

Particle Detectors – part 2

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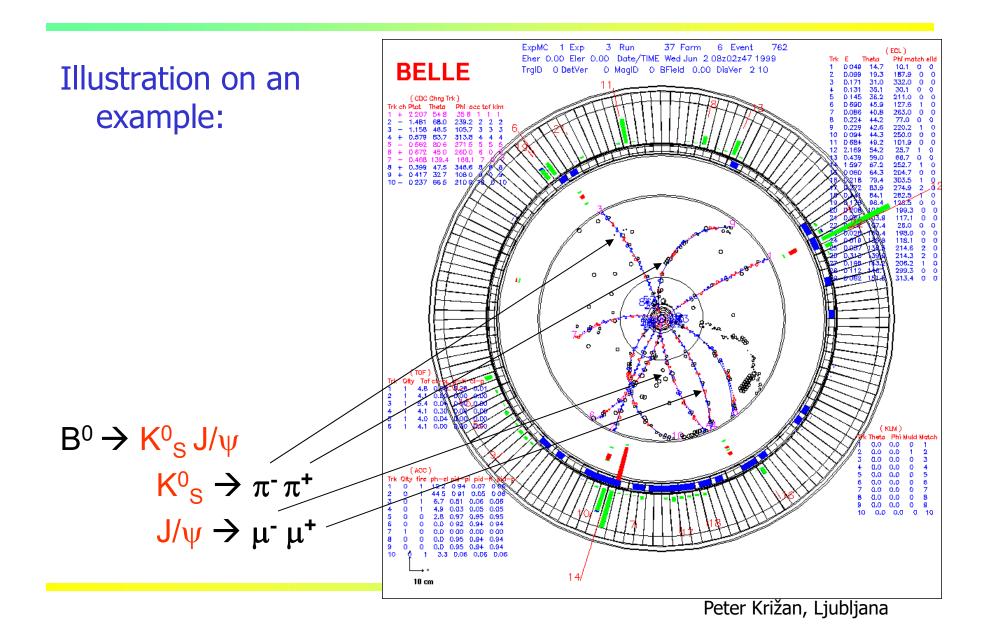


How to understand what happened in a collision?

- Measure the coordinate of the point ('vertex') where the reaction occured, and determine the positions and directions of particles that have been produced
- Measure momenta of stable charged particles by measuring their radius of curvature in a strong magnetic field (~1T)
- •Determine the identity of stable charged particles (e, μ , π , K, p)
- •Measure the energy of high energy photons γ
- Detect neutral hadrons

•Combine final state particles to form intermediate states that decayed too quickly to be directly detected

How to understand what happened in a collision?



Search for particles that decayed close to the production point

How do we reconstruct reaction products that decayed to several stable particles (e.g., 1, 2, 3)?

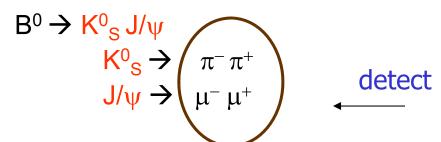
From the measured tracks calculate the invariant mass of the system (i = 1, 2, 3):

$$M = \sqrt{(\sum E_i)^2 - (\sum \vec{p}_i)^2}$$

The candidates for the $X\rightarrow 123$ decay show up as a peak in the distribution on (mostly combinatorial) background.

The name of the game: have as little background under the peak as possible without loosing the events in the peak (=reduce background and have a small peak width).

How do we know it was precisely this reaction?

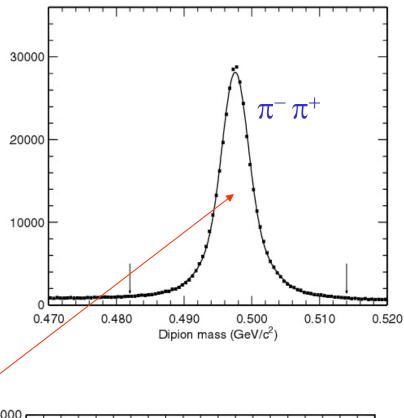


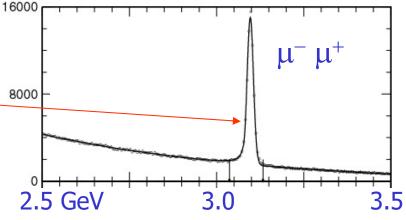
For $\pi^-\pi^+$ in $\mu^-\mu^+$ pairs we calculate the invariant mass:

$$M = \sqrt{\left(\sum E_i\right)^2 - \left(\sum \vec{p}_i\right)^2}$$

 Mc^2 must be for K_S^0 close to 0.5 GeV, for J/ψ close to 3.1 GeV.

Rest in the histrogram: random coincidences ('combinatorial background')





The name of the game: have as little background under the peak as possible without loosing the events in the peak (=reduce background and have a small peak width).

Components of an experimental apparatus ('spectrometer')

- Tracking and vertexing systems
- Particle identification devices
- Calorimeters (measurement of energy)

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Resolution in invariant mass

$$B^0 \rightarrow K_S^0 J/\psi, K_S^0 \rightarrow \pi^- \pi^+, J/\psi \rightarrow \mu^- \mu^+$$

$$M^2c^4 = (E_1 + E_2)^2 - (p_1 + p_2)^2c^2 \rightarrow M^2c^4 = 2 p_1 p_2 c^2 (1 - \cos\Theta_{12})$$
 (p >> m_{\(\mu\)}c)

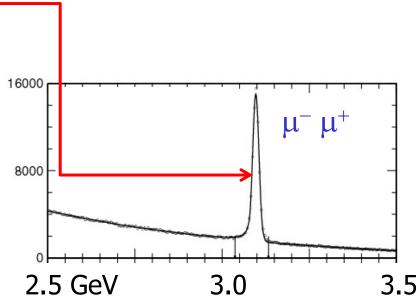
The J/ψ peak should be narrow to minimize the contribution of random coincidences ('combinatorial background') under the peak.

The required resolution in Mc²: about 10 MeV.

What is the corresponding momentum resolution?

For simplicity assume J/ ψ is at rest \rightarrow $\Theta_{12}=180^{\circ}$, $p_1=p_2=p=1.5$ GeV/c, Mc²=2pc $\rightarrow \sigma(\text{Mc}^2)=2$ $\sigma(\text{pc})$ at p=1.5 GeV/c

$$\rightarrow \sigma(p)/p = 10 \text{ MeV}/2/1.5 \text{GeV} = 0.3\%$$



Momentum resolution

Tracking system uncertainty

$$\frac{\sigma_{p_T}}{p_T} = \frac{\sigma_x p_T}{eBL^2} \sqrt{\frac{720}{N+4}}$$

For N measurements along the track of length L, each with a precision of σ_x , in a magnetic field B

Uncertainty from multiple scattering

$$\frac{\sigma_{p_T}}{p_T} = \frac{13.6 MeV}{eB\sqrt{LX_0}}$$

In a detector with an average radiation length X₀

Combined

$$\frac{\sigma_{p_T}}{p_T} = \sqrt{\left(\frac{\sigma_{p_T}}{p_T}\right)^2_{tracking} + \left(\frac{\sigma_{p_T}}{p_T}\right)^2_{msc}}$$

Momentum resolution - example

Tracking system uncertainty

$$\frac{\sigma_{p_T}}{p_T} = \frac{\sigma_x p_T}{eBL^2} \sqrt{\frac{720}{N+4}}$$

For B = 1.5T, L = 1m, N = 50, σ_x = 100 μ m, X_0 = 100m

$$\frac{\sigma_{p_T}}{p_T} = p_T \frac{0.1 \times 10^{-3} m}{0.3 (GeV/m) \times 1.5 \times 1m^2} \sqrt{\frac{720}{54}} = \frac{p_T \times 0.0008}{GeV}$$

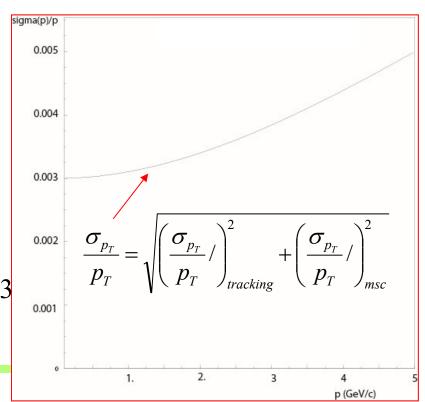
For $p_T = 1 \text{ GeV}$: $\sigma_{pT}/p_T = 0.08\%$

