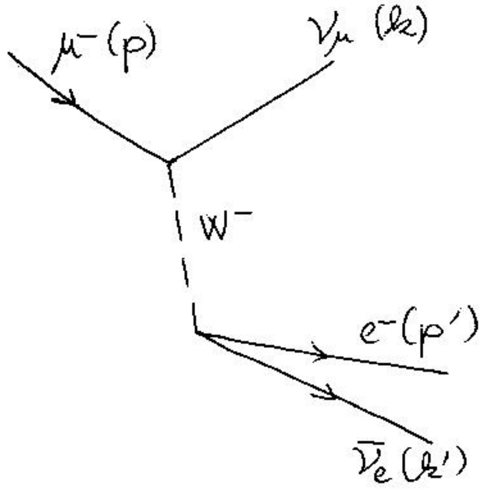
MUON DECAY



$$\mathcal{M} = \left(\frac{g}{\sqrt{2}}\, \bar{u}(k)\gamma^\mu \frac{1}{2}(1-\gamma^5)\, u(p)\right) \frac{1}{M_W^2 - q^2} \cdot \left(\frac{g}{\sqrt{2}}\, \bar{u}(p')\gamma_\mu \frac{1}{2}(1-\gamma^5) v(k')\right)$$

SINCE $q^2 \ll M_W^2$ IN MUON DECAY, WE CAN REWRITE THIS TO

$$\mathcal{M} = \frac{G}{\sqrt{2}} \left[\bar{u}(k)\gamma^\mu(1-\gamma^5)\, u(p)\right]\left[\bar{u}(p')\gamma_\mu(1-\gamma^5)\, v(k')\right]$$

WITH $\frac{G}{\sqrt{2}} = \left(\frac{g}{\sqrt{2}}\right)^2 \cdot \left(\frac{1}{2}\right)^2 \frac{1}{M_W^2}$, G = FERMI CONSTANT $= 1.166 \cdot 10^{-5} (\mathrm{GeV})^{-2}$

$$d\Gamma = \underset{\substack{\uparrow \\ \text{NORMALISATION OF THE INITIAL STATE } (E = E_\mu)}}{\frac{1}{2E}}\ \underset{\substack{\uparrow \\ \text{AVERAGED OVER SPINS}}}{\overline{|\mathcal{M}|^2}}\, dQ$$

$$dQ = \text{PHASE SPACE} = \underset{\substack{\uparrow \\ e^-}}{\frac{d^3p'}{(2\pi)^3 2E'}} \cdot \underset{\substack{\uparrow \\ \nu_\mu}}{\frac{d^3k}{(2\pi)^3 2\omega}} \cdot \underset{\substack{\uparrow \\ \bar{\nu}_e}}{\frac{d^3k'}{(2\pi)^3 2\omega'}} \times$$

$$\times (2\pi)^4 \delta^4(p - p' - k' - k)$$

$$\Gamma = \frac{G^2 M_\mu^5}{192\pi^3}$$

PION DECAY

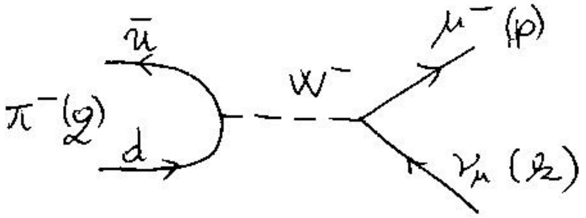
$\pi^- \to \mu^- \bar{\nu}_\mu \,,\quad e^- \bar{\nu}_e$

TWO BODY DECAY, $\Gamma(A \to 1+2) \propto |\vec{p}_f|$ (PHASE SPACE)

EXPECT $\to$ $\pi^- \to e^- \bar{\nu}_e$ MORE PROBABLE THAN $\mu^- \bar{\nu}_\mu$

$m_\pi = 139$ MeV $\quad m_\mu = 104$ MeV $\quad m_e = 0.5$ MeV

EXPERIMENT : $\Gamma(\pi^- \to e^- \bar{\nu}_e) \ll \Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)$



$$\mathcal{M} = \frac{G}{\sqrt{2}} \underset{\substack{\uparrow \\ \text{NOT PLANE WAVES}}}{(\ldots\ldots)^\mu} \left[\bar{u}(p)\gamma_\mu(1-\gamma^5)\, v(k)\right]$$

