$\Rightarrow j^\mu = \overline{\psi}\gamma^\mu\psi$ ŠTEVILSKI TOK. ELEKTROMAG. TOK: $j^\mu = -e_0\overline{\psi}\gamma^\mu\psi$

ŠE NEKAJ LASTNOSTI DIRACOVE E.

POLNOSTNA RELACIJA - LASTNOST BISPINORJEV

$$\boxed{\sum_{s=1,2} u^{(s)}(\vec{p})\,\overline{u}^{(s)}(\vec{p}) = \not{p} + m} \leftarrow 4\times4 \text{ MATRIKA}$$

134. INTERAKCIJA Z E. M. POLJEM, 2. DEL

$$\partial_\mu \rightarrow \partial_\mu - ieA_\mu = D_\mu$$

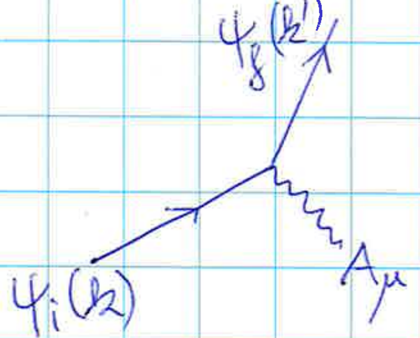
INVARIANTNOST NA $\psi(x) \rightarrow e^{i\alpha(x)}\psi(x)$
UMERITVENA TRANSFORMACIJI
(GAUGE)

$$[i\gamma^\mu(\partial_\mu - ieA_\mu) - m]\psi = 0$$

$$[i\gamma^\mu\partial_\mu - m]\psi = -e\gamma^\mu A_\mu\psi \equiv \gamma^0 V\psi \quad \Rightarrow \quad \gamma^0 V = -e\gamma^\mu A_\mu \quad |\gamma^0\cdot$$

$$\boxed{V = -e\gamma^0\gamma^\mu A_\mu}$$

V: POVZROČI $\psi_i \rightarrow \psi_f$



$$T_{fi} = -i\int \psi^\dagger(k',x)\,V\,\psi_i(k,x)\,d^4x$$

$$= ie\int \psi_f^\dagger\gamma^0\gamma^\mu A_\mu\psi_i\,d^4x = ie\int\overline{\psi}_f\gamma^\mu A_\mu\psi\,d^4x =$$

$$= ie\int\overline{\psi}_f\gamma^\mu\psi_i A_\mu\,d^4x = -i\int\underbrace{(-e)\overline{\psi}_f\gamma^\mu\psi_i}_{j^\mu_{fi}}A_\mu d^4x =$$

$$\boxed{T_{fi} = -i\int j^\mu_{fi}A_\mu d^4x}$$

A_μ! POLJE, KI GA USTVARJA DRUG DELEC (TISTI, NA KATEREM SE PRVI SIPLJE)

MAXWELLOVA ENAČBA $\partial^\nu \partial_\nu \underset{\uparrow}{A_\mu} = j_\mu \leftarrow$ TOK, KI GA USTVARJA DELEC ②

POTENCIAL KI GA ČUTI DELEC ①

$\psi(p) \to \psi(p')$

$j_\mu^{fi} = -e\bar{\psi}(p')\gamma_\mu \psi(p) =$

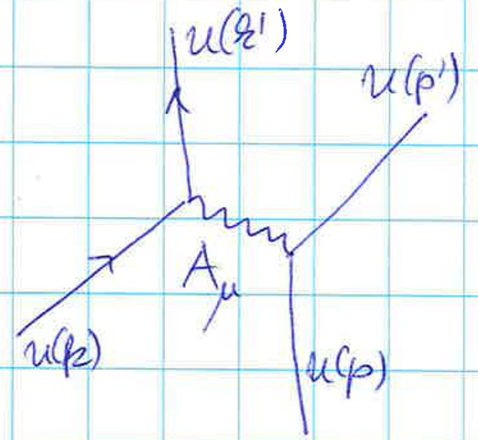
$= -e\bar{u}(p') e^{ip'_\nu x^\nu} \gamma_\mu u(p) e^{-ip_\nu x^\nu} = -e\bar{u}(p')\gamma_\mu u(p) e^{iq_\nu x^\nu}$

