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CP Violation and Quantum Mechanics in the B System [1]

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Abstract

We discuss the testing of the Standard Model of CP violation, and the search for CP-violating effects from beyond the Standard Model, in B decays. We then focus on the quantum mechanics of the experiments on CP violation to be performed at B factories. These experiments will involve very pretty Einstein-Podolsky-Rosen correlations. We show that the physics of these experiments can be understood without invoking the "collapse of the wave function," and without the mysteries that sometimes accompany discussions of EPR effects.

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Introduction

We are anticipating that future measurements of CP-violating asymmetries in B decays will cleanly and incisively test the Standard Model (SM) description of CP violation. Physics beyond the SM could reveal itself through failures of the SM predictions for these asymmetries. As we shall see, the future experiments will provide a beautiful example of the workings of quantum mechanics.

We first discuss the B-system test of the SM of CP violation. What quantities would one like to measure in order to carry out this test? How can these quantities be measured cleanly? Which B decay modes probe each quantity? Finally, how can physics beyond the SM affect the CP-violating effects to be studied?

We then turn to the quantum mechanics of the planned CP experiments at B factories. These experiments will involve a very pretty modern example of an Einstein–Podolsky–Rosen (EPR) correlation. We show that the experiments can be understood through an approach based entirely on amplitudes, rather than on wave functions. This approach is manifestly covariant, and does not entail the somewhat mysterious "collapse of the wave function" which is usually invoked to describe EPR effects.

CP Violation in the B System

According to the SM, CP violation is a consequence of the fact that in the Cabibbo–Kobayashi–Maskawa (CKM) quark mixing matrix,

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} , \tag{1}$$

some of the elements are not real, but complex. CP-violating effects in B decays can severely test this hypothesis by cleanly determining the phases of various products of CKM elements. In principle, one would like to determine all the independent phases of this kind. How may such independent phases exist, and what are they?

To answer this question, we recall that the SM requires the CKM matrix V to be unitary. This unitarity imposes, among other constraints, the orthogonality conditions

$$\sum_{\alpha=1}^{3} V_{\alpha i} V^*_{\alpha j} = 0; \quad ij = ds, sb, db \, . \tag{2}$$

Here, $V_{\alpha i}$ is an element of V. For given ij, the orthogonality condition (2) is conveniently pictured as the statement that the individual terms in the condition form the sides of a *closed* triangle (a "unitarity triangle") in the complex plane. From the existing information on the magnitudes of the various $V_{\alpha i}$, we expect that the triangle for $ij = db$ (the "db unitarity triangle") will have sides of comparable length, so that its interior angles α, β, and γ may all be large. This triangle is shown schematically in Fig. 1. By contrast, we expect that in the sb unitarity triangle, the $V_{us}V^*_{ub}$ side is only ~ 0.02 as long as the other two sides. Thus, the angle opposite the $V_{us}V^*_{ub}$ side, which we shall call ϵ, is $\lesssim 0.02$ radians. Similarly, we expect that in the ds triangle, the $V_{td}V^*_{ts}$ side is only ~ 0.002 as long as the other sides, so that the angle ϵ' opposite this short side is $\lesssim 0.002$ radians. Suppose, now, that ϕ is the phase of some convention-independent product of CKM elements. Then it can be shown[1] that

$$\phi = n_\alpha \alpha + \eta_\beta \beta + \eta_\epsilon \epsilon + \eta_{\epsilon'} \epsilon' \ , \tag{3}$$

where $\eta_{\alpha,\beta,\epsilon,\epsilon'}$ are integers. Thus, the four angles α, β, ϵ, and ϵ' in the unitarity triangles are the independent phases of all possible (convention-independent) products of CKM elements. Since the phases of CKM products are the quantities which will be determined by the experimental studies of CP violation in the B system, these studies may be thought of as probes of these four angles. Quite possibly, the angle ϵ', which is at most a few milliradians and leads to CP-violating effects which, correspondingly, are at most a few parts per 10^3, will prove to be beyond experimental reach. However, experiments which hopefully will determine the remaining angles, α, β, and ϵ, are actively being developed.

Wolfenstein has introduced a very good ($\sim 3\%$) approximation to the CKM matrix[2] in which $\epsilon = \epsilon' = 0$. In this approximation, the only nontrivial independent phase angles are α and β in the *db* triangle. For this reason, in the literature, attention has been properly focussed on this triangle.

The goals of the experiments aimed at testing the SM of CP violation through studies of the B system can be summarized in the following way: First, to measure the four independent angles of the unitarity triangles, or at least three of them $(\alpha, \beta, \epsilon)$. Attention will be focussed first on the angles α and β, since these may both be large. Secondly, to overconstrain the system as much as possible. To do so, one can: a) See if CP asymmetries in different decay modes, which all yield the same angle (say, β) *if* the SM of CP violation is correct, actually yield the same numerical result. (b) Measure independently the angles α, β, and the dependent angle γ in the *db* triangle and see whether these three angles actually add up to π. (c) Measure the lengths of the sides of the *db* triangle (via studies on non-CP-violating effects such as decay rates and neutral B mixing), and then see whether the interior angles implied by these lengths agree with those inferred directly from CP-violating asymmetries. Needless to say, overconstraining the system in these ways will enable one to test whether the SM provides a consistent picture of CP-violating phenomena, or leads to inconsistencies which point to physics beyond the SM.

