TOP performance checks - first results

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- Introduction
- Likelihood calculation (analytic)
- Simulation details
- TOP performance vs paramters
- Impact on the design
- Summary

Likelihood for TOP

Log likelihood probability for a given mass hyphothesis:

$$\log \mathcal{L} = \sum_{i=1}^{N} \log(\frac{S(x_{ch},t) + B(x_{ch},t)}{N_e}) + \log P_N(N_e)$$

Where

N is the measured number of photons,

$N_e = N_S^{exp} + N_B^{exp}$ is the expected number of photons (signal+background),

$S(x_{ch},t)$ is 2D distribution of signal photons,

$B(x_{ch},t)$ is 2D distribution of background photons and

$P_N(N_e)$ is the Poisson probability of mean N_e to get N photons.

Distributions S and B are normalised in the way:

$$\sum_{x_{ch}} \int_0^{t_m} S(x_{ch},t)dt = N_S^{exp}, \qquad \sum_{x_{ch}} \int_0^{t_m} B(x_{ch},t)dt = N_B^{exp}$$

Sum runs over all channels x_{ch} and integration over full TDC range.

Note: $S(x_{ch},t)$ and N_S^{exp} are mass hypothesis dependent.

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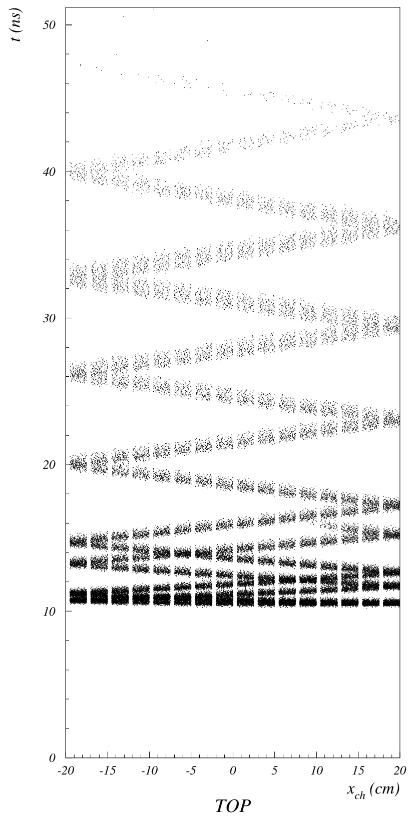
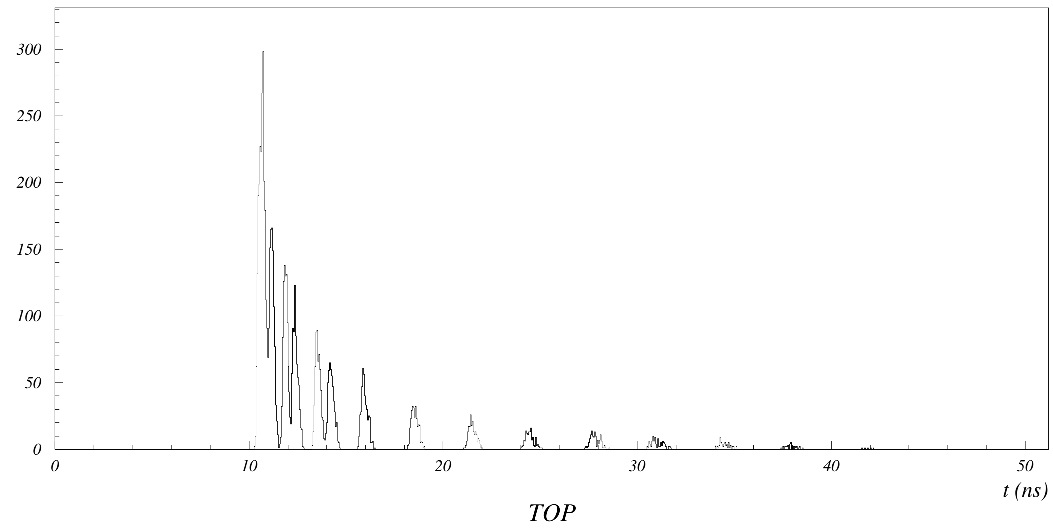
Parametrization of TOP signal distribution





Signal distribution for channel x_{ch} can be parametrized as:

$$S(x_{ch},t) = \sum_{k=1}^{m(x_{ch})} n_k(x_{ch}) g(t - t_k(x_{ch}); \sigma_k(x_{ch}))$$

Where

- n_k is the number of photons in k-th peak,
- $g(t - t_k; \sigma_k)$ is its shape ($\int g(t)dt = 1$),
- t_k is its position and
- σ_k is its width (r.m.s)

Parametrization of TOP distribution

- The goal: find analytical expressions for $n_k(x_{ch})$, $t_k(x_{ch})$ and $\sigma_k(x_{ch})$
- Geometric view of TOP detection: intersection of Čerenkov cone with a plane
  - $\rightarrow$ well known, quadratic equations
  - $\rightarrow$ analytical solutions should exist

Toward the solution ...

