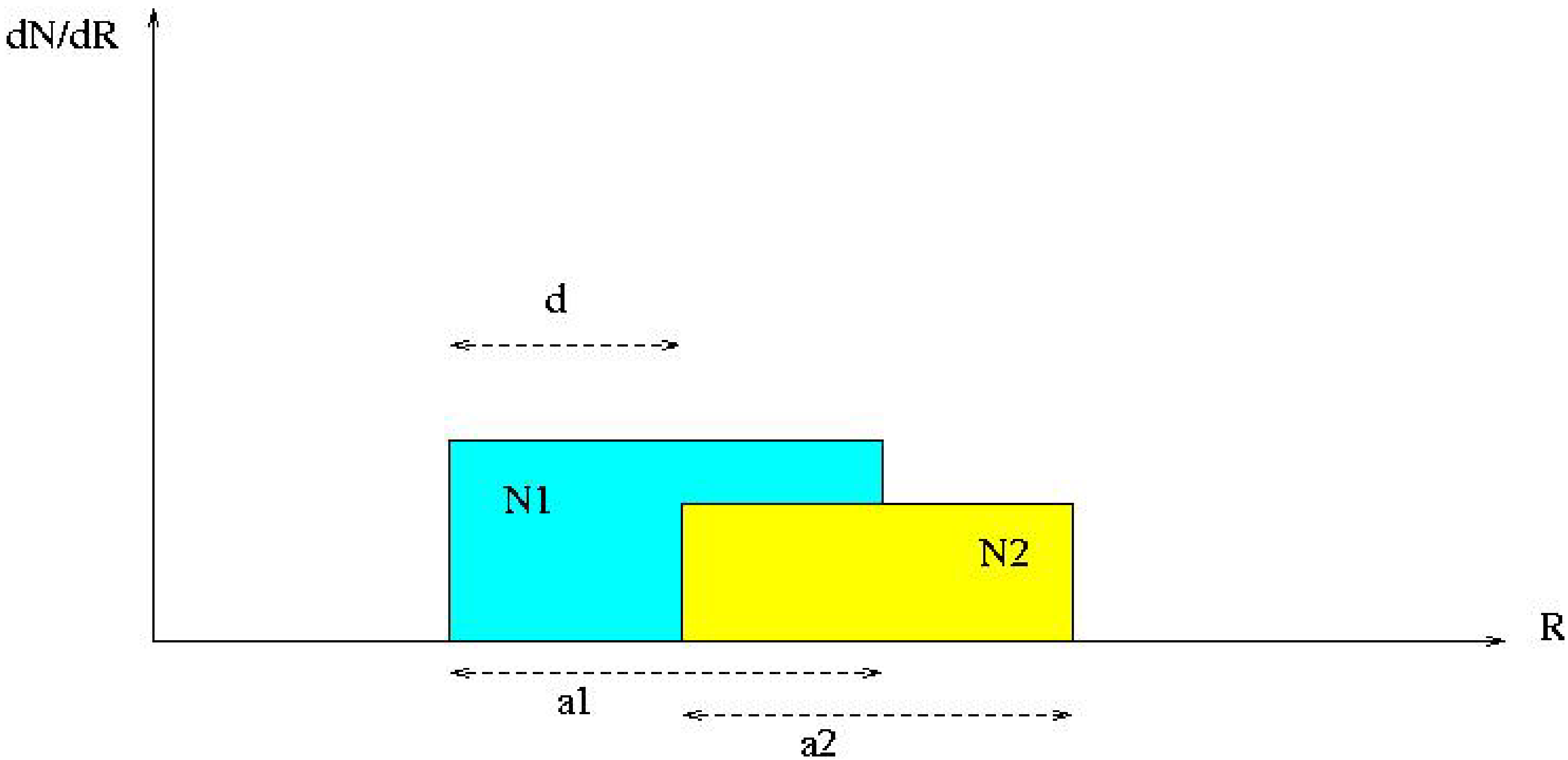
Multiple radiator: Optimisation of radiator parameters

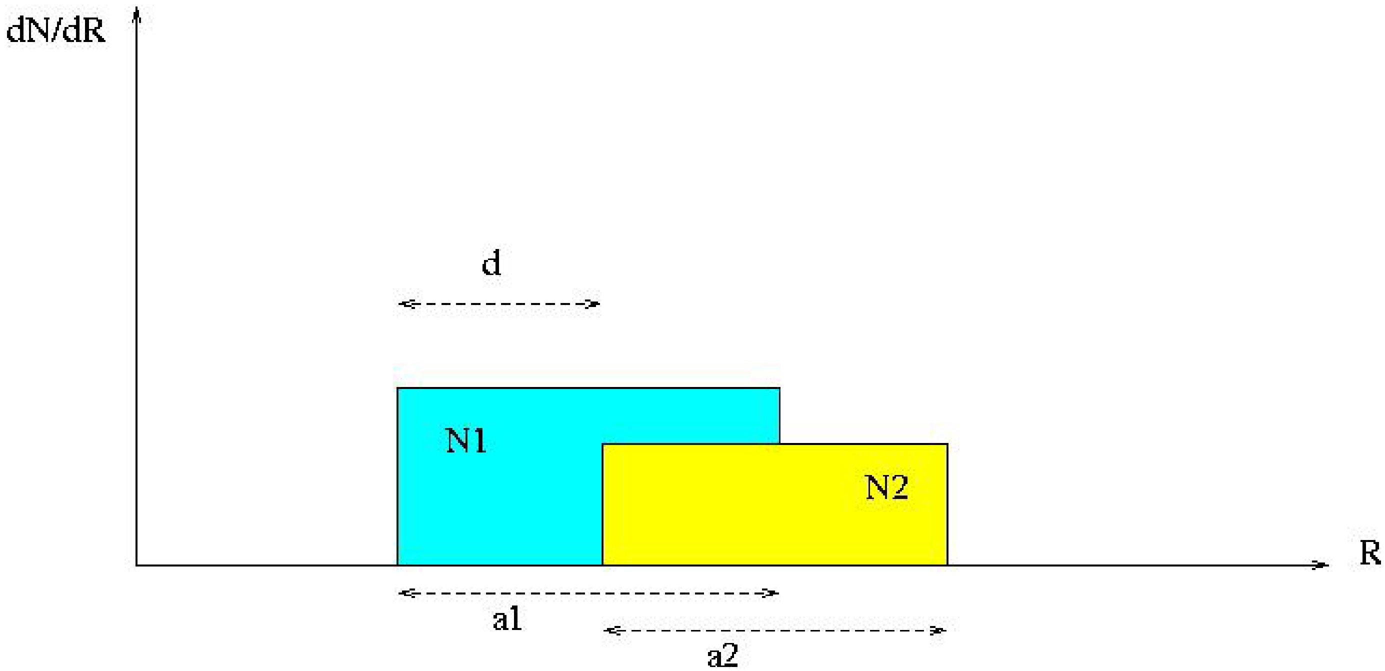


Dual radiator: three parameters (if fixed space available)

- Difference in θ
- Ratio of radiator thicknesses
- Total radiator thickness

Radial photon impact point distribution





$$\langle R \rangle = \frac{a1\,N1 + (a2 + 2\,d)\,N2}{2\,(N1 + N2)}$$

$$\langle R^2 \rangle = \frac{a1^2\,N1 + (a2^2 + 3\,a2\,d + 3\,d^2)\,N2}{3\,(N1 + N2)}$$

$$\sigma_R{}^2 = \langle R^2\rangle - \langle R\rangle^2 =$$

$$= -\frac{(a1\,N1 + (a2 + 2\,d)\,N2)^2}{4\,(N1 + N2)^2} + \frac{a1^2\,N1 + (a2^2 + 3\,a2\,d + 3\,d^2)\,N2}{3\,(N1 + N2)}$$

$$= \frac{1}{12\,(N1 + N2)^2}\,(-3\,(a1\,N1 + (a2 + 2\,d)\,N2)^2 + 4\,(N1 + N2)\,(a1^2\,N1 + (a2^2 + 3\,a2\,d + 3\,d^2)\,N2))$$

$$\sigma_{R,tot}{}^2 = \sigma_R{}^2 + \sigma_{R,padsize}{}^2$$