The B decays that can yield *clean* information about the angles in the unitarity triangles are, for the most part, decays of the neutral B mesons $B_d(\bar{b}d)$ and $B_s(\bar{b}s)$. The physics of the $B_s - \overline{B_s}$ system is similar to that of the $B_d - \overline{B_d}$ system,[3] so we shall discuss only the latter. The key feature of the $B_d - \overline{B_d}$ system is that the $B_d(\bar{b}d)$ and the $\overline{B_d}(b\bar{d})$ mix. In the SM, this mixing is due largely to the WW box diagram in Fig. 2. The $B_d \to \overline{B_d}$ amplitude induced by this higher-order diagram is a suppressed one, so that $B_d \to \overline{B_d}$ mixing mechanisms from beyond the SM could conceivably compete with or even dominate over the SM diagram. Thus, the modication of $B - \bar{B}$ mixing is perhaps the most promising route through which non-SM physics could modify CP violation in the B system.

The physics of the $B_d - \overline{B_d}$ system is well-known. However, it is an important background to the quantum mechanical discussion of the next Section, so we shall briefly review it. The $B_d - \overline{B_d}$ system has two mass eigenstates, B-heavy (B_H) and B-light (B_L), given by

$$\mid B_{H(L)} \rangle \ = \frac{1}{\sqrt{2}} \left[\mid B_d \ \rangle \ \overset{+}{(-)} \ \omega_{Mix} \mid \overline{B_d} \rangle \right] \ . \tag{4}$$

Here, $\omega_{Mix} \equiv \left[A\left(B_d \to \overline{B_d}\right) / A\left(\overline{B_d} \to B_d\right) \right]^{\frac{1}{2}}$, where, here and hereafter, we use the letter A to denote an amplitude. Empirically, $|\omega_{Mix}|$ is known to be very close to unity, so that ω_{Mix} is just a phase factor. Thus, only the *phase* of $B - \bar{B}$ mixing affects B_H and B_L, and, through them, CP violation in neutral B decay.

In the *SM*, $A(B_d \to \overline{B_d})$ is given by the box diagram in Fig. 2. The amplitude $A(\overline{B_d} \to B_d)$ is then given by the same box diagram, but with every quark (antiquark) replaced by an antiquark (quark). This replacement has the effect that every CKM factor in $A(B_d \to \overline{B_d})$ is replaced by its complex conjugate in $A(\overline{B_d} \to B_d)$ Thus,

$$\omega_{Mix} = \frac{V_{td}V_{tb}^*}{V_{td}^*V_{tb}} \equiv e^{-2i\delta_{CKM}^{Mix}} \quad , \tag{5}$$

where δ_{CKM}^{Mix} is the "$B_d - \overline{B_d}$ mixing phase."

We shall write the masses of B_H and B_L as

$$m_{H(L)} = m \overset{+}{(-)} \frac{\Delta m}{2} - i\frac{\Gamma}{2} \quad , \tag{6}$$

where m is their average mass, Δm is their mass difference, and Γ is their width, which they are expected to have in common.

Suppose that at a time $t = 0$ a free neutral B is a pure $|B_d>$. Due to the mixing, at a later time t it will no longer be a pure $|B_d>$, but will have evolved into a state $|B_d(t)>$ which is a superposition of $|B_d>$ and $|\overline{B_d}>$ given by

$$| B_d(t) > \; = e^{-i(m-i\frac{\Gamma}{2})t}\{c \mid B_d > -ie^{-2i\delta_{CKM}^{Mix}} \; s \mid \overline{B_d} >\} \quad . \tag{7}$$

Here, $c \equiv \cos(\frac{\Delta m}{2}t)$ and $s \equiv \sin(\frac{\Delta m}{2}t)$. Note from Eq. (7) that, until it decays into some final state, the B which at $t = 0$ was a pure $|B_d>$ oscillates back and forth between being a pure $|B_d>$ and a pure $|\overline{B_d}>$. This behavior will be important in the discussion of quantum mechanics in the next Section.

