

Data analysis

(from raw data to physics results)

➡ From raw data to summary data

("Raw data -> DST")
track fitting
momentum determination
calorimetry (cluster reconstr.)
particle identification (Cherenkov angle)

➡ Calibration

tracking detectors
data (RICH) and MC (tracking) calibration

➡ Analysis

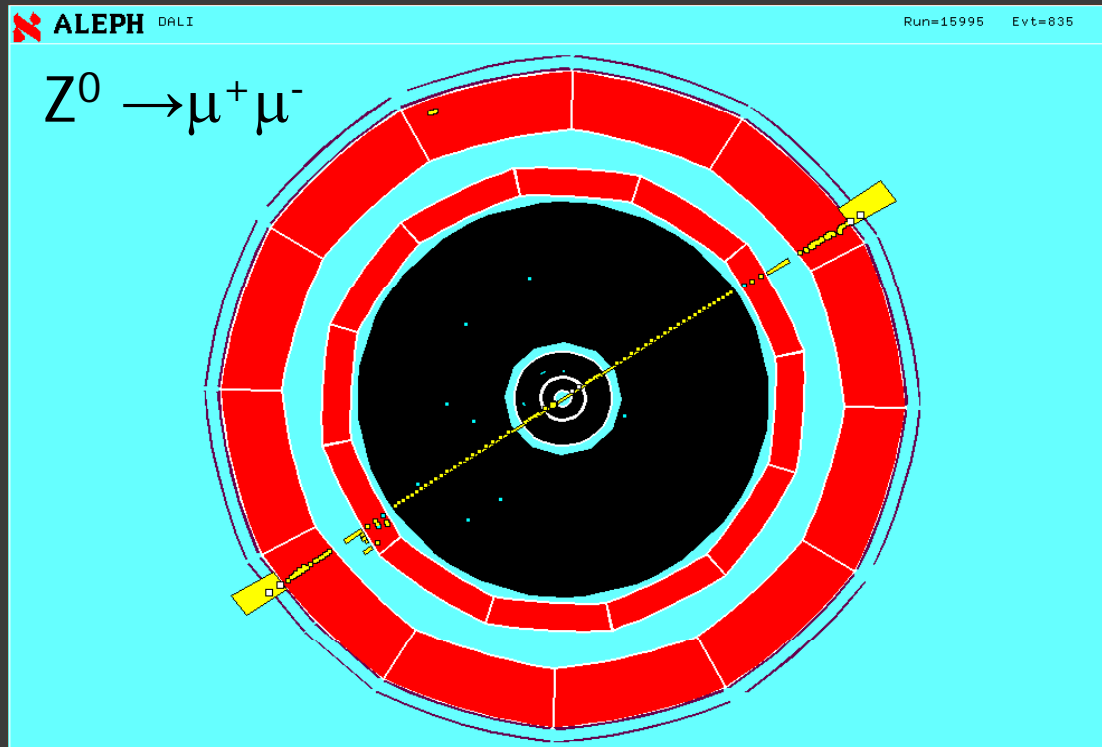
stat. methods → other lectures
jet reconstruction
b-quark tagging
flavour tagging
fitting using kinematic constraints
exclusive/inclusive channels
neural networks → other lectures

From raw data to summary data

➔ **Raw data:** digitized record of detector electronic signals;

directly used for graphical presentation;

detector part	signal value
---------------	--------------



for statistical analysis: need physics quantities p, E, q, m, \dots

➔ processed data, summary data, Data Summary Tape (DST)

From raw data to summary data reconstruction

➡ Procedure of processing raw data to summary data: **reconstruction**

example: to conclude
about $Z^0 \rightarrow \mu^+ \mu^-$ decay
one needs to

establish two tracks
of corresponding \mathbf{p}



association of signals
in tracking det. into
tracks; track fitting;
determination of \mathbf{p}

determine small energy
deposited in EM
calorimeter (μ)



association of signals in
calorim. into clusters;
association of clusters
to tracks

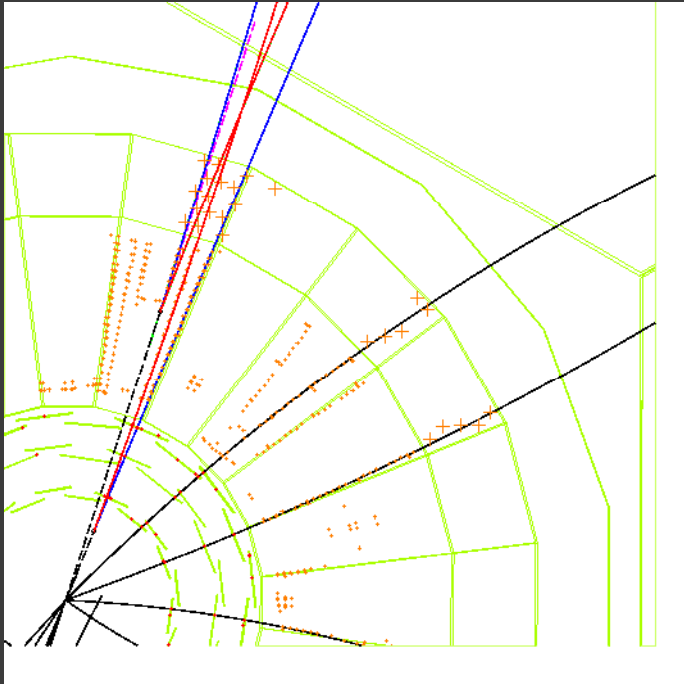
identify μ



hits in μ det.;
association to tracks
(different
procedures for
hadron ident.)

From raw data to summary data

track fitting

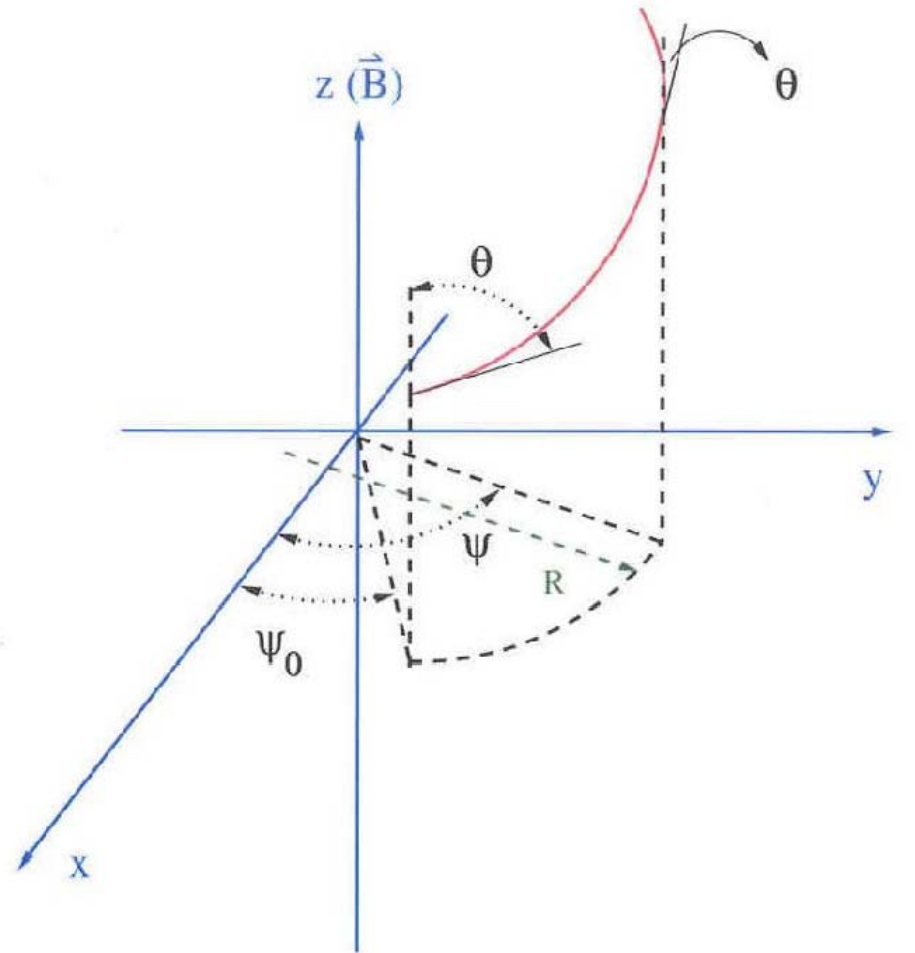


- ➔ charged track in **B** \Rightarrow **helix** ⓘ
- ➔ association of electronic signals in tracking detectors into groups - tracks
pattern recognition
- ➔ fitting of helix parameters to associated hits
track fitting

From raw data to summary data

helix

helix parameterization:



$$\begin{aligned}x &= x_0 + R(\sin \psi - \sin \psi_0) \\y &= y_0 - R(\cos \psi - \cos \psi_0) \\z &= z_0 + R \cot \theta (\psi - \psi_0)\end{aligned}$$

helix is parameterized with 5 parameters at chosen point, e.g.:

$y_0, z_0, \psi_0, \theta_0, 1/R$

$(x_0 = y_0 / \tan \psi_0)$



From raw data to summary data pattern recognition



pattern recognition:

high number of detector hits → difficult association with helix;
transformation of transversal helix projection



for parts of track (in most precise tracking detector) $x_{S'}$, $y_{S'}$, $z_{S'}$, $\psi_{S'}$, $\phi_{S'}$, $1/R_{S'}$
available - **TE, Track Element**

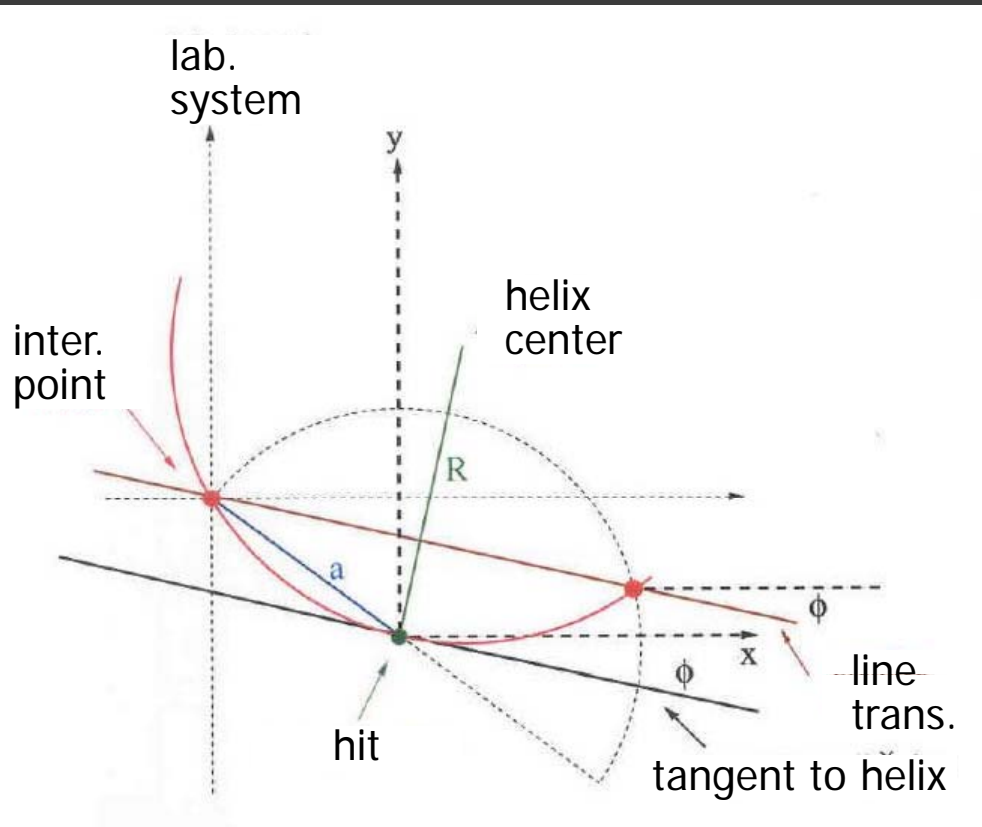
for each **other TE**: calculate transformed point x' , y' ;

calculate ϕ' ($\angle(x', y')$ and int. point);
check $|\phi' - \phi_S| < \alpha$;