Parameters of the counter

```
a1 = D1 * Tan[Θ1] ; a2 = D2 * Tan[Θ1 - δ] ;
d = L * Tan[Θ1] - (L + D1) * Tan[Θ1 - δ] ; D1 = k * D0;
D2 = D0 - D1; N1 = 50 * D1 * (Sin[Θ1]) ^2 * Exp[-D1 / 2 / Lam1] ;
N2 = 50 * D2 * (Sin[Θ1 - δ]) ^2 * Exp[-D2 / 2 / Lam2 - D1 / Lam1] ;
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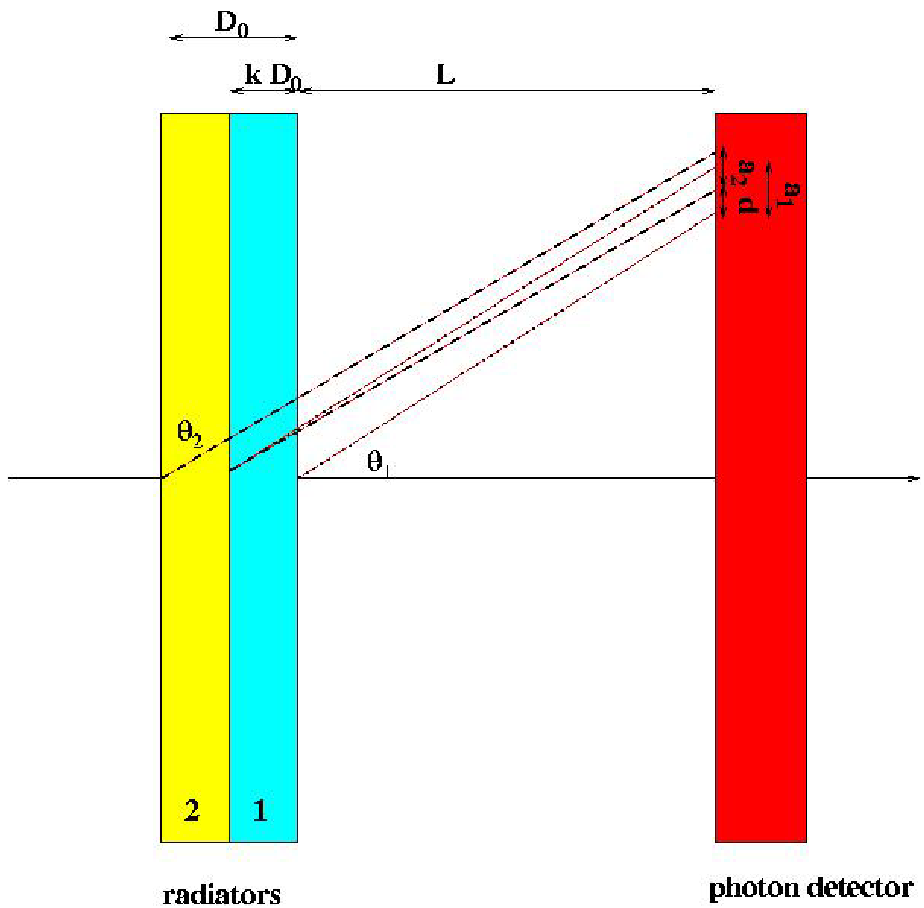
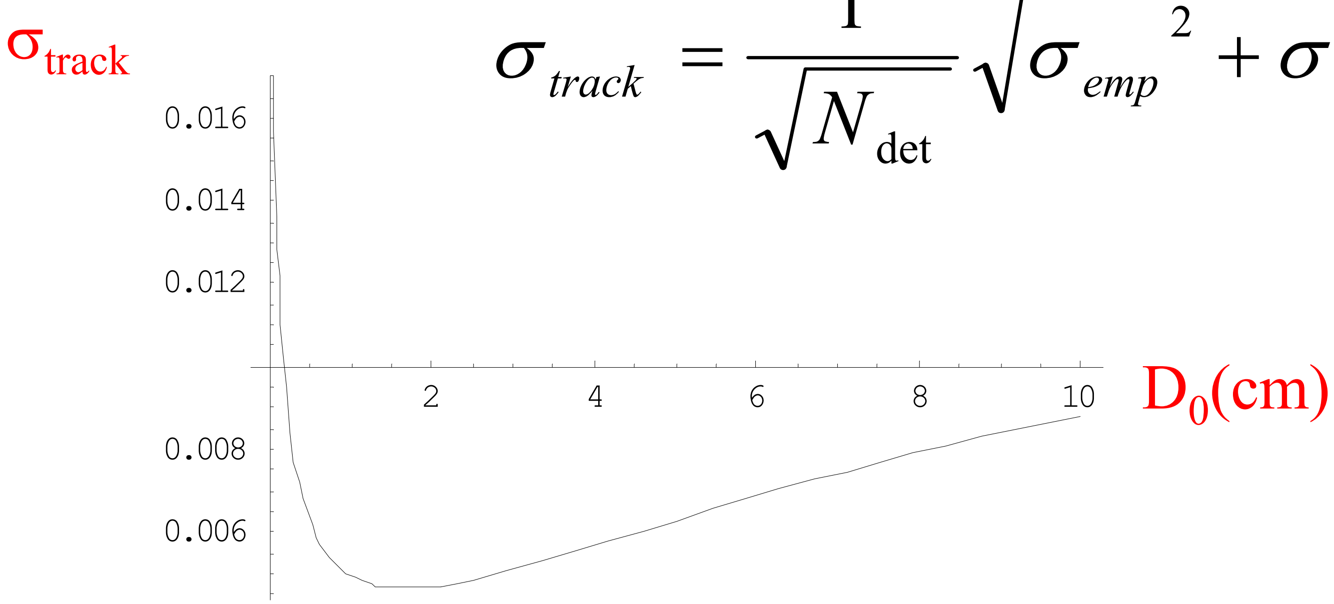


Check: plot track error vs. radiator thickness for the case both radiators have the same index

Input data:

- total length $L+D_0$ is fixed (20cm)
- attenuation length: 3cm
- Cherenkov angle = 0.3, pad size 6mm, $\sigma_{rest}=0$

$$\sigma_{track} = \frac{1}{\sqrt{N_{\det}}}\sqrt{\sigma_{emp}^{\;2} + \sigma_{\det}^{\;2} + \sigma_{rest}^{\;2}}$$



Minimized: error per track