$(\ldots)^\mu$ HAS TO BE 4-VECTOR (OR AXIAL), BUILT FROM QUANTITIES ASSOCIATED WITH THE PION. PION SPINLESS
$\to$ ONLY 4-MOMENTUM $\times$ SCALAR FUNCTION

$$\to (\ldots)^\mu = q^\mu f(q^2) = q^\mu f(m_\pi^2) = q^\mu f_\pi$$

$$\mathcal{M} = \frac{G}{\sqrt{2}} q^\mu f_\pi \bar{u}(p)\gamma_\mu(1-\gamma^5)v(k) = \frac{G}{\sqrt{2}}(p^\mu + k^\mu) f_\pi \bar{u}(p)\gamma_\mu(1-\gamma^5)v(k)$$

SINCE $\bar{u}(\not{p} - m) = 0, (\not{k} + m)v = 0$ (FOR ν: $\not{k} v = 0$)

$$\to \mathcal{M} = \frac{G}{\sqrt{2}} f_\pi m_\mu \left[\bar{u}(p)(1-\gamma^5)v(k)\right]$$

PION DECAY

$$|\mathcal{M}|^2 = \frac{G^2}{2} f_\pi^2 m_\mu^2 \left[\bar{u}(p)(1-\gamma^5)v(k)\right]\left[\bar{u}(p)(1-\gamma^5)v(k)\right]^\dagger$$

$$= \frac{G^2}{2} f_\pi^2 m_\mu^2 \left[\bar{u}(p)(1-\gamma^5)\, v(k)\, \bar{v}(k)\, (1+\gamma^5) u(p)\right]$$

SPIN AVERAGE : USE COMPLETNESS RELATIONS $\sum_{\text{SPINS}} u\bar{u} = (\not{p}+m)$

$$\overline{|\mathcal{M}|^2} = \frac{G}{2} f_\pi^2 m_\mu^2 \, \mathrm{Tr}\left[(\not{p}+m_\mu)(1-\gamma^5)\not{k}\,(1+\gamma^5)\right]$$

$$\not{k} = k^\mu \gamma_\mu \qquad \gamma_\mu \gamma^5 = -\gamma^5\gamma_\mu \qquad (1-\gamma^5)^2 = 2(1-\gamma^5)$$

$$\Rightarrow \left[(\not{p}+m_\mu)(1-\gamma^5)\not{k}\,(1+\gamma^5)\right] = 2\left[(\not{p}+m_\mu)(1-\gamma^5)\not{k}\right]$$

$$= 2\left(\not{p}\not{k} + m_\mu \not{k} + \not{p}(-\gamma^5)\not{k} - m_\mu \gamma^5 \not{k}\right)$$

Tr (odd number of γ_μ's) $= 0$

$\mathrm{Tr}(\not{p}\not{k}) = 4\, p\cdot k$

$\mathrm{Tr}(\gamma^5\not{p}\not{k}) = 0$

$p = (E, \vec{p}),\ k = (\omega, \vec{k})$

$$\overline{|\mathcal{M}|^2} = \frac{G}{2} f_\pi^2 m_\mu^2 \; 8\, p\cdot k$$

$$p\cdot k = E\omega - \vec{p}\cdot\vec{k} = $$
$$= E\omega + \vec{k}\cdot\vec{k} =$$
$$= (E+\omega)\,\omega$$

$$\Gamma = \frac{G^2 f_\pi^2 m_\mu^2}{(2\pi)^2\; 2 m_\pi} \cdot \int \frac{d^3p\, d^3k}{E\,\omega}\, \omega(E+\omega)\quad \delta(m_\pi - E - \omega)\, \delta^3(\vec{k}+\vec{p})$$

ALREADY USED FOR

$$= \frac{G^2 f_\pi^2 m_\mu^2}{8\pi^2 m_\pi} \int 4\pi\omega^2 dk \left(1+\frac{\omega}{E}\right) \delta(m_\pi - E - \omega)$$

$$E = \sqrt{\omega^2 + m_\mu^2}$$

$$0 = m_\pi - \sqrt{\omega_0^2 + m_\mu^2} - \omega_0$$
$$\omega_0^2 + m_\mu^2 = (m_\pi - \omega_0)^2$$
$$\omega_0 = (m_\pi^2 - m_\mu^2)/2m_\pi$$

PION DECAY

REWRITE δ FUNCTION $\delta(f(\omega)) = \delta(\omega-\omega_0)\dfrac{1}{\left|\frac{\partial f}{\partial \omega}\right|_{\omega_0}}$

$$f(\omega) = m_\pi - \sqrt{\omega^2+m_\mu^2} - \omega$$

$$\left.\frac{\partial f}{\partial \omega}\right|_{\omega_0} = -\frac{2\,\omega_0}{2\sqrt{\omega_0^2+m_\pi^2}} - 1 = -\left(1+\frac{\omega_0}{E}\right)$$

$$\int d\omega\, \omega^2 \left(1+\frac{\omega}{E}\right)\frac{1}{\left(1+\frac{\omega_0}{E}\right)}\cdot\delta(\omega-\omega_0) = \omega_0^{\,2}$$

