



CP violation and related issues

Part 10: Mixing

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D^0 mixing

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B oscillations

$$P_{u,m} = \frac{1}{2} \Gamma_q e^{-\Gamma_q t} [1 \pm \cos(\Delta m_q t)]$$

m =mixed, u =unmixed

(neglecting CP, CPT violation, $\Delta\Gamma/\Gamma=0$)

Motivation: proceeds through loop diagrams, could be a tool to discover new physical phenomena.



Mixing: estimates

$$\Delta m \propto$$

	$ V_{tb}^* V_{td} ^2 m_t^2 \propto \lambda^6 m_t^2 \approx 3$
	$ V_{cb}^* V_{cd} ^2 m_c^2 \propto \lambda^6 m_c^2 \approx 3 \cdot 10^{-4}$

	$ V_{tb}^* V_{ts} ^2 m_t^2 \propto \lambda^4 m_t^2 \approx 70$
	$ V_{cb}^* V_{cs} ^2 m_c^2 \propto \lambda^4 m_c^2 \approx 7 \cdot 10^{-3}$

	$ V_{bu}^* V_{bc} ^2 m_b^2 \propto \lambda^{10} m_b^2 \approx 7 \cdot 10^{-6}$
	$ V_{su}^* V_{sc} ^2 m_s^2 \propto \lambda^2 m_s^2 \approx 4 \cdot 10^{-3}$



B_{d,s} oscillations: refined theoretical predictions

B decay constant
193±29 MeV
(LQCD)
208±27 MeV
(QCD sum rules)

ren. group
inv. param.
1.34±0.12
(LQCD)
1.10±0.15
(QCD sum rules)

NLO QCD corr.
0.55±0.01

$$\Delta m_d = 0.50 ps^{-1} \left[\frac{F_{B_d} \sqrt{B_{B_d}}}{230 MeV} \right]^2 \left[\frac{m_t}{167 GeV} \right]^{1.52} \left[\frac{|V_{td}|}{7.8 \cdot 10^{-3}} \right]^2 \left[\frac{\eta_B}{0.55} \right]$$

$$\Delta m_s = 17.2 ps^{-1} \left[\frac{F_{B_s} \sqrt{B_{B_s}}}{260 MeV} \right]^2 \left[\frac{m_t}{167 GeV} \right]^{1.52} \left[\frac{|V_{ts}|}{0.040} \right]^2 \left[\frac{\eta_B}{0.55} \right]$$

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B oscillations

Experimental methods:
time dependent

time integrated

Lots of tagging
and reconstruction
methods

$$P_{u,m} = \frac{1}{2} \Gamma_q e^{-\Gamma_q t} [1 \pm \cos(\Delta m_q t)]$$

(neglecting CP, CPT violation,
 $\Delta\Gamma/\Gamma=0$), m =mixed, u =unmixed

need proper time measurement
 $\sigma_t/\tau_B = \sigma_L/L \oplus (t/\tau_B) (\sigma_p/p)$

flavor tagging @decay and
@production

$$\bar{\chi} = f_d \chi_d + f_s \chi_s$$

high energy $\Upsilon(4s)$

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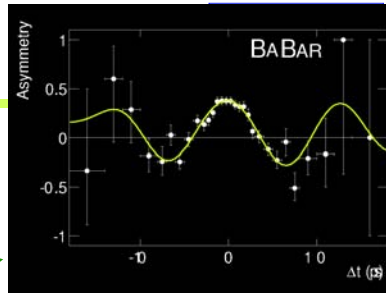


B_d oscillations

Examples of analyses:

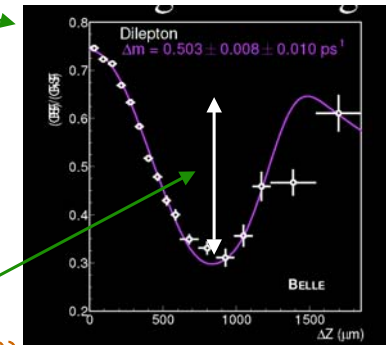
(BaBar, hep-ex/0212017(02))

exclusive method, BaBar;
 $B^0 \rightarrow D^* l \nu$, reconstruct D^* ;
 flavor tagging other side NN;
 asymmetry = $(P_u - P_m) / (P_u + P_m)$



$\Delta m_d = 0.492 \pm 0.018 \pm 0.013 \text{ ps}^{-1}$
 (20fb⁻¹)

semi inclusive, Belle;
 two fast leptons;
 flavor tagging - lepton charge;



$\Delta m_d = 0.503 \pm 0.008 \pm 0.010 \text{ ps}^{-1}$
 (29fb⁻¹)

large amplitude ↔ good tagging ε
 large statistics (Belle, PRD67, 052004(03))

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B_d oscillations

Oscillation probability including $\Delta\Gamma$

$$P_{u,m} = \frac{1}{2} \Gamma_q e^{-\Gamma_q t} \left[\cosh\left(\frac{\Delta\Gamma_q}{2} t\right) \pm \cos(\Delta M_q t) \right]$$

no assumption of CPT invariance:

$$|B_H\rangle = p |B^0\rangle + q |\bar{B}^0\rangle$$

$$|B_L\rangle = p' |B^0\rangle - q' |\bar{B}^0\rangle$$

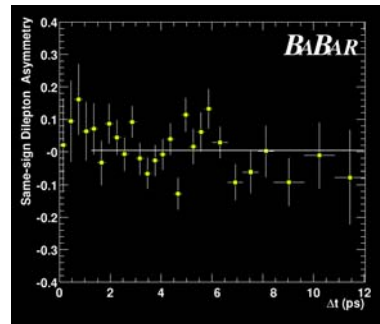
$$\frac{q}{p} = \tan\left(\frac{\theta}{2}\right) e^{i\phi}$$

$$\frac{q'}{p'} = \cot\left(\frac{\theta}{2}\right) e^{i\phi}$$

CP violated if $\text{Im}(\phi) \neq 0$
 dileptons: difference in 1^+1^+ and 1^-1^- rates;

CPT violated if $\theta \neq \pi/2$;

expressions for $P_{u,m}$ changed
 by dependence on θ, ϕ



Belle, dileptons:

$$|m_{B_0} - m_{\bar{B}_0}| / m_{B_0} < 1.16 \times 10^{-14}$$

$$|\Gamma_{B_0} - \Gamma_{\bar{B}_0}| / \Gamma_{B_0} < 0.11$$

@ 90% C.L.

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How to measure B_s mixing?

Measure: probability that a B_s (at $t=0$) turns into an anti- B_s at time t .

Need:

- a well defined final state with precisely measured vertex, momentum
- a tag to determine the initial B_s flavour (B_s or anti- B_s at $t=0$)

Final states:

- $\mu D_s \nu, D_s \rightarrow \phi \pi^+, K^+K_S, K^+K^*$
- $J/\psi K^* \rightarrow \mu \mu K \pi$
- $D_s^{(*)} \pi^+ (\pi^+ \pi^+ \pi^-)$

Tagging:

- charge of kaon and lepton from the associated B decay (opposite side tagging)
- charge of kaon from the same side

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B_s oscillations

Cannot be measured at a B factory: no B_s mesons!

First measurements were done at LEP (at E_{cms} of Z^0).

tagging

opposite side:

Neural net (NN) to separate tracks from primary and secondary vertex;
NN to compute charge estimators (jet charge, lepton charge, K charge, etc.) ;

same side:

wide b-jet (all B_s decay products + fragmentation products close in phase space) using large y_{cut} (JADE);
NN to compute charge estimator (from K, jet charge,...)