For $p_T = 2 \text{ GeV}$: $\sigma_{pT}/p_T = 0.16\%$

Uncertainty from multiple scattering

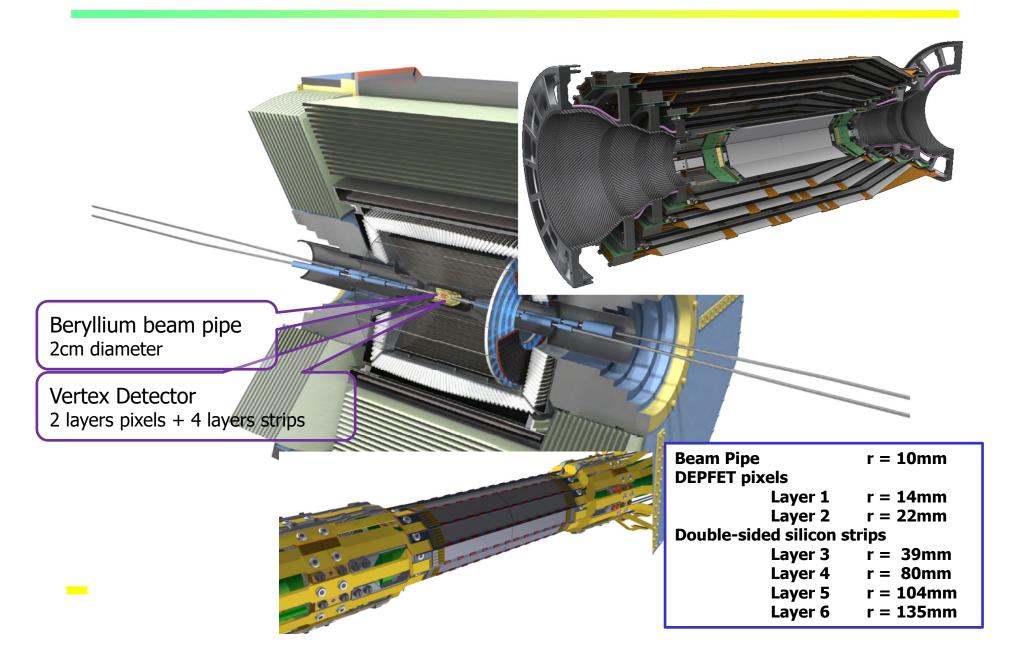
$$\frac{\sigma_{p_T}}{p_T} = \frac{13.6 MeV}{eB\sqrt{LX_0}}$$

$$\frac{\sigma_{p_T}}{p_T} = \frac{13.6 MeV}{0.3 (GeV/m) \times 1.5 \sqrt{1m \times 100m}} = 0.003$$

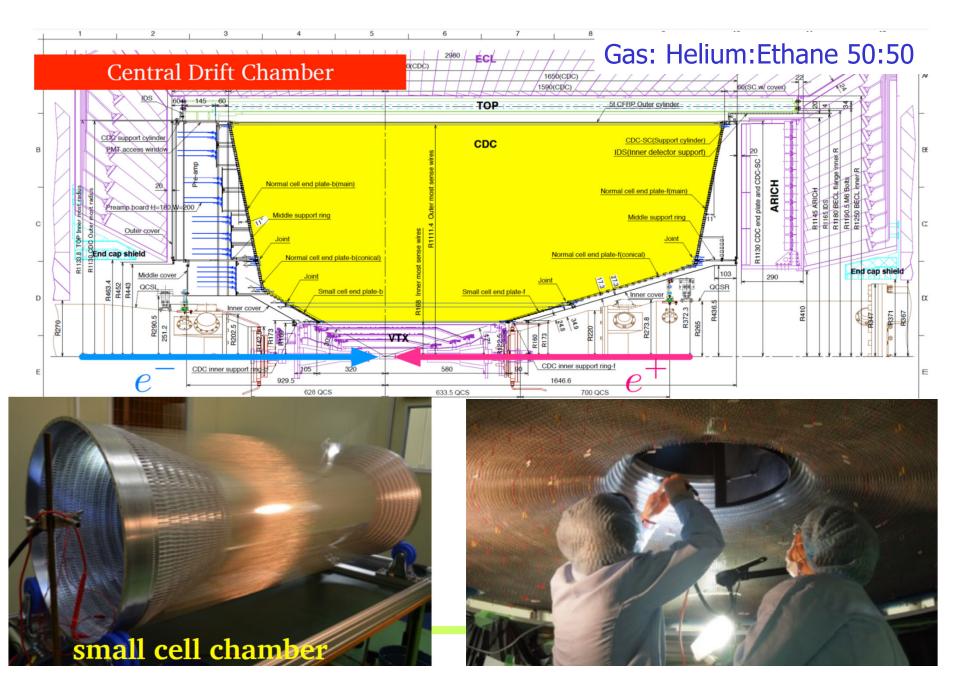


N.B. eB = 0.3 (B/T) (1/m) GeV/c

Belle II Detector – vertex region

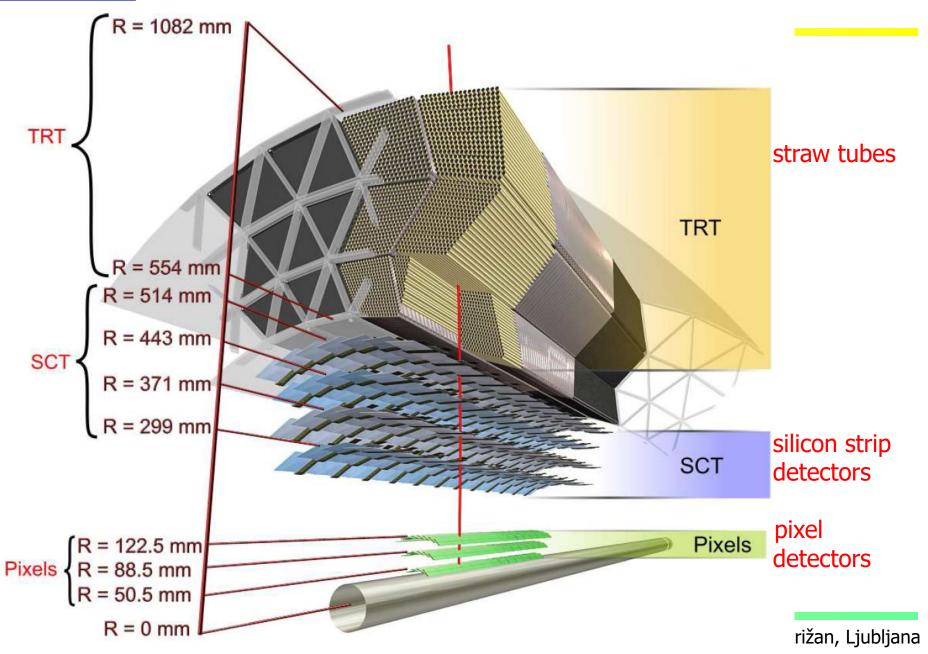


Belle II central drift chamber





Tracking system of the inner detector



What kind of momentum resolution do we need?

Reminder: example: we are looking for an unknown particle X, X $\rightarrow \mu^- \mu^+$

$$M^2c^4 = (E_1 + E_2)^2 - (p_1 + p_2)^2 \rightarrow M^2c^4 = 2 p_1 p_2 (1 - \cos\Theta_{12})$$

The X peak should be narrow to minimize the contribution of random

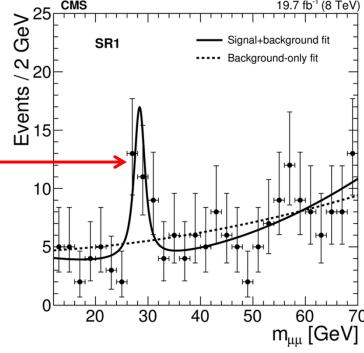
coincidences ('combinatorial background')

The required resolution in Mc²: about 1 GeV at 30 GeV.

What is the corresponding momentum resolution?

For simplicity assume X is at rest \rightarrow $\Theta_{12}=180^{0}$, $p_{1}=p_{2}=p=15$ GeV/c, Mc²=2pc $\rightarrow \sigma(Mc^{2})=2$ $\sigma(pc)$ at p=15 GeV/c

$$\rightarrow \sigma(p)/p = 1 \text{ GeV}/2/15\text{GeV} = 3\%$$



CMS could-be-particle (turned out to be a statistical fluctuation...)

Momentum resolution

$$\frac{\sigma_{p_T}}{p_T} = \frac{\sigma_x p_T}{eBL^2} \sqrt{\frac{720}{N+4}}$$

$$\frac{\sigma_{p_T}}{p_T} = \frac{13.6 MeV}{eB\sqrt{LX_0}}$$

$$\frac{\sigma_{p_T}}{p_T} = p_T \frac{0.1 \times 10^{-3} m}{0.3 (GeV/m) \times 2 \times 1m^2} \sqrt{\frac{720}{54}} = p_T \times 0.0006$$

eB = 0.3 (B/T) (1/m) GeV/c

For B=2T, L = 1m, σ_x = 0.1 mm

For $p_T = 1 \text{ GeV}$: $\sigma_{pT}/p_T = 0.06\%$

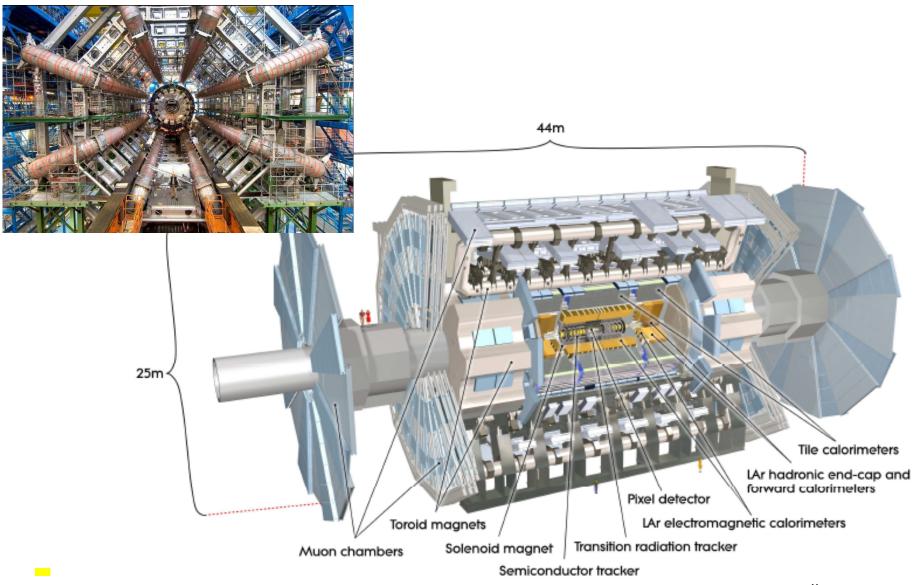
For $p_T = 10 \text{ GeV}$: $\sigma_{pT}/p_T = 0.6\%$

For $p_T = 100 \text{ GeV}$: $\sigma_{pT} / p_T = 6\%$

How to improve high momentum resolution?

