PREHOD NABITIH DELCEV SKOZI SNOV, NADALJEVANJE

→ KVANTNOMEHANSKA OBRAVNAVA

p : ČETVEREC GIB. K.

$$p' + p_\gamma = p \qquad p = (E, c\vec{p}) \qquad p_\gamma = (\hbar\omega, c\hbar\vec{k})$$

$$\vec{p} = \vec{p}' + \hbar\vec{k}$$
$$E = E' + \hbar\omega$$

$$\hbar k \ll p, \quad \hbar\omega \ll E \qquad \hbar\omega = \Delta E = \frac{p\Delta p}{E} = v\Delta p$$

$$E^2 = p^2c^2 + m^2c^4$$
$$E\,dE = p\,dp$$
$$\frac{pc}{E} = \frac{\beta\gamma m}{\gamma m} = v$$

$$\vec{p} - \vec{p}' = \Delta\vec{p} = \hbar\vec{k} \quad | \cdot \vec{v}$$

$$(\vec{p} - \vec{p}')\cdot\vec{v} = pv - p'v\cos\alpha \doteq v\Delta p \quad (\alpha \ll 1)$$

$$\Rightarrow \hbar\vec{k}\cdot\vec{v} = v\Delta p = \hbar\omega$$

$$\hbar k v\cos\vartheta = \hbar\omega \Rightarrow \omega = kv\cos\vartheta \quad (1)$$

V SNOVI: DISPERZIJSKA RELACIJA $\omega^2 = \frac{k^2c^2}{\varepsilon}$ (2)

$$\varepsilon(\omega) = \varepsilon_1(\omega) + i\,\varepsilon_2(\omega)$$

IZ (1) in (2): $\frac{k^2c^2}{\varepsilon} = \omega^2 = k^2v^2\cos^2\vartheta$

$$\Rightarrow \frac{c^2}{\varepsilon} = v^2\cos^2\vartheta \Rightarrow \boxed{\sqrt{\varepsilon}\cdot\frac{v}{c}\cos\vartheta = 1}$$

ODVISNOST $\varepsilon(\omega)$

TRI OBMOČJA

a) OPTIČNO $\varepsilon_2 = 0$ $\varepsilon_1 > 1$

~~b)~~ $\hbar\omega \lesssim 10\,eV$

ČE $v > \frac{c}{\sqrt{\varepsilon}} \Rightarrow \vartheta$ = REALEN ⇒ SEVANJE ČERENKOVA

b) ABSORPTIVNO $\varepsilon_2 > 0$, $\varepsilon_1 < 1$

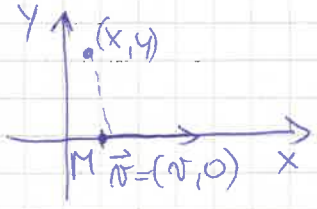
$10\,eV \lesssim \hbar\omega \lesssim 5\,keV$

ABSORPCIJA VIRTUALNIH FOTONOV → IONIZACIJA, VZBUJANJE

c) RENTGENSKO $\varepsilon_2 \ll 1$, $\varepsilon_1 \lesssim 1$

$\hbar\omega \gtrsim 5\,keV$

MALO IONIZACIJE, SEVANJE PRI PREHODU MEJE – PREHODNO SEVANJE



IZ (1): $v k_x = \omega$

(2): $k^2 = k_x^2 + k_y^2 = \frac{\varepsilon\omega^2}{c^2}$

$$k_y = \sqrt{k^2 - k_x^2} = \frac{\omega}{v}\sqrt{\left(\frac{v}{c}\right)^2\varepsilon - 1}$$

① $\left(\frac{v}{c}\right)^2\varepsilon > 1$: k_y REALEN, SEVANJE $e^{i(\vec{k}\vec{r} - \omega t)}$