(A1eph, CERN-EP-2002-16)

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B_s oscillations

$$P_{u,m} = \frac{1}{2} \Gamma_q e^{-\Gamma_q t} [1 \pm \cos(\Delta m_q t)]$$

m =mixed, u =unmixed

Fit the data in a different way: fix Δm_s and fit the oscillation amplitude A

$$P_m = \frac{1}{2} \Gamma_q e^{-\Gamma_q t} [1 - A \cos(\Delta m_q t)]$$

If A consistent with 0 \rightarrow no mixing.
 Mixing established if $A=1$, and $A=0$ excluded with high significance.

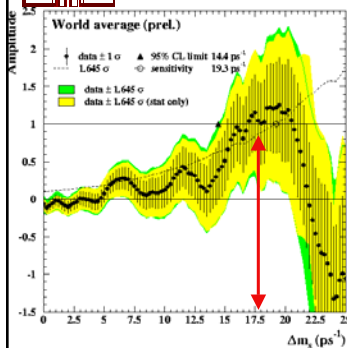
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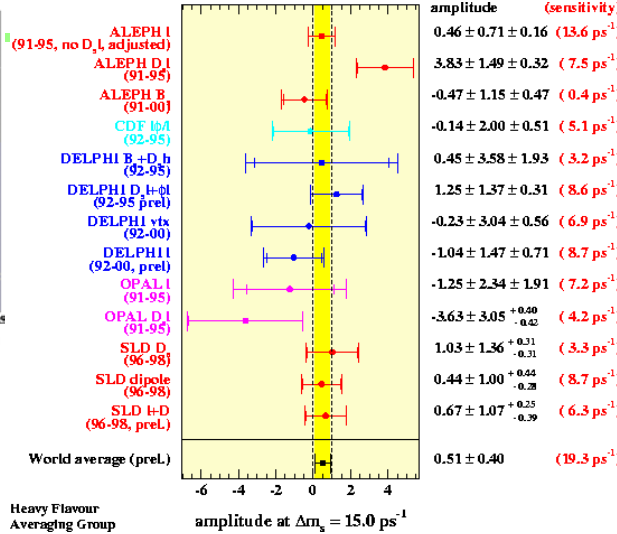
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B_s oscillations



data consistent with oscillations
 $\Delta m_s \sim 17.5 \text{ ps}^{-1}$
 $@ 2.2\sigma$



(HFAG, winter '03)

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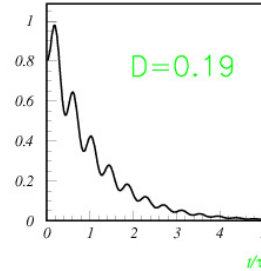
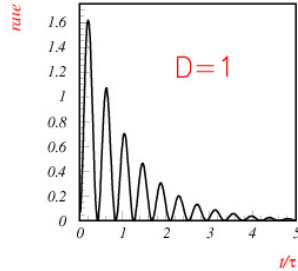
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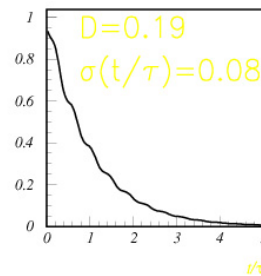
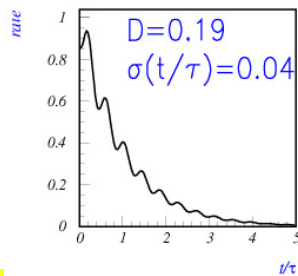
B_s mixing: dilution effects for $x_s=15$