- Better resolution: wire chamber → silicon strip detector (full CMS tracker, partly ATLAS)
- Higher field: CMS B=4T
- Longer lever arm for muons: additional tracking in the magnetic muon system (ATLAS)

Momentum measurement for very high energy muons - example ATLAS



Components of an experimental apparatus ('spectrometer')

- Tracking and vertexing systems
- Particle identification devices
- Calorimeters (measurement of energy)

Why Particle ID?

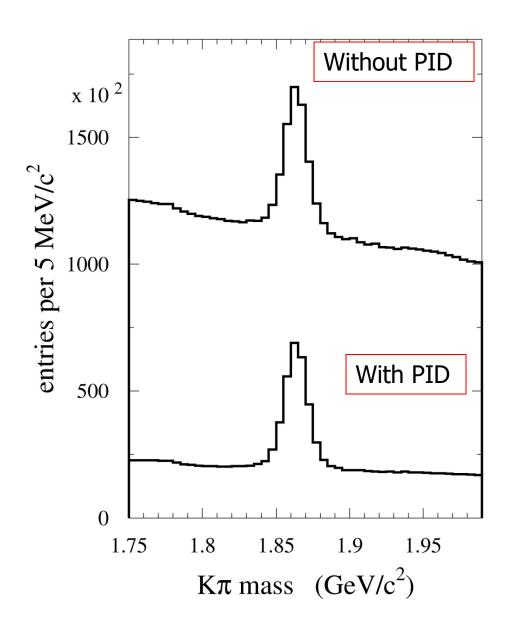
Particle identification is an important aspect of particle, nuclear and astroparticle physics experiments.

Some physical quantities in particle physics are only accessible with sophisticated particle identification (B-physics, CP violation, rare decays, search for exotic hadronic states).

Nuclear physics: final state identification in quark-gluon plasma searches, separation between isotopes

Astrophysics/astroparticle physics: identification of cosmic rays – separation between nuclei (isotopes), charged particles vs high energy photons

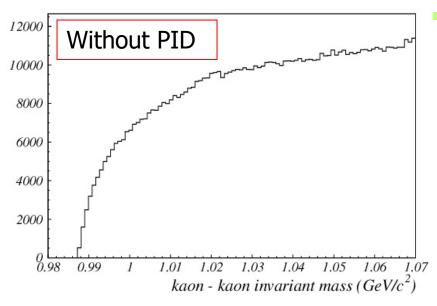
Introduction: Why Particle ID?

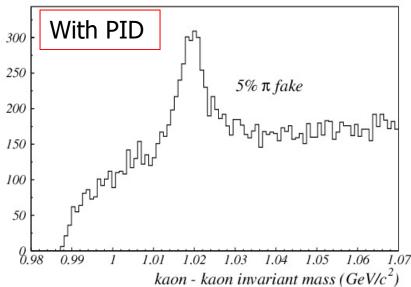


Example 1: B factories

Particle identification reduces combinatorial background by ~3x

Introduction: Why Particle ID?



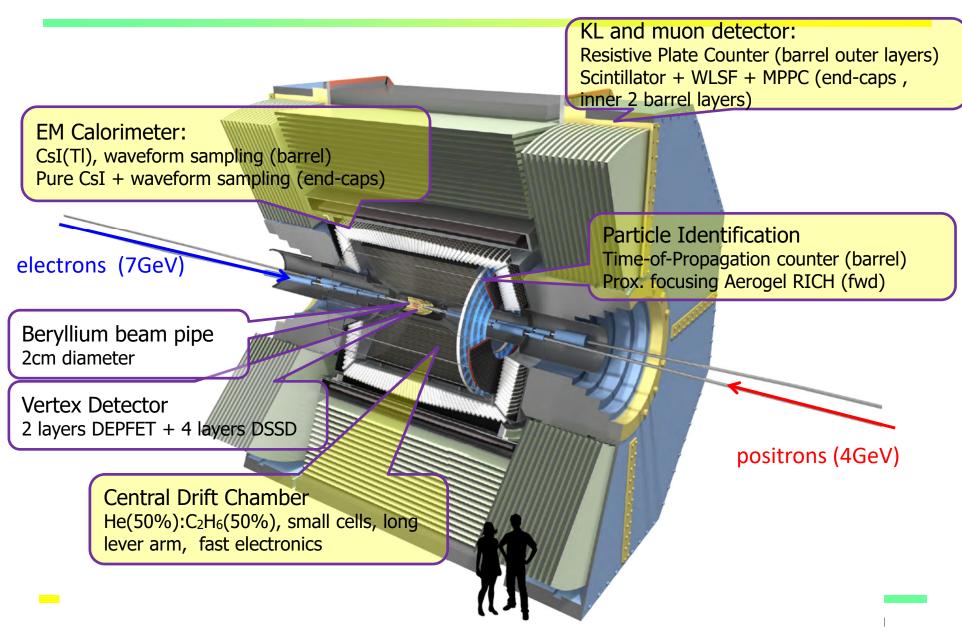


Example 2: HERA-B

K+K- invariant mass.

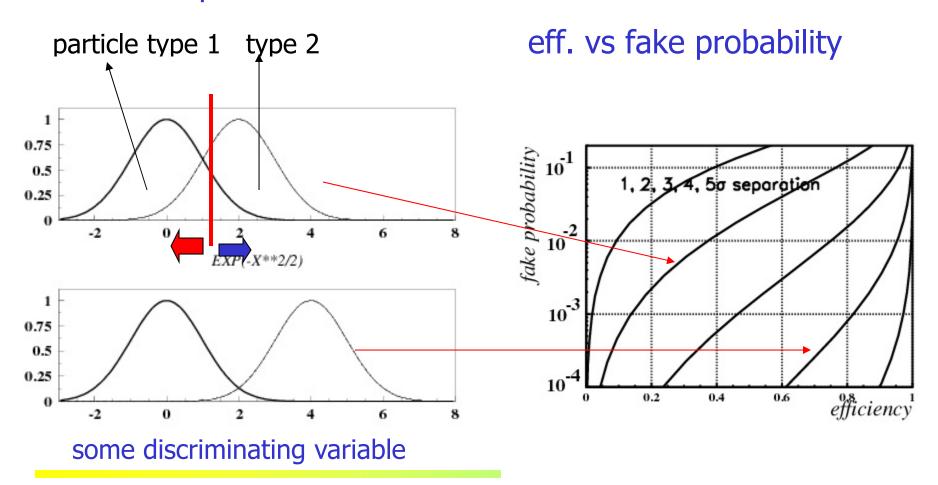
The $\phi \rightarrow K^+K^-$ decay only becomes visible after particle identification is taken into account.

Particle identification systems in Belle II



Efficiency and purity in particle identification

Efficiency and purity are tightly coupled! Two examples:



Identification of charged particles

Particles are identified by their mass or by the way they interact.

Determination of mass: from the relation between momentum and velocity, $p=\gamma mv$.

Momentum known (radius of curvature in magnetic field)

→ Measure velocity:

time of flight

ionisation losses dE/dx

Cherenkov angle

transition radiation

Mainly used for the identification of hadrons.

Identification through interaction: electrons and muons

Time-of-flight measurement 2

Required resolution, example:

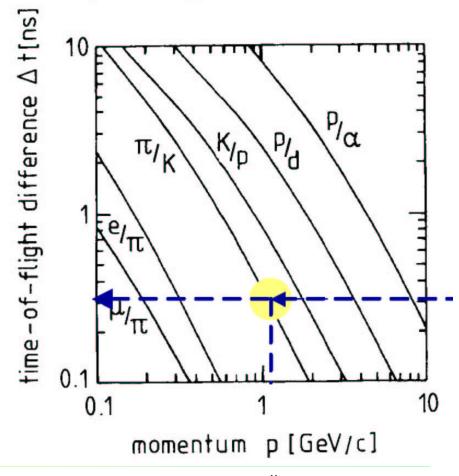
 π/K difference at 1GeV/c: 300ps

For a 3σ separation need $\sigma(TOF)=100ps$

Resolution contributions:

- •PMT: transient time spread (TTS)
- Path length variation
- Momentum uncertainty

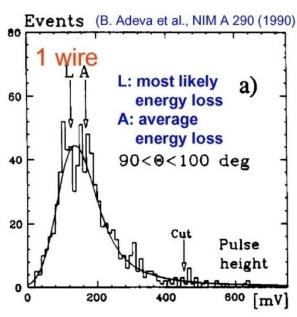
Time difference between two particle species for path length=1m



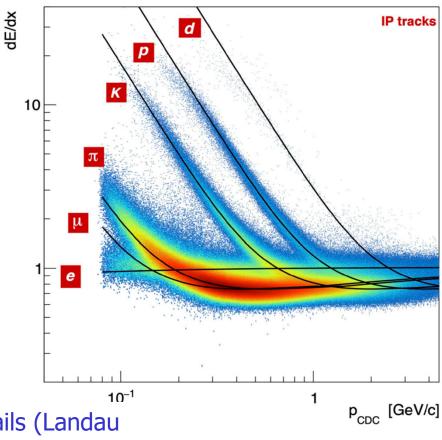
Identification with dE/dx measurement

dE/dx is a function of velocity.