$\partial^\nu \partial_\nu A_\mu = -e\bar{u}(p')\gamma_\mu u(p) e^{iq_\nu x^\nu} = j_\mu$ $\quad q_\mu = p'_\mu - p_\mu$

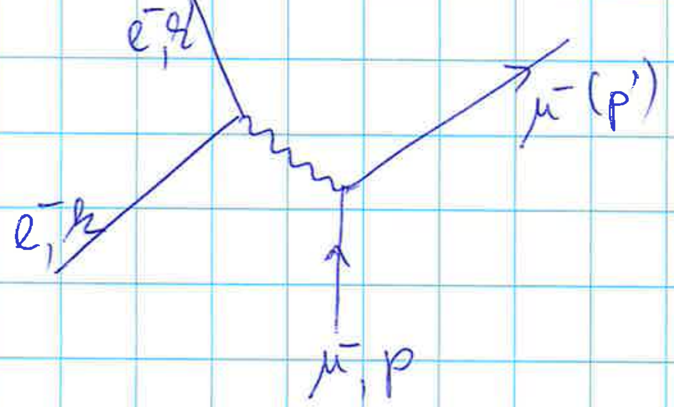
NASTAVEK $A_\mu = C e^{iq_\nu x^\nu} \Rightarrow -q^\nu q_\nu C e^{iq_\nu x^\nu} = -e\bar{u}(p')\gamma_\mu u(p) e^{iq_\nu x^\nu}$

$\Rightarrow \boxed{A_\mu = -\frac{1}{q^2} j_\mu^{f'i'}}$ $\quad q^2 = q^\nu q_\nu$

$T_{fi} = -i\int j^\mu_{fi} A_\mu \, d^4x = -i\int j^\mu_{fi} \left(\frac{-1}{q^2}\right) j_\mu^{f'i'} \, d^4x$



ELEKTROMAGNETNO SIPANJE $e^-\mu^- \to e^-\mu^-$



$T_{fi} = -i\int j_\nu^{(e)} \left(-\frac{1}{q^2}\right) j^\nu_{(\mu)} \, d^4x$

$j_\nu^{(e)} = -e\bar{u}(k')\gamma_\nu u(k) e^{i(k'-k^*)\cdot x}$

$j_\nu^{(\mu)} = -e\bar{u}(p)\gamma_\nu u(p) e^{i(p'-p^*)\cdot x}$

$$T_{fi} = -i\left[-e\bar{u}(k')\gamma_\nu u(k)\right]\frac{1}{q^2}\left[-e\bar{u}(p')\gamma^\nu u(p)\right]\underbrace{\int e^{i(k'+p'-k-p)\cdot x}\,d^4x}_{(2\pi)^4\delta^4(k'+p'-k-p)}$$

OHRANITEV $E, \vec{p}$

$$T_{fi} = -(2\pi)^4\delta^4(k'+p'-k-p)\,\mathcal{M}$$

$$-i\mathcal{M} = \left[-e\bar{u}(k')\gamma_\mu u(k)\right]\frac{g^{\mu\nu}}{q^2}\left[-e\bar{u}(p')\gamma_\nu u(p)\right]$$

$\mathcal{M}$ = INVARIANTNA AMPLITU

$$g^{\mu\nu}\gamma_\nu = \gamma^\mu$$

ČE POZNAMO SPINA NA ZAČETKU IN SPINA NA KONCU $\Rightarrow$ $\sigma \propto |\mathcal{M}|^2 =$

$$= |\mathcal{M}_{s_1, s_2, s_1', s_2'}|^2$$

NEPOLARIZIRAN PRESEK
- POVPREČIMO PO SPINIH NA ZAČETKU
- SEŠTEJEMO PO SPINIH NA KONCU

$$\sigma \propto \underbrace{\frac{1}{(2s_a+1)(2s_b+1)}}_{\text{ZA } e, \mu:\ \frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}}\sum_{s_1 s_2 s_1' s_2'}|\mathcal{M}_{s_1, s_2, s_1', s_2'}|^2 \equiv \overline{|\mathcal{M}|^2}$$