Ideal



Dilution:
Tagging

Dilution:
Tagging,
Vertex



Dilution:
Tagging,
Vertex

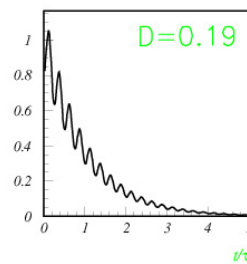
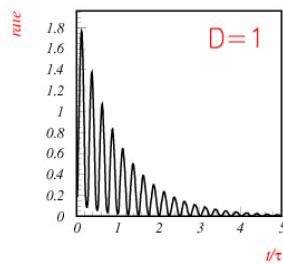
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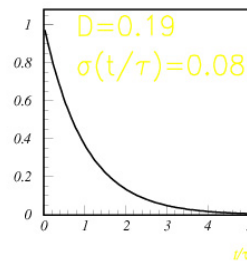
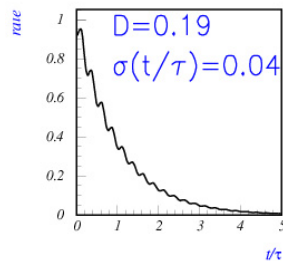
B_s mixing: dilution effects for $x_s=25$

Ideal



Dilution:
Tagging

Dilution:
Tagging,
Vertex



Dilution:
Tagging,
Vertex

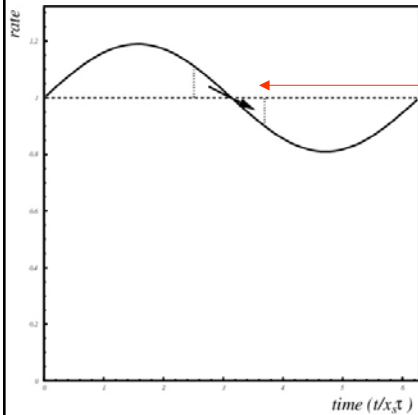
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B_s mixing: dilution effects due to vertexing



Full curve: dilution tagging only

Dilution due to finite vertex resolution: events move in the region $\pm \sim \sigma_t$ off the node

Simple estimate: linear approx around the node, fraction of events that move from the up part of the wave to the down part: $\sim (\Delta m_s \sigma_t)^2 / 2$

Amplitude reduced by a factor $(1 - (\Delta m_s \sigma_t)^2 / 2)$

Full calculation: convolution, $\exp(-(\Delta m_s \sigma_t)^2 / 2)$

Simple estimate: the first term in expansion.

H. G. Moser, A. Roussarie, NIM **A384** (1997)

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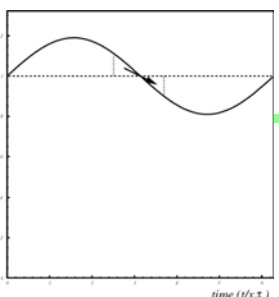
B_s mixing: dilution effects due to vertexing

- $1 - \cos(\Delta m_s t)$ ← No dilution
- $1 - D \cos(\Delta m_s t)$ ← Dilution, tagging only
- $1 - D' \cos(\Delta m_s t), D' = D e^{-\frac{(\Delta m_s \sigma_t)^2}{2}}$ ← Dilution due to finite proper time resolution σ_t (vertex and momentum resolution)

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B_s mixing: sensitivity in Δm_s

Simple estimate of the statistics required for a significant measurement: fix Δm_s , and divide the events in two classes, those from the 'up' part of the wave, and those from the 'down' part.

The measured oscillation amplitude for a given Δm_s differs from zero if the two classes are found to be differently populated.

The distribution over the two classes is binomial, with probability for the 'up' part equal to $p=1/2+cD'$, where c is a constant of order 1.

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B_s mixing: sensitivity in Δm_s

Error on p for a measurement with N reconstructed and tagged events:
 $\sigma(p) = \sqrt{p(1-p)/N}$.

For $p \sim 1/2$: $\sigma(p) \sim \frac{1}{2} 1/\sqrt{N}$

Error on the amplitude D' : $\sigma(D') \sim 1/(2c \sqrt{N})$

and the significance of the measurement equals to

$D'/\sigma(D') \sim 2c \sqrt{N} D \exp(-(\Delta m_s \sigma_t)^2/2)$

For a given required significance, the number of events needed is proportional to $\exp(+(\Delta m_s \sigma_t)^2)$.