For particles with different masses the Bethe-Bloch curves get displaced → separation is possible if the resolution is good enough − 5%



Belle II CDC: dE/dx vs p



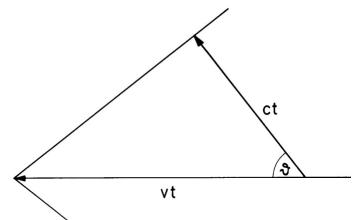
Problem: long tails (Landau distribution, not Gaussian), use truncated mean, exclude highest values

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Cherenkov datectors

A charged track with velocity v=βc exceeding the speed of light c/n in a medium with refractive index n emits polarized light at a characteristic (Cherenkov) angle,

$$\cos\theta = c/nv = 1/\beta n$$



Two cases:

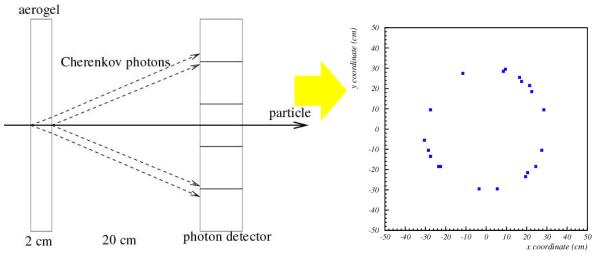
- $\rightarrow \beta < \beta_t = 1/n$: below threshold no Cherenkov light is emitted.
- $\rightarrow \beta > \beta_t$: the number of Cherenkov photons emitted over unit photon energy E=hv in a radiator of length L:

$$\frac{dN}{dE} = \frac{\alpha}{\hbar c} L \sin^2 \theta = 370(cm)^{-1} (eV)^{-1} L \sin^2 \theta$$

→ Few detected photons

Measuring the Cherenkov angle

Particles above threshold: measure θ

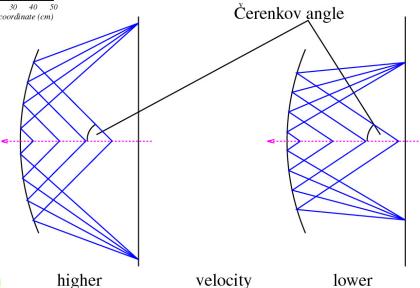


Idea: transform the direction into a coordinate → ring on the detection plane

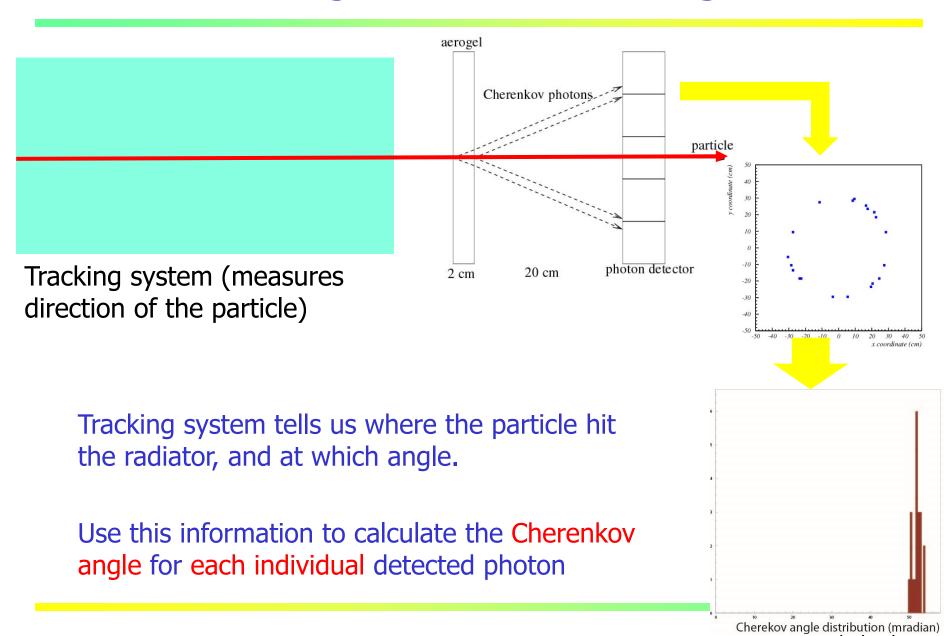
→ Ring Imaging Cherenkov (RICH) counter

Proximity focusing RICH

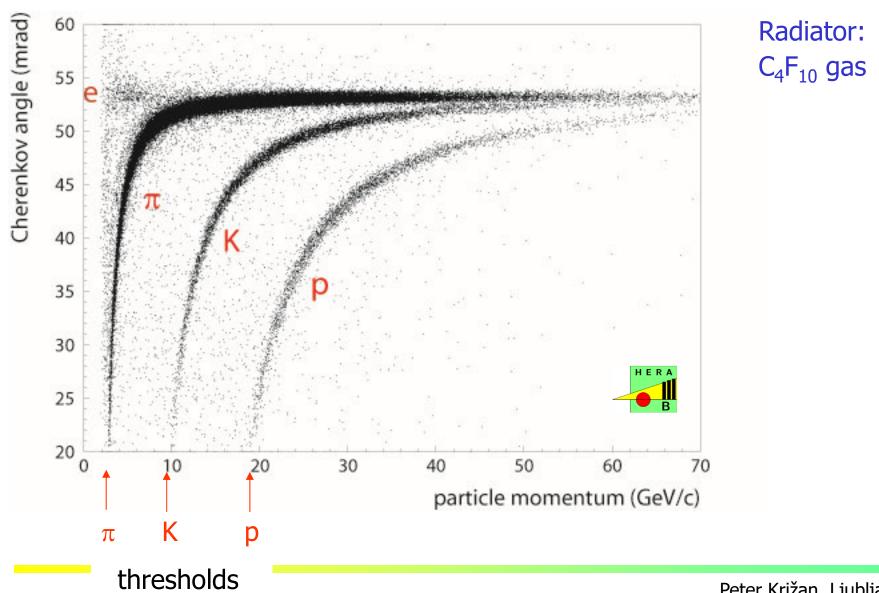
RICH with a focusing mirror



Measuring the Cherenkov angle



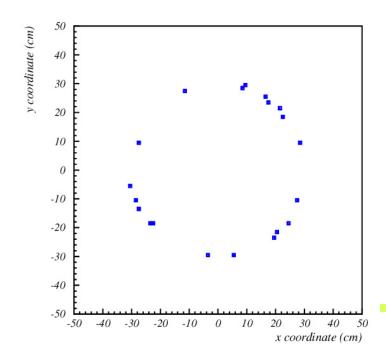
Measuring Cherenkov angle



Photon detection in RICH counters

RICH counter: measure photon impact point on the photon detector surface

- → detection of single photons with
- sufficient spatial resolution
- high efficiency and good signal-to-noise ratio (few photons!)
- over a large area (square meters)



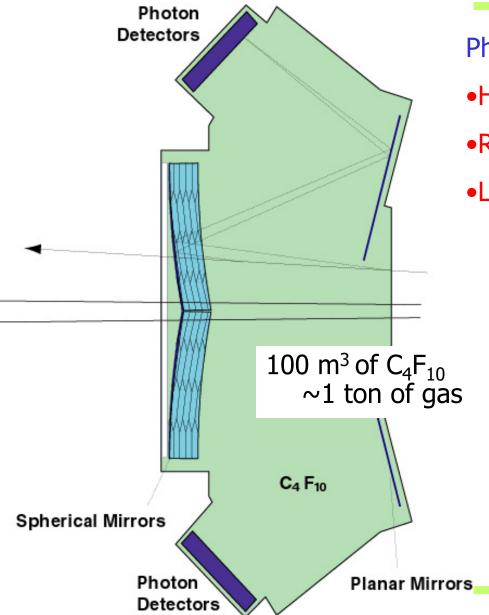
Special requirements:

- Operation in magnetic field
- High rate capability
- Very high spatial resolution
- Excellent timing (time-of-arrival information)

Photon detector is the most crucial element of a RICH counter



HERA-B RICH

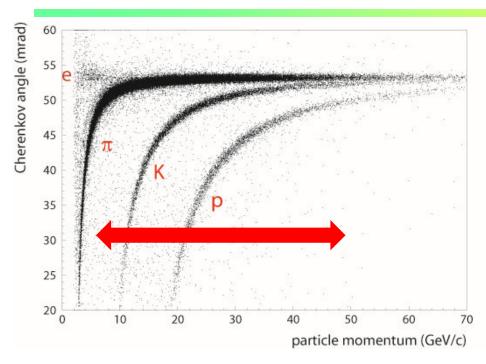


Photon detector requirements:

- •High QE over ~3m²
- •Rates ~1MHz
- Long term stability



Kinematic range of a RICH counter



Example: kinematic range for kaon/pion separation

Kinematic range for separation of two particle types:

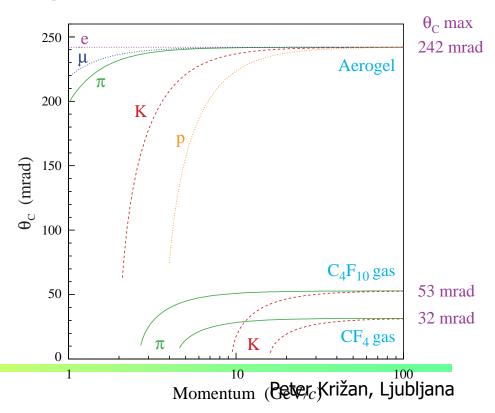
- •Lower limit p_{min}: sufficiently above lighter particle threshold
- •Upper limit p_{max}: given by Cherenkov angle resolution overlap of the two bands

