Dalitz plot

Pictorial representation in high-energy nuclear physics for data on the distribution of certain three-particle configurations. Many elementary-particle decay processes and high-energy nuclear reactions lead to final states consisting of three particles (which may be denoted by a, b, c, with mass values m_a, m_b, m_c). Well-known examples are provided by the K-meson decay processes, Eqs. (1) and (2), and by the K- and K-meson reactions with hydrogen, given in Eqs.

$$K^+ \rightarrow \pi^+ + \pi^+ + \pi^- \qquad (1)$$

$$K^+ \rightarrow \pi^0 + \mu^+ + \nu \qquad (2)$$

$$K^+ + p \rightarrow K^0 + \pi^+ + p \qquad (3)$$

$$K^- + p \rightarrow \Lambda + \pi^+ + \pi^- \qquad (4)$$

(3) and (4). For definite total energy E (measured in the barycentric frame), these final states have a continuous distribution of configurations, each specified by the way this energy E is shared among the three particles. (The barycentric frame is the reference frame in which the observer finds zero for the vector sum of the momenta of all the particles of the system considered.) See also: Elementary particle

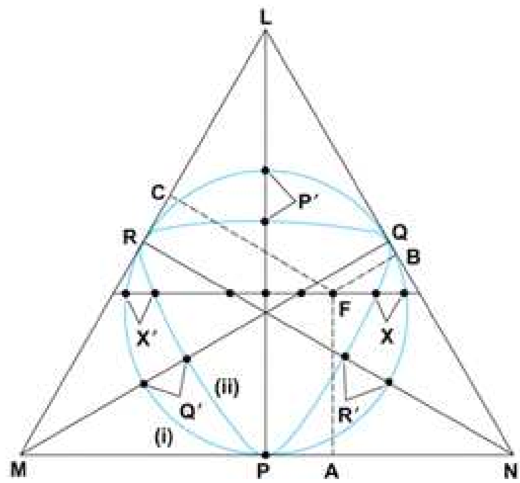
Equal mass representation

If the three particles have kinetic energies T_a, T_b, and T_c (in the barycentric frame), Eq.

$$T_a + T_b + T_c = E - m_a c^2 - m_b c^2 m_c c^2 = Q \qquad (5)$$

(5) is obtained. As shown in Fig. 1, this energy sharing may be represented uniquely by a point F within an equilateral triangle LMN of side 2Q/3, such that the perpendiculars FA, FB, and FC to its sides are equal in magnitude to the kinetic energies T_a, T_b, and T_c. This exploits the property of the equilateral triangle that (FA + FB + FC) has the same value (Q, the height of the triangle) for all points F within it. The most important property of this representation is that the area occupied within this triangle by any set of configurations is directly proportional to their volume in phase space. In other words, a plot of empirical data on this diagram gives at once a picture of the dependence of the square of the matrix element for this process on the a, b, c energies.

Fig. 1 A three-particle system (abc) in its barycentric frame is specified by a point F so that perpendiculars FA, FB, and FC to the sides of an equilateral triangle LMN (of height Q) are equal in magnitude to the kinetic energies T_a, T_b, T_c, where Q denotes their sum. See Eq. (5). Curve (i) encloses all points F which correspond to physically allowed configurations, for equal masses and nonrelativistic kinematics; curve (ii) corresponds to curve (i) when relativistic kinematics are used, appropriate to the decay process ω(785 MeV) $\rightarrow \pi^+\pi^-\pi^0$.



Not all points F within the triangle LMN correspond to configurations realizable physically, since the a, b, c energies must be consistent with zero total momentum for the three-particle system. With nonrelativistic kinematics (that is, $T_a = p^2_a/2m_a$, etc.) and with equal masses m for a, b, c, the only allowed configurations are those corresponding to points F lying within the the circle inscribed within the triangle, shown as (i) in Fig. 1. With unequal masses the allowed domain becomes an inscribed ellipse, touching the side MN such that MP:PN equals m_b:m_c (and cyclical for NL and LM).