$$\sigma_{track} = \frac{1}{\sqrt{N_{\det}}}\sqrt{\sigma_{emp}^{\ 2} + \sigma_{\det}^{\ 2} + \sigma_{rest}^{\ 2}}$$

Distance to photon detector

$$\frac{1}{(L + D0 / 2)}$$

Number of photons

$$\sigma_{tot}^{\ 2} = \sigma_{emp}^{\ 2} + \sigma_{\det}^{\ 2} + \sigma_{rest}^{\ 2}$$

$$\sqrt{\left(\frac{1}{(N1 + N2)}\right.}$$

Emission point error

$$\left(\frac{1}{12\,(N1 + N2)^2}\right.$$

$$\left(-3\,(a1\,N1 + (a2 + 2\,d)\,N2)^2 + \right.$$

$$4\,(N1 + N2)\,\left(a1^2\,N1 + \left(a2^2 + 3\,a2\,d + 3\,d^2\right)\,N2\right)\Big) +$$

$$\left.\left. pad\text{^}2 / 12\right)\right)$$

Pad size contribution

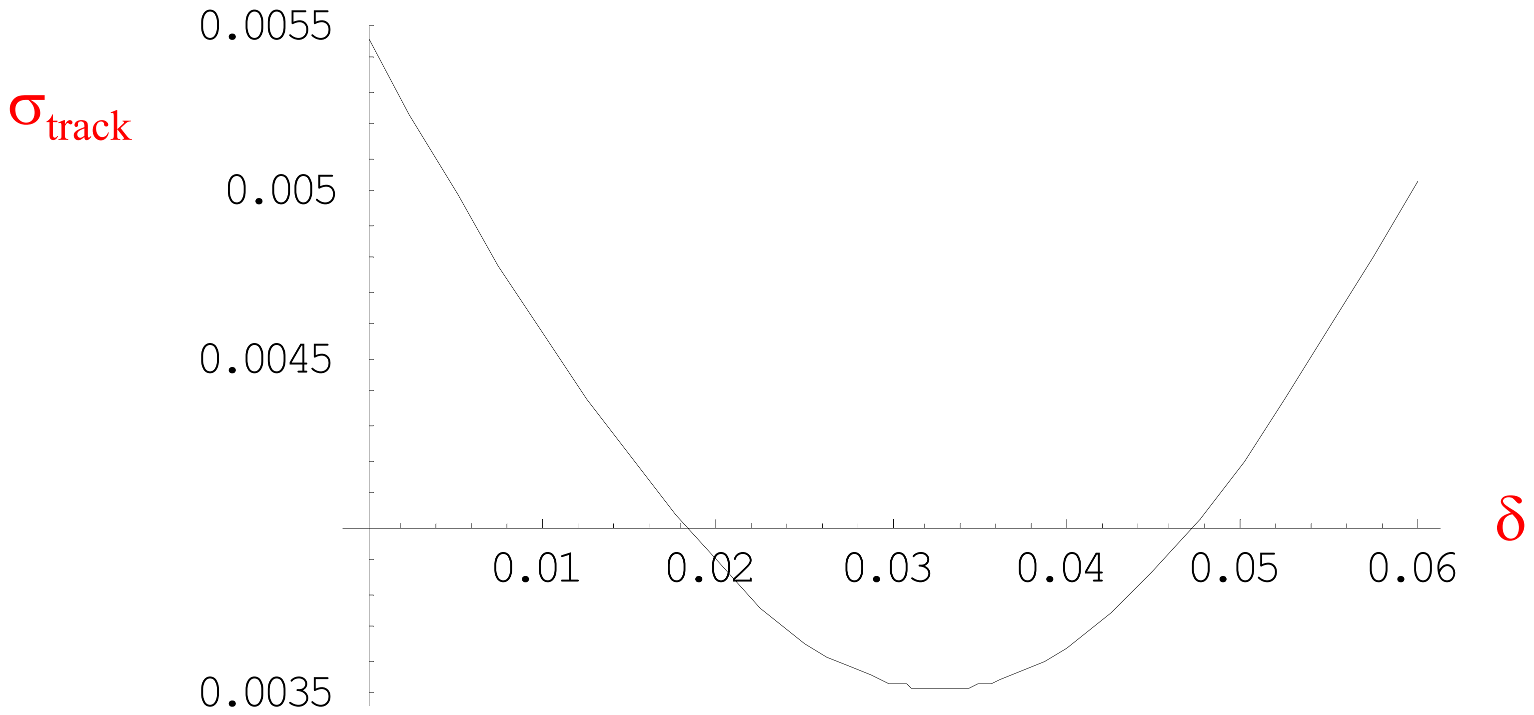
Until futher notice assume σ_{rest}=0

Vary Cherenkov angle difference δ

Track error vs. δ for the case both radiators have the same thickness (k=0.5)

Input data:

- total length $L+D_0$ is fixed (20cm), D_0=4cm
