素粒子宇宙起源研究機構

Kobayashi-Maskawa Institute for the Origin of Particles and the Universe

ANTIDELCI

KLEIN GORDONOVA E. ZA PROST DELEC

$$\phi = \frac{1}{\sqrt{V}} e^{i p^\mu x_\mu} \qquad x_\mu = (t, -\vec{x}) \qquad p_\mu = (E, -\vec{p})$$

$$E^2 = p^2 + m^2$$

$$\Rightarrow E = \pm\sqrt{p^2 + m^2}$$

KAJ SO REŠITVE Z − = NEGATIVNE ENERGIJE

$$j^\mu = \frac{2}{V}(E, \vec{p})$$

OBVESTILO: 30.4. VAJE → PREDAVANJA

$$j^\mu = -e_0 \frac{2}{V}(E, \vec{p})$$ ELEKTROM. TOK Z e^-

$$j^\mu = e_0 \frac{2}{V}(E, \vec{p}) = -e_0 \frac{2}{V}(-E, -\vec{p})$$ POZITRON e^+

FEYNMAN-STÜCKELBERGOVA INTERPRETACIJA

DIRACOVA ENAČBA

ENAČBA S PRVIMI ODVODI, IN $H^2\psi = (p^2 + m^2)\psi$

$$\hat{H}\psi = i\hbar \frac{\partial \psi}{\partial t}$$

POSKUSIMO

$$\hat{H}\psi = [\vec{\alpha}\cdot\vec{p} + \beta m]\psi$$

$$[\vec{\alpha}\vec{p} + \beta m]^2 = (\vec{\alpha}\cdot\vec{p})^2 + \vec{\alpha}\cdot\vec{p}\,\beta m + \beta m\,\vec{\alpha}\cdot\vec{p} + \beta^2 m^2 =$$

$$(\alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3)(\alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p) + \alpha_1 p_1 \beta m + \alpha_2 p_2 \beta m + \alpha_3 p_3 \beta m + \beta m \alpha_1 p_1 + \beta m \alpha_2 p_2 + \beta m \alpha_3 p_3 + \beta^2 m^2 = \alpha_1^2 p_1^2 + \alpha_2^2 p_2^2 + \alpha_3^2 p_3^2 + (\alpha_1\alpha_2 + \alpha_2\alpha_1) p_1 p_2 +$$

$$+ (\alpha_2\alpha_3 + \alpha_3\alpha_2) p_2 p_3 + (\alpha_1\alpha_3 + \alpha_3\alpha_1) p_1 p_3 +$$

$$+ (\alpha_1\beta + \beta\alpha_1) p_1 m + (\alpha_2\beta + \beta\alpha_2) p_2 m + (\alpha_3\beta + \beta\alpha_3) p_3 m + \beta^2 m^2$$

$\Rightarrow \quad \alpha_i^2 = 1, \beta^2 = 1, \quad \alpha_i \alpha_j + \alpha_j \alpha_i = 0 \quad i \neq j \qquad \alpha_i \beta + \beta \alpha_i = 0, \; i = 1,2,3$

$$\vec{\alpha} = \begin{bmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{bmatrix} \qquad \sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\beta = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix} \qquad I = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

$\vec{\alpha}, \beta$ 4x4 MATRIKE $\rightarrow$ ψ IMA 4 KOMPONENTE

$$\hat{H}\psi = [\vec{\alpha} \cdot \vec{p} + \beta m]\psi \Rightarrow i \frac{\partial}{\partial t}\psi = [-i\vec{\alpha} \cdot \vec{\nabla} + \beta m]\psi$$

$$i\beta \frac{\partial}{\partial t}\psi = [-i\beta\vec{\alpha} \cdot \vec{\nabla} + \underbrace{\beta^2}_{1} m]\psi$$

DEFINIRAMO

$$\gamma^\mu = (\beta, \beta\vec{\alpha}) \quad \Rightarrow \quad \boxed{[i\gamma^\mu \partial_\mu - m]\psi = 0}$$

KOVARIANTNA OBLIKA DIRACOVE E.

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2 g^{\mu\nu} \qquad g^{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