from $\Delta z = z - z'$ and ψ_S calculate ψ' ($\Delta z = R_S \cot \theta (\psi' - \psi_S)$);
check $|\psi'(\text{calculated}) - \psi'(\text{measured})| < \beta$

join consistent TE's

From raw data to summary data pattern recognition



$$x' = \frac{a^2 x}{x^2 + y^2} \quad \text{transformation}$$

$$y' = \frac{a^2 y}{x^2 + y^2}$$

$$\text{circle} \quad (x - x_c)^2 + (y - y_c)^2 = R^2$$

$$\text{line} \quad y' = -\frac{x_c}{y_c} x' + \frac{a^2}{2y_c}$$

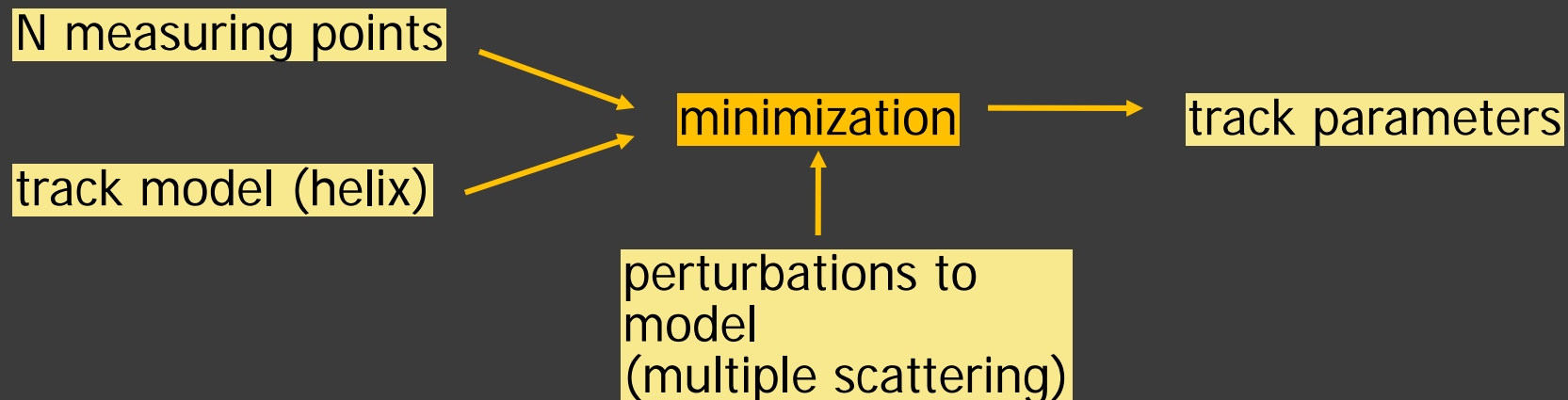
$$\frac{dy}{dx} \Big|_{x=0, y=0} = -\frac{x_c}{y_c}$$

easier to check consistency
of hits with straight line than
with helix



From raw data to summary data pattern recognition / track fitting

- ➔ **algorithm properties:**
 - minimal number of loops;
 - α , β determined individually for each sub-detector;
 - using int. point - not applicable to secondary tracks;
 - each TE can be associated to several tracks;
 - additional info can be included (energy, direction, ...)
- ➔ **track fit:**
 - from multiple TE's determine best helix parameters in chosen point (closest approach to int. point)



From raw data to summary data track fitting

- ➔ **Track fitting algorithms:**
divided according to
track model usage, inclusion of model distortions (mult. scatt., energy losses)

Global Methods

Progressive Methods

Break Point Methods

- ➔ **Global Methods:**
simultaneous minimization of χ^2 of all measurement points;
mult. scatt. included in the error matrix



properties:

- all meas. points used simultaneously;
- simultaneous pattern recognition not possible
(as opposed to Progressive methods);
- calculation expensive (NxN matrix inversion);

From raw data to summary data track fitting

Global method - track model:
expected coordinate values

$$\begin{pmatrix} x_{\text{exp}}^n \\ y_{\text{exp}}^n \\ z_{\text{exp}}^n \end{pmatrix} = \begin{pmatrix} x_0 + R_0^{-1} [\sin \psi_n - \sin \psi_0] \\ y_0 - R_0^{-1} [\cos \psi_n - \cos \psi_0] \\ z_0 + R_0^{-1} \cot \theta_0 [\psi_n - \psi_0] \end{pmatrix}$$

5 free parameters: $\mathbf{p}_0 = (y_0, z_0, \psi_0, \theta_0, 1/R)$
($x_0 = y_0 / \tan \psi_0$)

N measured 3-dimensional points \Rightarrow N 3-dimensional functions
depending on 5 parameters $\mathbf{f}(\mathbf{p}_0)$

global χ^2 minimization:

$$\chi^2(\vec{p}_0) = (\vec{f}(\vec{p}_0) - \vec{m})^T \vec{C}^{-1} (\vec{f}(\vec{p}_0) - \vec{m})$$

From raw data to summary data track fitting

Global method - example:
straight line fit

model: $y_n = kx_n + y_0$
N meas. of y at x_n

N	$k\Delta x$	$\sigma_k \Delta x$
2	$y_2 - y_1$	$\sqrt{2}\sigma$
3	$(y_3 - y_1)/2$	$\sigma/\sqrt{2}$
4	$(3y_4 + y_3 - y_2 - 3y_1)/10$	$\sigma/\sqrt{5}$



$$\chi^2 = \sum_{n=1}^N \frac{(y_n - kx_n - y_0)^2}{\sigma_n^2}$$

minimization yields

$$k \sum_{n=1}^N \frac{x_n^2}{\sigma_n^2} + y_0 \sum_{n=1}^N \frac{x_n}{\sigma_n^2} - \sum_{n=1}^N \frac{y_n x_n}{\sigma_n^2} = 0$$

$$k \sum_{n=1}^N \frac{x_n}{\sigma_n^2} + y_0 \sum_{n=1}^N \frac{1}{\sigma_n^2} - \sum_{n=1}^N \frac{y_n}{\sigma_n^2} = 0$$

for $x_n = n\Delta x$ and $\sigma_n = \sigma \Rightarrow$

$$k = \frac{1}{\Delta x} \frac{N \sum n y_n - \sum n \sum y_n}{N \sum n^2 - (\sum n)^2}$$

From raw data to summary data track fitting



Progressive method (Kalman filter):

parameters after n measurement points known;
extrapolate (track model) to $(n+1)^{\text{st}}$ point;
parameters after $n+1$ points = average of extrapolated and
measured parameters at $(n+1)^{\text{st}}$ point;



properties:

enables simultaneous pattern recognition and track fitting;
specific scattering regions inherently included;

From raw data to summary data track fitting

Progressive method:

vector of parameters after n measurement points
 error matrix after n measurement points
 vector of extrapolated parameters
 extrapolated error matrix

\mathbf{p}_n^F ;
 \mathbf{W}_n^F ;
 \mathbf{p}_n^{Fe} ;
 \mathbf{W}_n^e

$$\mathbf{W}_n^e = \mathbf{D}^T \mathbf{W}_n \mathbf{D}, \quad \mathbf{D} = \frac{\partial \vec{p}}{\partial \vec{p}^e}$$

χ^2 : sum of contribution from extrapolation and measurement:

$$\chi^2(\vec{p}_{n+1}) = \underbrace{\chi^2(\vec{p}_n^F)}_{\chi^2 \text{ from } n \text{ points}} + \underbrace{[\vec{p}_{n+1} - \vec{p}_n^{Fe}]^T \mathbf{W}_n^e [\vec{p}_{n+1} - \vec{p}_n^{Fe}]}_{\chi^2 \text{ from extrapolat.}} + \underbrace{[\vec{p}_{n+1} - \vec{p}_{n+1}^{izmer}]^T \mathbf{U} [\vec{p}_{n+1} - \vec{p}_{n+1}^{izmer}]}_{\chi^2 \text{ of } (n+1)^{\text{st}} \text{ point}}$$

after minimization: set of equations for \mathbf{p}_{n+1}^F ;
 if χ^2 from extrapol. larger than chosen value for specific point
 \Rightarrow point not assigned to track

From raw data to summary data track fitting

Progressive method - example:

straight line;

y_n^F and k_n^F after n measurement points;

extrapolation to $(n+1)^{\text{st}}$ point:

$$\vec{p}_n^{Fe} = \begin{pmatrix} y_n^F + k_n^F \Delta x \\ k_n^F \end{pmatrix} = \begin{pmatrix} y_n^{Fe} \\ k_n^{Fe} \end{pmatrix}$$

extrapolated error matrix:

$$D = \frac{\partial \vec{p}}{\partial \vec{p}^e} = \begin{pmatrix} \frac{\partial y_n}{\partial y_n^e} & \frac{\partial y_n}{\partial k_n^e} \\ \frac{\partial k_n}{\partial y_n^e} & \frac{\partial k_n}{\partial k_n^e} \end{pmatrix} = \begin{pmatrix} 1 & -\Delta x \\ 0 & 1 \end{pmatrix}$$
$$W_n^e = D^T W_n D$$

From raw data to summary data track fitting

Progressive method - example:

straight line;

start with first point,

$$y_1^F = y_1^{Fe} = y_1^m,$$

$$k_1^F = k_1^{Fe} = k_1^m = 0$$

starting error matrix:

$$W_1 = \begin{bmatrix} 1/\sigma^2 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\Rightarrow W_1^e = \begin{bmatrix} 1/\sigma^2 & -\Delta x/\sigma^2 \\ -\Delta x/\sigma^2 & \Delta x^2/\sigma^2 \end{bmatrix}$$

$$W_2 = W_1^e + U = \begin{bmatrix} 2/\sigma^2 & -\Delta x/\sigma^2 \\ -\Delta x/\sigma^2 & \Delta x^2/\sigma^2 \end{bmatrix}$$

etc.

N	$k^F \Delta x$	$\sigma_k^F \Delta x$
2	$y_2 - y_1$	$\sqrt{2}\sigma$
3	$(3y_3 - y_2 - 2y_1)/5$	$\sqrt{(14/25)}\sigma = 0.748\sigma$
4	$(30y_4 - y_3 - 18y_2 - 11y_1)/70$	0.524σ

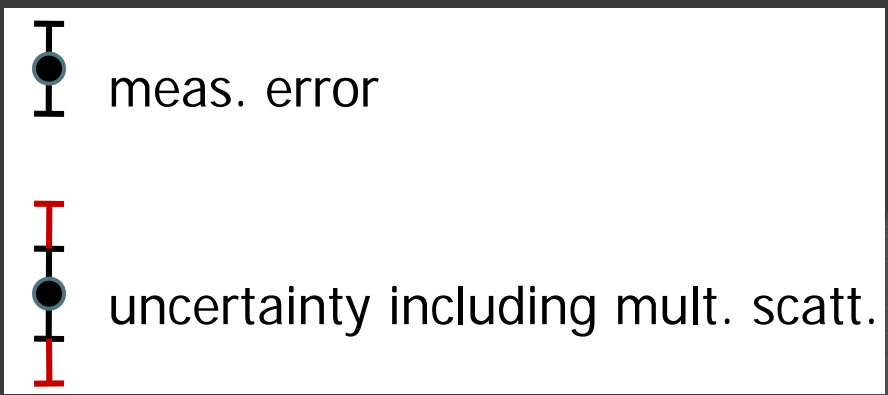
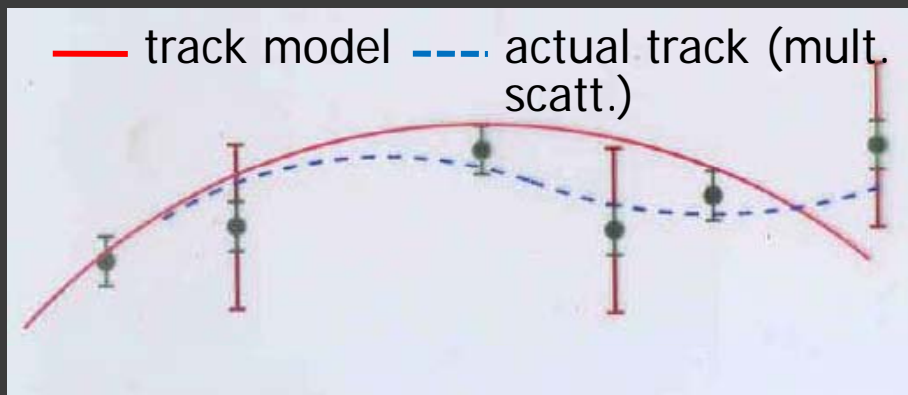
From raw data to summary data track fitting