- Coordinate system of Q-bar:
  z-axis along Q-bar, parallel to z-axis of Belle
  y-axis perpendicular to Q-bar (along smalles dimension)
  origin in the centre of Q-bar
- Particle traversing the Q-bar at polar angles θ and ϕ
- Čerenkov photon emitted at point $\vec{r}_0 = (x_0, y_0, z_0)$ with polar angles θ_c and ϕ_c in respect to particle direction.
- The photon directional vector, expressed in Q-bar system, is:

$$\vec{k} = \begin{pmatrix} k_x \\ k_y \\ k_z \end{pmatrix} = \begin{pmatrix} \cos\phi(\cos\theta\sin\theta_c\cos\phi_c + \sin\theta\cos\theta_c) - \sin\phi\sin\theta_c\sin\phi_c \\ \sin\phi(\cos\theta\sin\theta_c\cos\phi_c + \sin\theta\cos\theta_c) + \cos\phi\sin\theta_c\sin\phi_c \\ \cos\theta\cos\theta_c - \sin\theta\sin\theta_c\cos\phi_c \end{pmatrix}$$

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Toward analytical solution

- Photon straight line of flight: $\vec{r} = \vec{r}_0 + l\vec{k}$ (l is distance from $\vec{r}_0$ to $\vec{r}$).
- Intersection with detector plane at $z = z_D$:

$$z_D = z_0 + lk_z \quad \Rightarrow \quad l = \frac{z_D - z_0}{k_z}$$

  if length of flight $l > 0$ the intersection is in photon's forward direction and the coordinates of photon hit are:

$$x_D = x_0 + lk_x, \quad y_D = y_0 + lk_y$$

- Time of propagation of photon is

$$t_{TOP} = \frac{l}{c_0} n_g$$

- Total reflections:
  Imagine the detector plane divided into cells of a size of Q-bar transverse dimensions ($a \times b$)
  total reflections - the same as folding the detector plane at cell bounderies

Toward analytical solution

- Number of reflections

$$n_x = nint(x_D/a)$$
$$n_y = nint(y_D/b)$$

- Coordinates at the middle cell (Q-bar exit window)

$$x = \begin{cases} x_D - an_x \ , & n_x = 0, \pm 2, \pm 4, ... \\ an_x - x_D \ , & n_x = \pm 1, \pm 3, ... \end{cases} \quad y = \begin{cases} y_D - bn_y \ , & n_y = 0, \pm 2, \pm 4, ... \\ bn_y - y_D \ , & n_y = \pm 1, \pm 3, ... \end{cases}$$

- Total reflection requirement

$$|k_x| < \sqrt{1 - 1/n^2} \ , \qquad |k_y| < \sqrt{1 - 1/n^2}$$

- In summary - we've found:

$$t_{TOP}(\phi_c) = \frac{(z_D - z_0)n_g}{k_z(\phi_c)c_0} \qquad x_D(\phi_c) = x_0 + \frac{k_x(\phi_c)}{k_z(\phi_c)}(z_D - z_0)$$

$\rightarrow$ eliminate ϕ_c to get $t_{TOP}(x_D)$

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The analytical solution

- Detector plane coordinate of a channel x_{ch} for k-th reflection is

$$x_k = \begin{cases} ka + x_{ch} \ , & k = 0, \pm 2, \pm 4, ... \\ ka - x_{ch} \ , & k = \pm 1, \pm 3, ... \end{cases}$$

- By defining:

$$a_k = \frac{x_0 - x_k}{z_0 - z_D} \cos\theta \cos\theta_c$$
$$b_k = \frac{x_0 - x_k}{z_0 - z_D} \sin\theta \sin\theta_c$$
$$c = \cos\phi \cos\theta \sin\theta_c$$
$$d = \sin\phi \sin\theta_c$$
$$e = \cos\phi \sin\theta \cos\theta_c$$