The decay $B_d(t) \to f$ of the time-evolved particle $B_d(t)$ into some final state f has a time-dependent rate, $\Gamma_f(t)$, which from Eq. (7) is given by

$$\Gamma_f(t) = |< f \mid T \mid B_d(t) >|^2 = e^{-\Gamma t} \mid c < f \mid T \mid B_d > - \, ie^{-2i\delta_{CKM}^{Mix}} s < f \mid T \mid \overline{B_d} >|^2 \quad . \tag{8}$$

Let us assume that in this expression, the amplitude $< f|T|B_d >$ for the pure $|B_d>$ decay is dominated by a single Feynman diagram. This assumption, which is expected to be a good one for some of the more important decay modes, is essentially the only assumption that the standard analysis of neutral B decays entails. With this assumption, we may write

$$< f \mid T \mid B_d > \; = Me^{i\delta_{CKM}^f}e^{i\alpha_{ST}} \tag{9}$$

Here, M is the magnitude of the dominating diagram, δ_{CKM}^f is the phase of the product of CKM elements to which this diagram is proportional, and α_{ST} is a phase due to strong-interaction effects. Suppose, now, that f is a CP eigenstate, so that $CP|f> = \eta_f|f>$, with η_f the CP parity of f. Then

$$\begin{aligned} < f \mid T \mid \overline{B_d} > &= \eta_f < CP[f] \mid T \mid CP[B_d] > \\ &= \eta_f Me^{-i\delta_{CKM}^f}e^{i\alpha_{ST}} \quad . \end{aligned} \tag{10}$$

Here, we have used the fact that amplitudes for CP-mirror-image processes, such as $< f|T|B_d >$ and $< CP[f]|T|CP[B_d] >$, have opposite CKM phase. This is due to the circumstance that every quark in a process is replaced by its antiquark in the CP- mirror-image process, so that, as previously mentioned, every CKM factor is replaced by its complex conjugate. We have also used the fact that, apart from CKM phases, SM amplitudes are CP invariant, so that $< f|T|B_d >$ and $< CP[f]|T|CP[B_d] >$ have the same magnitude and strong phase. From Eqs. (8),(9), and (10), we have

$$\Gamma_f(t) = M^2e^{-\Gamma t}\left\{1 - \eta_f \sin\phi \sin(\Delta m t)\right\} \quad , \tag{11}$$

with

$$\phi \equiv 2(\delta_{CKM}^{Mix} + \delta_{CKM}^f) \quad . \tag{12}$$

When f is a CP eigenstate, the CP mirror image of $B_d(t) \to f$ is $\overline{B_d}(t) \to f$, where the time–evolved state $\overline{B_d}(t)$ was a pure $\overline{B_d}$ at $t = 0$. Since SM amplitudes for CP-mirror-image processes are identical except for having opposite CKM phases, Eq. (11) implies that the rate for $\overline{B_d}(t) \to f, \bar{\Gamma}_f(t)$, is given by

$$\bar{\Gamma}_f(t) = M^2 e^{-\Gamma t} \left\{ 1 + \eta_f \sin\phi \sin(\Delta m t) \right\} \quad . \tag{13}$$

The CP-violating asymmetry between $\bar{\Gamma}_f(t)$ and $\Gamma_f(t)$ is then

$$\frac{\bar{\Gamma}_f(t) - \Gamma_f(t)}{\bar{\Gamma}_f(t) + \Gamma_f(t)} = \eta_f \sin\phi \sin(\Delta m t) \quad . \tag{14}$$

The mass difference Δm is known (at least for the B_d system), and η_f will be known for any chosen final state f, so a measurement of this asymmetry will cleanly determine $\sin\phi$.[4] Note from Eq. (12) that the angle ϕ that is determined in this way is, as previously stated, the phase of a product of CKM elements.

While the case where f is a CP eigenstate is, both theoretically and experimentally, the simplest one, clean information on the phases of products of CKM elements can also be extracted from many hadronic $\overset{(-)}{B_d}(t)$ and $\overset{(-)}{B_s}(t)$ decays where f is not a CP eigenstate.[5] The angle ϕ determined by decay into a final state which is not a CP eigenstate is no longer given by Eq. (12), but is still the relative CKM phase of the two interfering amplitudes in the expression (8) for the decay rate.

In Table 1, we list some decay modes that are being considered as possible probes of the various angles in the unitarity triangles. The last column of this Table gives the angle that can be cleanly determined via study of each mode. The one charged B decay listed in the Table illustrates the fact that occasionally even charged B decays can provide clean CKM phase information.

Experiments based on the decay modes of Table 1 and others will be carried out both at dedicated high–luminosity e^+e^- colliders("B factories")[11] and at hadron facilities.[12] These experiments, with their differing strong points, should prove to be quite complementary.

How could physics beyond the SM (PBSM) affect CP violation in B decays? As we have seen, $B - \bar{B}$ mixing, being suppressed in the SM, is perhaps the ingredient of CP violation in the B system most susceptible to the effects of PBSM. Let us briefly mention three examples of non-SM physics which, conceivably, could modify CP violation in B decays by altering $B - \bar{B}$ mixing.