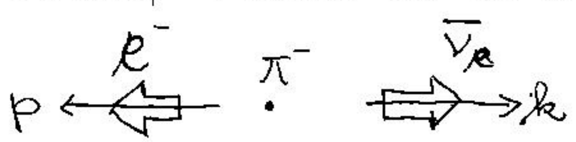
$$\Gamma = \frac{G^2 f_\pi^{\,2} m_\mu^{\,2}}{8\pi^2\, m_\pi}\cdot 4\pi\,\frac{(m_\pi^{\,2}-m_\mu^{\,2})^2}{4\,m_\pi^{\,2}} = \frac{G^2 f_\pi^{\,2} m_\mu^{\,2} m_\pi}{8\pi}\left(1-\frac{m_\mu^2}{m_\pi^2}\right)^2$$

$$\frac{\Gamma_e}{\Gamma_\mu} = \underbrace{\frac{m_e^{\,2}}{m_\mu^{\,2}}}_{\uparrow}\cdot\underbrace{\frac{\left(1-\overset{\sim 0}{\overset{\downarrow}{\frac{m_e^{\,2}}{m_\pi^{\,2}}}}\right)^2}{\left(1-\frac{m_\mu^{\,2}}{m_\pi^{\,2}}\right)^2}} = 1.2\cdot 10^{-4}$$

↑ PHASE SPACE RATIO - IN FAVOUR OF e

↑ STRUCTURE OF WEAK INTERACTION
-LEFT HANDED PARTICLES

PION SPINLESS, ANGULAR MOMENTUM CONSERVED

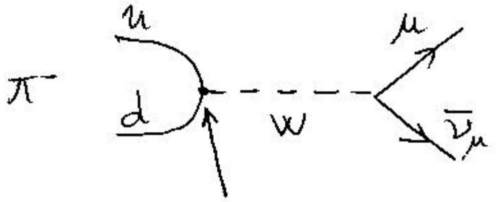


↓ ANTI-NEUTRINO: ALWAYS RIGHT HANDED, POSIT. HEL.
SPIN $\parallel \vec{k}$

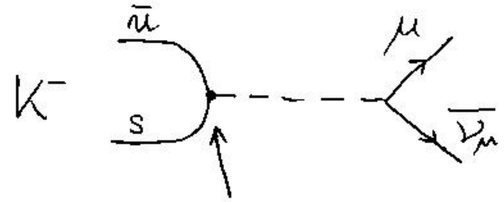
e^- FORCED TO BE IN THE ~~RIGHT~~ POSITIVE HELICITY STATE
FOR ENERGETIC ~~NEO~~ ELECTRON HIGHLY SUPPRESSED
(THERE POSITIVE HELICITY ~ RIGHT HANDEDNESS)

$\frac{1}{2|\vec{p}|}\left(\vec{\sigma}\cdot\vec{p} \quad \vec{\sigma}\cdot\vec{p}\right) u \qquad \frac{1}{2}(1+\gamma^5)\, u$

PION DECAY, KAON, B



$$V_{ud} = \cos\theta_c$$

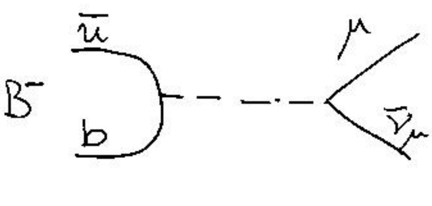


$$V_{us} = \sin\theta_c$$

$$\frac{\Gamma(K^- \to \mu^- \bar{\nu}_\mu)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} = \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{f_K^2}{f_\pi^2} \cdot \frac{m_K}{m_\pi} \cdot \frac{\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2}{\left(1 - \frac{m_\mu^2}{m_K^2}\right)^2}$$

EXERCISE: DETERMINE θ_c FROM KNOWN
$\tau_K = 1.24 \cdot 10^{-8}$ s, $\tau_\pi = 2.60 \cdot 10^{-8}$ s, $BR(K^- \to \mu^- \bar{\nu}_\mu) = 0.635$
$BR(\pi^- \to \mu^- \bar{\nu}_\mu) = 1$, $m_\pi = 0.1396$ GeV, $m_K = 0.4937$ GeV
+ ASSUME $f_K / f_\pi = 1$ (WAVE FUNCTIONS ~ SIMILAR)

B^- DECAY



$$\Gamma(B^- \to \mu^- \bar{\nu}_\mu) = |V_{ub}|^2 \cdot \frac{G^2 f_B^2 m_\mu^2 m_B}{8\pi} \underbrace{\left(1 - \frac{m_\mu^2}{m_B^2}\right)^2}_{1}$$