- The cosine of ϕ_c for k-th peak in channel x_{ch} is:

$$\cos\phi_c^{(k)} = \frac{-(b_k + c)(e - a_k) \pm d\sqrt{d^2 + (b_k + c)^2 - (e - a_k)^2}}{(b_k + c)^2 + d^2}$$

- and the peak position (using mean values for θ_c and n_g):

$$t_k = \frac{z_D - z_0}{(\cos\theta \cos\theta_c - \sin\theta \sin\theta_c \cos\phi_c^{(k)})} \frac{n_g}{c_0}$$

The analytical solution

- Number of photons in the k-th peak:

$$n_k = N_0 l_{track} \sin^2\theta_c \frac{\Delta\phi_c^{(k)}}{2\pi}, \qquad \Delta\phi_c^{(k)} = |\phi_c(x_k + \Delta x_{ch}/2) - \phi_c(x_k - \Delta x_{ch}/2)|$$

- Width of the k-th peak due to dispersion is proportional to t_k:

$$\sigma_k^{disp} = t_k \cdot \left| f(\phi_c^{(k)}) \frac{1}{n}\frac{dn}{de} + \frac{1}{n_g}\frac{dn_g}{de} \right| \sigma_e$$

where

$$f(\phi_c^{(k)}) = \frac{(\cos\theta\sin\theta_c + \sin\theta\cos\theta_c\cos\phi_c^{(k)})}{(\cos\theta\cos\theta_c - \sin\theta\sin\theta_c\cos\phi_c^{(k)})} \cdot \frac{\cos\theta_c}{\sin\theta_c}$$

σ_e is the r.m.s. of Čerenkov photon energy distribution (given by QE of PMT) and e is photon energy.

Basics data for TOP used in simulation

- Refractive index of quartz (Toru Iijima):

$$n(\lambda) = 1.44 + \frac{8.20\text{nm}}{\lambda - 126\text{nm}} \qquad n_g(\lambda) = n(\lambda) + \frac{8.20\text{nm} \ \ \lambda}{(\lambda - 126\text{nm})^2}$$

- Absorption length of quartz(J. Vav'ra):

$$\lambda_{abs} = 500\text{m}\left(\frac{\lambda}{442\text{nm}}\right)^4$$

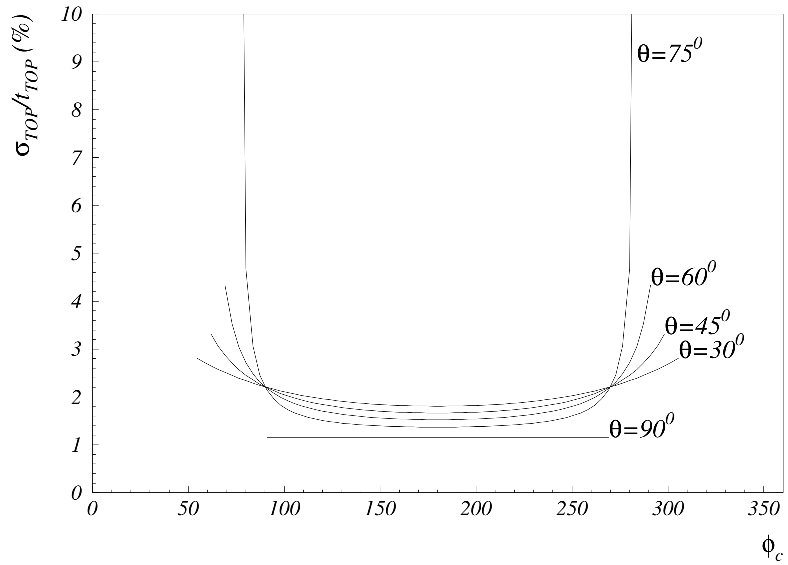
- Quantum efficiency as for Hamamatsu R5900-M16
- 70% collection efficiency
- Using above data the basic TOP parameters are:
  - N_0 = 105 cm^{-1}
  - $< e >$ = 3.3 eV $\Rightarrow < n >$ = 1.473, $< n_g >$ = 1.522
  - σ_e =0.5556 eV
  - $\frac{1}{n}\frac{dn}{de}$ = 1.02%, $\frac{1}{n_g}\frac{dn_g}{de}$ = 2.96%
- PMT time resolution: σ_{PMT} = 50ps
- Q-bar dimensions: 40cm $\times$ 2cm $\times$ 255cm
- Coverage: Δx_{ch}=5mm, 64 active channels out of 80 per Q-bar exit window