➔ Global method – multiple scattering:
error matrix:

$$C_{ij} = \sigma_i \sigma_j \delta_{ij} + \overline{\epsilon_i^{MS} \epsilon_j^{MS}}$$

σ_i : uncertainty of ind. measurement;
 ϵ_i : contr. to uncertainty due to mult. scatt.
(Molière formula:

$$\begin{aligned} \overline{\theta_i^{MS}} &= 0 \\ \sqrt{(\overline{\theta_i^{MS}})^2} &= \frac{13,6 \text{ MeV}}{cp\beta} \sqrt{\frac{L}{X_0}} \left[1 + 0.038 \ln \frac{L}{X_0} \right] \end{aligned}$$



distribution of $(y_{\text{meas}} - y_{\text{fit}}) / \sigma_y$ ("pull") is a measure of understanding the effect of mult. scatt. rather than of understanding the meas. errors

From raw data to summary data track fitting

- ➔ Progressive method – multiple scattering:
mult. scatt. between n^{th} and $(n+1)^{\text{st}}$ point:

$$W_n^e = \left[\left[D^T W_n D \right]^{-1} + W_{\text{MS}}^{-1} \right]^{-1}$$

included in the error matrix extrapolation;

using a corresponding mult. scatt. matrix W_{MS} one can include specifics of material between n^{th} and $(n+1)^{\text{st}}$ point



From raw data to summary data track fitting



Break points method:

appropriate for detectors with a limited number of regions with significant scattering;

scattering angles included in χ^2 as free parameters

$$\chi^2(\mathbf{p}_n^F) \rightarrow \chi^2(\mathbf{p}_n^F, \theta_n)$$

From raw data to summary data momentum measurement



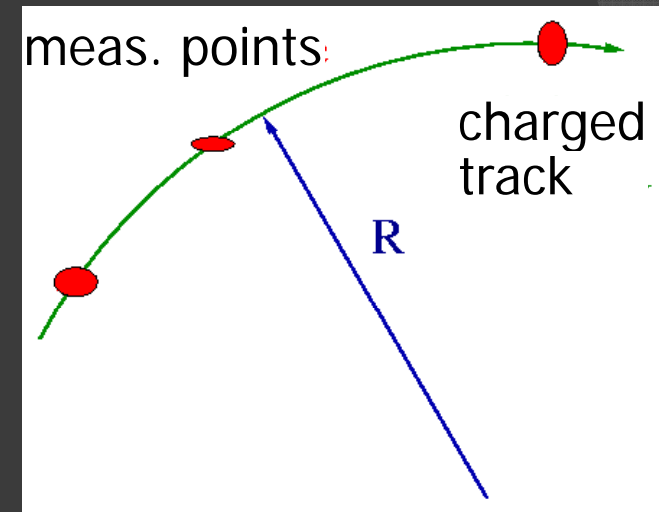
Magnetic field:

$p_t = qBR$;
from curvature R one determines the
transverse (w.r.t. \mathbf{B}) component of \mathbf{p} ;
actual meas. is curvature R ;

accuracy depends on:

of meas. points;
spatial resolution of each point;
mag. field integral BL ;
momentum p ;

multiple scattering;



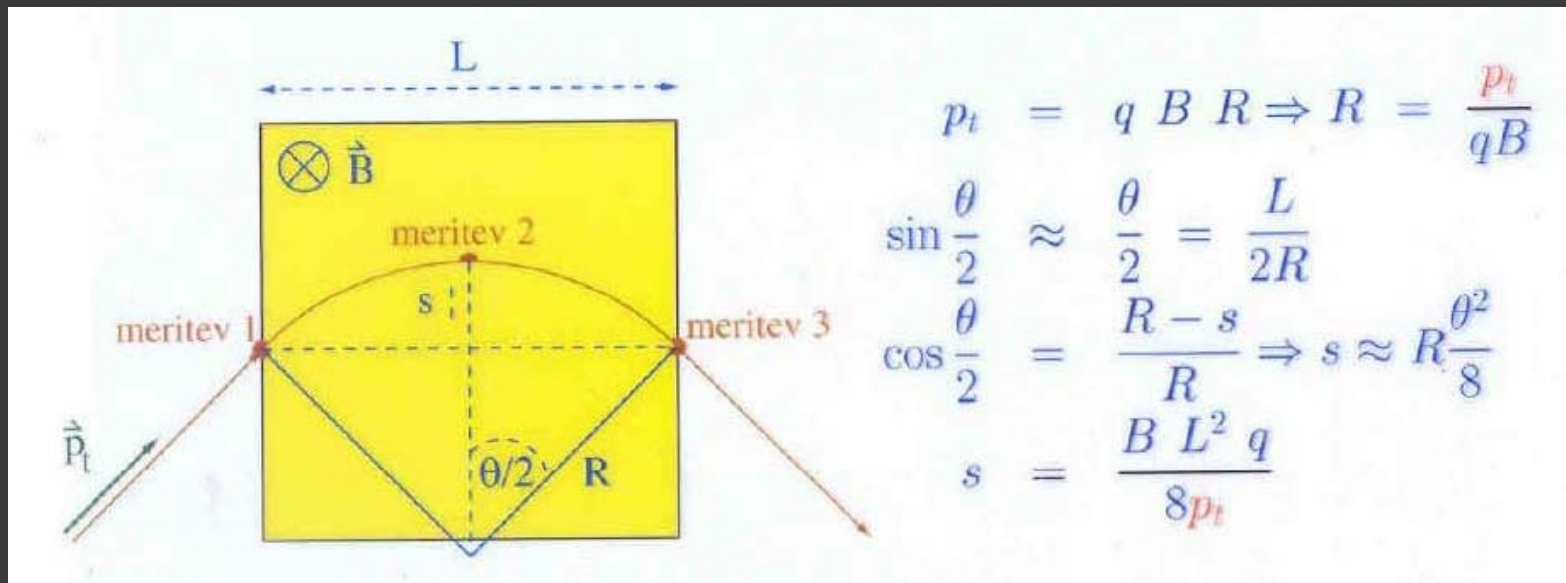
$$\frac{\sigma_{p_t}}{p_t} = \sqrt{ap_t^2 + b}$$

intrinsic resol.

mult. scatt.

From raw data to summary data momentum measurement

Example of momentum determination:



if s determined by
3 measurement points:

$$s = x_2 - \frac{x_1 + x_3}{2}$$

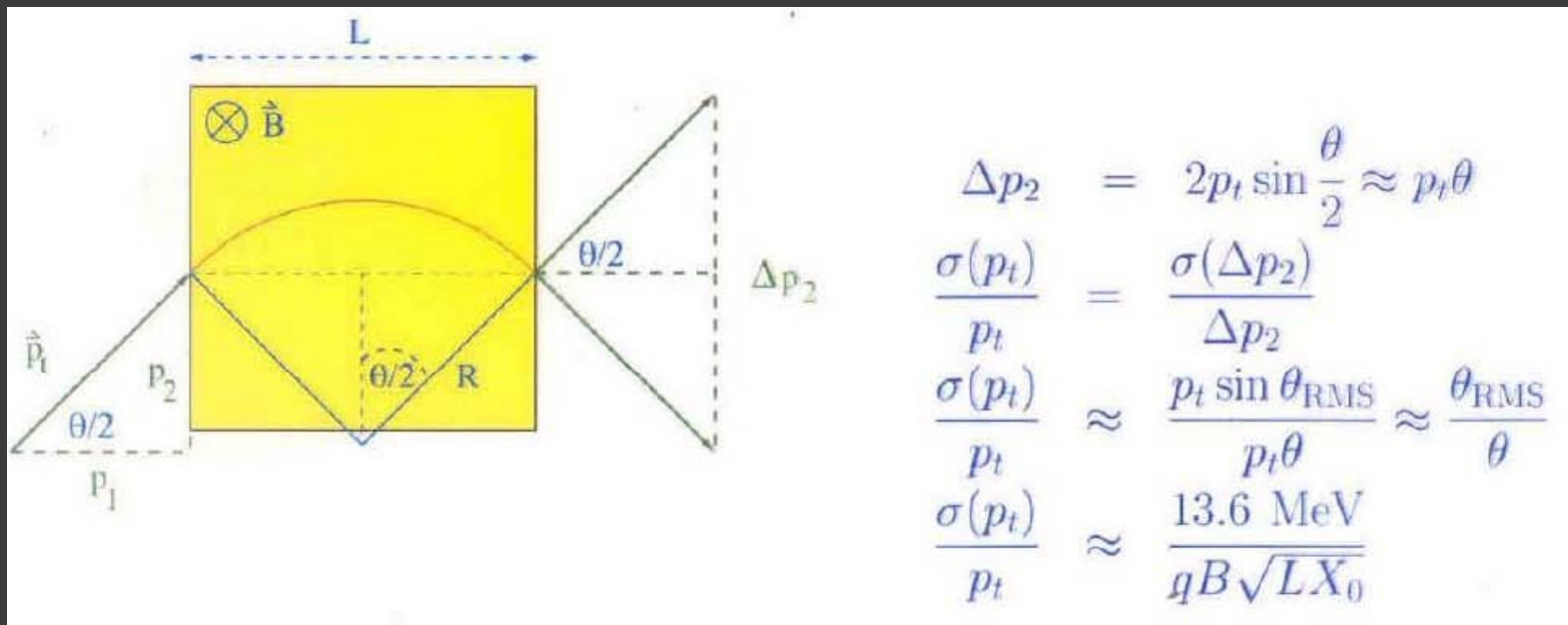
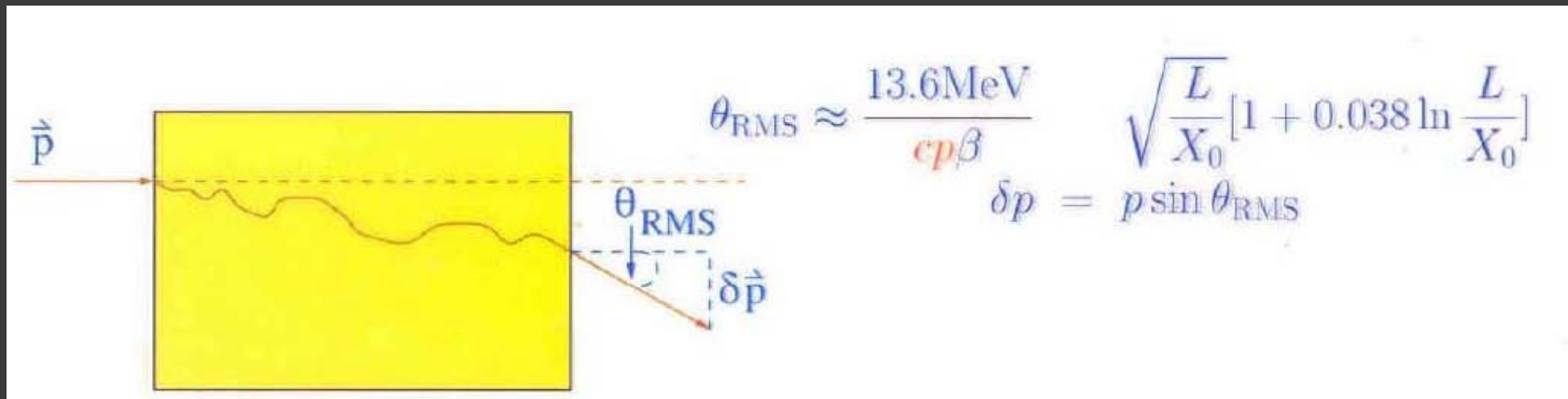
$$\frac{\sigma(p_t)}{p_t} = \frac{\sigma(s)}{s} = \frac{\sqrt{\frac{3}{2}} \sigma(x) 8 p_t}{B L^2 q}$$

for N measurement points:

$$\frac{\sigma(p_t)}{p_t} = \frac{\sigma(x) 8 p_t}{B L^2 q} \sqrt{\frac{720}{N + 4}}$$