1. Suppose that, in addition to the three known quark $SU(2)_L$ doublets, there is also a charge -1/3 singlet.[13] Then, the Z boson can have quark couplings which, in mass-eigenstate basis, are nondiagonal. These nondiagonal couplings can carry phases beyond those in the 3 x 3 CKM matrix of the SM. Thus, one can have at tree level the process $\bar{b}d \to Z \to \bar{d}b$, and this non-SM contribution to $B_d - \overline{B_d}$ mixing can carry a non - SM phase. By modifying the phase of $B_d - \overline{B_d}$ mixing, this contribution can modify CP violation in $\overset{(-)}{B_d}(t)$ decay.

2. Suppose that nature contains not just one Higgs multiplet, as in the SM, but several. Then there can be spontaneous CP violation - a condition in which different neutral Higgs fields develop vacuum expectation values which, relative to one another, are not real. New contributions to $B - \bar{B}$ mixing can include tree-level processes of the type $\bar{b}d \to H \to \bar{d}b$, where H is some neutral Higgs boson. The spontaneous CP violation can impart to these processes nontrivial, non-SM phases. Then, the phase of $B - \bar{B}$ mixing, and consequently CP violation in neutral B decay, is altered.[14]

3. Suppose that the world is described by some version of supersymmetry (SUSY). In non-minimal SUSY models, there can be new phases beyond those in the CKM matrix, and CP violation in the B system can be altered substantially.[15] However, there are also minimal SUSY models in which the avoidance of potentially large flavor changing neutral currents has the consequence that there are *no* extra phases beyond those in the CKM matrix. Now, these models do contain non-SM contributions to $B - \bar{B}$ mixing, such as the gluino-squark box diagram in Fig. 3. Naively, one might imagine that these extra contributions have a different dependence on the CKM phases than does the SM box diagram of Fig. 2, so that the phase of $B - \bar{B}$ mixing will differ from its SM value. However, in reality the CKM phases of the extra contributions are to a very good approximation identical to the CKM phase of the SM box diagram, so that the phase of $B - \bar{B}$ mixing is *the same* as in the SM.[16, 15] Now, we have seen that it is only the phase of the $B - \bar{B}$ mixing amplitudes, and not their magnitudes, that influences CP violation. Thus, CP violation in neutral B decay will also be the same as in the SM.

It is interesting to ask whether the SUSY contributions to $B - \bar{B}$ mixing, while not affecting CP violation, could still be uncovered by overconstraining the B system. It is estimated that these contributions could change the magnitudes of the mixing amplitudes, and consequently the neutral B mass differences Δm, by (10-20)%.[17, 18] In the SM, the difference Δm_d between the masses of the mass eigenstates of the $B_d - \overline{B_d}$ system arises from the box diagram in Fig. 2, and so is proportional to $|V_{td}|^2$. The analogous mass difference Δm_s in the $B_s - \overline{B_s}$ system arises from a similar diagram in which the d quarks have been replaced by s quarks, and so is proportional to $|V_{ts}|^2$. Thus, in the SM,

$$\sqrt{\frac{\Delta m_d}{\Delta m_s}} = C \frac{| V_{td} |}{| V_{ts} |} \quad , \tag{15}$$

where the coefficient C is expected to be approximately unity, and, more precisely, is estimated to be 0.86 ± 0.1.[19] Now, we know from unitarity that $|V_{ts}| \cong |V_{cb}|$, and $|V_{cb}|$ is known to $\sim 15\%$. Thus, if $B - \bar{B}$ mixing comes only from the SM box diagram, a measurement of $\Delta m_d / \Delta m_s$ would determine $|V_{td}|$. Since $|V_{tb}| \cong 1$, we would then know the length of the $V_{td}V_{tb}$ side of the unitarity triangle of Fig. 1. If we had also learned, from other sources, the lengths of the other two sides, we could then deduce the interior angles.

Now, if $B - \bar{B}$ mixing actually contains significant non-SM contributions from SUSY, then one might expect the SM relation (15) to fail. The length $|V_{td}V_{tb}| \cong |V_{td}|$ of the $V_{td}V_{tb}$ side of the unitarity triangle deduced from $\Delta m_d / \Delta m_s$ by using the relation (15) would then be wrong. Thus, the interior angles of the triangle inferred from the alleged lengths of the sides would be wrong as well, and would disagree with the true interior angles determined by measurements of CP - violating asymmetries. This disagreement would be the signal of physics beyond the SM.

Unfortunately, it appears that the SUSY correction to $\Delta m_q (q = d \text{ or } s)$ is proportional to its SM value. Thus, although SUSY may change Δm_d and Δm_s individually by $\sim 20\%$, it does not visibly alter their ratio from its SM size.[17, 20] Hence,unlike the other models we mentioned, this minimal version of SUSY is an interesting example of non-SM physics whose presence could not be detected by studies of CP violation or mixing in neutral B decays.