More generally, with relativistic kinematics ($T_a = (m^2_a c^4 + p^2_a c^2) - m_a c^2$, etc.), the limiting boundary is distorted from a simple ellipse or circle. This is illustrated in Fig. 1 by the boundary curve (ii), drawn for the $\omega \rightarrow 3\pi$ decay process, where the final masses are equal. This curve was also calculated by E. Fabri for Eq. (1), and this plot is sometimes referred to as the Dalitz-Fabri plot. In the high-energy limit $E \rightarrow \infty$, where the final particle masses may be neglected, the boundary curve approaches a triangle inscribed in LMN.

The following points (and the regions near them) are of particular interest:

1. All points on the boundary curve. These correspond to collinear configurations, where a, b, c have parallel momenta.

2. The three points of contact with the triangle LMN. For example, point P corresponds to the situation where particle c is at rest (and therefore carries zero orbital angular momentum).

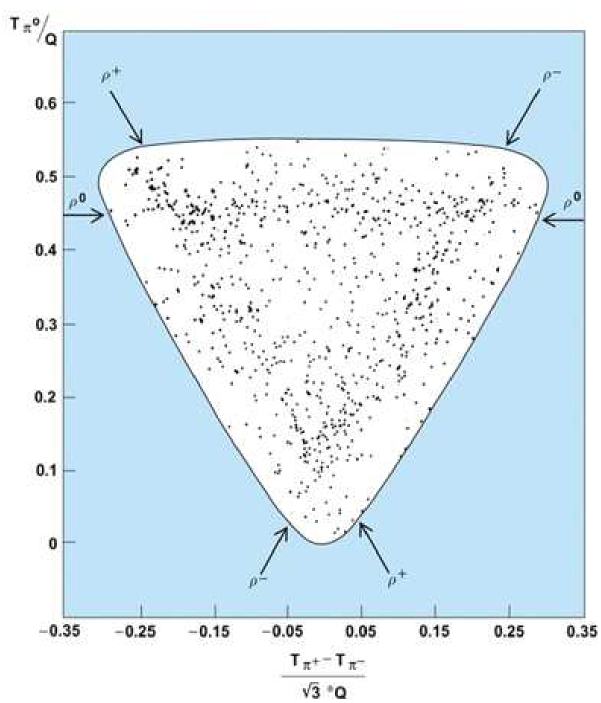
3. The three points which are each farthest from the corresponding side of the triangle LMN. For example, point P′ on Fig. 1 corresponds to the situation where b and c have the same velocity (hence zero relative momentum, and zero orbital angular momentum in the bc rest frame).

If the process occurs strongly through an intermediate resonance state, say a + (bc)* where (bc)* → b + c, there will be observed a "resonance band" of events for which T_a has the value appropriate to this intermediate two-body system. Such a resonance band runs parallel to the appropriate side of the triangle [the side MN for the case (bc)*, and cyclically] and has a breadth related with the lifetime width for the resonance state.

Antiproton annihilation

The Dalitz plot shown for equal masses in Fig. 1 has been especially useful for three-pion systems, since it treats the three particles on precisely the same footing. Points placed symmetrically with respect to the symmetry axis PL represent configurations related by the interchange of π_b and π_c. The symmetry axes PL, QM, and RN divide the allowed regions into six sectors; the configurations in each sector can be obtained from those corresponding to one chosen sector (for example, the sector such that $T_a \geq T_b \geq T_c$) by the six operations of the permutation group on three objects. These operations are of particular interest for three-pion systems, since pions obey Bose statistics; the intensities in the six sectors are related with the permutation symmetry of the orbital motion in the three-pion final state. The Dalitz plot shown in Fig. 2, for the antiproton capture reaction pp → $\pi^+\pi^-\pi^0$, illustrates these points. Three ρ -meson bands [ρ(765 MeV) → ππ] are seen, corresponding to intermediate states $\pi^+\rho^-$, $\pi^0\rho^0$, and $\pi^-\rho^0$; the six sectors have equal intensity. See also: Bose-Einstein statistics