- Cherenkov angle = 0.3, pad size 6mm, no attenuation

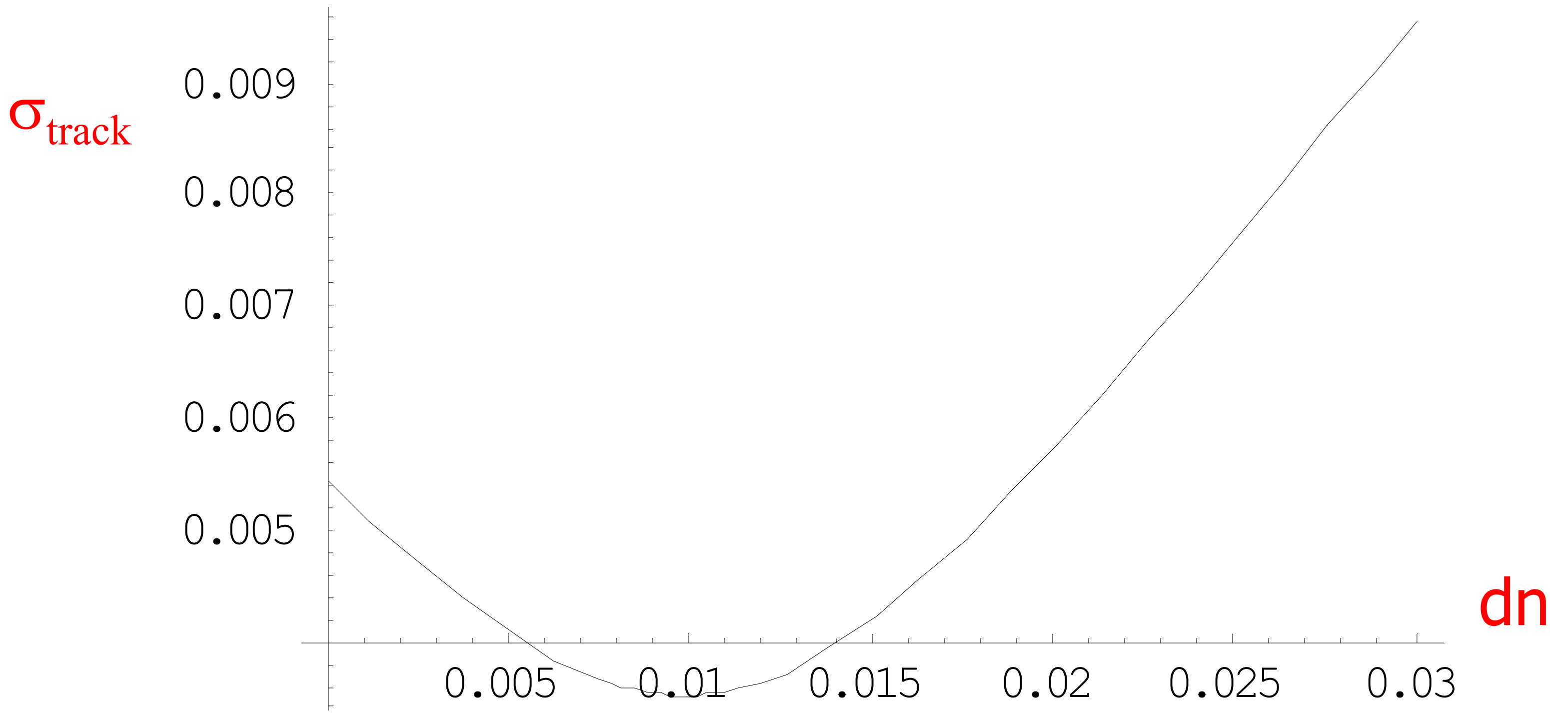


Vary refractive index difference dn

Track error vs. dn for the case both radiators have the same thickness (k=0.5)

Input data:

- total length $L+D_0$ is fixed (20cm), D_0=4cm
- Cherenkov angle = 0.3, pad size 6mm, no attenuation

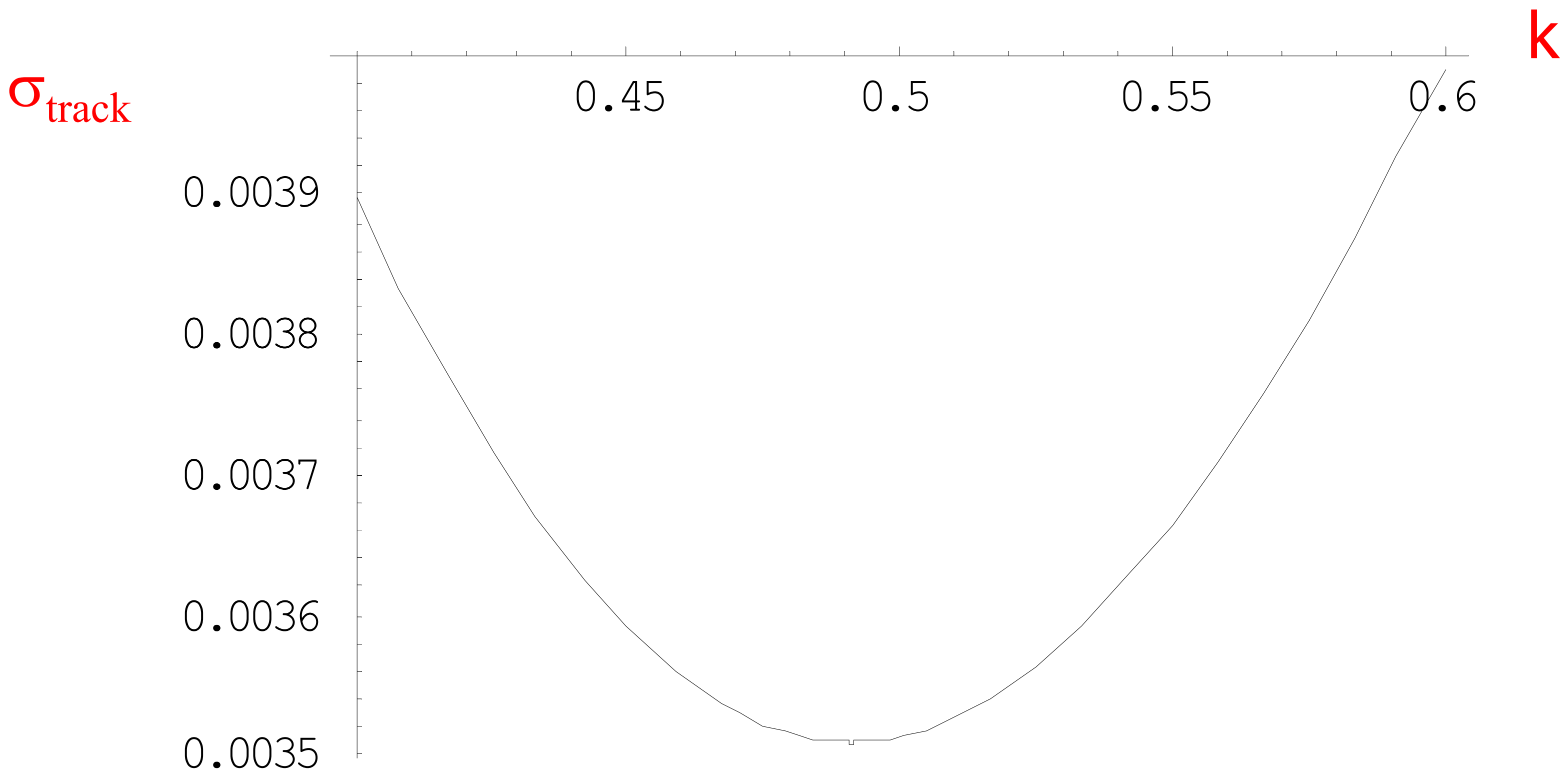


Vary relative radiator thickness k

Track error vs. k for the case dn=0.01

Input data:

- total length $L+D_0$ is fixed (20cm), D_0=4cm
- Cherenkov angle = 0.3, pad size 6mm, no attenuation

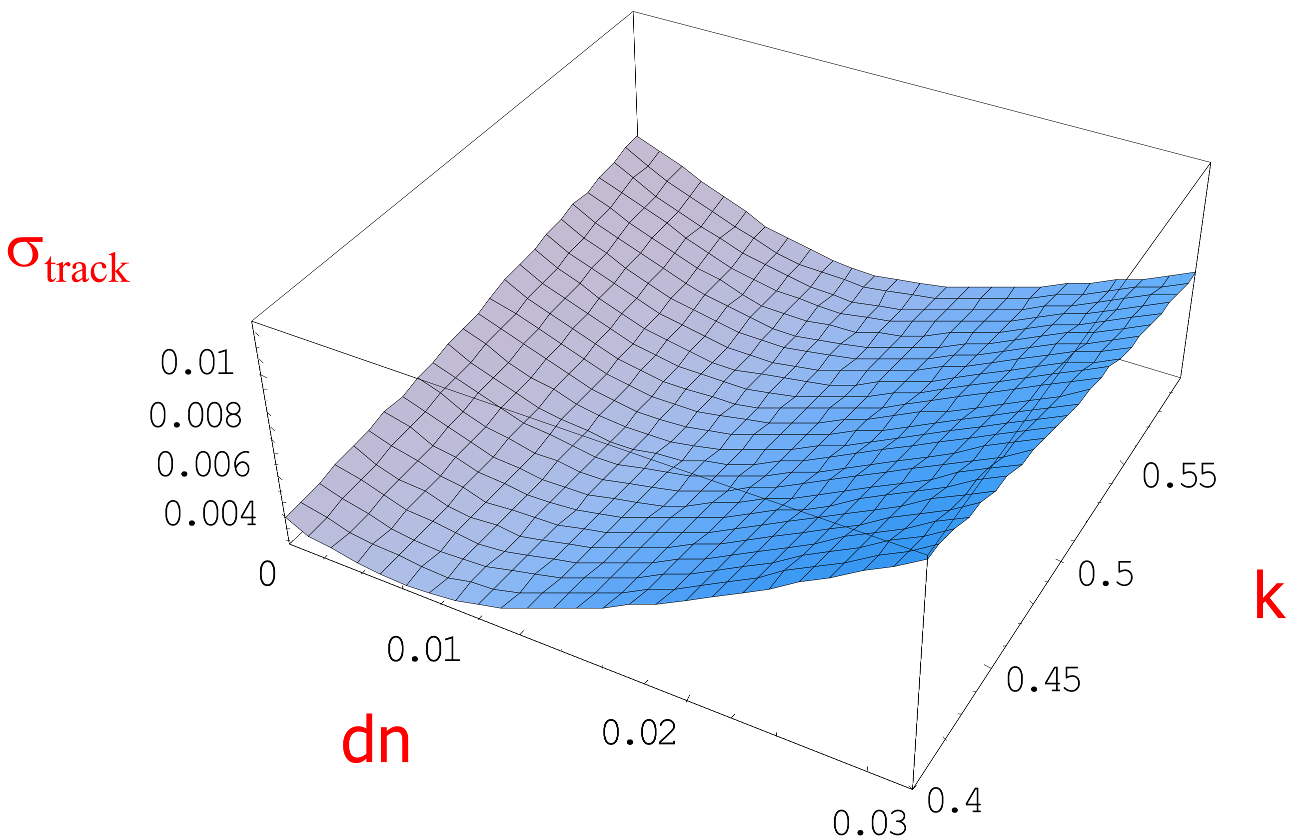


Vary radiator thickness k and refractive index difference dn

Track error vs. (k, dn): correllation is no surprise

Input data:

- total length $L+D_0$ is fixed (20cm), D_0=4cm
- Cherenkov angle = 0.3, pad size 6mm, no attenuation