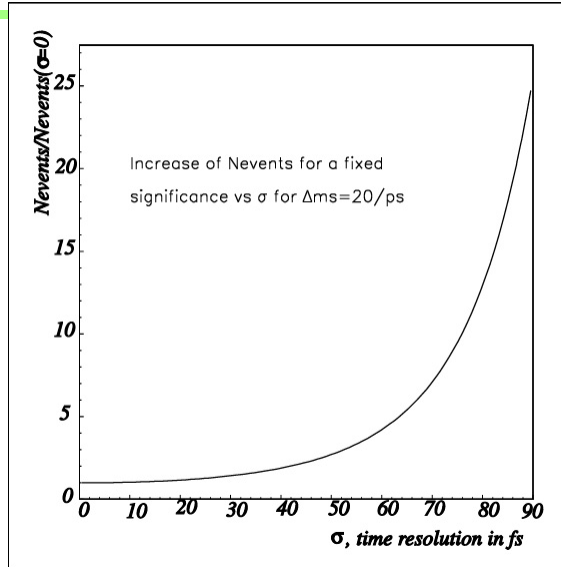
- > a very steep function of the proper time resolution and the mixing parameter above $\Delta m_s \sigma_t = 1$
- > If $\Delta m_s = 20/\text{ps}$, need $\sigma_t < 50 \text{ fs}$ to stay below this limit.

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B_s mixing: sensitivity in Δm_s

Increase in the number of events needed for a given significance vs resolution.



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D⁰ mixing

Δm very small, $(\Delta m t) \ll 1 \rightarrow$

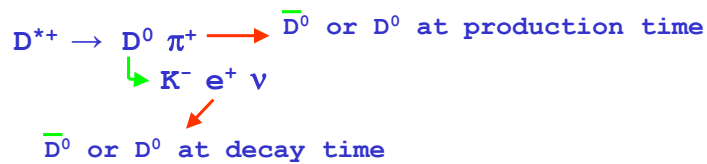
Time evolution:

$$P_m = \frac{1}{2} \Gamma_q e^{-\Gamma_q t} [1 - \cos(\Delta m_q t)] \rightarrow \frac{1}{2} \Gamma_q e^{-\Gamma_q t} \frac{(\Delta m_q t)^2}{2}$$

Almost nothing happens before the D meson decays.

The method: search for D mixing in the decay sequence:

$D^{*+} \rightarrow D^0 \pi^+$, $D^0 \rightarrow$ flavour specific final state.



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D⁰ mixing in D⁰→Kπ and Klν decays

The method: search for D mixing in the decay sequence: D^{*+}→D⁰π⁺, D⁰→flavour specific final state.

Semileptonic decay:

- K⁻ e⁺ ν : no mixing (RS, Right Sign)
- K⁺ e⁻ ν : mixing (WS, Wrong Sign)

→ measure WS rate

Hadronic decay:

- K⁻ π⁺ : no mixing
- K⁺ π⁻ : mixing or doubly Cabbibo suppressed (DCSD)

→ measure WS time evolution

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D⁰ mixing in D⁰→Kπ decays

D⁰→Kp time evolution

$$dN/dt \propto \{ \underbrace{R_D}_{\text{interference}} + \underbrace{R_D^{1/2} y' t}_{\text{mixing}} + (x'^2 + y'^2) t^2/4 \} e^{-t}$$