Quantum Mechanics at B Factories

The studies of CP violation to be performed at B factories will involve some very interesting quantum mechanics, to which we now turn.[21]

At the B factories, the B mesons whose decays are to be studied will be produced via the reaction $e^+e^- \to \Upsilon(4S) \to BB$. The $\Upsilon(4S)$ is a $\bar{b}b$ bound state which decays into B pairs essentially 100% of the time. Roughly half of its decays yield B^+B^-, and the other half $B_d\overline{B_d}$. (The $\Upsilon(4S)$ is not heavy enough to decay to $B_s\bar{B}_s$.)

Since it is mostly the decays of neutral B mesons which can yield clean CKM phase information, we shall be interested in events where $\Upsilon(4S) \to B_d\overline{B_d}$. Now, the $\Upsilon(4S)$ has spin of unity, and B mesons are spinless, so the B pair produced by $\Upsilon(4S) \to B_d\overline{B_d}$ is in a p wave . Let us view this pair in the $\Upsilon(4S)$ rest frame, where the B mesons are moving outward in opposite directions from the $\Upsilon(4S)$ decay point. Due to $B_d - \overline{B_d}$ mixing, each of the two B mesons is oscillating back and forth between pure B_d and pure $\overline{B_d}$ (see Eq. (7)). However, at no time may we have two identical spinless bosons in a p wave. Thus, if at some time t one of the B mesons is found to be, say, a $\overline{B_d}$, then at this time the other B meson must be a B_d. This is a modern example of an Einstein-Podolsky-Rosen (EPR) correlation.[22]

This EPR correlation plays a crucial role in the traditional description[23] of a typical B factory experiment. Let us recall this description. The sequence of events in the experiment, viewed from the $\Upsilon(4S)$ rest frame, is shown in Fig. 4. At a time we shall call $t = 0$, the $\Upsilon(4S)$ decays into a pair of neutral B mesons, which move outward back to back. At a subsequent time t_ℓ, one of the B mesons decays semileptonically, and we shall suppose that it yields, in particular, a negatively-charged lepton ℓ^-, plus other particles X. At another time t_{CP}, the other B meson decays into a hadronic CP eigenstate, which we shall take for illustration to be $\pi^+\pi^-$.

Let us suppose first that the semileptonic decay occurs before the one to $\pi\pi$: $t_\ell < t_{CP}$. Now, only a $\overline{B_d}$, but not a B_d, can decay to a *negative* lepton, so the charge of the ℓ^- in Fig. 4 indentifies its parent as being a $\overline{B_d}$ at the instant of decay, t_ℓ. Since, at any given time, one cannot have two identical bosons in a p wave, this means that *at the same instant* t_ℓ the other B, on the right in Fig. 4, must be a pure B_d. The decay of one B into an ℓ^- has "collapsed the wave function" for the BB state, leaving behind a single B whose state, at the time of collapse, is known precisely. Of course, subsequent to the time t_ℓ, the surviving B will oscillate between pure B_d and pure $\overline{B_d}$ because of mixing. Taking advantage of the fact that, in the $\Upsilon(4S)$ rest frame, the B mesons are rather nonrelativistic ($\frac{v}{c} \sim 0.06$), we may neglect their motion. The probability for the B which survives beyond t_ℓ to decay into the CP eigenstate $\pi^+\pi^-$ is then given by the B-rest-frame expression (11), in which to a very good approximation we need not distinguish between time in the B rest frame and in the $\Upsilon(4S)$ frame. However, we must note that the time variable t in expression (11) represents the time of the decay to the CP eigenstate (here, t_{CP}) relative to the time when the parent B was known to be pure B_d. In the present case, the latter time is not $t = 0$, the instant when the B was born, but $t = t_\ell$, the instant when the *other* B decayed to an ℓ^-. That is, in applying Eq. (11), we must take $t = t_{CP} - t_\ell$. The joint probability for one B to decay to ℓ^-X at time t_ℓ and the other to decay to $\pi^+\pi^-$ at time t_{CP}, $\Gamma\left[\text{One } B \to \ell^-X \text{ at } t_\ell \text{ ; Other } B \to \pi^+\pi^- \text{ at } t_{CP}\right]$, is then given by

$$\begin{aligned}
&\Gamma\left[\text{One } B \to \ell^-X \text{ at } t_\ell;\ \text{Other } B \to \pi^+\pi^- \text{ at } t_{CP}\right] \\
&\propto e^{-\Gamma t_\ell} e^{-\Gamma t_\ell} e^{-\Gamma(t_{CP}-t_\ell)} \{1 - \sin\phi \sin[\Delta m(t_{CP} - t_\ell)]\} \\
&= e^{-\Gamma(t_{CP}+t_\ell)} \left\{ 1 - \sin\phi \sin[\Delta m(t_{CP} - t_\ell)] \right\} \ .
\end{aligned} \tag{16}$$

In the first part of Eq. (16), the first factor, $e^{-\Gamma t_\ell}$, is the probability for the B which yields ℓ^-X to survive until time t_ℓ; the second factor, $e^{-\Gamma t_\ell}$, is the probability for the B which will eventually yield $\pi^+\pi^-$ to survive until time t_ℓ; and the remaining factors are, from Eq. (11), the probability for the latter B to decay to $\pi^+\pi^-$, given that at time t_ℓ it was a pure B_d. In writing Eq. (16), we have used the fact that for $\pi^+\pi^-$, $\eta_f = +1$.