Fig. 2 Dalitz plot for 823 examples of the antiproton annihilation process, p + p → π^+ + π^+ + π^0. Arrows show positions expected for ρ(765)-meson resonance bands, which appear clearly for the ($\pi^+\pi^-$), ($\pi^-\pi^0$), and ($\pi^0\pi^+$) systems. Distribution is symmetrical between the six sectors obtained by drawing the three axes of symmetry. The authors interpret this distribution as being due to (1) the reaction pp → ρπ occurring in the I = 0 S_1^3 initial state, (2) the reaction pp → 3π (s-wave pions) occurring in the I = 1 S_0^1 initial state, with roughly equal intensities. (After C. Baltay et al., Annihilation of antiprotons in hydrogen at rest, Phys. Rev., 140:B1039, 1965)



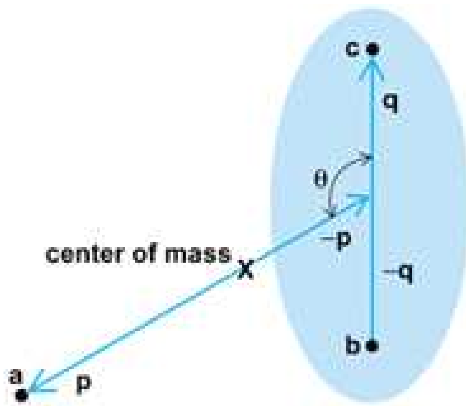
Relativistic three-particle system

Figure 3 depicts a less symmetric specification for a three-particle system. The momentum of c in the (bc) barycentric frame is denoted by q, the momentum of a in the (abc) barycentric frame by p, and the angle between p and q by θ. For fixed energy T_a, the points F on Fig. 1 lie on a line parallel to MN; as cos θ varies from +1 to -1, the point F representing the configuration moves uniformly from the left boundary X' to the right boundary X. If cartesian coordinates are used for F, with origin P and y axis along NM (as has frequently been found useful in the literature), then Eqs. (6)

$$x = T_a/Q \qquad y = (T_b - T_c)/Q\sqrt{3} \tag{6}$$

hold. Note that, for fixed x, y is linearly related with cos θ.

Fig. 3 Coordinate system for relativistic three-particle system. For given total energy E, the two momenta, q and p, are related in magnitude by the following equations: $E = (m^2_a+p^2) + (M^2_{bc}+p^2)$ and $M_{bc} = (m^2_b+q^2) + (m^2_c+q^2)$.



Unsymmetrical plot

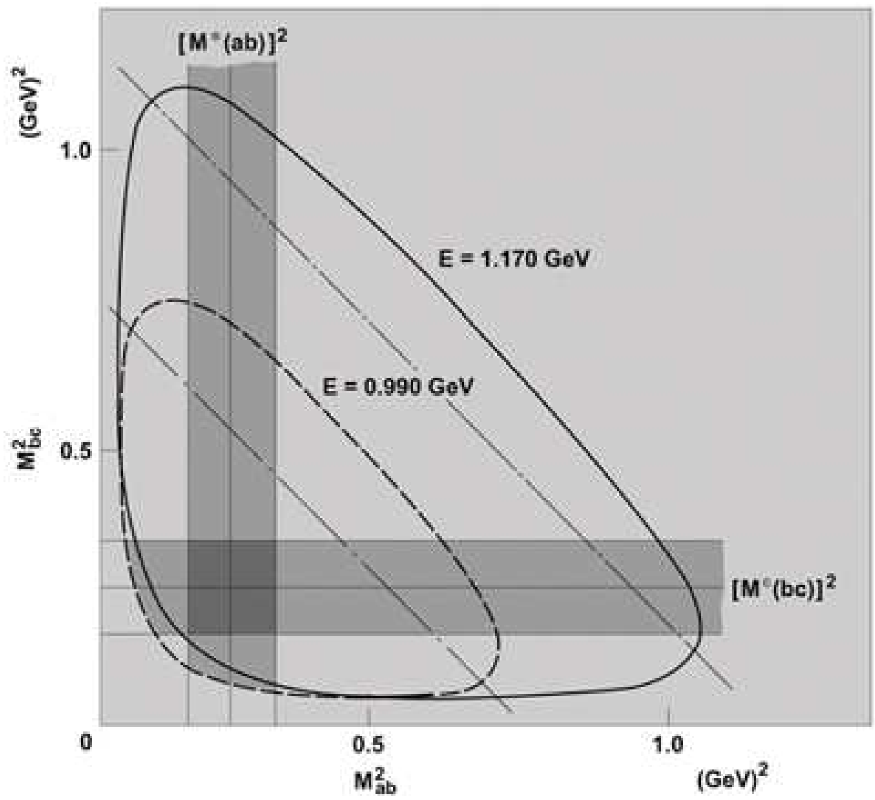
The Dalitz plot most commonly used is a distorted plot in which each configuration is specified by a point with coordinates (M^2_{ab}, M^2_{bc}) with respect to right-angled axes. This depends on the relationship given in Eq.