Rule of thumb: $p_{max}/p_{min} < 10$

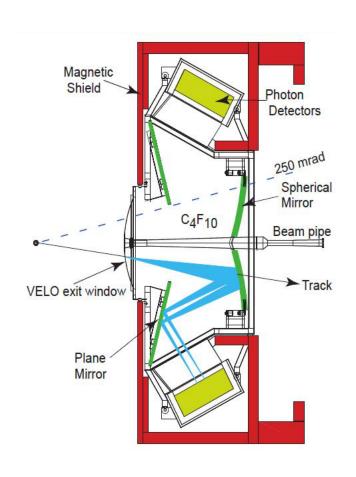
LHCb RICHes

Need:

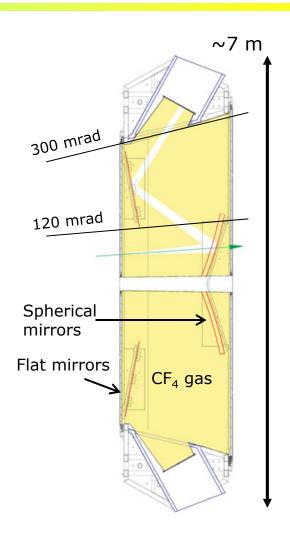
- •Particle identification for momentum range ~2-100 GeV/c
- •Granularity 2.5x2.5mm²
- •Large area (2.8m²) with high active area fraction
- •Fast compared to the 25ns bunch crossing time
- •Have to operate in a small B field
- →3 radiators
- Aerogel
- $\bullet C_4 F_{10}$ gas
- •CF₄ gas



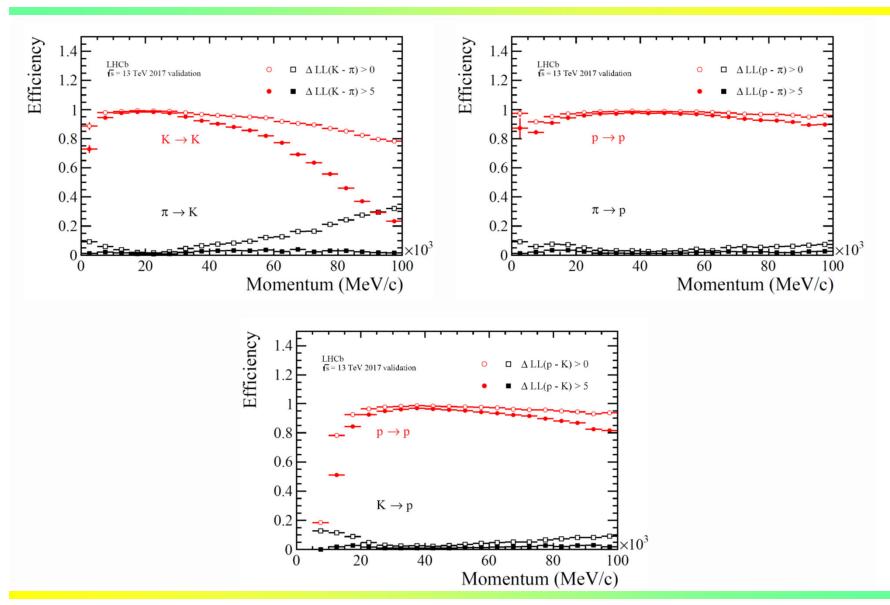
LHCb RICHes



RICH 1 + 2

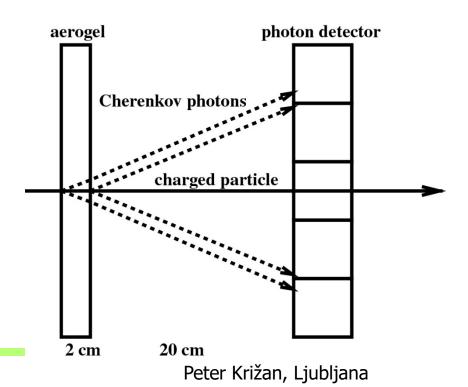


LHCb RICHes: performance



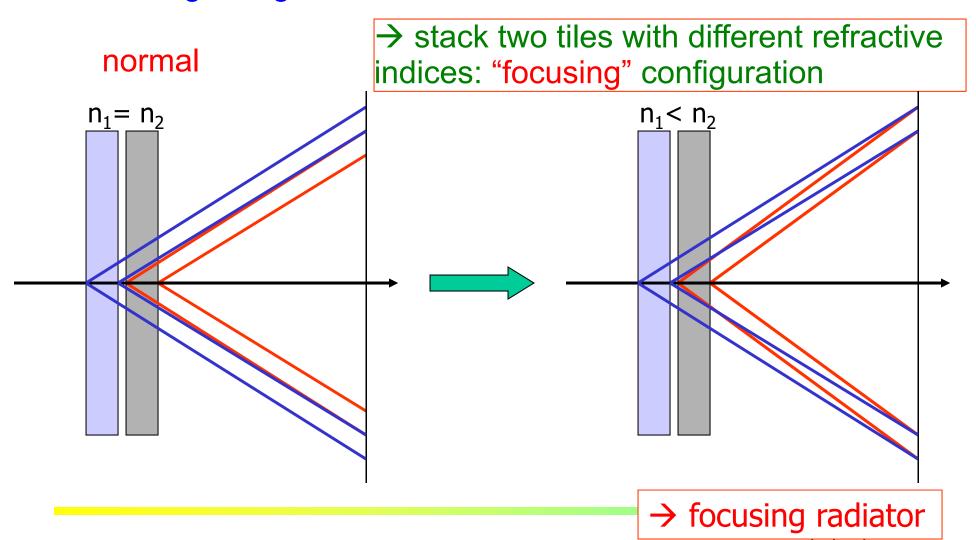
Resolution of a RICH counter

- Photon impact point resolution (photon detector resolution
- Emission point uncertainty
- Dispersion: $n=n(\lambda)$ in $\cos\theta = 1/\beta n$
- Track parameters
- Errors of the optical system

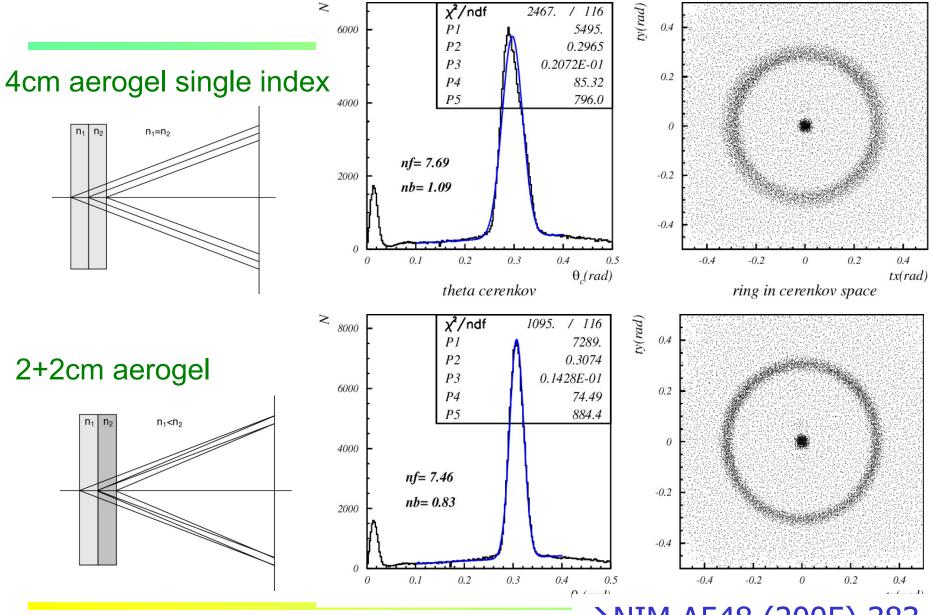


Radiator with multiple refractive indices

How to increase the number of photons without degrading the resolution?