TOP time resolution

Relative time resolution due to dispersion, calculated with derived formulas

$$\sigma_k^{disp}/t_k \approx 1\% \; - \; 2\%$$

depends on track angle θ $\longrightarrow$



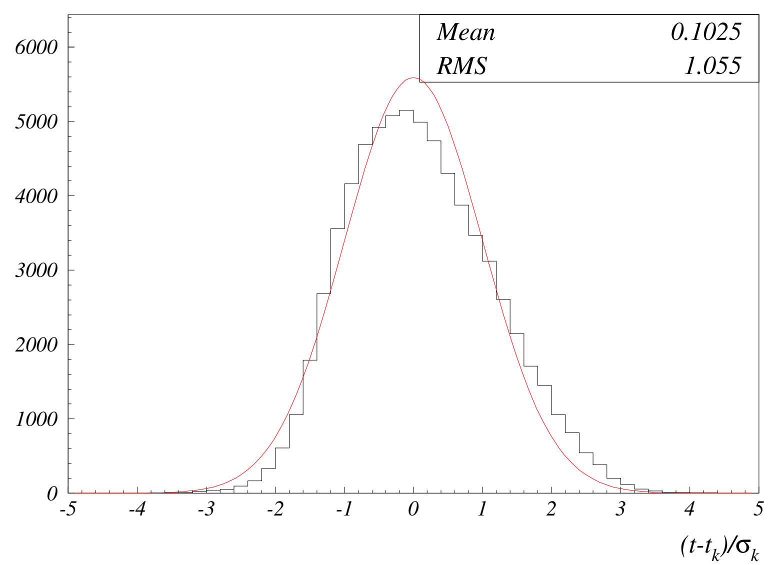
Peak shape ($B \to \pi\pi$ tracks)

Slightly asymmetric but can be well approximated by Gaussian function

$$g(t - t_k; \sigma_k) = \frac{n_k}{\sqrt{2\pi}\sigma_k} e^{-\frac{(t-t_k)^2}{2\sigma_k^2}}$$

with

$$\sigma_k = \sigma_k^{disp} \oplus \sigma_{PMT}$$



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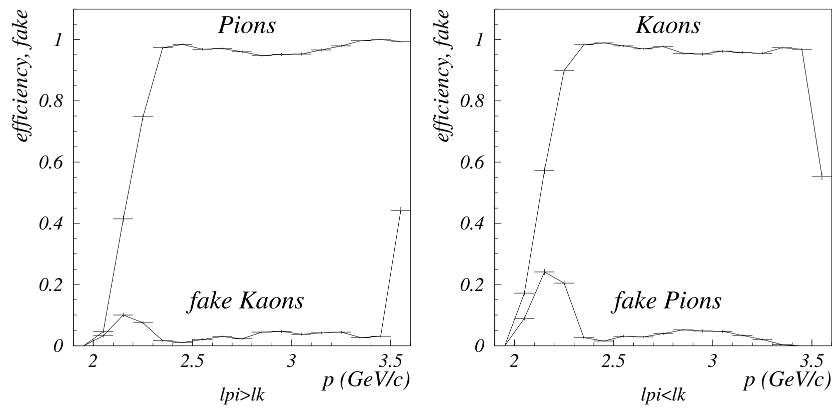
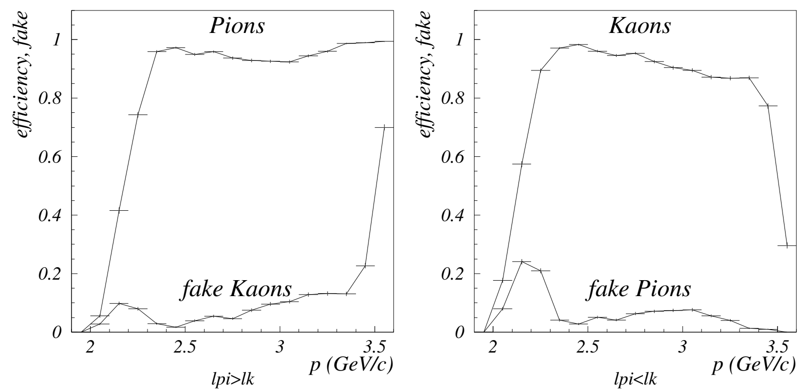
TOP counter performance 1