$$M^2_{bc} = (E - m_a c^2)^2 - 2ET_a \tag{7}$$

(7) and its cyclic permutations, for the total barycentric energy M_{bc} of the two-particle system bc. This plot may be obtained from Fig. 1 by shearing it to the left until LM is perpendicular to MN, and then contracting it by the factor 3/2 parallel to MN [which leads to a cartesian coordinate system (T_c, T_a), finally reversing the direction of the axes [required by the minus sign in Eq. (7)] and moving the origin to the point M^2_{ab} = M^2_{bc} = 0. The plots shown in Fig. 4 correspond in this way to the relativistic curve (ii) in Fig. 1 for two values of the total energy E. This distorted plot retains the property that phase-space volume is directly proportional to the area on the plot. As shown in Fig. 4, the (ab)* and (bc)* resonance bands have a fixed location on this plot; data from experiments at different energies E can then be combined on the same plot to give a stronger test concerning the existence of some intermediate resonance state. On the

other hand, the (ca)* resonance bands run across the plot at 135° and move asE varies, so that a different choice of axes [say (M^2_{ca}, m^2_{ab})] is more suitable for their presentation.

Fig. 4 An unsymmetrical Dalitz plot. Configuration of system (abc) is specified by a point (M^2_{ab}, M^2_{bc}) in a rectangular coordinate system, where M_{ij} denotes the barycentric energy of the two-particle system (ij). Kinematic boundaries have been drawn for equal masses $m_a = m_b = m_c = 0.14$ GeV and for two values of total energy E, appropriate to a three-pion system ($\pi^+\pi^-\pi^+$). Resonance bands are drawn for states (ab) and (bc) corresponding to a (fictitious) π-π resonance mass 0.5 GeV and full width 0.2 GeV. The dot-dash lines show the locations a (ca) resonance band would have for this mass of 0.5 GeV, for the two values of the total energy E.



Conclusion

It must be emphasized that the Dalitz plot is concerned only with the internal variables for the system (abc). In general, especially for reaction processes such as Eqs. (3) and (4), there are other variables such as the Euler angles which describe the orientation of the plane of (abc) relative to the initial spin direction or the incident momentum, which may carry additional physical information about the mechanism for formation of the system (abc). The Dalitz plots usually presented average over all these other variables (because of the limited statistics available); this sometimes leads to a clearer picture in that there are then no interference terms between states with different values for the total spin-parity. However, it is quite possible to consider Dalitz plots for fixed values of these external variables, or for definite domains for them. See also: Euler angles; Goldhaber triangle

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How to cite this article

Suggested citation for this article:

Richard H. Dalitz, "Dalitz plot", in AccessScience@McGraw-Hill, http://www.accessscience.com, DOI 10.1036/1097-8542.179930, last modified: November 14, 2001.

DOI 10.1036/1097-8542.179930