What if the decay to $\pi^+\pi^-$ occurs before the semileptonic one ($t_{CP} < t_\ell$)? To answer this question, we note that there is always a linear combination of $|B_d>$ and $|\overline{B_d}>, |B_{no}> \propto <\pi^+\pi^-|T|\overline{B_d}>|B_d> - <\pi^+\pi^-|T|B_d>|\overline{B_d}>$, which has vanishing amplitude for decay to $\pi^+\pi^-$. Now, it is obvious that if the B meson on the right in Fig. 4 decays to $\pi^+\pi^-$, then at the time of its decay, t_{CP}, it is not in the state $|B_{no}>$. But then, since one cannot have two identical bosons in a p wave, at this same time, t_{CP}, the B meson on the left in Fig. 4 *is* in the state $|B_{no}>$. More generally, if $\Upsilon(4S) \to BB$ (where the B mesons are neutral), and one of the B mesons decays to some final state f, then the other B meson cannot decay to the same final state *at the same time*.[24, 25] Thus, the decay of one B to $\pi^+\pi^-$ "collapses the BB wave function" and fixes the state of the remaining B as $|B_{no}>$ at time t_{CP}. The state of this remaining B may then be evolved forward in time from the instant t_{CP} using the Schrödinger equation for the $B_d - \overline{B_d}$ system, and one may calculate the amplitude for the time-evolved state to decay to $\ell^- X$ at time t_ℓ. Interestingly, one finds that the joint probability for one B to decay to $\pi^-\pi^-$ at time t_{CP}, and the other to $\ell^- X$ at time $t_\ell > t_{CP}$, is given by the same expression as before, Eq. (16). This equation holds regardless of the order of events.[26]

Having reviewed the traditional description of this typical B-factory experiment, let us ask how our picture of the experiment is modified if we require consistency with relativity and take the motion of the B mesons fully into account. When relativity is not neglected, several issues arise: First, in relativity, the simultaneity of two events depends on the frame of reference. Thus, if in the traditional treatment one asserts that the decay of one B fixes the state of the other B *at the same time,* one must specify in which frame of reference this assertion is true. Which frame is it, and why? Secondly, in some events, the separation between the two B decays in Fig. 4 will be spacelike. Then, which B decays first, collapsing the wave function and fixing the state of the other B, will depend on the frame of reference. That is, in the traditional treatment, the same collection of occurences will be described quite differently in different frames. It would appear that an alternative treatment which is not so strongly frame dependent would be advantageous. Thirdly, what are the relativistic corrections to Eq. (16), the expression which will be used to extract the values of CKM phases ϕ from B-factory experiments?

Let us generalize the decay sequence of Fig. 4 to include any chain of the form,

$$\begin{array}{llll} \Upsilon(4S) \to B & + & B & , \\ & \hookrightarrow f_1(t_1, \vec{x}_1) & \hookrightarrow f_2(t_2, \vec{x}_2) & \end{array} \tag{17}$$

where the B mesons are neutral, the f_j are arbitrary final states, and $(t_j, \vec{x_j})$ is the spacetime point where the decay to f_j occurs. To take relativity into account and address the issues just raised, let us treat any chain of this type by directly calculating the amplitude for it, [21] without introducing the wave function for the BB state, or invoking the collapse of this wave function. To calculate the amplitude for (17), it is convenient to work in the B mass eigenstate basis. The amplitude has two terms. One of these describes the process in which the B meson which decays to f_1 is a B_H, while the one which decays to f_2 is a B_L. The other term describes a process in which the roles of B_H and B_L are interchanged. Since the $B_H - B_L$ mass difference is tiny, these two processes are experimentally indistinguishable, so their amplitudes must be added coherently. Owing to the antisymmetry of the amplitude for $\Upsilon(4S) \to BB$, it is not possible for both B mesons to be B_H or B_L.

The amplitude A_{HL} for the process where it is B_H which decays to f_1 and B_L which decays to f_2 is given by

$$A_{HL} = A(B_H \ \text{to}\ 1; B_L \ \text{to}\ 2) e^{-im_H\tau_1} e^{-im_L\tau_2} A(B_H \to f_1) A(B_L \to f_2) \ . \tag{18}$$