② < 1: k_y IMAG., $e^{i(\vec{k}\cdot\vec{r} - \omega t)} = e^{i\omega\left(\frac{x}{v} - t\right)} \cdot e^{-\frac{y}{y_0}}$

$$y_0 = \frac{v}{\omega}\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2\varepsilon}} = \frac{1}{\omega\sqrt{\frac{1}{v^2}-\frac{\varepsilon}{c^2}}}$$

y_0 RASTE Z v → DELEC INTERAGIRA Z VEČ e^-

→ RAZLOG ZA LOG. NARAŠČANJE $-\frac{dE}{dx}$ PRI VIŠJIH β

PREHOD DELCEV SKOZI SNOV – BETHE–BLOCH

$$-\frac{dE}{dx} = K z_p^2 \frac{\rho Z}{A}\frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_e c^2\beta^2\gamma^2 T_{MAX}}{I^2} - \beta^2 - \frac{\delta}{2}\right]$$

$K = 4\pi N_A r_e^2 m_e c^2 = 0{,}307 \ \frac{MeV\ cm^2}{g}$

δ = EFEKT GOSTOTE – SENČENJE POLJA

I = POVPREČNI IONIZACIJSKI POTENCIAL, $\frac{I}{Z} \sim 10\,eV$

T_{MAX} = MAKSIMALNI PRENOS ENERGIJE

$$= \frac{2\gamma^2 m_e c^2 \beta^2}{1 + 2\gamma\frac{m_e}{M} + \left(\frac{m_e}{M}\right)^2}$$

ENERGIJSKE IZGUBE $e^{+/-}$

B–B → VELIK ODKLON, IDENTIČNI DELCI, $T_{MAX} = \frac{I}{2}$

ZAVORNO SEVANJE

$W \propto \ddot{p}_e^2 \propto a^2 \propto \left(\frac{Z}{M}\right)^2$

ZA μ $4\cdot10^4$ x MANJŠE KOT ZA e

$$E = E_0\, e^{-\frac{x}{X_0}}$$

X_0 RADIACIJSKA DOLŽINA

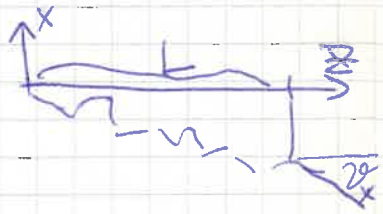
	X_0
C	18,8 cm
Pb	0,56 cm
W	0,35 cm

$$X_0 \propto \frac{A}{\rho Z^2}$$

KRITIČNA ENERGIJA $\left(\frac{dE}{dx}\right)_{TRKI} = \left(\frac{dE}{dx}\right)_{ZAVOR.\ SEV.}$

$$E_c \sim \frac{800\,MeV}{Z+1{,}2}$$

VEČKRATNO SIPANJE COULOMBSKO



SIPANJE NA E. POT. JEDRA $M_j \gg M$

RUTHERFORD $\propto \frac{1}{\sin^4\frac{\vartheta}{2}}$

GAUSOVA SREDICA

$$P(\vartheta_x)\,d\vartheta_x = \frac{1}{\sqrt{2\pi\vartheta_0'^2}}\, e^{-\frac{\vartheta_x^2}{2\vartheta_0'^2}}$$

$$\vartheta_0' = z_p \frac{15\,MeV}{p\beta c}\sqrt{\frac{L}{X_0}}\left(1 + \frac{1}{9}\log_{10}\frac{L}{X_0}\right)$$

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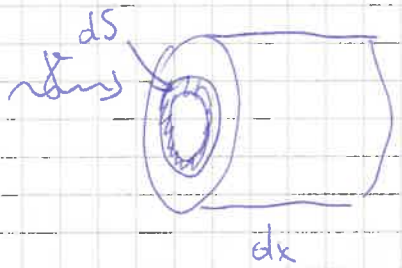
PREHOD FOTONOV SKOZI SNOV

PROCESI:
- FOTOEFEKT
- COMPTONSKO SIPANJE
- TVORBA PAROV

DRUGAČE KOT PRI DELCIH: FOTON IZGINE
→ MANJŠA SE TOK $I = I_0 e^{-\mu x}$

$\sigma \ll \sigma_{DELCI}$ → „DOSEG" VEČJI

VERJETNOST ZA INTERAKCIJA NA DOLŽINI dx



$dp_{INT} = \sigma \frac{dn_e}{dS}$ ZA 1 FOTON

$[\sigma] = m^2$

$\frac{dn_e}{dS} = dx \frac{dn_e}{dV} = dx\, Z_A \frac{dn_a}{dV} = dx\, Z_A \rho_A \frac{dn_e}{dm}$