$$x' = x \cos \delta + y \sin \delta$$

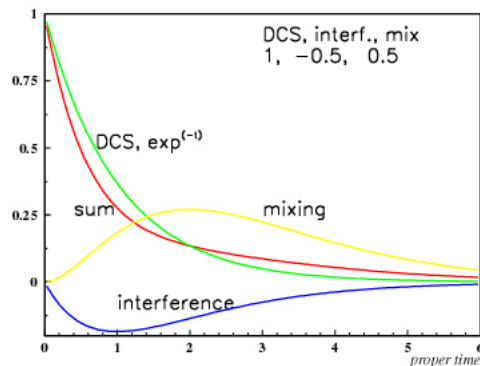
$$y' = y \cos \delta - x \sin \delta$$

$$x = \Delta M/\Gamma \quad y = \Delta\Gamma/2\Gamma$$

δ= strong phase difference

SM: x < 10⁻³, y < 10⁻³ (long dist. effects);

new physics: x >> y, CPV



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C



D⁰ mixing in D⁰→Kπ decays

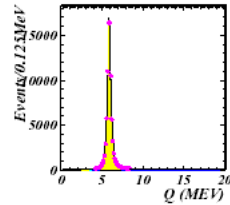
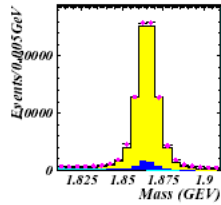


Signal extraction

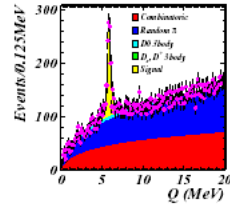
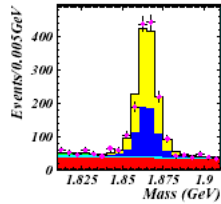
$$M = M(K, \pi)$$

$$Q = M(K^+, \pi^-, \pi_{\text{slow}}) - M(K^+, \pi^-) - M_\pi$$

Right-Sign



Wrong-Sign



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D⁰ mixing in D⁰→Kπ decays



Free fit

$$R_D = (0.287 \pm 0.037)\%$$

$$y^2 = (2.54^{+1.11}_{-1.02})\%$$

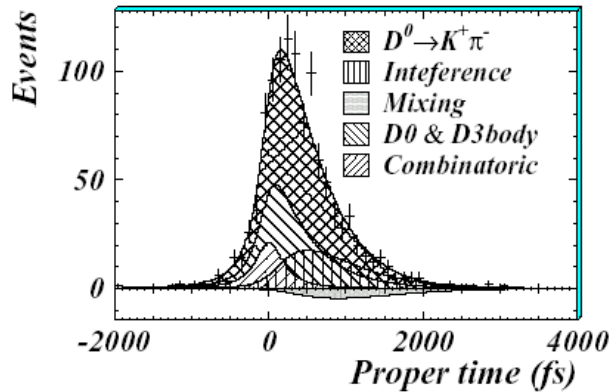
$$x^2 = -(0.153^{+0.08}_{-0.10})\%$$

Physical region

$$R = (0.343^{+0.027}_{-0.026})\%$$

$$y^2 = (0.60 \pm 0.33)\%$$

$$x^2 = 0\%$$



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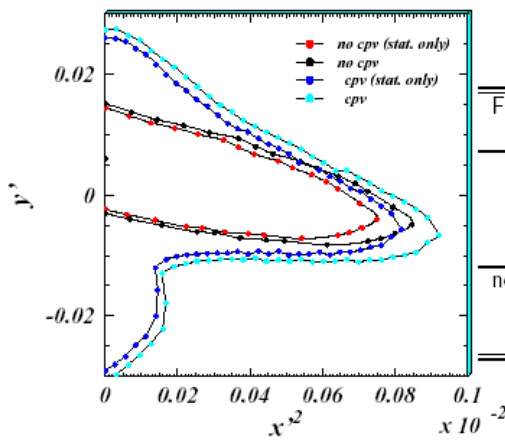
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D⁰ mixing in D⁰→Kπ decays



Results: 95% contour in x² and y' plane (with 90 fb⁻¹)