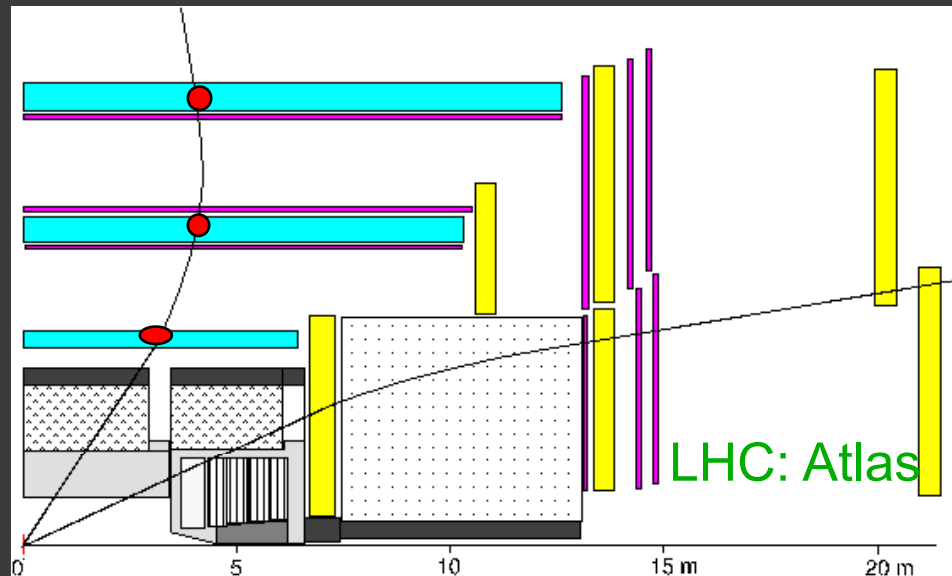
From raw data to summary data momentum measurement

Multiple scattering:



From raw data to summary data momentum measurement

Momentum meas. ATLAS (μ):

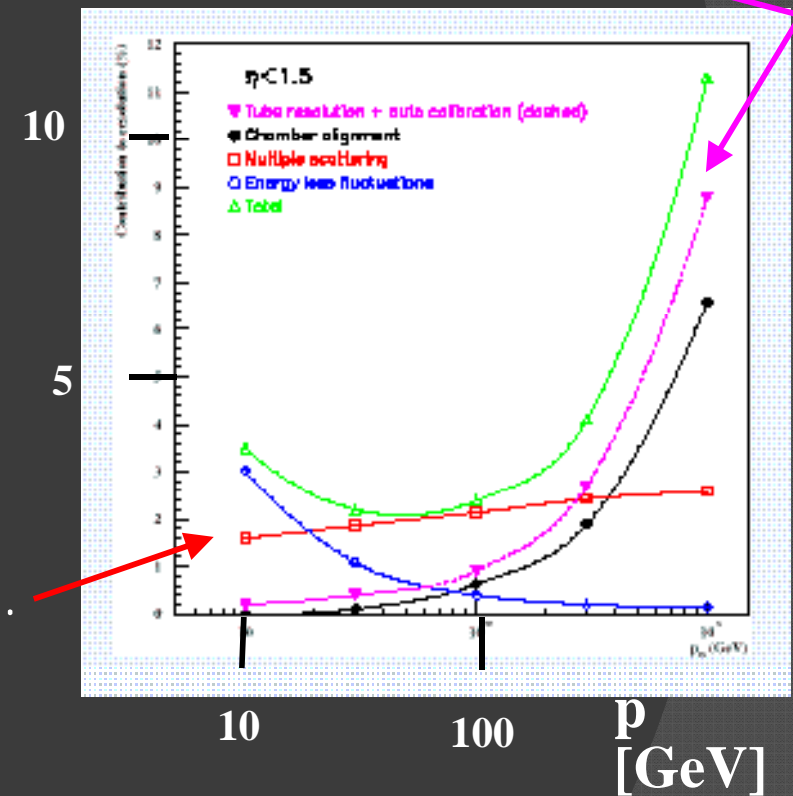


Muon Drift Tube chambers (MDT)
 3 meas. points in barrel;
 $\sigma(x) = 50 \mu\text{m}$
 $L = 4 \text{ m}$
 $B = 1 \text{ T}$ ($BL = 3 - 9 \text{ Tm}$)
 1000 GeV μ from W', Z'
 $\Rightarrow \sigma(p)/p \sim 10\%$

mult. scatt.
 ($\sim \text{const.}$)

$\sigma(p)/p$
 [%]

intrinsic resol.

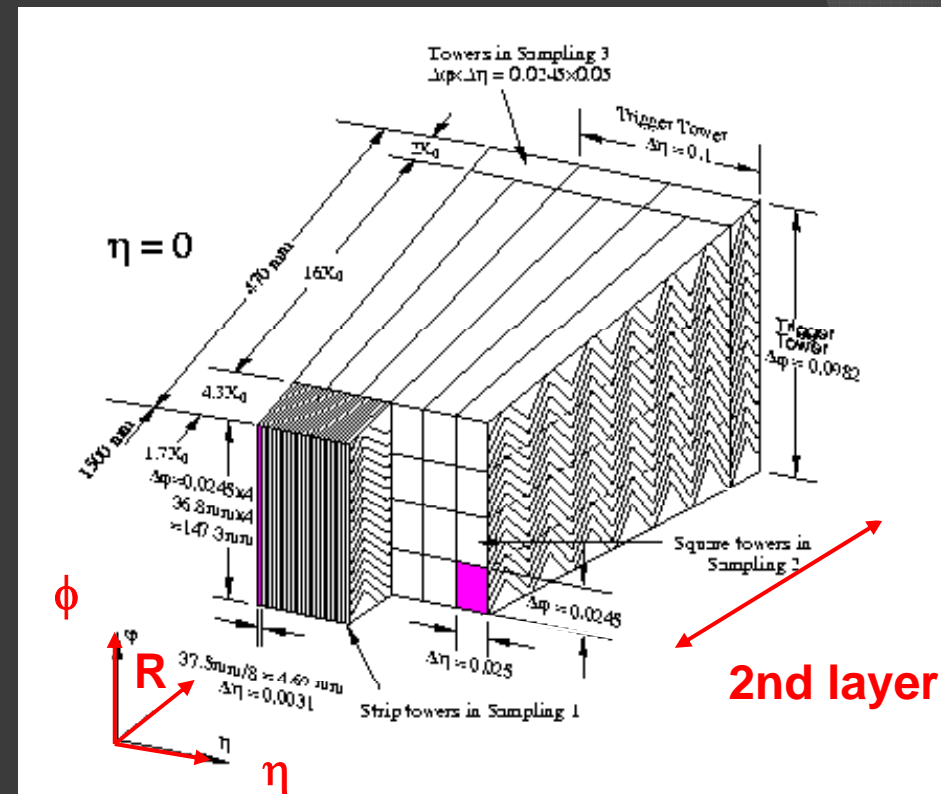


From raw data to summary data calorimeter showers

Calorimeters

are granulated
(composed of individual cells);
charged and neutral particles deposit
energy in several cells;
to measure E of particle (or even
hadronic jet) need method of associating
individual cell energy deposit to
particles ("clustering")

purpose of clustering:
improved signal/noise (considering
correlations among cells);
separation of EM/hadronic showers;
search for isolated particles (e, γ, μ, \dots)



ATLAS LiAr EM calorim.:
accordion geom.;
3 layers in radial direction;
2nd layer: $\Delta\eta \times \Delta\phi = 0.025 \times 0.025$
 $\eta = -\ln(\text{tg } \Theta/2)$ ($\Rightarrow 4 \text{ cm} \times 4 \text{ cm}$)

From raw data to summary data calorimeter showers



Reconstruction in few steps:

basic selection of cells

rejection of cells with known noise (online);

selection of cells with high signal ("seed") and neighbouring cells with lower signal;

$E_i/E_{\text{noise},i} > a$ ($a=3,4,\dots$) and $E_{i+1}/E_{\text{noise},i+1} > b$ ($b=2,3,\dots$);

association of cells into showers

several known algorithms, e.g. Mulguisin algorithm



Hadron calorim.

for hadronic jets energy meas.;

precision of reconstruction reflects in invariant mass resolution

optimization of clustering depends on

luminosity;

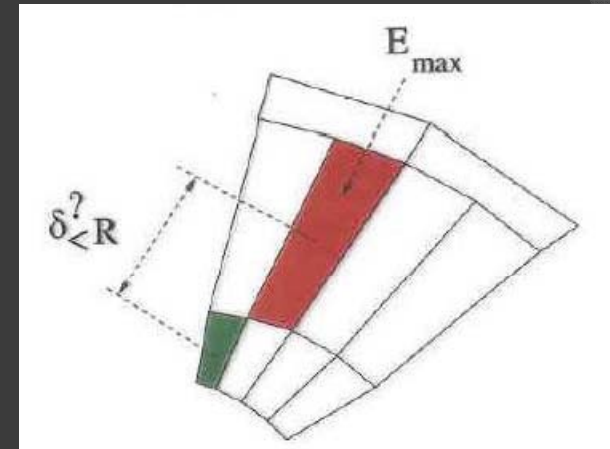
process under study;

From raw data to summary data calorimeter showers



Mulguisin algorithm:

- search for cell with largest E deposit – represents initial shower; dimension set to calorim. spatial resolution R_0 ;
- search for cell with 2nd largest E deposit;
- calculate distance between two cells;
 - if smaller than shower dimension \Rightarrow assoc. cell to shower;
 - (can calculate new shower center (weighted);
new shower dimension can be set to max. dist. between shower center and each assoc. cell;)
 - if larger than shower dimension \Rightarrow start of new shower with dimension R_0
- repeat until all cells taken into account;



From raw data to summary data calorimeter showers

Mulguisin algorithm, implementations:

$$\begin{aligned}
 E_{k+1} &= E_k^{\text{pljusk}} + E_k^{\text{celica}} && \text{cell} \\
 \eta_{k+1}^{\text{pljusk}} &= \eta_k^{\text{pljusk}} && \text{shower} \\
 \phi_{k+1}^{\text{pljusk}} &= \phi_k^{\text{pljusk}} \\
 R_{k+1}^{\text{pljusk}} &= R_0 && \text{size and direction of shower const.}
 \end{aligned}$$

$$\begin{aligned}
 E_{k+1} &= E_k^{\text{pljusk}} + E_k^{\text{celica}} \\
 \eta_{k+1}^{\text{pljusk}} &= (E_k^{\text{pljusk}} \eta_k^{\text{pljusk}} + E_k^{\text{celica}} \eta_k^{\text{celica}}) / (E_k^{\text{pljusk}} + E_k^{\text{celica}}) \\
 \phi_{k+1}^{\text{pljusk}} &= (E_k^{\text{pljusk}} \phi_k^{\text{pljusk}} + E_k^{\text{celica}} \phi_k^{\text{celica}}) / (E_k^{\text{pljusk}} + E_k^{\text{celica}}) \\
 R_{k+1}^{\text{pljusk}} &= R_0
 \end{aligned}$$

size of shower const.,
direct. recalculated
at each step

$$\begin{aligned}
 E_{k+1} &= E_k^{\text{pljusk}} + E_k^{\text{celica}} \\
 \eta_{k+1}^{\text{pljusk}} &= (E_k^{\text{pljusk}} \eta_k^{\text{pljusk}} + E_k^{\text{celica}} \eta_k^{\text{celica}}) / (E_k^{\text{pljusk}} + E_k^{\text{celica}}) \\
 \phi_{k+1}^{\text{pljusk}} &= (E_k^{\text{pljusk}} \phi_k^{\text{pljusk}} + E_k^{\text{celica}} \phi_k^{\text{celica}}) / (E_k^{\text{pljusk}} + E_k^{\text{celica}}) \\
 R_{k+1}^{\text{pljusk}} &= \max(R_k, \delta)
 \end{aligned}$$

size and direct. of
shower recalculated
at each step

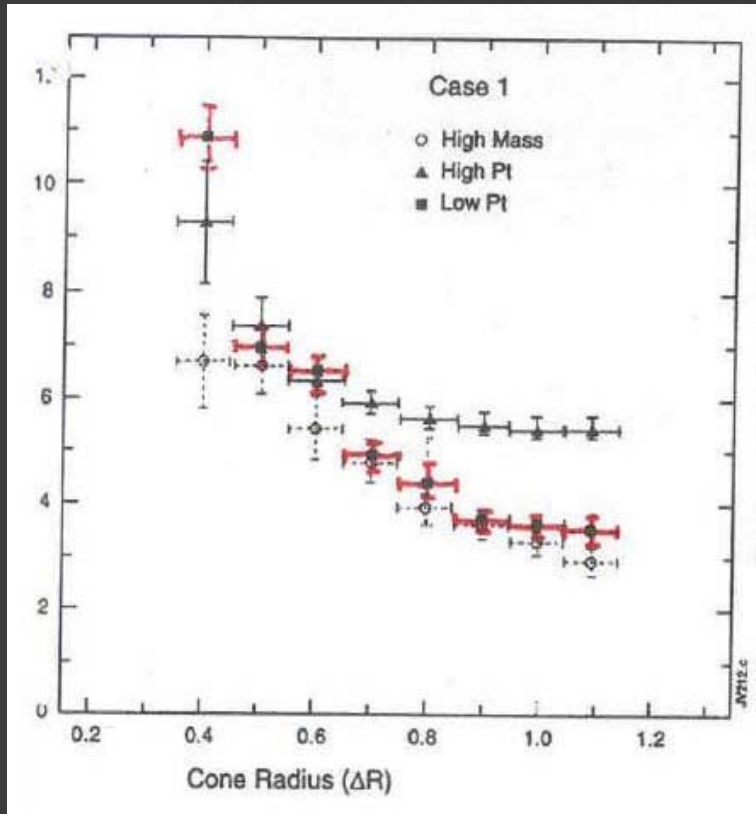


From raw data to summary data calorimeter showers

Resolution on dijet invariant mass:

LHC simulation;
individual event:

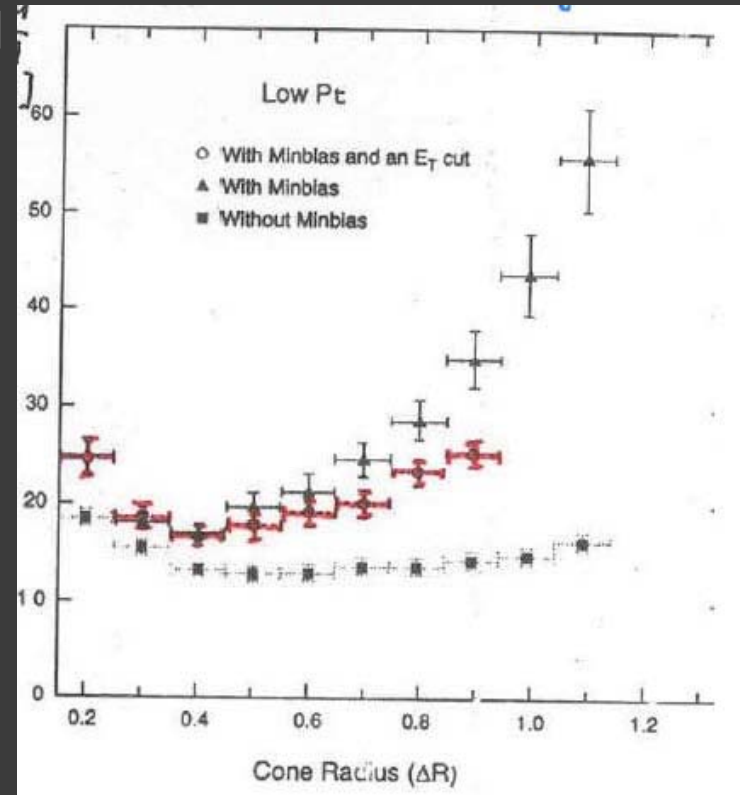
σ_M/M
[%]



→ increasing allowed shower size ⇒
larger fraction of E reconstr. ⇒ better resol.

event + 30 min. bias events:

σ_M/M
[%]



← decreasing allowed shower size ⇒
smaller fraction of E reconstr. ⇒
worse resol.;
→ increasing allowed shower size ⇒
larger fract. of E from other events ⇒
worse resol.

From raw data to summary data particle identification

Hadron identification

most detectors use some variation of **Cherenkov light** detection;

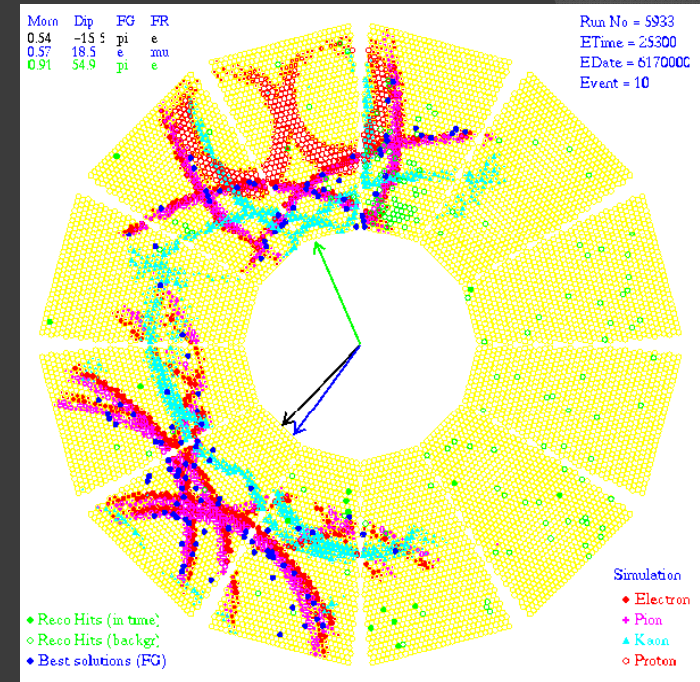
Cherenkov ring detectors:
photons in detector \Rightarrow radius of ring
 \Rightarrow Cherenkov angle \Rightarrow particle velocity \Rightarrow mass

large number of γ 's – impossible to consider all combinations;

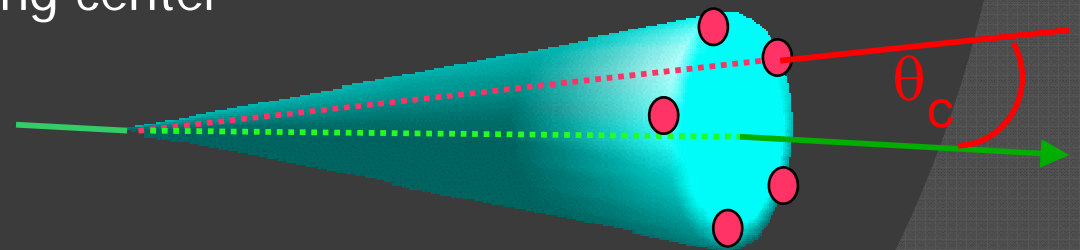
charged track (through geometry dependent equations) determines ring center;

consider only γ 's consistent with ring center

$$\theta_C = \theta_C^{\text{exp}}(m_i) \pm N\sigma_{\theta_C}$$



BaBar - DIRC



From raw data to summary data particle identification

Cherenkov ring detectors
likelihood function:

$$\mathcal{L}(N_{exp}, N_{bg}) = \mathcal{P}_{\text{Poisson}} \prod_{i=1}^N \mathcal{P}_i$$

$$\mathcal{P}_{\text{Poisson}} = \frac{(N_{exp} + N_{bg})^N e^{-(N_{exp} + N_{bg})}}{N!}$$

$$\mathcal{P}_i = \frac{N_{exp}}{N_{exp} + N_{bg}} \underbrace{\frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{(\theta_i - \theta_C)^2}{2\sigma_i^2}}}_{\text{Gaussova porazdelitev } \gamma \text{ okoli } \theta_C(p_i, m_i)} + \frac{N_{bg}}{N_{exp} + N_{bg}} \underbrace{\frac{\theta_i}{6\theta_C\sigma_i}}_{\text{enakomerna porazdelitev po površini detekt. x } \theta_i}$$

$$N_{exp}, \theta_C = f(p_i, m_i)$$

background:
uniform distrib. over
detector surface

$\mathcal{L}(N_{exp}, N_{bg}) / \mathcal{L}(N_{exp}=0, N_{bg})$: measure of probability for set of γ 's to originate from a particle with (p_i, m_i)

From raw data to summary data particle identification

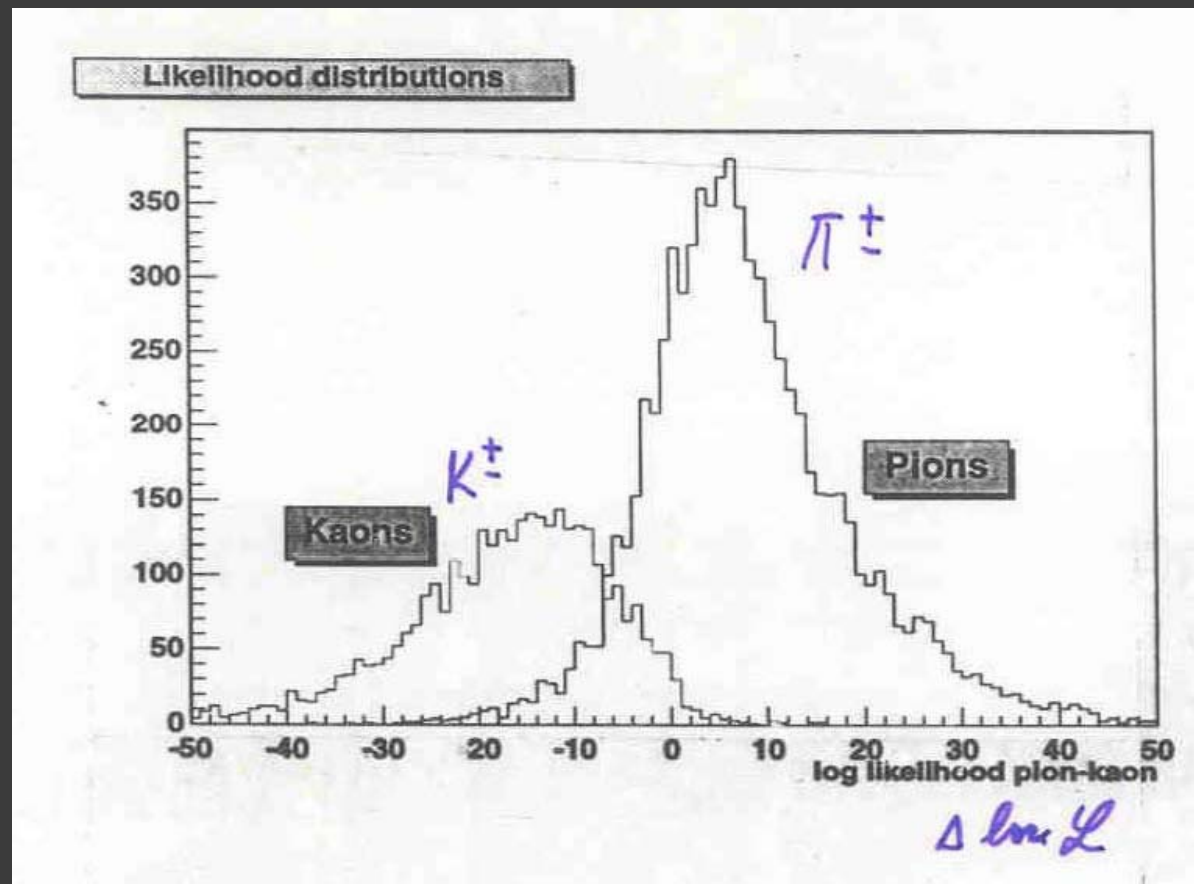
Cherenkov ring detectors

particle separation:

from $\mathcal{L}(N_{exp}, N_{bg}) / \mathcal{L}(N_{exp}=0, N_{bg}) \Rightarrow P(m_i)$