Minimize track error vs. radiator thickness k and refractive index difference dn, no attenuation

dn=0.0082

k=0.455

σ at minimum 0.0035

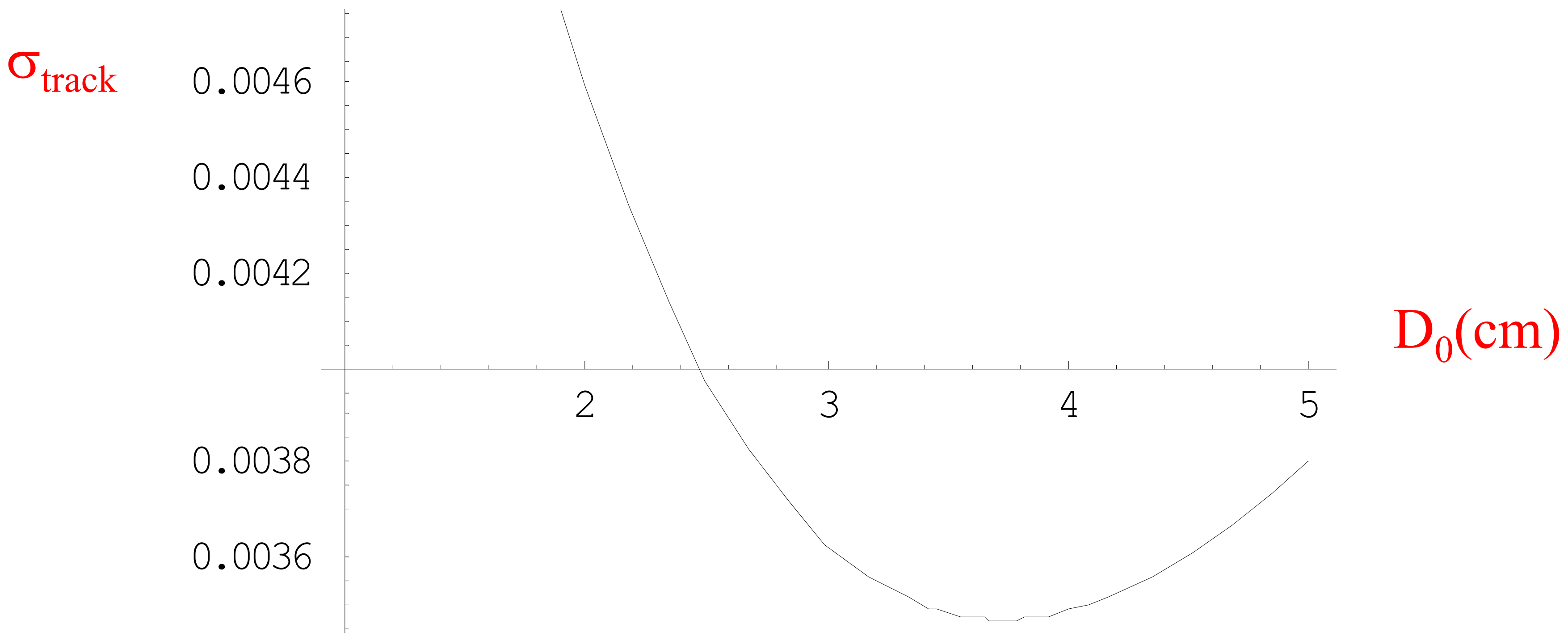
Very shallow minimum...

Vary total radiator thickness D_0

Track error vs. k for the case dn=0.0082, k=0.455

Input data:

- total length $L+D_0$ is fixed (20cm)
- Cherenkov angle = 0.3, pad size 6mm, no attenuation



Minimize track error vs. radiator thickness k and refractive index difference dn, attenuation lengths 3cm

dn=0.007

k=0.42

σ at minimum 0.0045

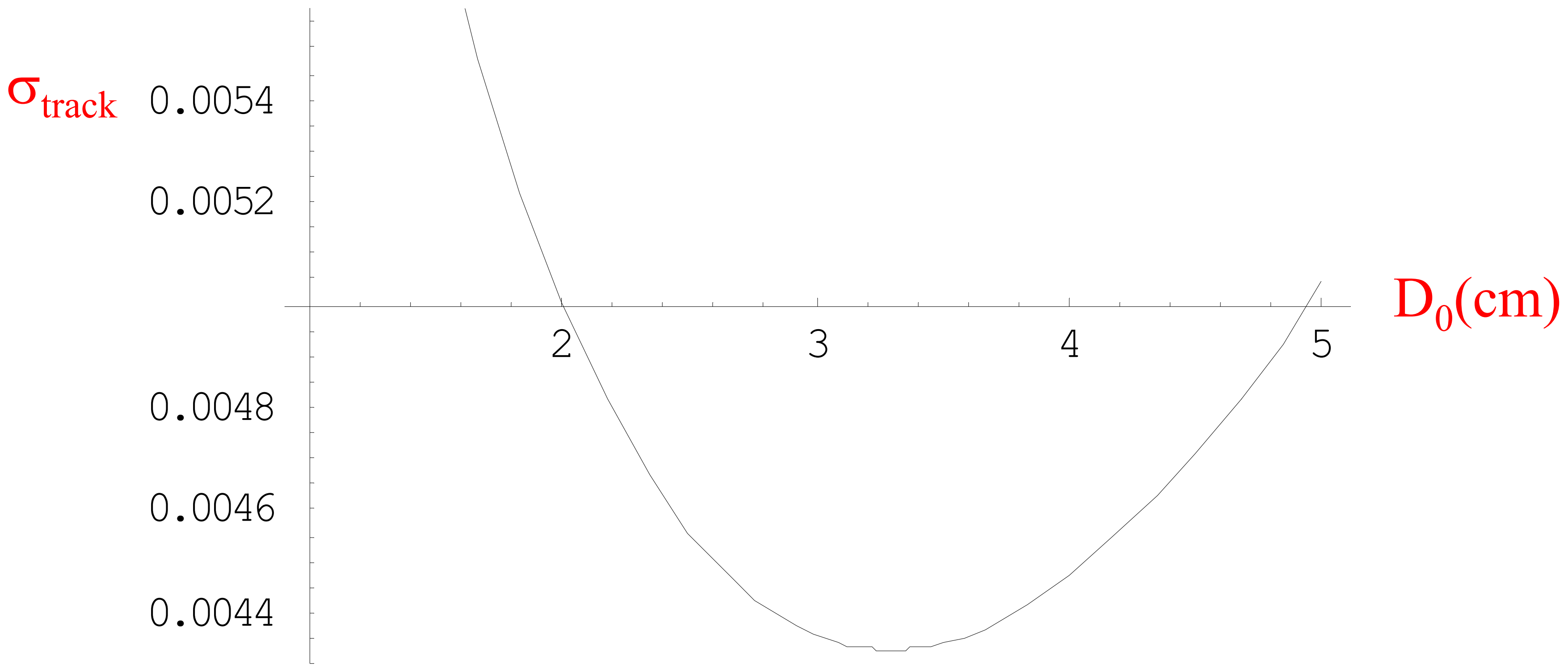
Vary total radiator thickness D_0

Track error vs. k for the case dn=0.007, k=0.42

Input data:

•total length $L+D_0$ is fixed (20cm)

•Cherenkov angle = 0.3, pad size 6mm, attenuation lengths 3cm



Minimize track error vs.