$= dx\, Z_A \rho_A \frac{N_A}{M_A}$

N FOTONOV: $dN_{int} = N\, dp_{INT} = N \mu\, dx$

$\Rightarrow N = N_0 e^{-\mu x}$ $\qquad \mu = \sigma Z_A \rho_A \frac{N_A}{M_A}$

a) FOTOEFEKT: VEZANI e^- V ATOMU

VELIKA VERJETNOST ZA E_γ ENAKA E_v ZA K ELEKTRON

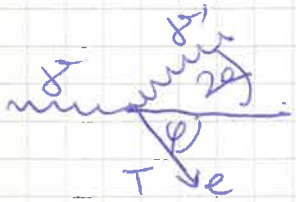
Pb

$\sigma \propto \frac{Z^5}{E_\gamma^n}$ $\quad n = \frac{7}{2}$ $\quad E_k < E_\gamma \ll m_e c^2$

$\sigma \propto \frac{Z^5}{E}$ $\quad E_\gamma \gg m_e c^2$

b) COMPTONSKO SIPANJE

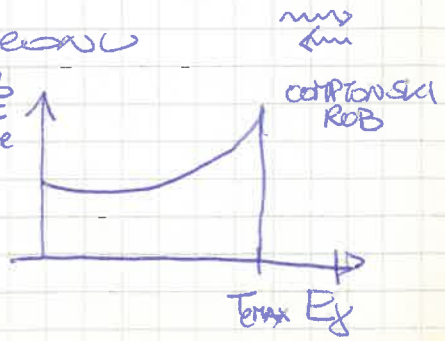
- NA ~ PROSTEM ELEKTRONU



$E_\gamma' = \frac{E_\gamma}{1 + \frac{E_\gamma}{m_e c^2}(1-\cos 2\theta)}$

$T_e = E_\gamma \frac{E_\gamma (1-\cos 2\theta)}{m_e c^2 + E_\gamma (1-\cos 2\theta)}$

$T_{e\,max} = E_\gamma \frac{1}{1 + \frac{m_e c^2}{2E_\gamma}}$



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4.

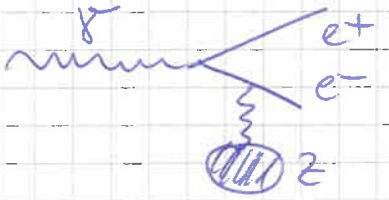
COMPTONSKO SIPANJE – NADALJEVANJE

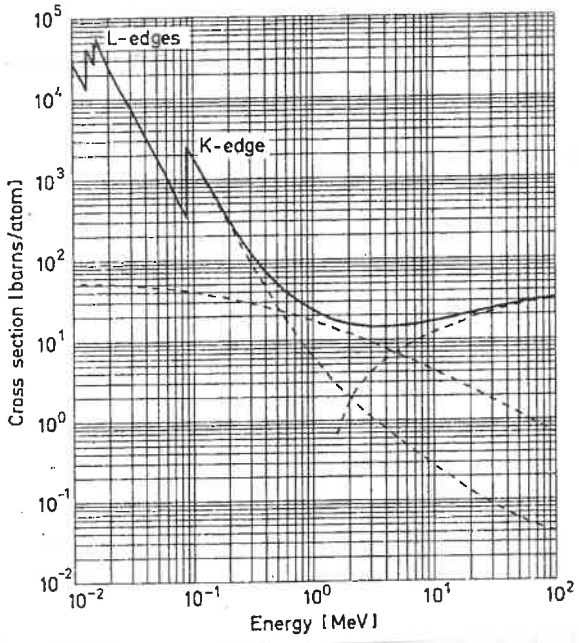
PRESEK: $\frac{d\sigma}{d\Omega}$: KLEIN–NISHINA FORMULA (QED)

INTEGRALNI PRESEK $\sigma \propto \frac{1}{E_\gamma}$ (NA e^-), $\frac{Z}{E_\gamma}$ NA ATOM

c) TVORBA PAROV

SIPANJE NA JEDRIH



PRESEK N KONSTANTEN ZA $E_\gamma \gg m_e c^2$

$\sigma_p \propto Z^2$