Here, $A(B_H \ \text{to}\ 1; B_L \ \text{to}\ 2)$ is the amplitude for an $\Upsilon(4S)$ to decay to a B_H moving towards the point $(t_1, \vec{x_1})$ and a B_L moving towards $(t_2, \vec{x_2})$. The only feature of this amplitude which will be relevant is its antisymmetry under $B_H \leftrightarrow B_L$. The factor $\exp[-im_H\tau_1]$ is the amplitude for the B_H, which has complex mass m_H(including its width), to propagate from the spacetime point where it is produced by the $\Upsilon(4S)$ decay to the point $(t_1, \vec{x_1})$ where it decays. In this factor, τ_1 is the proper time, in the B_H rest frame, which elapses during this propagation. Similarly, the factor $\exp[-im_L\tau_2]$ is the amplitude for the B_L to propagate to $(t_2, \vec{x_2})$. That the amplitude for a particle of mass M to propagate for a proper time τ is $\exp[-iM\tau]$ may be understood by solving Schrödinger's equation for the time evolution of such a particle in its rest frame, and then noticing that the solution, $\exp[-iM\tau]$, is Lorentz-invariant.[27] Finally, the factor $A(B_H \to f_1)$ in A_{HL} is the amplitude for B_H to decay to f_1, and similarly for $A(B_L \to f_2)$. If the various A's on the right-hand side of Eq. (18) are invariants, A_{HL} is an invariant.

Adding to the A_{HL} of Eq. (18) the analogous expression for $A_{LH} \equiv A_{HL}(B_H \leftrightarrow B_L)$, and taking into account the antisymmetry of $A(B_H \text{ to } 1; B_L \text{ to } 2)$ under $B_H \leftrightarrow B_L$, we find that the complete amplitude A for the decay chain (17) is given by

$$\begin{aligned} A \;\propto\; & e^{-im_H\tau_1}e^{-im_L\tau_2}A(B_H \to f_1)A(B_L \to f_2) \\ & -e^{-im_L\tau_1}e^{-im_H\tau_2}A(B_L \to f_1)A(B_H \to f_2) \ . \end{aligned} \tag{19}$$

Obviously, the amplitude approach which has yielded this simple result is quite general. It can be applied, for example, to multibody sequences of the form

$$\begin{array}{cccccccc} P \to & A & + & B & + & C & + \cdots \ , \\ & \hookrightarrow f_1 & & \hookrightarrow f_2 & & \hookrightarrow f_3 & \end{array} \tag{20}$$

in which P can be a single particle or two particles which have collided, and the "decays" $A \to f_1$, etc., can alternatively be measurements of various properties of the particles A,B,...

In the traditional collapsing wave function description of decay sequences of the type (17), one invokes the fact that if one of the B mesons decays to some final state f at a time t, then the other B cannot decay to the same final state *at the same time*. This fixes the state of the surviving B at time t. However, if relativity is not neglected, then, as we have noted, one must ask *in which Lorentz frame* the simulaneous decay of the two B mesons to the same final state is impossible, so that the decay of the one fixes the state of the other. The amplitude (19) answers this question. For, if we take $f_1 = f_2$, then this amplitude vanishes for $\tau_1 = \tau_2$. That is, the two B mesons cannot decay in the same way at equal *proper* times in their respective rest frames. This implies that it is the $\Upsilon(4S)$ rest frame in which they cannot decay to the same final state simultaneously. For, in the $\Upsilon(4S)$ frame the two B mesons have equal speed, so that, after time dilation, equal proper times in the two B rest frames correspond to a single, common time in the $\Upsilon(4S)$ frame.

Let us now apply our general amplitude (19) to the specific decay chain of most interest to B factories,

$$\Upsilon(4S) \to \underset{\llcorner\!\!\to \ell^- X}{B} \quad + \quad \underset{\llcorner\!\!\to f_{CP}}{B} \qquad , \tag{21}$$

where f_{CP} is a CP eigenstate. Taking $f_1 = \ell^- X$ in (19) and using Eqs.(4) and (5) and the fact that only a $\overline{B_d}$, but not a B_d, can decay to $\ell^- X$, we have

$$\begin{aligned} A(B_{H(L)} \to f_1) \quad &= \quad < \ell^- X \mid T \mid B_{H(L)} > \\ &= \overset{+}{(-)}\, \frac{1}{\sqrt{2}} \; e^{\,-2i\delta^{Mix}_{CKM}} < \ell^- X \mid T \mid \overline{B_d} > \quad . \end{aligned} \tag{22}$$

Taking $f_2 = f_{CP}$, assuming as before that one diagram dominates $B_d \to f_{CP}$, and using Eqs. (4), (5), (9), and (10), we have

$$\begin{aligned} A(B_{H(L)} \to f_2) &= < f_{CP} \mid T \mid B_{H(L)} > \\ &= \frac{1}{\sqrt{2}} M e^{i\delta^f_{CKM}} e^{i\alpha_{ST}} (1 \; \overset{+}{(-)} \; \eta_f e^{-i\phi}) \quad . \end{aligned} \tag{23}$$