radiator thickness k and refractive index difference dn and total thickness D_0

attenuation lengths 3cm

dn=0.004

k=0.44

D_0 = 2.35cm

σ at minimum 0.0040

no attenuation

dn=0.006

k=0.46

D_0 = 3.1cm

σ at minimum 0.0034

Available space in front of photon detector: 20cm

Minimize track error vs.

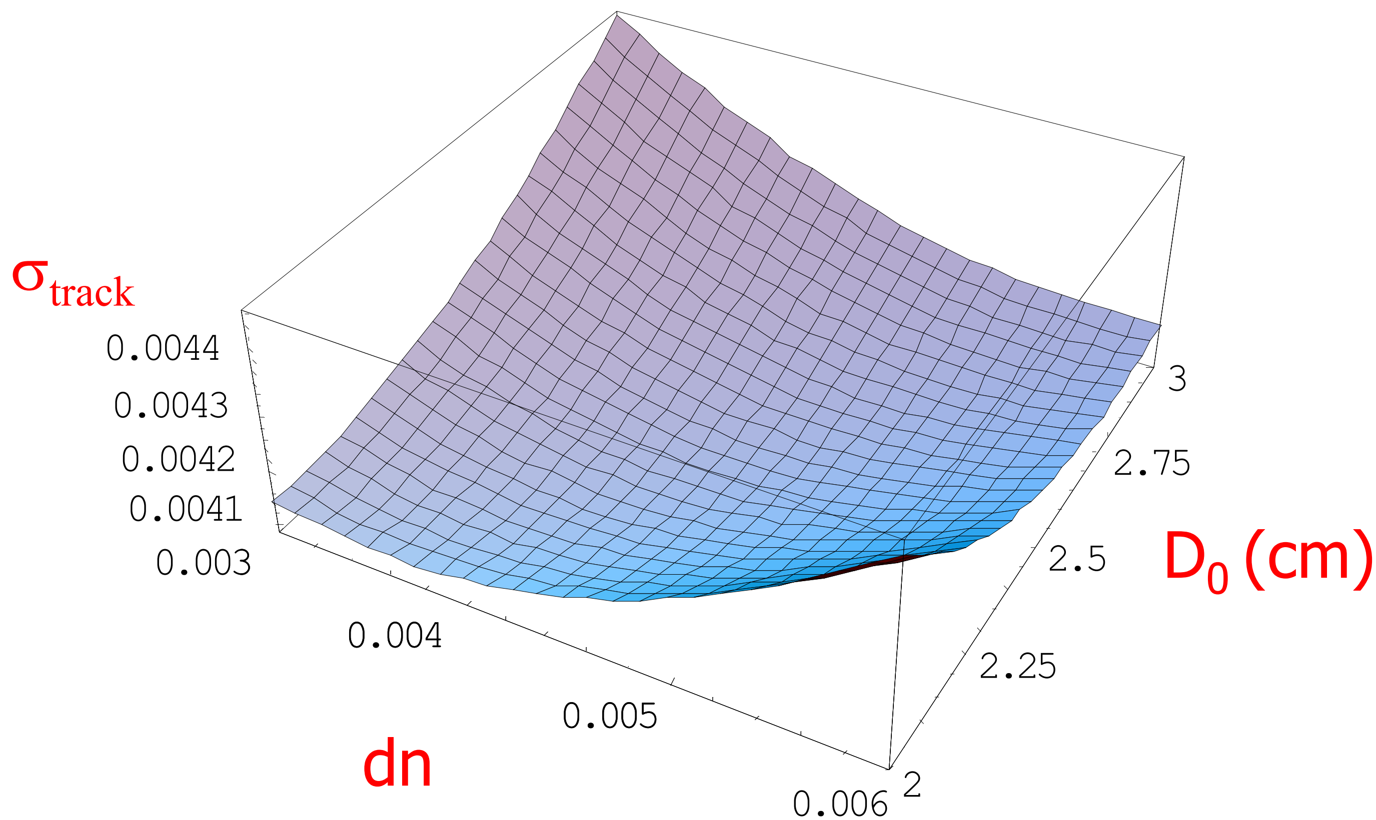
radiator thickness k and refractive index difference dn and total thickness D_0

attenuation lengths 3cm	no attenuation
dn=0.0034	**dn=0.006**
k=0.44	**k=0.47**
D_0 = 2.4cm	**D_0 = 3.1cm**
σ at minimum 0.0032	σ at minimum 0.0025

Available space in front of photon detector: 25cm

Track error vs.