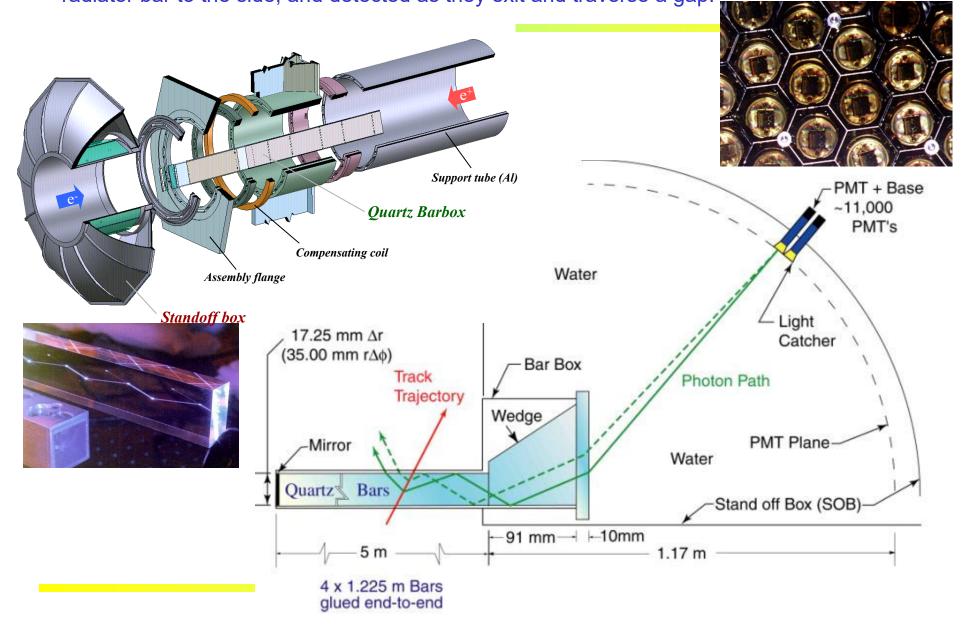


Focusing configuration – data



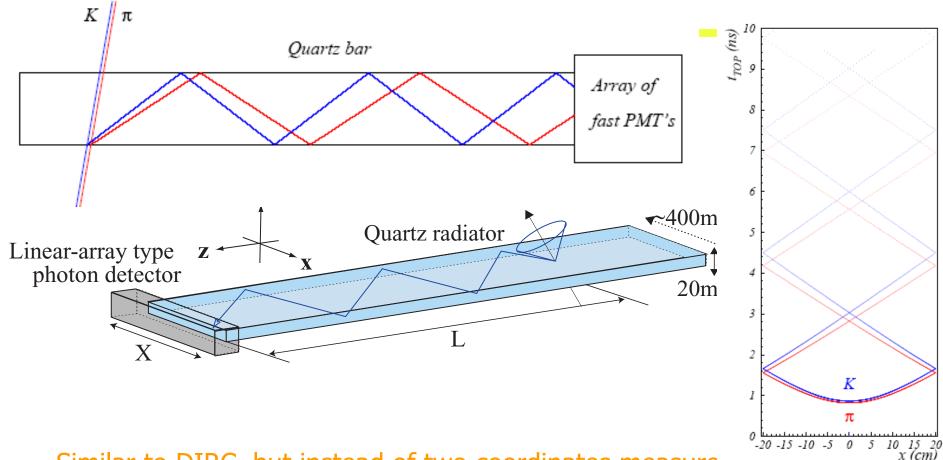
→NIM A548 (2005) 383

DIRC - Detector of Internally Reflected Cherekov photons - a special kind of RICH where Cherenkov photons trapped in a solid radiator (e.q. quartz) are propagated along the radiator bar to the side, and detected as they exit and traverse a gap.





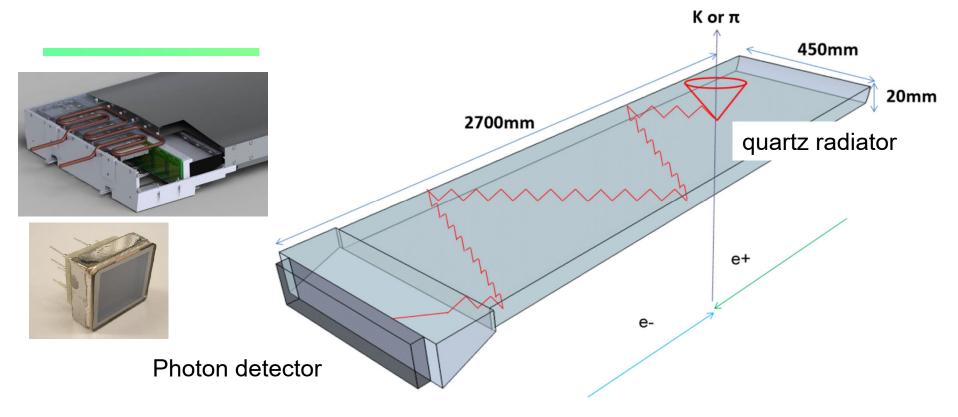
Time-Of-Propagation (TOP) counter



Similar to DIRC, but instead of two coordinates measure

- One (or two coordinates) with a few mm precision
- Time-of-arrival

Belle II Barrel PID: Time of propagation (TOP) counter



- Cherenkov ring imaging with precise time measurement.
- Reconstruct Cherenkov angle from two hit coordinates and the time of propagation of the photon
 - Quartz radiator (2cm thick)
 - Photon detector (MCP-PMT)
 - Excellent time resolution ~ 40 ps
 - Single photon sensitivity at 1.5 T

Transition radiation detectors

X rays emitted at the boundary of two media with different refractive indices, emission angle $\sim 1/\gamma$

Emission rate depends on γ (Lorentz factor): becomes important at $\gamma \sim 1000$

Electrons at 0.5 GeV

Pions, muons above 100 GeV

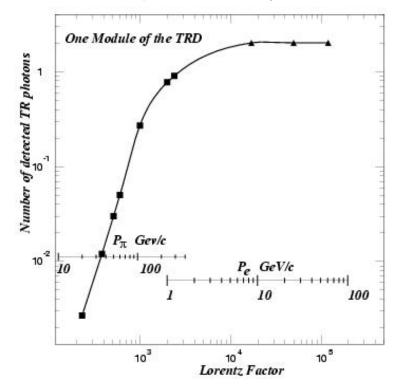
In between: discrimination e vs pions, mions

Detection of X rays: high Z gas – Xe

Few photons per boundary can be detected Need many boundaries

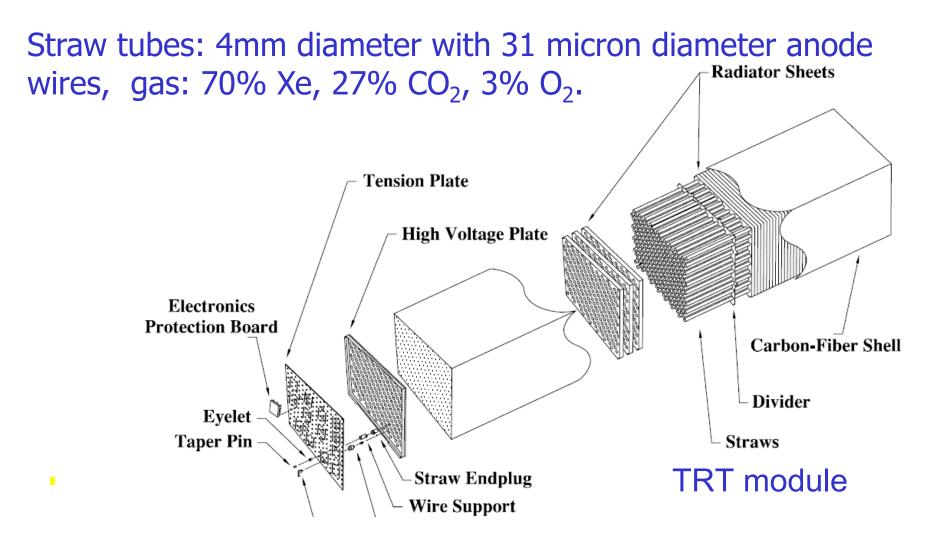


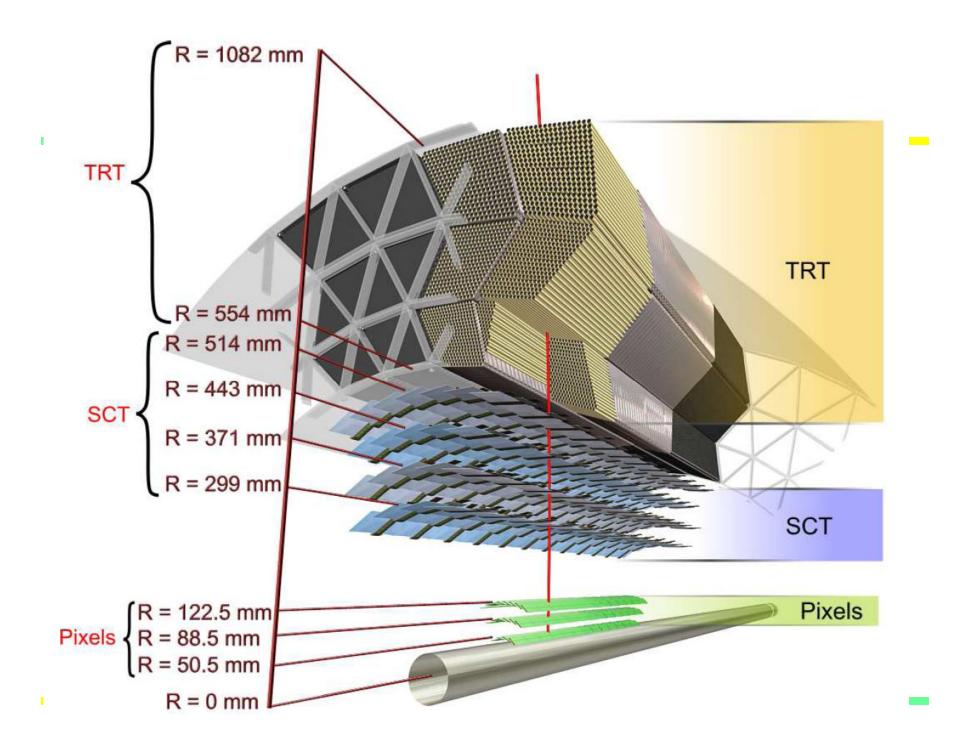
Porous materials – foam with many boundaries of individual 'bubbles'