Here, η_f is the CP parity of the final state, and the CKM phase $\phi \equiv 2(\delta^{Mix}_{CKM} + \delta^f_{CKM})$, as before, (cf. Eq. (12)). Let us write the proper time τ_1 of the decay to $\ell^- X$ as τ_ℓ, the proper time τ_2 of the decay to f_{CP} as τ_{CP}, and the masses $m_{H(L)}$ of $B_{H(L)}$ as in Eq. (6). Then, omitting irrelevant overall factors, we find from Eqs. (19), (22), and (23) that the amplitude A for the decay chain (21) is given by

$$\begin{aligned} A \quad \propto \quad & e^{-\frac{\Gamma}{2}(\tau_{CP}+\tau_\ell)} \{ \cos[\frac{\Delta m}{2}(\tau_{CP} - \tau_\ell)] \\ & - i\eta_f e^{-i\phi} \sin[\frac{\Delta m}{2}(\tau_{CP} - \tau_\ell)] \} \quad . \end{aligned} \tag{24}$$

Taking the absolute square of this expression, we find for the joint probability for one B to decay to $\ell^- X$ at proper time τ_ℓ, and the other to decay to f_{CP} at proper time τ_{CP},

$$\begin{aligned} & \Gamma \;\; [\text{One } \; B \to \ell^- X \; \text{ at } \; \tau_\ell ; \;\; \text{Other } \; B \to f_{CP} \; \text{ at } \; \tau_{CP}] \\ & \propto \quad e^{-\Gamma(\tau_{CP}+\tau_\ell)} \{ 1 - \eta_f \sin\phi \sin[\Delta m(\tau_{CP} - \tau_\ell)] \} \quad . \end{aligned} \tag{25}$$

This result agrees with the one of Eq. (16), obtained by collapsing the BB wave function. (The factor η_f is absent from Eq. (16) because that expression was derived for the illustrative case where $f_{CP} = \pi^+\pi^-$, a CP eigenstate with $\eta_f = +1$.) However, the relativistically precise Eq. (25) makes it clear that if the expression (16) for the joint decay probability is to be totally accurate, then the B decay times in it must be taken to be *proper* times, not times in the $\Upsilon(4S)$ rest frame. This is a negligible distinction for $\Upsilon(4S) \to BB$, but for the similar process $\phi \to KK \to (\pi^+\pi^-)(\pi^\circ\pi^\circ)$,[25] to be studied at the ϕ factory DAΦNE,[28] use of ϕ-frame times rather than K-frame proper times would cause a (2-3)% error. It is possible to refine the "wave function collapse" approach so that it takes the motion of the B mesons properly into account and yields exactly the same result, Eq. (25), as the "amplitude" approach, with no ambiguity about the time variables.[29] However, we[29] would not have known how to do so without using the "amplitude" result for guidance.

The amplitude approach has also been applied to $\Upsilon(4S) \to BB \to (\ell X)(f)$, where f is not a CP eigenstate, and to processes where the B meson pair is in a symmetric state, rather than the antisymmetric one of $\Upsilon(4S)$ decay.[29] These applications, like the one to $\Upsilon(4S) \to BB \to (\ell X)(f_{CP})$ which we have discussed, confirm the results of the collapse approach, while taking relativity fully into account, and avoiding the puzzles associated with collapse of the wave function.

Summary

Study of CP violation in B decay will provide a powerful test of the SM of CP violation, and a probe of physics beyond the SM. Both to test, and to look for physics beyond, the SM, it will be very important to overconstrain the B system as much as possible, through measurement of a variety of quantities at different experimental facilities. The measurements to be carried out at the e^+e^- B factories will involve some very pretty quantum mechanics, which can be simply understood via an amplitude approach which does not entail the "collapse of the wave function."

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Figure 1: The db unitarity triangle.

Figure 2: The SM diagram for $B_d \to \overline{B_d}$ mixing.

Figure 3: A non-SM contribution to $B_d \to \overline{B_d}$ mixing in SUSY. The $\tilde{g}$ is a gluino, $\tilde{d}$ and $\tilde{b}$ are squarks, and the symbol X stands for squark mixing.

Figure 4: The events in a typical B factory experiment, viewed in the $\Upsilon(4S)$ rest frame.

Table 1: Decay modes and the phase angle ϕ which they probe. In the final state $\psi K^{*\circ}$, the $K^{*\circ}$ is required to decay as shown. Similarly for the final state $\overset{(-)}{D^\circ}\ K^+$; g_{CP} is a CP eigenstate, such a $\pi^+\pi^-$ or K^+K^-. References are given in the first column.

Ref.	Decay Mode	ϕ
4,6	$B_d(t) \to \pi^+\pi^-, \pi^+\rho^-, \pi^+ \mathrm{a}_1^-$	2α
4,7	$B_d(t) \to \psi K_s, \psi K^{*\circ}$	2β
	$\hookrightarrow K_s\pi^\circ$	
8	$B_s(t) \to D_s^+ K^-$	γ
9	$B^+ \to \overset{(-)}{D^\circ}\ K^+$	γ
	$\hookrightarrow g_{CP}$	
10	$B_s(t) \to \psi\phi$	2ϵ