For two different time resolutions



50 ps t_{TOP} resolution



100 ps t_{TOP} resolution

Both with 20 backg. hits per bar, one bar side equipped, $B \to \pi\pi$ tracks

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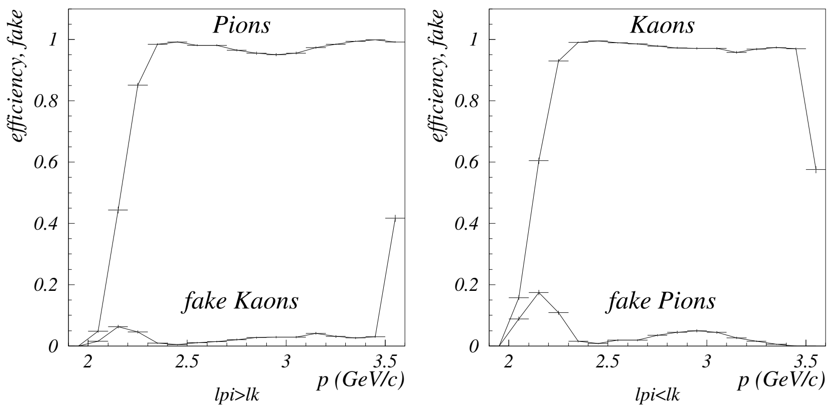
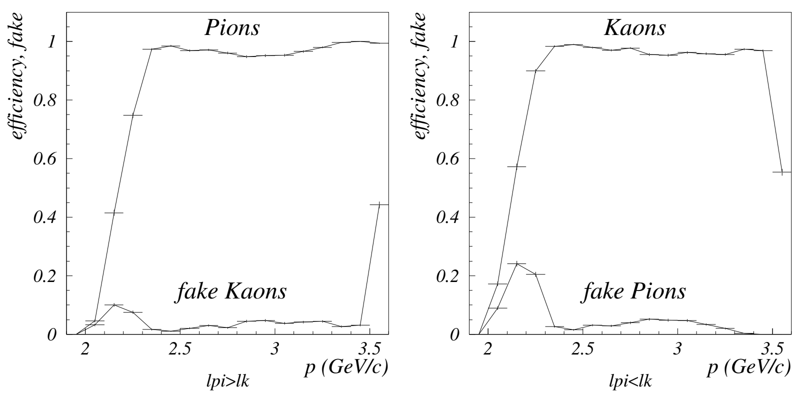
TOP counter performance 2