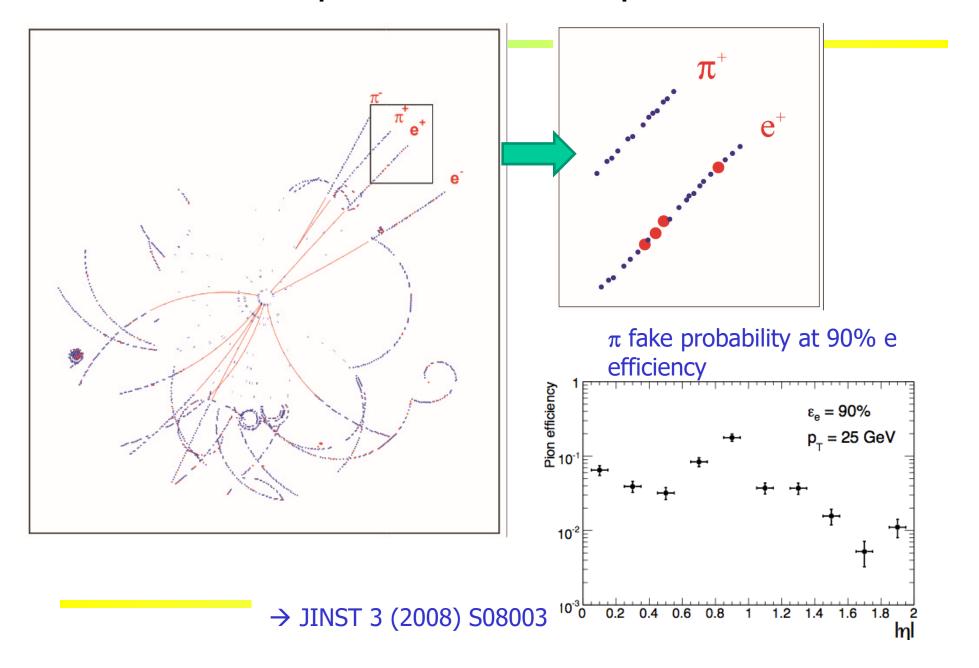
ATLAS TRT: combination of Transition Radiation detector and a Tracker

Radiator: 3mm thick layers made of polypropylene-polyethylene fibers with ~19 micron diameter, density: 0.06 g/cm³





TRT: pion-electron separation



Muon and K_I detector at B factories

Separate muons from hadrons (pions and kaons): exploit the fact that muons interact only electromag., while hadrons interact strongly → need a few interaction lengths to stop hadrons (interaction lengths – stong interaction analog to radiation length = about 10x radiation length in iron, 20x in CsI).

A particle is identified as muon if it penetrates the material.



Detect K_I interaction (cluster): again need a few interaction lengths.

Example: Muon and K_I detection at Belle II

KL and muon detector:14/15 layers RPC+Fe
Resistive Plate Counter (barrel outer layers)
Scintillator + WLSF + MPPC (end-caps, inner 2 barrel layers)

EM Calorimeter:

CsI(Tl), waveform sampling (barrel)

Pure CsI + waveform sampling (end-caps)

electrons (7GeV)

Beryllium beam pipe 2cm diameter

Vertex Detector

2 layers DEPFET + 4 layers DSSD

Central Drift Chamber He(50%):C₂H₆(50%), small cells, long lever arm, fast electronics Particle Identification

Time-of-Propagation counter (barrel)
Prox. focusing Aerogel RICH (fwd)

positrons (4GeV)

Muon and K_L detector

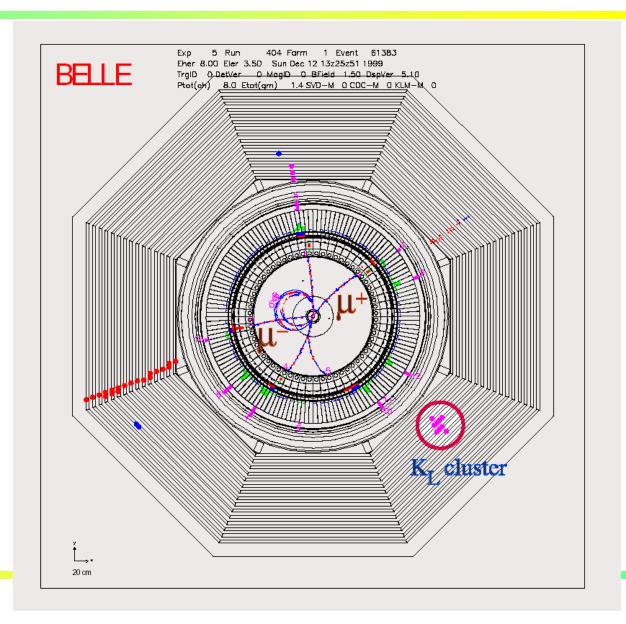
Example:

event with

•two muons and a

•K_L

and a pion that partly penetrated



Muon and K_L detector performance

Muon identification: efficient for p>800 MeV/c

2.5

3

2

efficiency 0.75 0.75 E(µ) 0.25

Fig. 109. Muon detection efficiency vs. momentum in KLM.

1.5

P(GeV/c)

0.5

fake probability

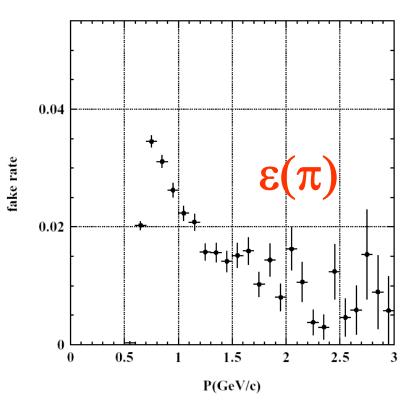
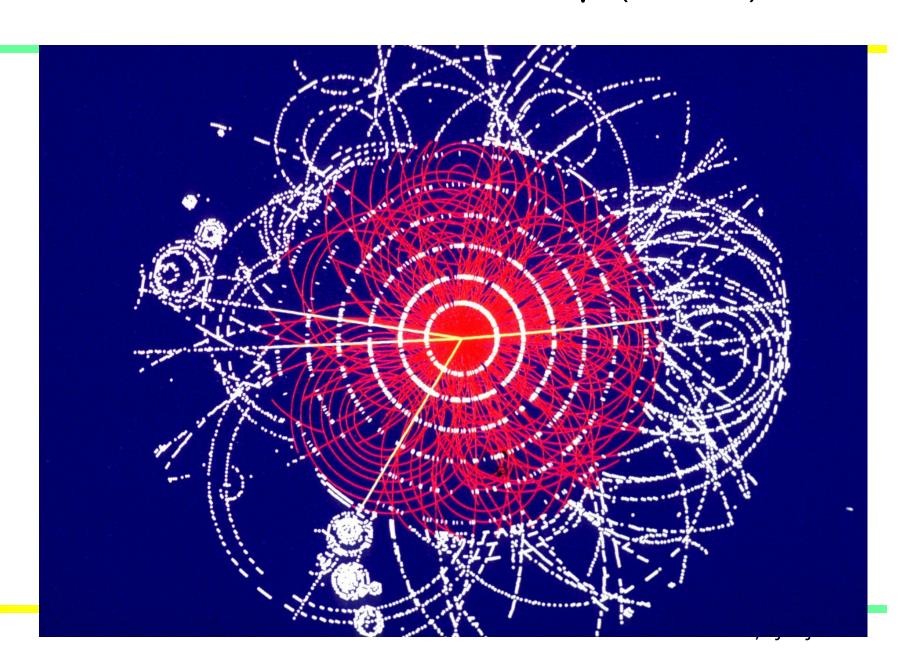
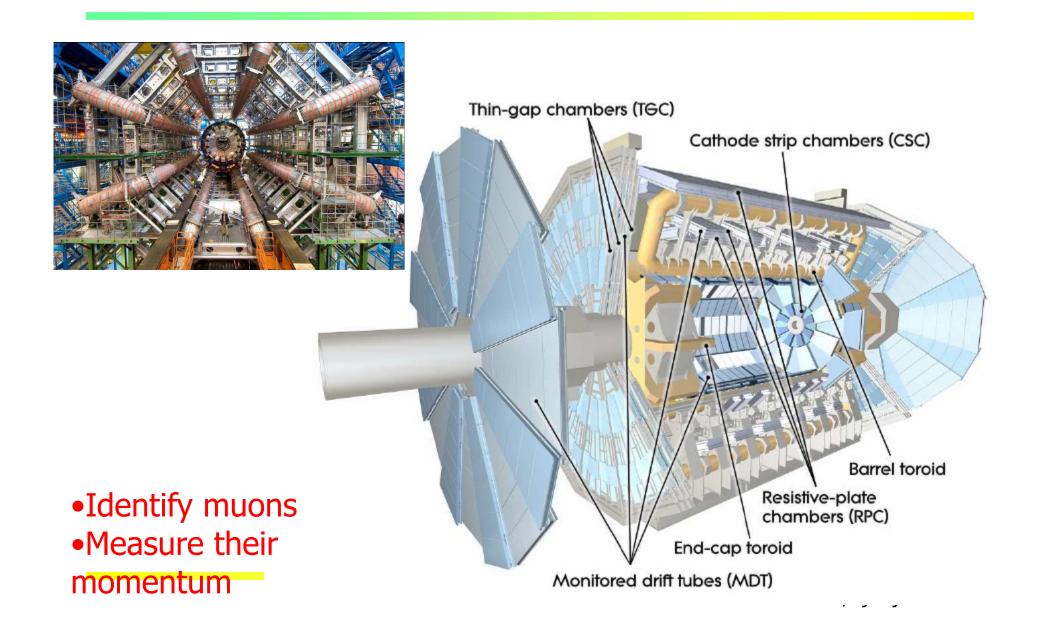


Fig. 110. Fake rate vs. momentum in KLM.