$P(m_i) / P(m_j)$ particle separation

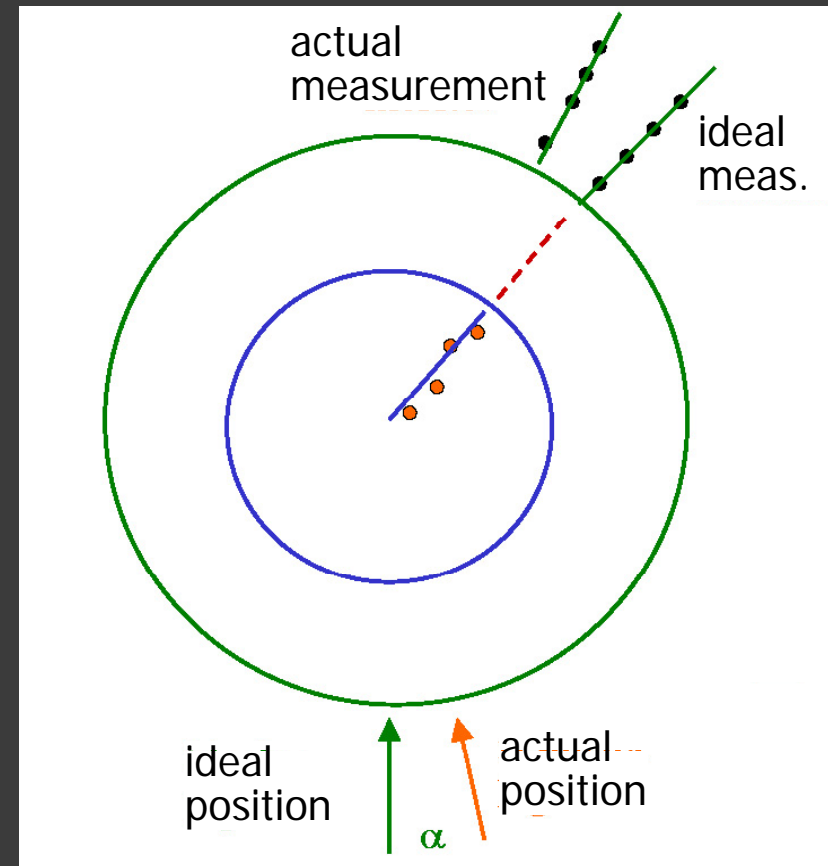
e.g. HERA-B $P(\pi, \text{not } K) / P(K, \text{not } \pi)$:



Calibration

Tracking detectors

- Tracking detectors calibration
 - individual subdetectors must be properly inter-oriented, otherwise tracks distorted;
 - for any calibration need sample (tracks, decays, ...) with precisely known detector response



Calibration

Tracking detectors

Description of detector (mis)alignment

position of individual subdetector w.r.t. reference
(most precisely mechanically positioned detector)

described by set of small parameters α
(translation, rotation, t-delay,...)

assume linear relation

$$\vec{q}^{meas} - \vec{q}^{ext} = S \vec{\alpha}$$

\mathbf{q}^{meas} : vector of measured coordinates
 \mathbf{q}^{ext} : vector of extrapolated coord.
(from the reference detector)

S: matrix depending on measuring
coord., track model, detector
geometry

simplest case:

α composed of 3 translations and 3 rotations

$$\alpha = (\eta_x, \eta_y, \eta_z, \varepsilon_x, \varepsilon_y, \varepsilon_z)$$

Calibration

Tracking detectors

Determination of position
minimization:

$$\chi^2 = \sum_{k \in \text{meas. points}} [\vec{q}_k^{\text{meas}} - \vec{q}^{\text{ext}} - S_k \vec{\alpha}]^T W_k^{-1} [\vec{q}_k^{\text{meas}} - \vec{q}^{\text{ext}} - S_k \vec{\alpha}]$$

result:

$$\left(\sum_{k \in \text{meas. points}} S_k^T W_k^{-1} S_k \right) \vec{\alpha} = \sum_{k \in \text{meas. points}} S_k^T W_k^{-1} (\vec{q}_k^{\text{meas}} - \vec{q}^{\text{ext}})$$

vector of displacements α

coordinates meas. in subdetector are corrected by α

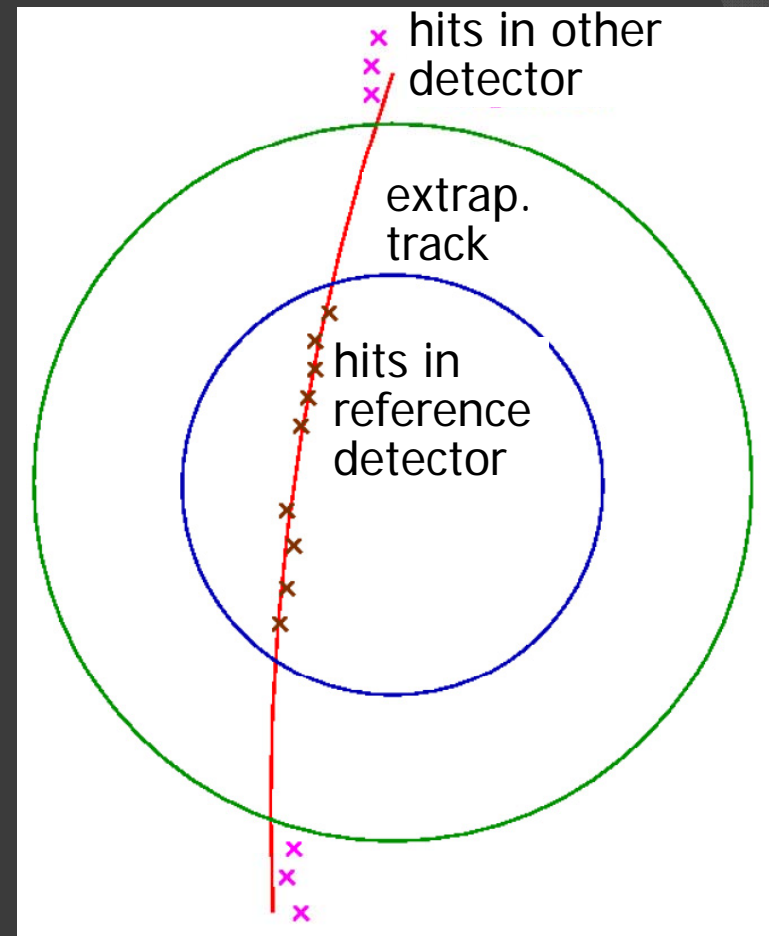
Calibration

Tracking detectors

Appropriate sample

often cosmic rays;
other decays observed,
e.g. $Z^0 \rightarrow \mu^+\mu^-$ (LEP);

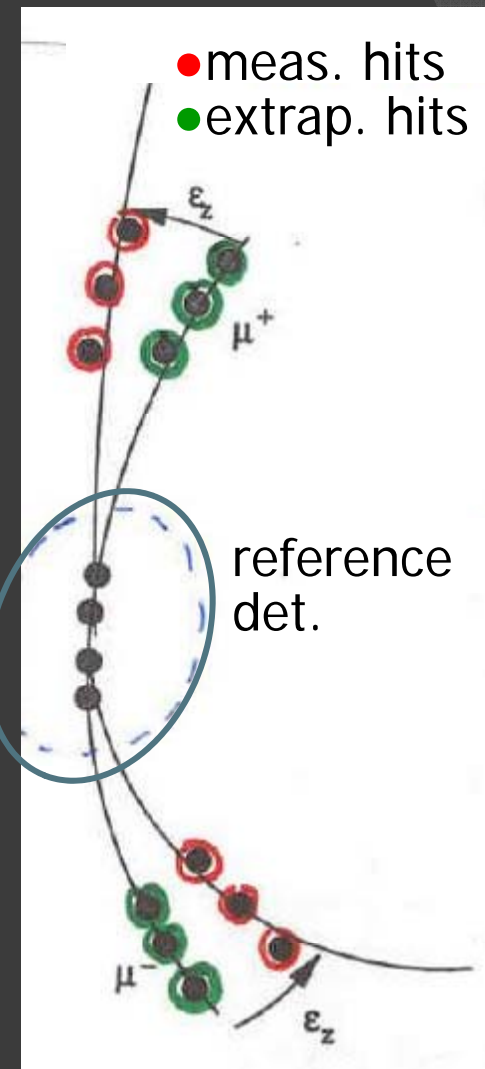
(needed also to check
the alignment method)



Calibration Tracking detectors

Appropriate sample

e.g. $Z^0 \rightarrow \mu^+\mu^-$ (LEP);

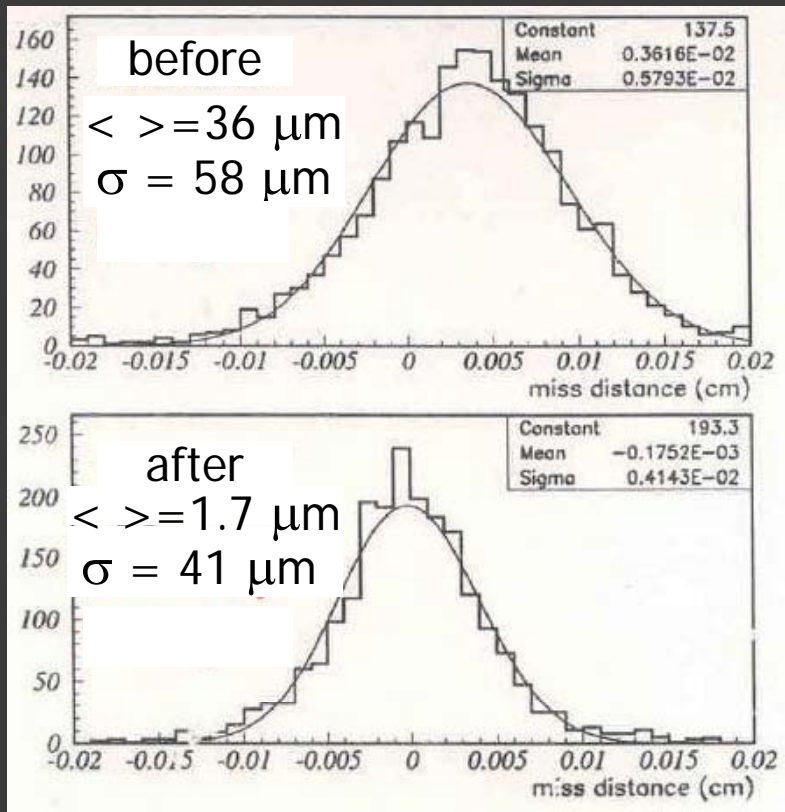


Calibration Tracking detectors

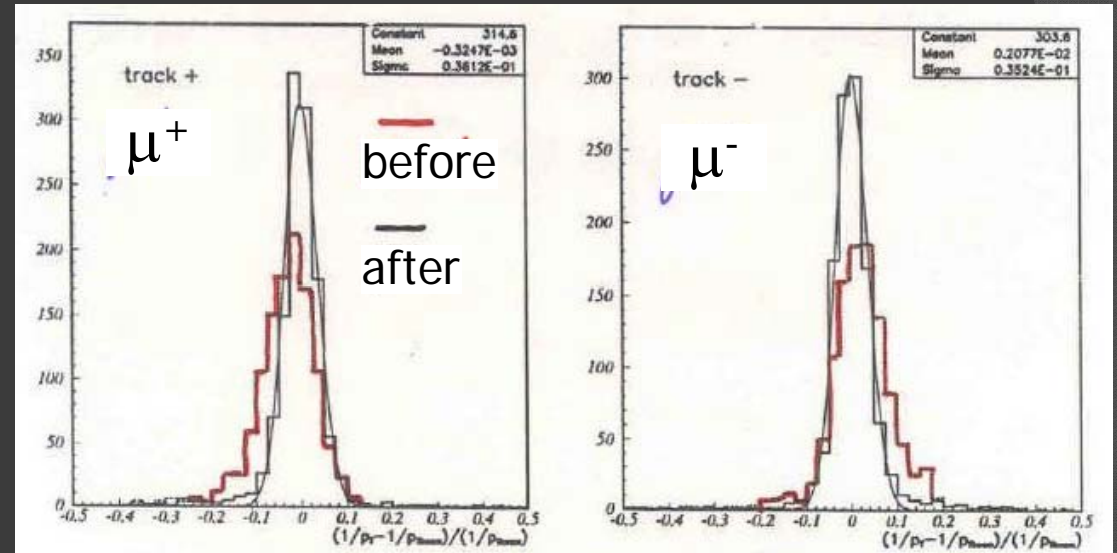
Example

Delphi detector at LEP

δ [cm]



$$\left[\left(\frac{1}{p_t} \right) - \left(\frac{1}{p_t^{\text{ext}}} \right) \right] / \left(\frac{1}{p_t^{\text{ext}}} \right)$$