SEŠTEVANJE KVADRATOV AMPLITUD: KO U PRINCIPU LAHKO LOČIM PROCESE

SEŠTEVANJE AMPLITUD: KO NE MOREM LOČITI (U PRINCIPU) PROCESOV
(ZA PRIMER GLEJ $e^-e^- \to e^-e^-$)

$$|\mathcal{M}|^2 \propto \left[-e\bar{u}(k')\gamma^\kappa u(k)\right]\left[-e\bar{u}(k')\gamma^\lambda u(k)\right]^* = \left[-e\bar{u}(k')\gamma^\kappa u(k)\right]\left[-e\underset{u^\dagger\gamma^0}{\bar{u}(k')}\gamma^\lambda u(k)\right]^\dagger =$$

$$(u^\dagger)^\dagger = u$$

$$= \left[-e\bar{u}(k')\gamma^\kappa u(k)\right]\left[-e\,u^\dagger(k)\gamma^{\lambda\dagger}\gamma^{0\dagger}u(k')\right] =$$

$$= e^2\left[\bar{u}(k')\gamma^\kappa u(k)\right]\left[\bar{u}(k)\gamma^\lambda u(k')\right]$$

$$\gamma^{0\dagger} = \gamma^0,\quad \gamma^{\lambda\dagger}\gamma^0 = \gamma^0\gamma^\lambda$$

$$\sum_{s_1,s_1'}' \left[\bar{u}^{(s_1')}(k')\gamma^{\kappa} u^{(s_1)}(k)\right]\left[\bar{u}^{(s_1)}(k)\gamma^{\sigma} u(k')\right] = \sum_{s_1,s_1'}' \left[\bar{u}_{\alpha}^{(s_1')}(k')\gamma^{\kappa}_{\alpha\beta} u_{\beta}^{(s_1)}(k)\right]\left[\bar{u}_{\delta}^{(s_1)}(k)\gamma^{\sigma}_{\delta\varepsilon} u_{\varepsilon}^{(s_1')}(k')\right] =$$

← ŠTEVILKE, LAHKO ZAMENJA

← VRSTNI RED FAKTORJEV

$$= \sum_{s_1,s_1'}' u_{\varepsilon}^{(s_1')}(k')\,\bar{u}_{\alpha}^{(s_1')}(k')\,\gamma^{\kappa}_{\alpha\beta}\,u_{\beta}^{(s_1)}(k)\,\bar{u}_{\delta}^{(s_1)}(k)\,\gamma^{\sigma}_{\delta\varepsilon} =$$

$$= \left(\sum_{s_1'}' u_{\varepsilon}(k')\,\bar{u}_{\alpha}^{(s_1')}(k')\right)\gamma^{\kappa}_{\alpha\beta}\left(\sum_{s_1}' u_{\beta}^{(s_1)}(k)\,\bar{u}_{\delta}^{(s_1)}(k)\right)\gamma^{\sigma}_{\delta\varepsilon} =$$

$$(\not{k}' + m_e)_{\varepsilon\alpha}\,\gamma^{\kappa}_{\alpha\beta}\,(\not{k} + m_e)_{\beta\delta}\,\gamma^{\sigma}_{\delta\varepsilon} = \underbrace{\left[(\not{k}' + m_e)\gamma^{\kappa}(\not{k} + m_e)\gamma^{\sigma}\right]_{\varepsilon\varepsilon}}$$

$$= \mathrm{Tr}\left[(\not{k}' + m_e)\gamma^{\kappa}(\not{k} + m_e)\gamma^{\sigma}\right]$$

SLED MATRIKE 4x4

PODOBNA IZPELJAVA ZA MIONE ⇒

$$\boxed{\overline{|\mathcal{M}|^2} = \frac{1}{4}\frac{e^4}{q^4}\mathrm{Tr}\left[(\not{k}' + m_e)\gamma^{\kappa}(\not{k} + m_e)\gamma^{\sigma}\right]\mathrm{Tr}\left[(\not{p}' + m_\mu)\gamma_{\kappa}(\not{p} + m_\mu)\gamma_{\sigma}\right]}$$