REŠITVE
IŠČEM V OBLIKI $\psi = u(\vec{p})\, e^{-ip^\mu x_\mu}$

$u(\vec{p})$ BISPINOR

$$\Rightarrow [\gamma^\mu p_\mu - m]\, u(\vec{p}) = 0$$

$$\hat{H} u(\vec{p}) = (\vec{\alpha} \cdot \vec{p} + \beta m]\, u(\vec{p}) = E\, u(\vec{p})$$

$$\begin{bmatrix} 0 & \vec{\sigma} \cdot \vec{p} \\ \vec{\sigma} \cdot \vec{p} & 0 \end{bmatrix} u(\vec{p}) + \begin{bmatrix} m & 0 \\ 0 & -m \end{bmatrix} u(\vec{p}) = E\, u(\vec{p})$$

$$u(\vec{p}) = \begin{bmatrix} u_A \\ u_B \end{bmatrix}$$

$$\begin{bmatrix} 0 & \vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p} & 0 \end{bmatrix}\begin{bmatrix} u_A \\ u_B \end{bmatrix} + \begin{bmatrix} m & 0 \\ 0 & -m \end{bmatrix}\begin{bmatrix} u_A \\ u_B \end{bmatrix} = E\begin{bmatrix} u_A \\ u_B \end{bmatrix}$$

$$\vec{\sigma}\cdot\vec{p}\, u_B + m\, u_A = E\, u_A \qquad \vec{\sigma}\cdot\vec{p}\, u_B = (E-m)u_A$$
$$\vec{\sigma}\cdot\vec{p}\, u_A + (-m)u_B = E\, u_B \qquad \vec{\sigma}\cdot\vec{p}\, u_A = (E+m)u_B$$

$$u_A^{(1)} = \chi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{SPINOR} \qquad u_A^{(2)} = \chi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$\boxed{\text{ZA } E>0}$

$$u_B^{(s)} = \frac{1}{E+m}\vec{\sigma}\cdot\vec{p}\, u_A^{(s)} \qquad s = 1,2$$

$$u^{(s)} = N\begin{bmatrix} \chi^{(s)} \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi^{(s)} \end{bmatrix} \qquad s=1,2 \quad \chi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix},\ \chi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$\boxed{E<0}$

$$u_B^{(s)} = \chi^{(s)} \qquad u_A^{(s)} = \frac{\vec{\sigma}\cdot\vec{p}}{E-m}\, u_B^{(s)}, \quad u_B^{(s)} = \chi^{(s)}$$

$$u^{(s+2)} = N\begin{bmatrix} -\frac{\vec{\sigma}\cdot\vec{p}}{|E|+m}\chi^{(s)} \\ \chi^{(s)} \end{bmatrix}$$

ODKOD DVOJNA DEGENERACIJA? 2 REŠ. Z $E>0$
2 REŠ. Z $E<0$

KOMUTATOR H IN $\hat{L}$

$$\hat{H} = \vec{\alpha}\cdot\hat{\vec{p}} + \beta m\,, \qquad \hat{\vec{L}} = \hat{\vec{r}}\times\vec{p} \qquad p_i = -i\frac{\partial}{\partial x_i}$$

$$[\hat{H}, \hat{L}_1] = [\vec{\alpha}\cdot\vec{p} + \beta m\,, x_2 p_3 - x_3 p_2] = [\vec{\alpha}\cdot\vec{p}\,, x_2p_3 - x_3p_2] +$$
$$+ \underbrace{[\beta m\,, x_2p_3 - x_3p_2]}_{=0} \qquad [x_i, p_j] = i\delta_{ij}$$

$$[\vec{\alpha}\cdot\vec{p}, x_2 p_3 - x_3 p_2] = [\alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3, x_2 p_3 - x_3 p_2] = [\alpha_2 p_2, x_2 p_3] - [\alpha_3 p_3, x_3 p_2]$$
$$= -i\alpha_2 p_3 + i\alpha_3 p_2$$

$$[\hat{H}, \hat{L}_1] = -i(\vec{\alpha}\times\vec{p})_1 \quad \Rightarrow \quad [\vec{H}, \vec{L}] = -i(\vec{\alpha}\times\vec{p})$$

$\Rightarrow \vec{L}$ NE KOMUTIRA S $\hat{H}$, $\vec{L}$ NI VEČ DOBRO KV. ŠT.

UPOŠTEVAMO

$$\vec{\Sigma} = \begin{bmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{bmatrix} \qquad [\vec{\alpha}\cdot\vec{p}, \Sigma_1] = \Big[\begin{matrix} 0 & [\sigma, \sigma_1 \end{matrix}$$

$$[\alpha_i, \Sigma_1] = \begin{bmatrix} 0 & [\sigma_i, \sigma_1] \\ [\sigma_i, \sigma_1] & 0 \end{bmatrix}$$

$$\sigma_i^2 = 1 \qquad [\sigma_1, \sigma_2] = 2i\sigma_3 \qquad [\sigma_2, \sigma_3] = 2i\sigma_1 \qquad [\sigma_3, \sigma_1] = 2i\sigma_2$$

$$[\vec{\alpha}\cdot\vec{p}, \Sigma_1] = p_2 \begin{bmatrix} 0 & -2i\sigma_3 \\ -2i\sigma_3 & \end{bmatrix} + p_3 \begin{bmatrix} 0 & 2i\sigma_2 \\ 2i\sigma_2 & 0 \end{bmatrix} =$$
$$= 2i\left[-p_2\alpha_3 + p_3\alpha_2\right] = 2i(\vec{\alpha}\times\vec{p})_1$$