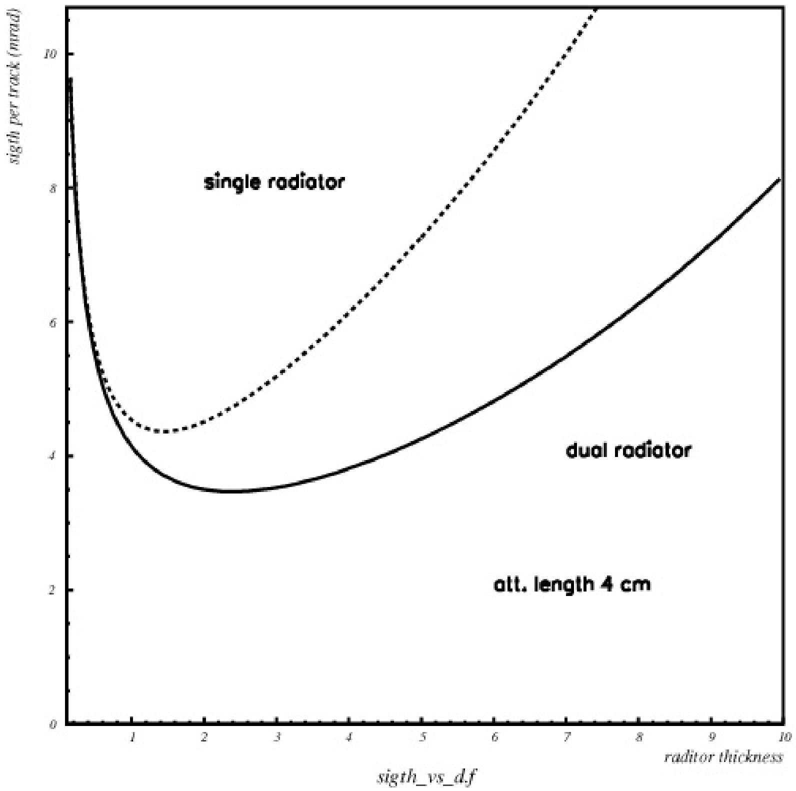
refractive index difference dn and total thickness D_0



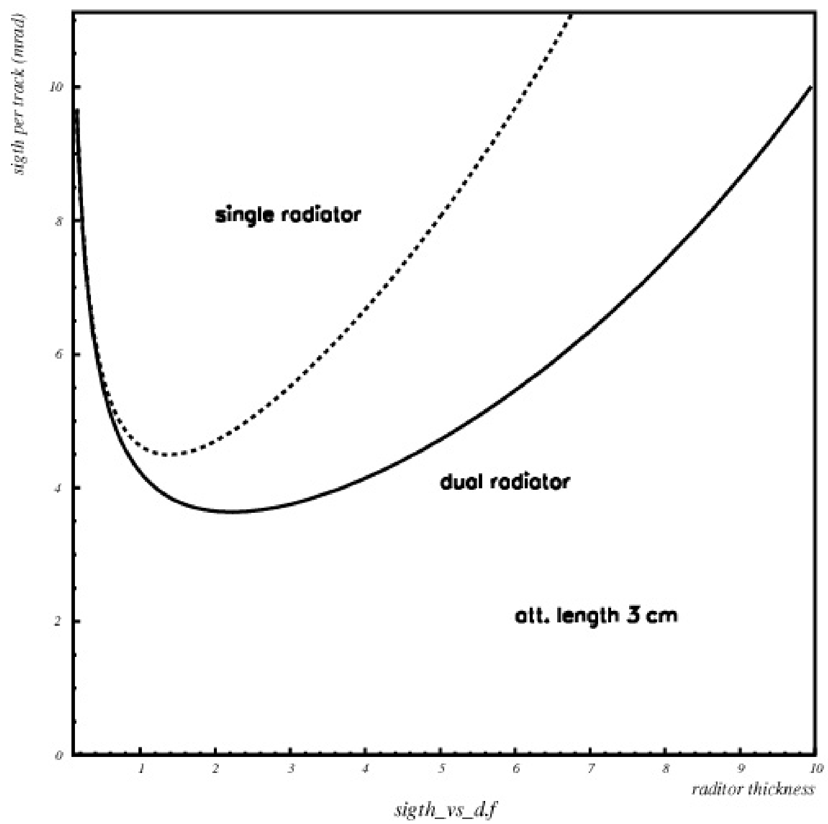
Can we make a simple model of what happens when we go from the single layer to focusing multilayer arrangement?

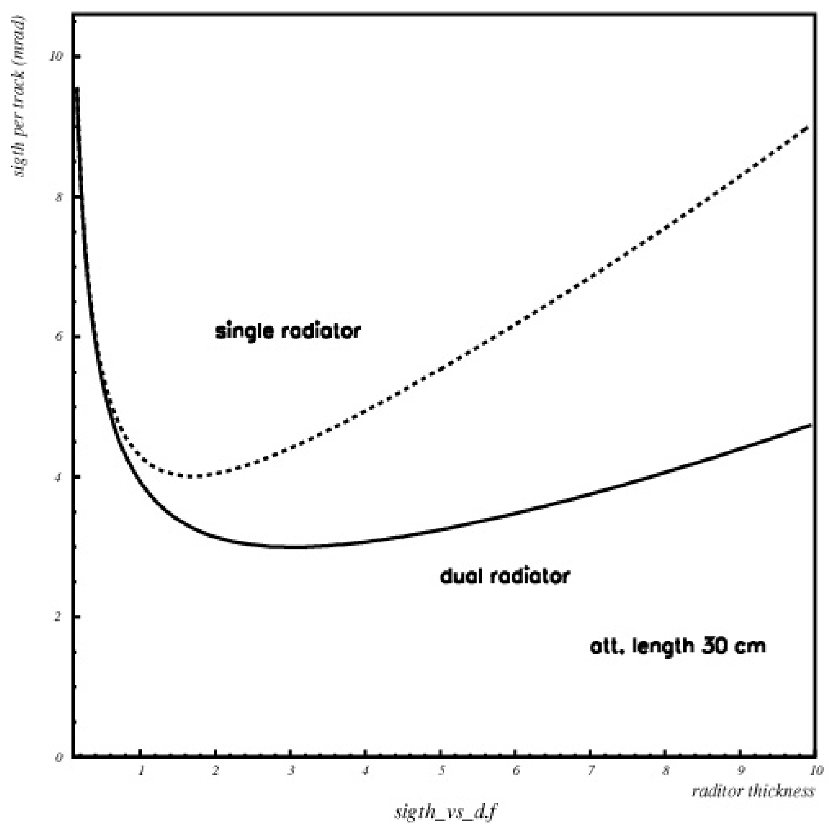
If the indices are well adjusted, the error in emission point goes down by a factor of two in the dual radiator case.

The optimal thickness D_0 increases.



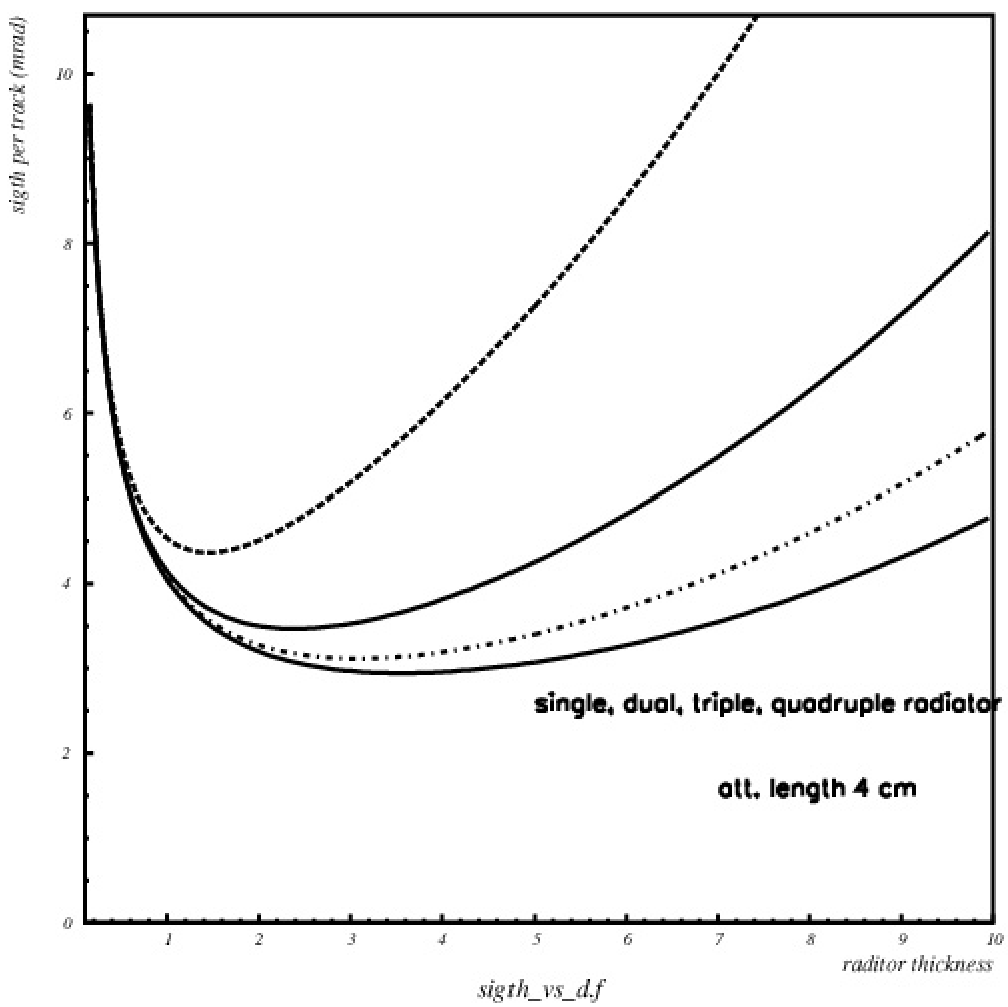
Simple model





Simple model

What happens if we increase the segmentation - number of layers?