Calibration

Data and simulation

- ➔ Calibration of data and MC simulation
example of RICH (Delphi at LEP)

sample with known detector response:

$$\cos\theta_c = 1/\beta n$$

tracks with $p > 6$ GeV; even protons at $p > 6$ GeV $\beta = 1 - 10^{-2}$
sample yields value of n ;

expected error on θ_c , $\sigma(\theta_c)$ needed for fits;

$(\theta_c^{\text{meas}} - \theta_c^{\text{exp}})/\sigma(\theta_c)$ "pull" examined;

pull distribution properties:

for gaussian distribution of θ_c^{meas} :

$$\langle \rangle = 0$$

$$\sigma = 1 \quad \text{if } \sigma \neq 1 \Rightarrow \text{correct } \sigma(\theta_c)$$

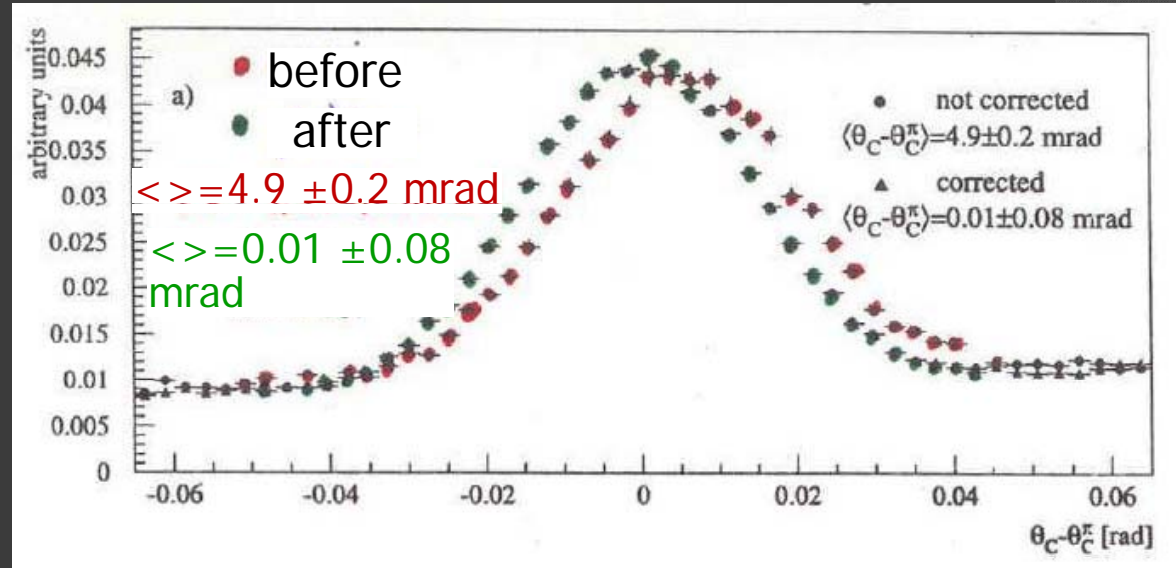
Calibration

Data and simulation

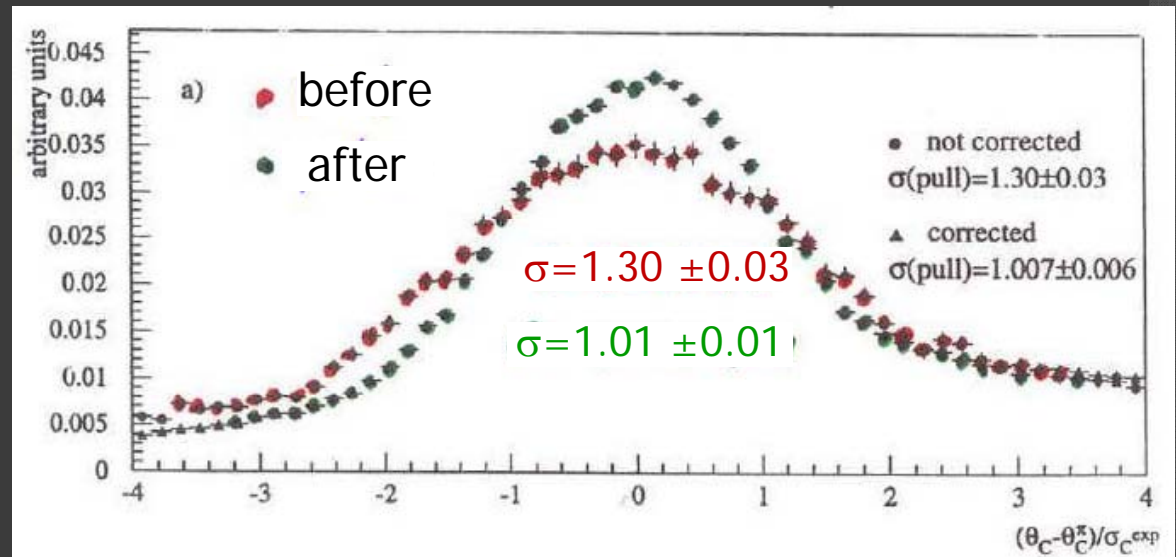
example of RICH
(Delphi at LEP)

same method and
corrections applied
to MC simulation
to match the data

$$\theta_C^{\text{meas}} - \theta_C^{\text{exp}} \text{ [rad]}$$



$$(\theta_C^{\text{meas}} - \theta_C^{\text{exp}}) / \sigma(\theta_C)$$



Analysis of data

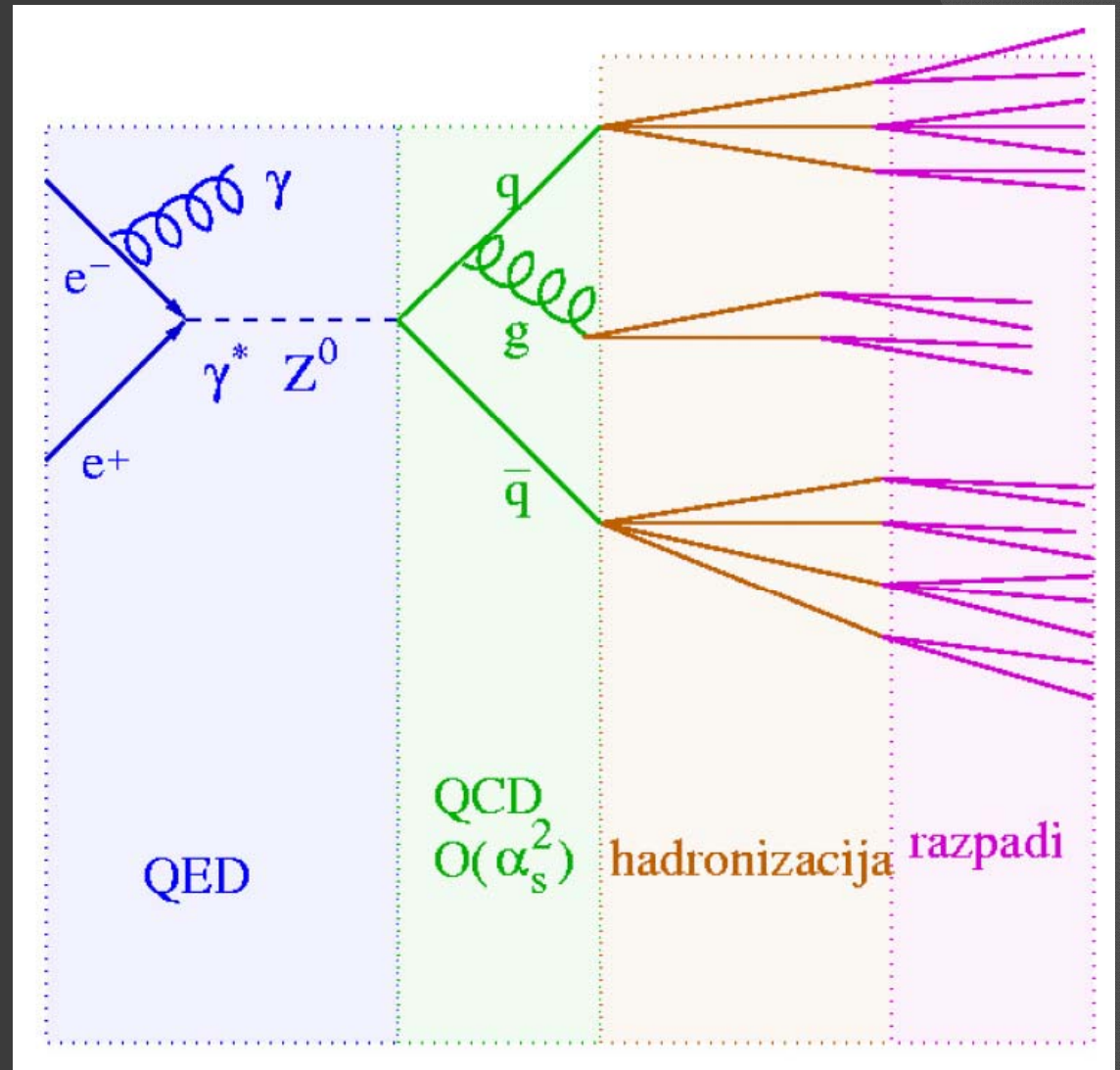
Hadronic jets reconstruction



quark production
observable jets

example of jet formation

reconstructed jets \Rightarrow
interpretation of
processes at parton level



energy scale (LEP) 91 10 1 E [GeV]

Analysis of data

Hadronic jets reconstruction

jet reconstruction

experimental method

(assigning tracks to jets, calculation of energy,);

observables (# jets, # tracks in jet, angular distributions,...)

must be expressed in terms of theory parameters

in order to test predictions;

definition of jet

should be appropriate for exp. usage and theoretical calculations in order to confront theory & experiments

Algorithm for track association

resolution parameter

combination of tracks

y_{cut} ;

if $y_{ij} < y_{\text{cut}}$ two tracks in
same jet

e.g. $\mathbf{p}_{\text{jet}} = \mathbf{p}_i + \mathbf{p}_j$;

Analysis of data

Hadronic jets reconstruction

jet reconstruction algorithms

name	resolution parameter	combination	comment
JADE	$y_{\text{cut}} = M_{ij}^2 / E_{\text{mer}}^2 = 2E_i E_j (1 - \cos \theta_{ij}) / E_{\text{mer}}^2$	$p_k = p_i + p_j$	preserves E, \mathbf{p}
p	$y_{\text{cut}} = (p_i + p_j)^2 / E_{\text{mer}}^2$	$\vec{p}_k = \vec{p}_i + \vec{p}_j$ $E_k = \vec{p}_k $	preserves \mathbf{p}
DURHAM	$y_{\text{cut}} = 2 \min(E_i^2, E_j^2) (1 - \cos \theta_{ij}) / E_{\text{mer}}^2$	$p_k = p_i + p_j$	preserves E, \mathbf{p} ; resummable NLO log's

in Table: $p_i \rightarrow 4\text{-momentum}$
 $\vec{p}_i \rightarrow 3\text{-momentum}$

higher order calculations in perturbative QCD performed for massless partons
 \Rightarrow resolution parameters calculated for massless partons;
 summing two jets 4-momenta in general leads to a non-zero mass object (new jet);
 several algorithms exist to avoid the problem

Analysis of data

Hadronic jets reconstruction

algorithms comparison

for all algorithms perturbative calculations exist to $\mathcal{O}(\alpha_s^2)$,
e.g. relative rate of n jets:

$$\begin{aligned}R_2 &= 1 - A(y_{\text{cut}}) \frac{\alpha_s(\mu)}{2\pi} - [B(y_{\text{cut}}, \mu) + C(y_{\text{cut}})] \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2 \\R_3 &= A(y_{\text{cut}}) \frac{\alpha_s(\mu)}{2\pi} + B(y_{\text{cut}}, \mu) \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2 \\R_4 &= C(y_{\text{cut}}) \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2\end{aligned}$$

using above predictions + models of hadronization
comparison of parton and hadron distributions

Analysis of data

Hadronic jets reconstruction

algorithms comparison

n jet rates vs. y_{cut}

before hadronization
(parton level)