Impact of background level



5 backg. hits per bar

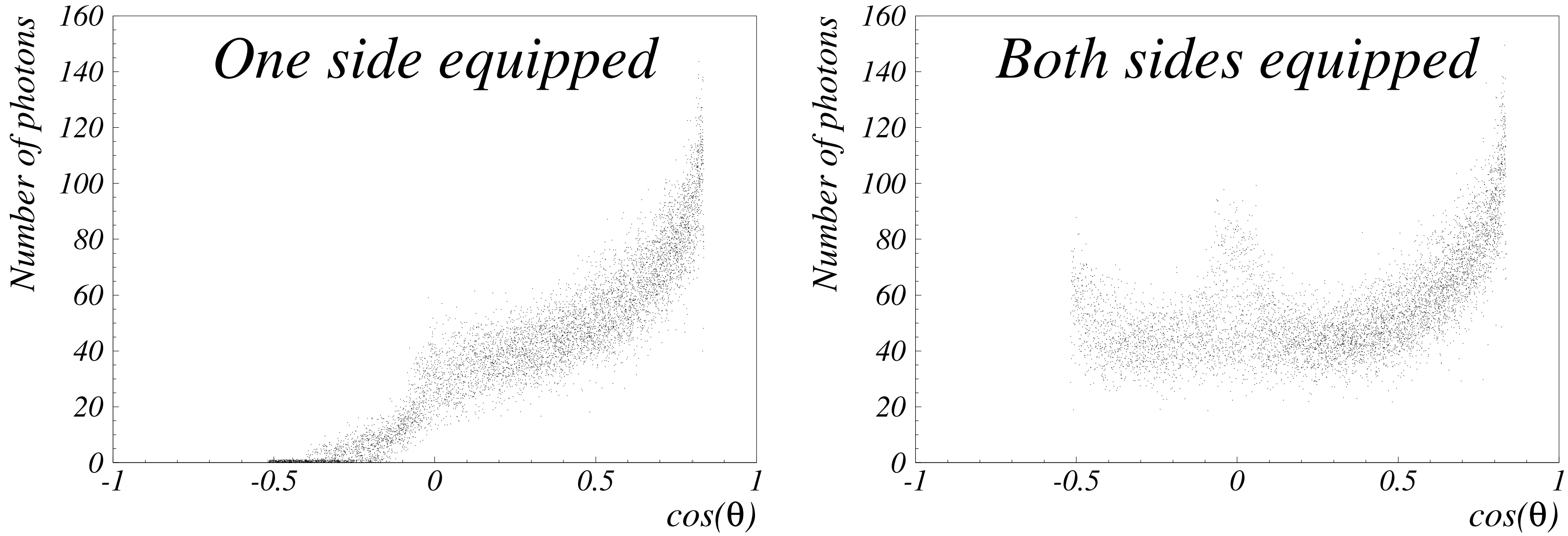


20 backg. hits per bar

Both with 50 ps t_{TOP} resolution, one bar side equipped, $B \to \pi\pi$ tracks

Design issues: Number of photons

Number of photons versus $\cos\theta$ for $B \to \pi\pi$ tracks (and a fully active PMT surface)



$\to$ both ends of Q-bar should be equipped with PMT's

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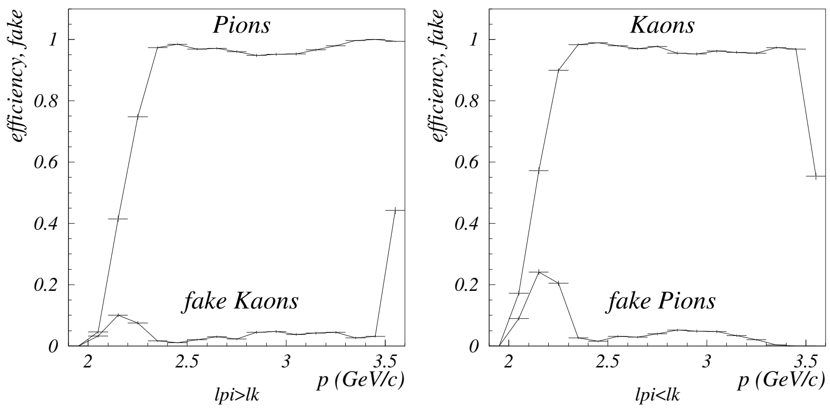
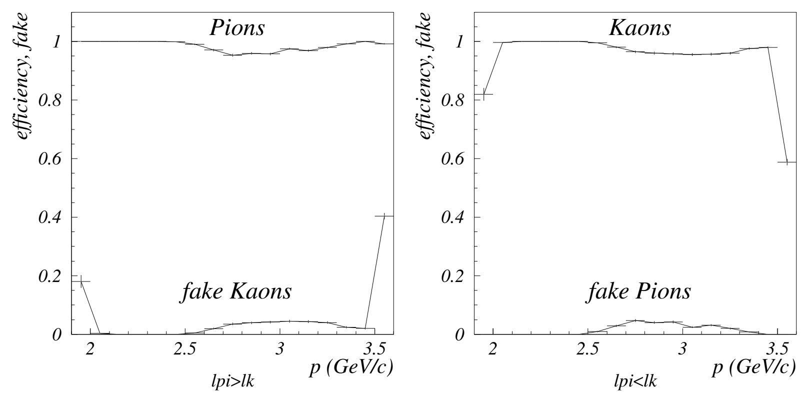
TOP counter performance 3

