A TWO STATE SYSTEM

SOLUTION FOR A SINGLE POTENTIAL WELL

STATE $|1\rangle$, ENERGY E_0

ANOTHER, DISPLACED WELL
STATE $|2\rangle$, ENERGY E_0

COMBINED SYSTEM

NEITHER $|1\rangle$ NOR $|2\rangle$ IS
A SOLUTION FOR THIS CASE,
MUST BE A LINEAR COMBINATION
- DENOTE BY $a|1\rangle + b|2\rangle$
OR $\begin{pmatrix} a \\ b \end{pmatrix}$

THE HAMILTON OPERATOR
IS IN THIS BASE: $H = \begin{pmatrix} E_0 & V \\ V & E_0 \end{pmatrix}$

V: RELATED TO TUNELING PROBABILITY OF $|1\rangle \leftrightarrow |2\rangle$.

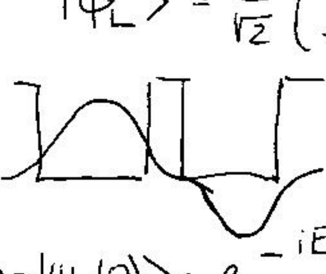
SEARCH FOR EIGENVALUES E : $\det \begin{pmatrix} E_0 - E & V \\ V & E_0 - V \end{pmatrix} = 0$

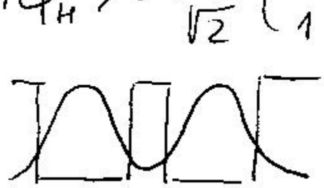
$\Rightarrow \quad (E_0 - E)^2 - V^2 = 0$

$\Rightarrow \quad E = E_0 \pm V \qquad E_L = E_0 - V \quad E_H = E_0 + V$

EIGENFUNCTION:

$|\psi_L\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad |\psi_H\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$





$|\psi_L(t)\rangle = |\psi_L(0)\rangle \cdot e^{-iE_0 t + iVt} \qquad |\psi_H(t)\rangle = |\psi_H(0)\rangle e^{-iE_0 t - iVt}$

WHAT HAPPENS TO $|\psi(t)\rangle$ WHICH IS A $|1\rangle$ AT $t=0$

$|\psi(0)\rangle = |1\rangle = \frac{1}{\sqrt{2}} \left(|\psi_L\rangle + |\psi_H\rangle \right)$

A TWO STATE SYSTEM

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(|\psi_L\rangle e^{-iE_0 t + iVt} + |\psi_H\rangle e^{-iE_0 t - iVt}\right)$$

WHAT IS THE PROBABILITY $P(1 \to 2)$ THAT AN INITIAL $|1\rangle$ TURNS INTO $|2\rangle$ AT TIME t?

$$P(1 \to 2, t) = |\langle 2|\psi(t)\rangle|^2$$

$$\langle 2|\psi(t)\rangle = \left(\frac{1}{\sqrt{2}}\right)^2 \left(\langle\psi_H| - \langle\psi_L|\right)\left(|\psi_L\rangle e^{-iE_0 t + iVt} + |\psi_H\rangle e^{-iE_0 t - iVt}\right)$$

$$= \frac{-1}{2} e^{-iE_0 t}\left(e^{iVt} - e^{-iVt}\right) = -i \sin Vt \, e^{-iE_0 t}$$

$$P(1 \to 2, t) = \sin^2 Vt$$

SIMILARLY: $P(1 \to 1, t) = \cos^2 Vt$