Fit case	Parameter	95% C.L. interval ($\times 10^{-3}$)
CPV	A_D	$-250 < A_D < 110$
	A_M	$-991 < A_M < 1000$
	x'^2	$x'^2 < 0.89$
no CPV	y'	$-30 < y' < 27$
	x'^2	$x'^2 < 0.81$
	R_D	$2.7 < R_D < 4.0$

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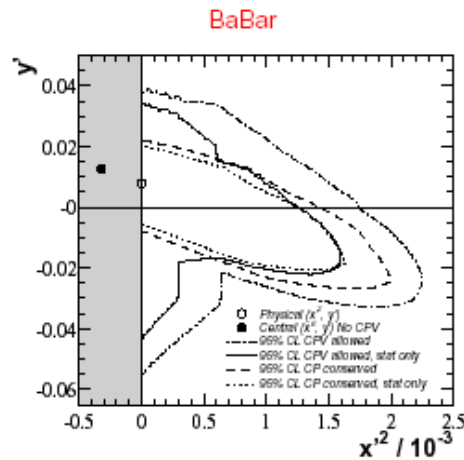
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D⁰ mixing in D⁰→Kπ decays



Results: 95% contour in x² and y' plane (with 57.1 fb⁻¹)



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D⁰ mixing in D⁰→Kev decays



Selection criteria:

- c.m.s. momentum of the Ke system > 2 GeV (bb, combinatorial background)
- Invariant mass of e⁻e⁺ (e⁺→p⁺) > 0.15GeV (γ conversions)
- Cut on decay time (backgrounds δ(t) + e^{-t}, signal t² e^{-t})

Neutrino reconstruction: hermiticity of the spectrometer, kinematic constraints.

Main observable: Δm = m(π,Kev) - m(Kev)

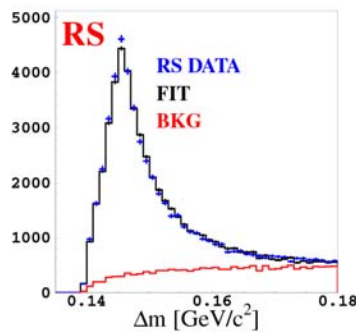
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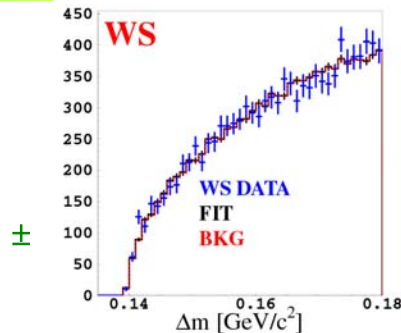
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D⁰ mixing in D⁰→Kev decays



$$N_{RS} = 40198 \pm 329$$



$$N_{WS} = 19 \pm 67$$

$$r_D = (N_{WS} / N_{RS}) (\epsilon_{RS} / \epsilon_{WS}) = (0.20 \pm 0.70) 10^{-3}$$

$$r_D < 1.4 10^{-3} \quad (90\% \text{ conf. level})$$

$$r_D = (x^2 + y^2) / 2$$

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D⁰ mixing in D⁰→Kev decays



BaBar: employs neural net techniques to reconstruct the D⁰ momentum vector (including again the neutrino), and to reject background events.

Yield: fit to Δm, t distributions.

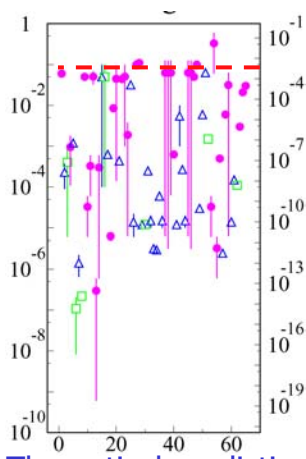
$$N_{RS} = 49620 \pm 265$$

$$N_{WS} = 114 \pm 61$$

$$r_D = (2.3 \pm 1.2(\text{stat})) 10^{-3}$$
$$r_D < 4.2 10^{-3} \text{ (90\% conf. level)}$$



D⁰ mixing in D⁰→Kev decays



BaBar: $r_D < 4.2 10^{-3}$ (90% conf. level)

Belle: $r_D < 1.4 10^{-3}$ (90% conf. level)

Theoretical predictions for r_D (right scale)



Backup slides

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How about assumptions on CP, CPT violation, $\Delta\Gamma/\Gamma=0$?