$$[\beta m, \Sigma_1] = 0$$

$$\Rightarrow [\hat{H}, \vec{\Sigma}] = 2i(\vec{\alpha}\times\vec{p})$$

$$\hat{\vec{J}} = \underset{\substack{\uparrow \\ \text{TIRNA} \\ \text{VRTILNA KOLIČINA}}}{\hat{\vec{L}}} + \underbrace{\tfrac{1}{2}\vec{\Sigma}}_{\text{SPIN DELCA}} \quad \Rightarrow \quad [\hat{H}, \hat{\vec{J}}] = 0$$

VIJAČNOST $\quad \vec{\Sigma}\cdot\frac{\vec{p}}{p}$

$$\left(\vec{\Sigma}\cdot\frac{\vec{p}}{p}\right)\psi^{(1)} = +\psi^{(1)}$$
$$\left(\vec{\Sigma}\cdot\frac{\vec{p}}{p}\right)\psi^{(2)} = -\psi^{(1)}$$

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VERJETNOSTNA GOSTOTA IN TOK ZA REŠITVE D.E.

$$[i\gamma^\mu \partial_\mu - m]\psi = 0 \qquad i\gamma^0 \frac{\partial\psi}{\partial t} + i\gamma^k \frac{\partial\psi}{\partial x^k} - m\psi = 0 , \quad k = 1,2,3$$

$$(\gamma^0)^+ = \gamma^0 ; \quad (\gamma^k)^+ = -\gamma^k \qquad -i\frac{\partial\psi^+}{\partial t}\gamma^0 + (-i)(-1)\frac{\partial\psi^+}{\partial x^k}\gamma^k - m\psi^+ = 0 \quad / \cdot \gamma^0$$

$$-i\frac{\partial\psi^+}{\partial t}\gamma^0\gamma^0 + i\frac{\partial\psi^+}{\partial x^k}\gamma^k\gamma^0 - m\psi^+\gamma^0 = 0$$

$$i\frac{\partial\psi^+}{\partial t}\gamma^0\gamma^0 + i\frac{\partial\psi^+}{\partial x^k}\gamma^0\gamma^k + m\psi^+\gamma^0 = 0$$

ADJUNGIRAN BISPINOR $\psi^+\gamma^0 = \overline{\psi}$

$$i\frac{\partial\overline{\psi}}{\partial t}\gamma^0 + i\frac{\partial\overline{\psi}}{\partial x^k}\gamma^k + m\overline{\psi} = 0$$

$$i\,\partial_\mu\overline{\psi}\gamma^\mu + m\overline{\psi} = 0$$

DIRAK. E. ZA $\overline{\psi}$

$$i(\partial_\mu\overline{\psi})\gamma^\mu\psi + m\overline{\psi}\psi = 0$$

$$i\overline{\psi}\gamma^\mu(\partial_\mu\psi) - m\overline{\psi}\psi = 0$$

SEŠTEJEM, DELIM Z i

$$(\partial_\mu\overline{\psi})\gamma^\mu\psi + \overline{\psi}\gamma^\mu(\partial_\mu\psi) = 0$$

$$\partial_\mu(\overline{\psi}\gamma^\mu\psi) = 0$$

KONTINUITETNA ENAČBA

$$\Rightarrow j^\mu = \overline{\psi}\gamma^\mu\psi$$

ELEKTROMAGNETNI TOK $j^\mu = -e_0\overline{\psi}\gamma^\mu\psi$

INTERAKCIJA Z E.M. POLJEM

$$\partial_\mu \to \partial_\mu - ieA_\mu = D_\mu$$ KOVARIANTNI ODVOD

$$A_\mu = (A_0, \vec{A}) \qquad E = -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla} A_0, \quad \vec{B} = \vec{\nabla} \times \vec{A}$$

DROBNI TISK: INVARIANTNOST $\psi(x) \to e^{i\alpha(x)}\psi(x)$

UMERITVENA TRANSFORMACIJA

$$[i\gamma^\mu(\partial_\mu - ieA_\mu) - m]\psi = 0$$

$$[i\gamma^\mu \partial_\mu - m]\psi = -e\gamma^\mu A_\mu \psi \equiv \gamma^0 V \psi$$

$$\gamma^0 V = -e\gamma^\mu A_\mu$$

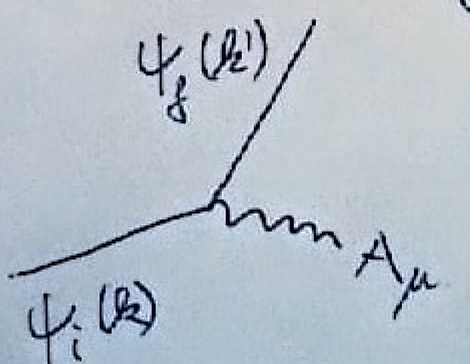
$$\underline{V = -e\gamma^0\gamma^\mu A_\mu}$$