TEOREMI O SLEDEH: RECIMO Tr LIHEGA ŠTEVILA MATRIK $\gamma = 0$, ITD

$$\mathrm{Tr}\left[(\not{k}' + m_e)\gamma^{\kappa}(\not{k} + m_e)\gamma^{\sigma}\right] = 4\left[k'^{\kappa}k^{\sigma} + k'^{\sigma}k^{\kappa} - (k'\cdot k - m_e^2)g^{\kappa\sigma}\right]$$

ZA MIONE:

$$4\left[p'_{\kappa}p_{\sigma} + p'_{\sigma}p_{\kappa} - (p'\cdot p - m_\mu^2)g_{\kappa\sigma}\right] \qquad k'\cdot k \equiv k'^{\mu}k_{\mu}$$

$$\overline{|\mathcal{M}|^2} = \hookrightarrow 8\frac{e^4}{q^4}\left[(k'\cdot p')(k\cdot p) + (k'\cdot p)(k\cdot p') - m_e^2\, p'\cdot p - m_\mu^2\, k'\cdot k + 2m_e^2 m_\mu^2\right]$$

LORENTZOVA INVARIANTA

SIPALNI PRESEK $\frac{d\sigma}{d\Omega} = \frac{dW_{fi}}{d\Omega}\frac{1}{\rho_i v_i}$ $\frac{dW_{fi}}{d\Omega} = \frac{2\pi}{\hbar}|T_{fi}|^2 \frac{d\rho_f}{d\Omega}$

$d^3N = V\frac{d^3p}{(2\pi\hbar)^3} = \frac{1}{\rho}\frac{d^3p}{(2\pi\hbar)^3}$; $\rho = \frac{1}{V}$ NERELATIVISTIČNI PRIMER

RELATIVISTIČNI DELCI $d^3N = \frac{V}{2E}\frac{d^3p}{(2\pi\hbar)^3}$ $\rho_f \propto \frac{d^3p}{E}$

KAKO SE TO TRANSFORMIRA, ČE LOR. TRANSF. V SMERI x

$dp_y' = dp_y$, $dp_z' = dp_z$, $dp_x' = \gamma(dp_x - \beta dE)$, $dE' = \gamma(dE - \beta dp_x)$

$$\frac{d^3p'}{E'} = \frac{\gamma(dp_x - \beta dE)\cdot dp_y dp_z}{\gamma E(1-\beta\frac{p_x}{E})} = \frac{\gamma dp_x(1-\beta\frac{p_x}{E})dp_y dp_z}{\gamma E(1-\beta\frac{p_x}{E})} =$$

$E^2 = p_x^2 + p_y^2 + p_z^2 + m^2$

$2E\,dE = 2p_x dp_x \Rightarrow \frac{dE}{dp_x} = \frac{p_x}{E}$

$= \frac{d^3p}{E}$ LORENTZOVO INVARIANTNO

$\rho_i v_i = \underbrace{\frac{2E_a}{V} v_a}_{\text{UPADNI DELCI}} \cdot \underbrace{\frac{2E_b}{V}}_{\text{GOSTOTA V TARČI}}$ ČE b MIRUJE, a SE GIBLJE

GOSTOTA TOKA

FLUKS FAKTOR

ČE SE GIBLJETA OBA $\rho_i v_i = F = \frac{2E_a}{V}\frac{2E_b}{V}|\vec{v}_a - \vec{v}_b|$

$\frac{v}{c} = \beta = \frac{\gamma m v c}{\gamma m c^2} = \frac{cp}{E} \Rightarrow \vec{v}_a - \vec{v}_b = \frac{|\vec{p}_a E_b - \vec{p}_b E_a|}{E_a E_b}$

$|\vec{p}_a E_b - \vec{p}_b E_a| = \sqrt{(p_a\cdot p_b)^2 - m_a^2 m_b^2}$

DIFERENCIALNI PRESEK $d\sigma = \frac{|\mathcal{M}|^2}{F}dQ$ $dQ = (2\pi)^4\delta^4(p_c+p_d-p_a-p_b)\frac{d^3p_c}{(2\pi\hbar)^3E_c}\frac{d^3p_d}{(2\pi\hbar)^3E_d}$

ZA $ab \rightarrow cd$