Single side equipped vs both sides



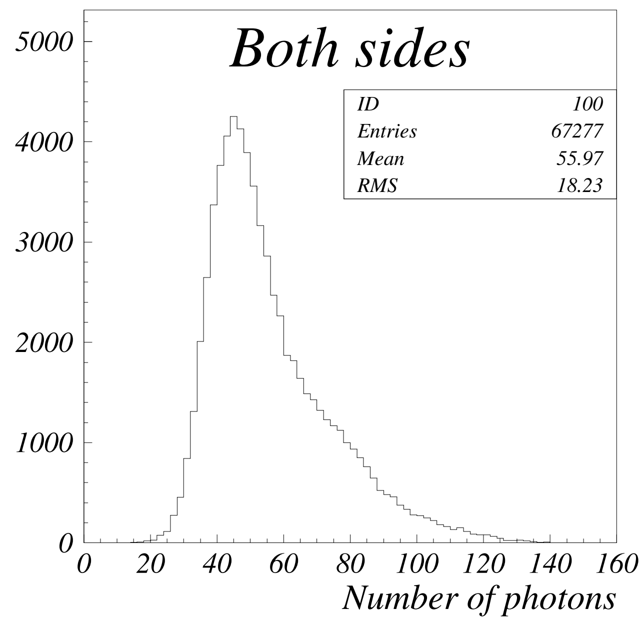
one bar side equipped



both sides equipped

Both with 50 ps t_{TOP} resolution, 20 backg. hits per bar, $B \to \pi\pi$ tracks

Design issues: Segmentation in y?



Number of photons ≈ Number of channels in x

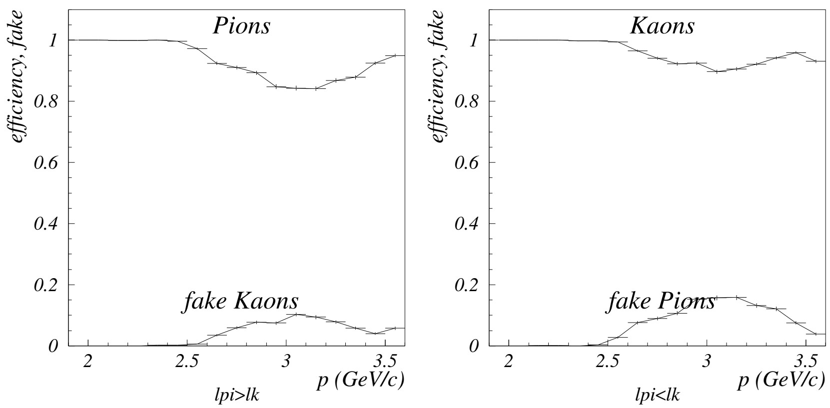
Pile-up within the same event becomes highly probable.

Assuming that the electronics will not be fast enough to handle double pulses with few ns separation: what to do?

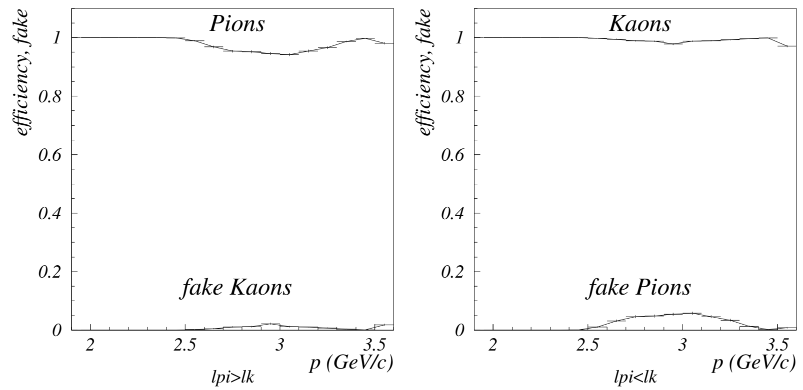
.... segmentation in y ? ....

Design issues: pile-up impact

Simulate pile-up; assumed: 50 ps t_{TOP} resolution, 20 background hits per bar, constant fraction discrimination interval 5 ns.



Single channel in y



Four channels in y

→ Clear advantage of segmentation in y

→ PMT with 5 mm x 5 mm pads instead of 5 mm x 20 mm strips.

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Summary

- ✦ Likelihood calculated analytically
- ✦ Simulation code set-up, running
- ✦ Add also a simple electronics pile-up simulation
- ✦ TOP performance was studied vs various parameters
- ✦ Impact on the design:
  - advisable to equip both bar sides (more photons, uniform response)
  - segmentation in y would be welcome (less pile-up)