Simple model

What happens if we take into account the σ_{rest}=7mrad?

Expect that the minimum in σ_{track} will come later, at higher values of radiator thickness D_0.

Find the minimum of σ_{track} vs D_0 as a function of the number of layers x.

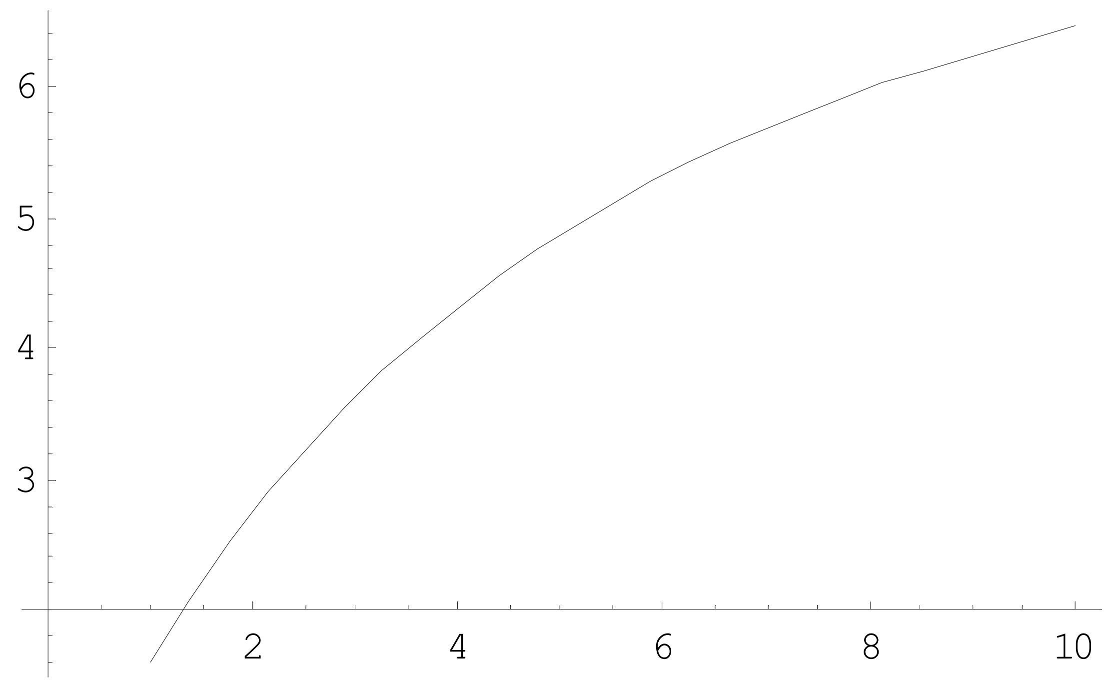
Plot D_0 vs x.

$$D_0^{opt} = -\frac{2\,\mathrm{Lam}}{3} - \Big(2^{1/3}\,\mathrm{Csc}[\Theta]^2\,\mathrm{Sec}[\Theta]^2\,\big(3\,x^2\,\mathrm{Cos}[\Theta]^2\,(12\,L\,\mathrm{sigre}^2 + a^2\,\mathrm{Cos}[\Theta]^4)\,\mathrm{Sin}[\Theta]^2 - 4\,\mathrm{Lam}^2\,\mathrm{Cos}[\Theta]^4\,\mathrm{Sin}[\Theta]^4\big)\Big) \Big/ \Big(3\,\Big(864\,L\,\mathrm{Lam}\,\mathrm{sigre}^2\,x^2\,\mathrm{Cos}[\Theta]^4\,\mathrm{Sin}[\Theta]^4 + 72\,a^2\,\mathrm{Lam}\,x^2\,\mathrm{Cos}[\Theta]^8\,\mathrm{Sin}[\Theta]^4 - 16\,\mathrm{Lam}^3\,\mathrm{Cos}[\Theta]^6\,\mathrm{Sin}[\Theta]^6 + \sqrt{\Big(4\,\big(3\,x^2\,\mathrm{Cos}[\Theta]^2\,(12\,L\,\mathrm{sigre}^2 + a^2\,\mathrm{Cos}[\Theta]^4)\,\mathrm{Sin}[\Theta]^2 - 4\,\mathrm{Lam}^2\,\mathrm{Cos}[\Theta]^4\,\mathrm{Sin}[\Theta]^4\big)^3 + \big(864\,L\,\mathrm{Lam}\,\mathrm{sigre}^2\,x^2\,\mathrm{Cos}[\Theta]^4\,\mathrm{Sin}[\Theta]^4 + 72\,a^2\,\mathrm{Lam}\,x^2\,\mathrm{Cos}[\Theta]^8\,\mathrm{Sin}[\Theta]^4 - 16\,\mathrm{Lam}^3\,\mathrm{Cos}[\Theta]^6\,\mathrm{Sin}[\Theta]^6\big)^2\Big)}\Big)^{1/3}\Big) + \frac{1}{3\,2^{1/3}}\Big(\mathrm{Csc}[\Theta]^2\,\mathrm{Sec}[\Theta]^2\,\Big(864\,L\,\mathrm{Lam}\,\mathrm{sigre}^2\,x^2\,\mathrm{Cos}[\Theta]^4\,\mathrm{Sin}[\Theta]^4 + 72\,a^2\,\mathrm{Lam}\,x^2\,\mathrm{Cos}[\Theta]^8\,\mathrm{Sin}[\Theta]^4 - 16\,\mathrm{Lam}^3\,\mathrm{Cos}[\Theta]^6\,\mathrm{Sin}[\Theta]^6 + \sqrt{\Big(4\,\big(3\,x^2\,\mathrm{Cos}[\Theta]^2\,(12\,L\,\mathrm{sigre}^2 + a^2\,\mathrm{Cos}[\Theta]^4)\,\mathrm{Sin}[\Theta]^2 - 4\,\mathrm{Lam}^2\,\mathrm{Cos}[\Theta]^4\,\mathrm{Sin}[\Theta]^4\big)^3 + \big(864\,L\,\mathrm{Lam}\,\mathrm{sigre}^2\,x^2\,\mathrm{Cos}[\Theta]^4\,\mathrm{Sin}[\Theta]^4 + 72\,a^2\,\mathrm{Lam}\,x^2\,\mathrm{Cos}[\Theta]^8\,\mathrm{Sin}[\Theta]^4 - 16\,\mathrm{Lam}^3\,\mathrm{Cos}[\Theta]^6\,\mathrm{Sin}[\Theta]^6\big)^2\Big)}\Big)^{1/3}\Big)$$

Simple model

How does the optimal thickness D_0 depend on the number of layers?

D_0 opt (cm)



Number of layers

σ_{rest}=7mrad, fixed: distance aerogel–photon detector,

att.len.=4cm

Simple model

What comes next?

- Repeat the optimisation study with σ_{rest}
- Extend the full calculation to the multilayer case
- Study the robustness of the optimum