V POVZROČI PREHOD IZ $\psi_i \to \psi_f$

$$T_{fi} = -i\int \psi_f^\dagger(k',x)\, V\, \psi_i(k,x)\, d^4x =$$

$$= ie\int \psi_f^\dagger \gamma^0 \gamma^\mu A_\mu \psi_i\, d^4x = ie\int \bar{\psi}_f \gamma^\mu A_\mu \psi_i\, d^4x =$$

$$= -i\int \underbrace{(-e\bar{\psi}_f \gamma^\mu \psi_i)}_{j_{fi}^\mu} A_\mu\, d^4x = -i\int j_{fi}^\mu A_\mu\, d^4x$$



A_μ: USTVARJA GA DRUG DELEC (ČE GLEDAM SIPANJE ENEGA DELCA NA DRUGEM)

ZA A_μ VELJA MAX. ENAČBA

$$\cancel{\partial^\mu \partial_\mu A} \quad \partial^\nu \partial_\nu A_\mu = j^\mu$$

POTENCIAL, KI GA ČUTI DELEC 1

TOK, KI GA USTVARI DELEC 2

DELEC $\psi(p) \to \psi(p')$

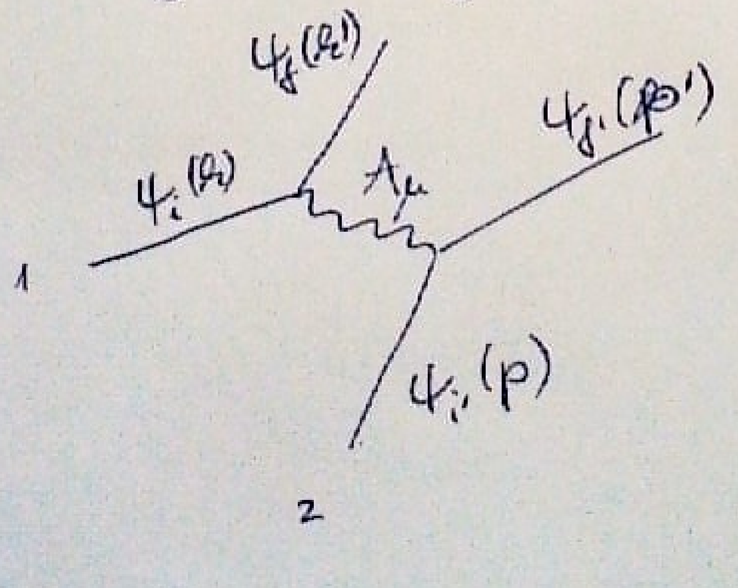
TOK $j_\mu = -e\bar{\psi}(p')\gamma_\mu \psi(p) = -e\bar{u}(p')e^{ip'x}\gamma_\mu u(p)e^{-ipx}$

$= -e\bar{u}(p')\gamma_\mu u(p)e^{iqx}$ $\quad q = p' - p$

$\Rightarrow A_\mu = -q^{-2} j^\mu$

$$\boxed{A_\mu = -\frac{j^\mu}{q^2}}$$

$$T_{fi} = -i\int j^\mu_{fi} \left(-\frac{1}{q^2}\right) j_\mu^{f'i'} \, d^4x$$



E.M. INTERAKCIJA V NERELATIVISTIČNI LIMITI

$$[\gamma^\mu p_\mu + e\gamma^\mu A_\mu - m]\psi = 0 \qquad \gamma^\mu = (\beta, \beta\vec{\alpha})$$

$$A^\mu = (A_0, \vec{A})$$

$$[\beta E - \beta\vec{\alpha}\vec{p} + e\beta A_0 - e\beta\vec{\alpha}\vec{A} - m]u = 0 \qquad 1\cdot\beta \text{ z leve}$$

$$[E - \vec{\alpha}\vec{p} + eA_0 - e\vec{\alpha}\cdot\vec{A} - \beta m]u = 0$$

$$[\vec{\alpha}\cdot(\vec{p} + e\vec{A}) - eA_0 + \beta m]u = Eu$$

$$\begin{bmatrix} m - eA_0 & \vec{\sigma}\cdot(\vec{p} + e\vec{A}) \\ \vec{\sigma}(\vec{p} + e\vec{A}) & -m - eA^0 \end{bmatrix}\begin{bmatrix} u_A \\ u_B \end{bmatrix} = E\begin{bmatrix} u_A \\ u_B \end{bmatrix}$$