Back-up slides

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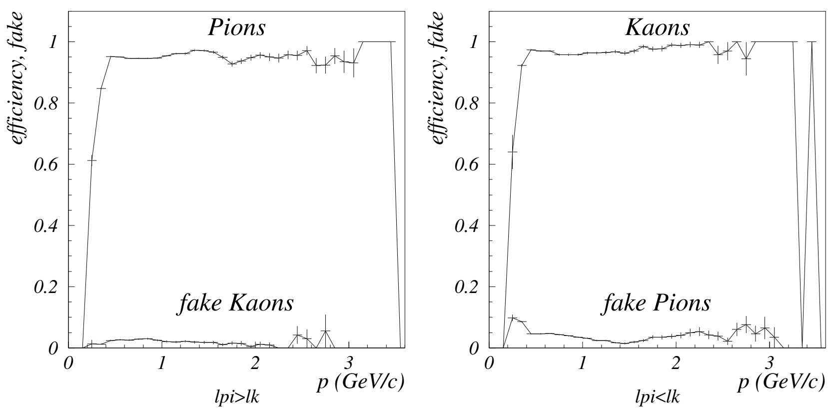
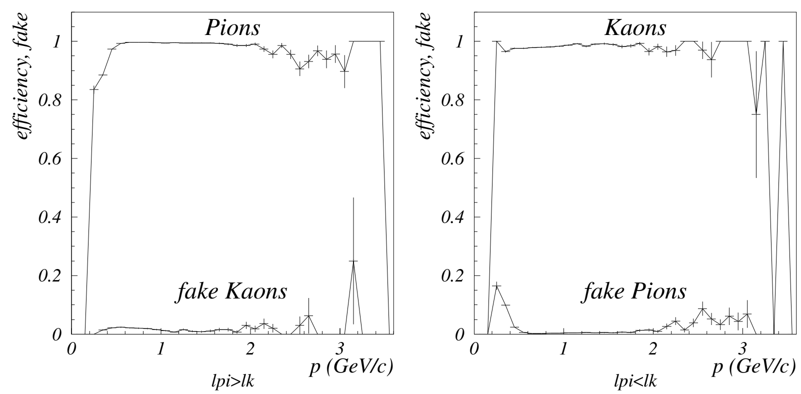
TOP counter performance 4

Single side equipped vs both sides



one bar side equipped



both sides equipped

Both with 50 ps t_{TOP} resolution, 20 backg. hits per bar, pile-up 5n s, QQ98 generated B decays

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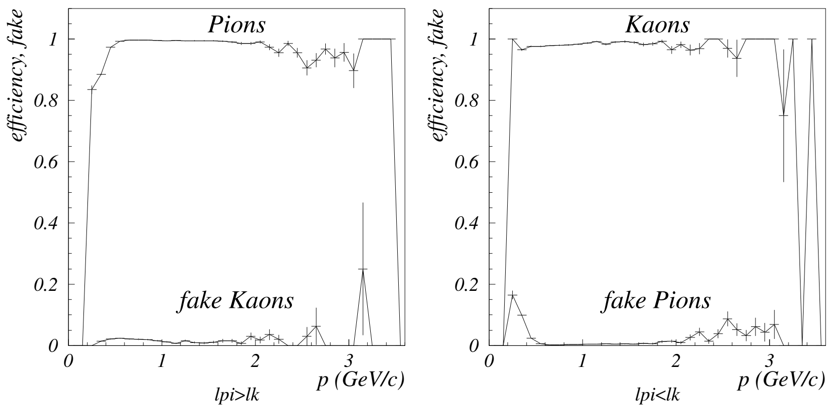
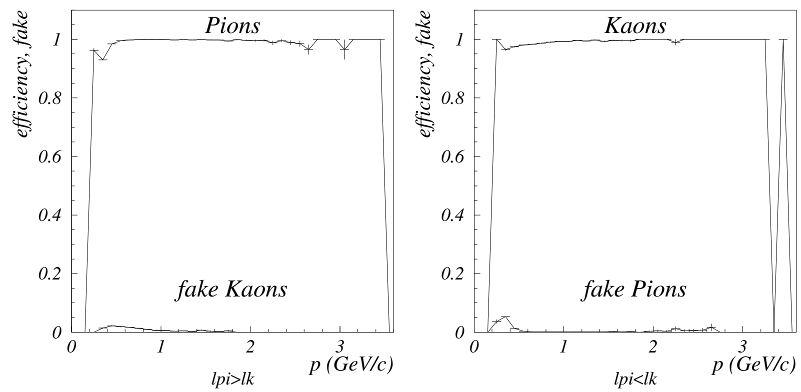
TOP counter performance 5

Simulate pile-up; assumed: 50 ps t_{TOP} resolution, 20 background hits per bar, constant fraction discrimination interval 5 ns.



Single channel in y



Four channels in y

Both with 50 ps t_{TOP} resolution, 20 backg. hits per bar, pile-up 5n s, QQ98 generated B decays

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