B_d oscillations

Belle, dilepton analysis:

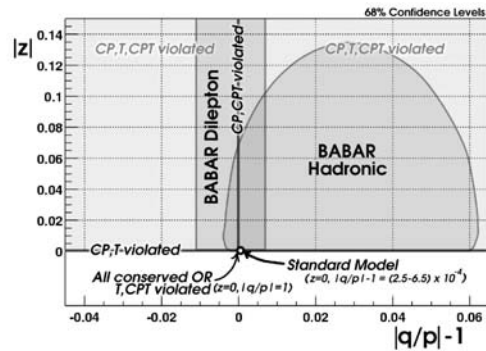
$$\begin{aligned}
 |B_H\rangle &= p|B^0\rangle + q|\bar{B}^0\rangle & \frac{q}{p} &= \tan\left(\frac{\theta}{2}\right)e^{i\phi} \\
 |B_L\rangle &= p'|B^0\rangle - q'|\bar{B}^0\rangle & \frac{q'}{p'} &= \cot\left(\frac{\theta}{2}\right)e^{i\phi}
 \end{aligned}$$

CP violated if $Im(\phi) \neq 0$
 CPT violated if $\theta \neq \pi/2$;
 CP violation and $\Delta\Gamma/\Gamma$ small:
 $Re(\cos\theta) = 0.00 \pm 0.12$
 $Im(\cos\theta) = 0.03 \pm 0.03$

BaBar:

fully reconstructed B in flavor or CP eigenstate;
 different tagging categories;
 multiparameter fit including $|q/p|$, $\Delta\Gamma/\Gamma$, λ_{CP} ;

$$z = \frac{\delta M - \frac{i}{2}\delta\Gamma}{\frac{1}{2}(\Delta m - \frac{i}{2}\Delta\Gamma)} \quad z \neq 0 \leftrightarrow \text{CPT}$$



(BaBar, hep-ex/0303043(03))

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B_d oscillations

BaBar:
 general time dependent
 decay rates of B⁰B⁰;
 CPT violation in mixing:

$$z = \frac{\delta M - \frac{i}{2} \delta \Gamma}{\frac{1}{2} (\Delta m - \frac{i}{2} \Delta \Gamma)}$$

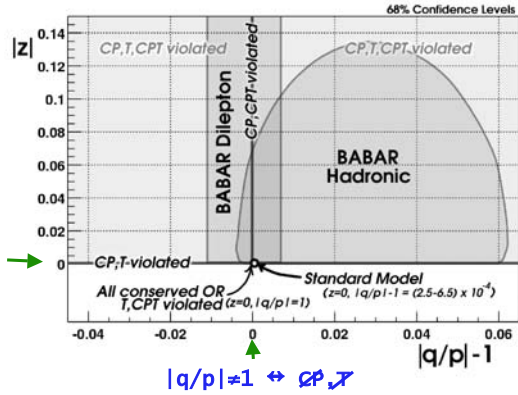
$$\delta M = \frac{M_{11} - M_{22}}{2} \quad \delta \Gamma = \frac{\Gamma_{11} - \Gamma_{22}}{2}$$

z=0 ↔ CP, CPT

fully reconstructed B in
 flavor or CP eigenstate;
 different tagging categories;
 multiparameter fit including
 |q/p|, ΔΓ/Γ, λ_{CP}, Z...

(BaBar, hep-ex/0303043(03))

|z|≠0 ↔ CP, CPT →



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