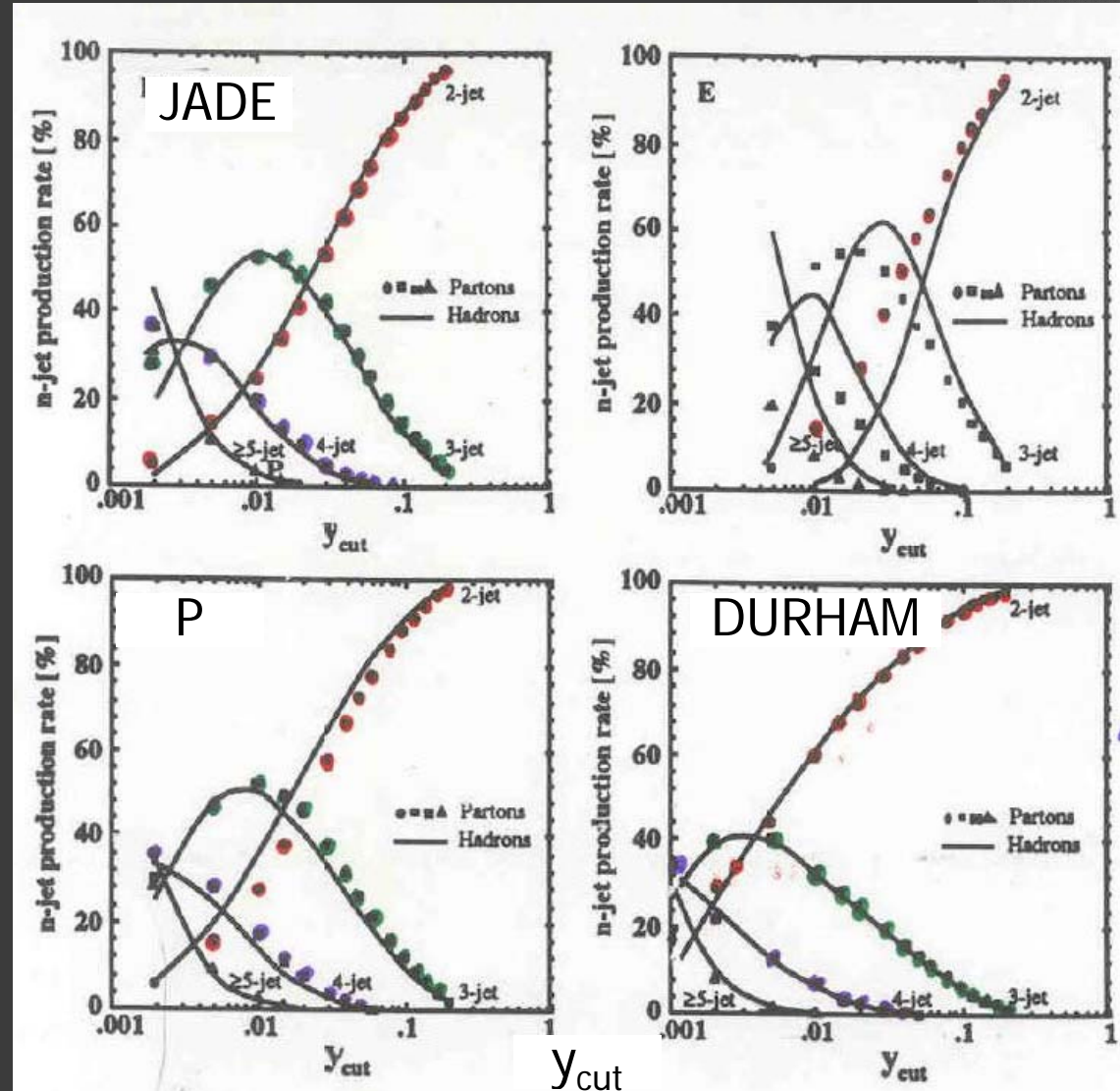
● 2 jets

● 3 jets

● 4 jets

-- after hadronization

smallest hadronization
corrections for
JADE and DURHAM

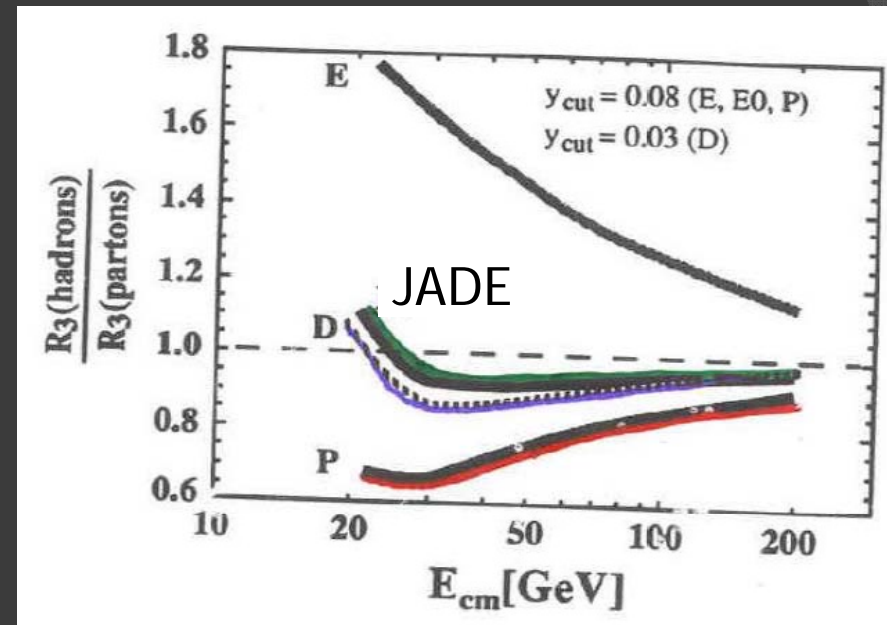


Analysis of data

Hadronic jets reconstruction

algorithms comparison
n jet rates vs. E

smallest hadronization
corrections for
JADE



Analysis of data

Heavy quark tagging

Heavy (b) quark tagging

$H(m > 150 \text{ GeV}) \rightarrow b\bar{b} > 50\%$; CPV in B system;

try to discriminate b initiated jets from others;
use properties of hadrons composed of b quarks:

lifetime

1.6 ps

mass

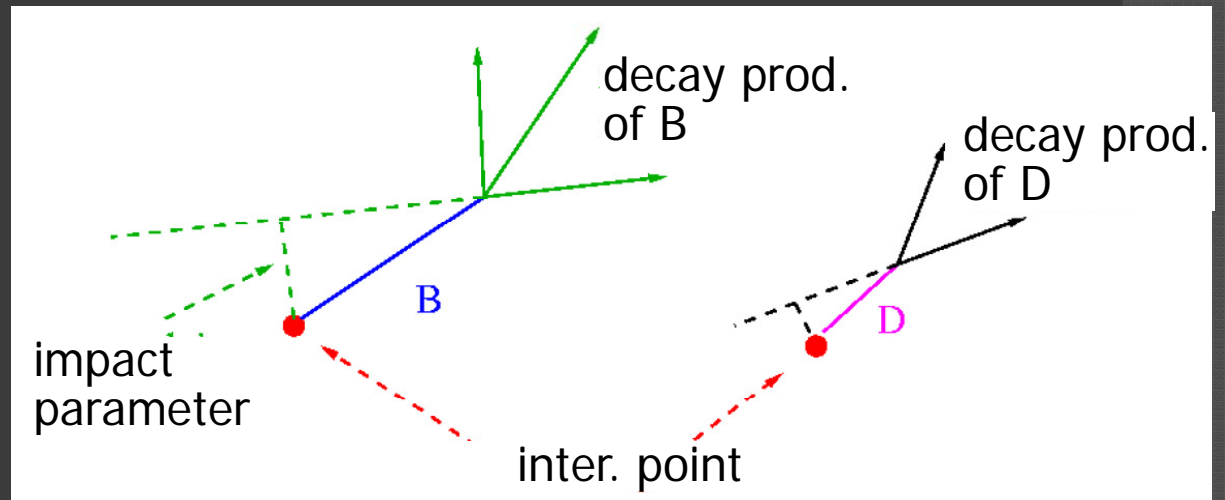
5.3 GeV

energy

in fragmentation higher than for other hadrons

lifetime:

$$\gamma c \tau_B > \gamma c \tau_D \quad \delta_B > \delta_D$$



Analysis of data

Heavy quark tagging

Heavy (b) quark tagging

mass:

example of
rapidity for

$B(M) \rightarrow X(E_1)Y(E_2)$;

$$\begin{aligned} \text{rapidity : } y &= \frac{1}{2} \ln \left[\frac{E + p_z}{E - p_z} \right] \\ M^2 &\approx 2E_1 E_2 (1 - \cos \theta) \\ E_1 &\approx E_2 \approx E/2 \\ \cos \theta &\approx 1 - \frac{2M^2}{E^2} \\ y &\approx \frac{1}{2} \ln \left[\frac{1 + \cos \theta/2}{1 - \cos \theta/2} \right] \underset{M/E \ll 1}{\approx} \frac{1}{2} \ln \left[\frac{4E^2}{M^2} - 1 \right] \end{aligned}$$

large $M \Rightarrow$ small y ;

average # of decay products higher for B than D

\Rightarrow y even smaller

Analysis of data

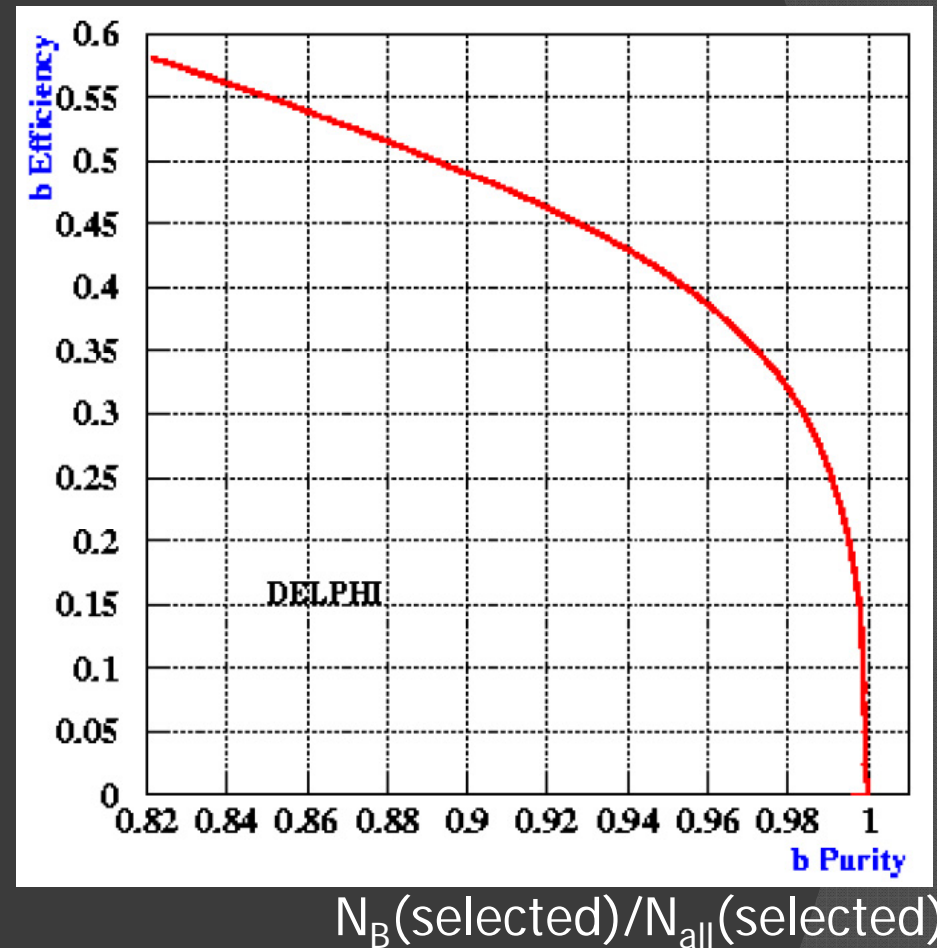
Heavy quark tagging

→ Heavy (b) quark tagging
combination of several
discriminating variables
into single one
(likelihood ratio);

example for Delphi
at LEP;

actual method for
b-tagging depends on
specific experimental
conditions

$N_B(\text{selected})/N_B(\text{generated})$



Analysis of data

Summary

Path from electronic signal detection to result for measured physical quantities involves a number of steps

Each of those represents a specific problem and requires specific methods and solutions (some of those illustrated here)

Quality (correctness and accuracy) of the final results depends crucially on the quality of reconstruction of raw data