MC simulation: H \rightarrow 4 μ (ATLAS)



Identification of muons in ATLAS



Components of an experimental apparatus ('spectrometer')

- Tracking and vertexing systems
- Particle identification devices
- Calorimeters (measurement of energy)

Calorimeters (measurement of energy)

Calorimetry:

Energy measurement by total absorption, combined with spatial reconstruction.

Calorimetry is a "destructive" method

Detector response α E

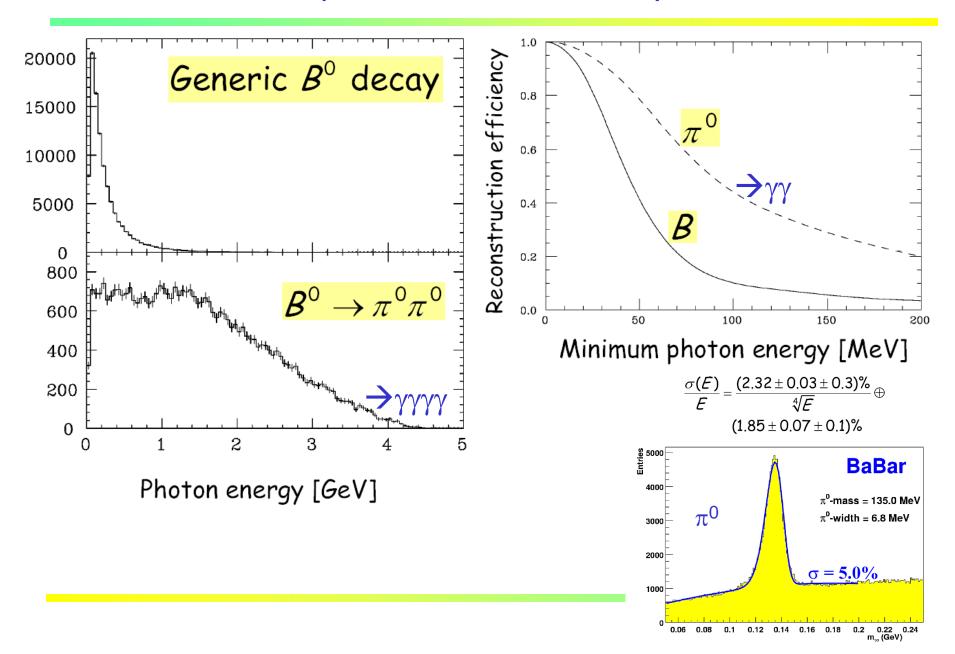
Calorimetry works both for

- charged (e± and hadrons) and
- neutral particles (n,γ)

Basic mechanism: formation of electromagnetic or hadronic showers.

Finally, the energy is converted into ionization or excitation of the matter.

Requirements at a B factory: Photons



Calorimetry Design: B factories

Requirements

- •Best possible energy and position resolution: 11 photons per Y(4S) event; 50% below 200 MeV in energy
- Acceptance down to lowest possible energies and over large solid angle
- Electron identification down to low momentum

Constraints

- Cost of raw materials and growth of crystals
- Operation inside magnetic field
- Background sensitivity

Implementation

Thallium-doped Cesium-Iodide crystals with 2 photodiodes per crystal

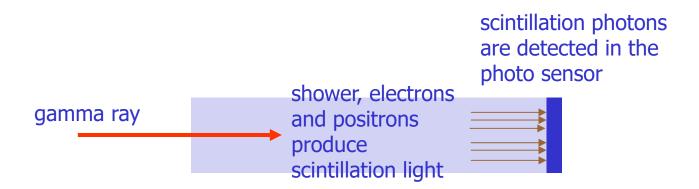
Thin structural cage to minimize material between and in front of crystals

Requirements: Photons

 $\pi^0 \rightarrow \gamma \gamma$ Need to reconstruct neutral pions from gamma pairs

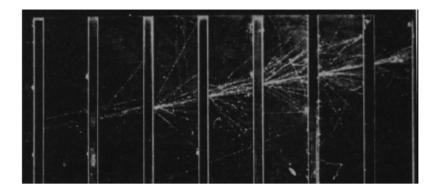
- Should also work for low energy gammas (photons)
- Excellent energy resolution

Detection of photons: scintillator crystal + photosensor

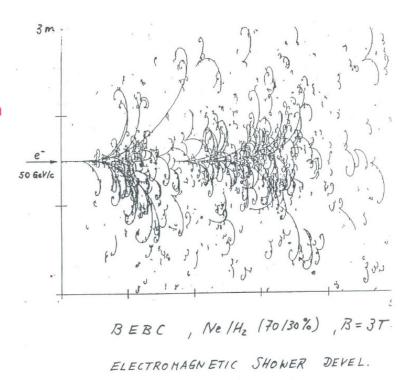


How does a shower develop? Gamma \rightarrow e+e- pair production \rightarrow bremstrahlung gammas \rightarrow e+e- pair production \rightarrow

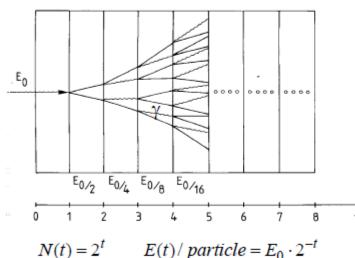
Electromagnetic Cascades (showers)



Electron shower in a cloud chamber with lead absorbers



Simple qualitative model



Consider only Bremsstrahlung and pair production.

Assume:

$$X_0 = \lambda_{pair}$$

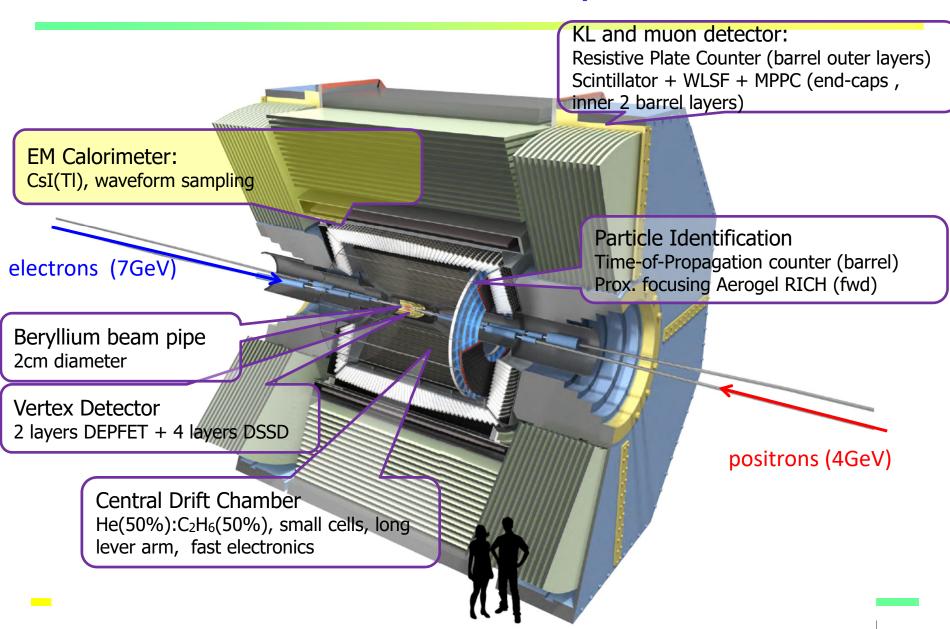
Process continues until $E(t) \le E_c$

$$t_{\max} = \frac{\ln E_0 / E_c}{\ln 2} \qquad N^{total} = \sum_{t=0}^{t_{\max}} 2^t = 2^{(t_{\max} + 1)} - 1 \approx 2 \cdot 2^{t_{\max}} = 2 \frac{E_0}{E_c}$$

After $t = t_{max}$ the dominating processes are ionization, Compton effect and photo effect \rightarrow absorption. \rightarrow Calorimeter size depends only logarithmically on E_{θ}

Peter Križan, Ljubljana

Calorimetry in Belle II



Energy resolution of a calorimeter (intrinsic limit)

$$N^{total} \propto \frac{E_0}{E_c}$$
 total number of track segments

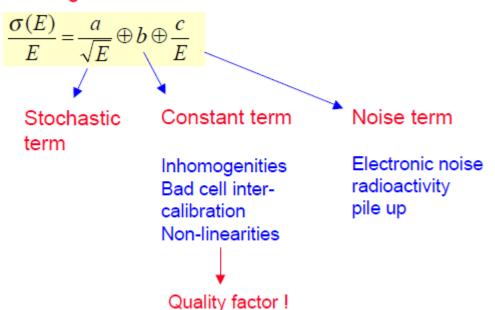
$$\frac{\sigma(E)}{E} \propto \frac{\sigma(N)}{N} \propto \frac{1}{\sqrt{N}} \propto \frac{1}{\sqrt{E_0}}$$
 ho

holds also for hadron calorimeters

Also spatial and angular resolution scale like $1/\sqrt{E}$

Relative energy resolution of a calorimeter improves with E_0

More general:



Irižan, Ljubljana

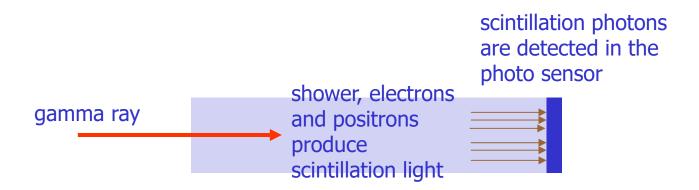
Requirements: Photons

 $\pi^0 \rightarrow \gamma \gamma$

Need to reconstruct neutral pions from gamma pairs

- Also gammas (photons) with low energy
- Excellent energy resolution

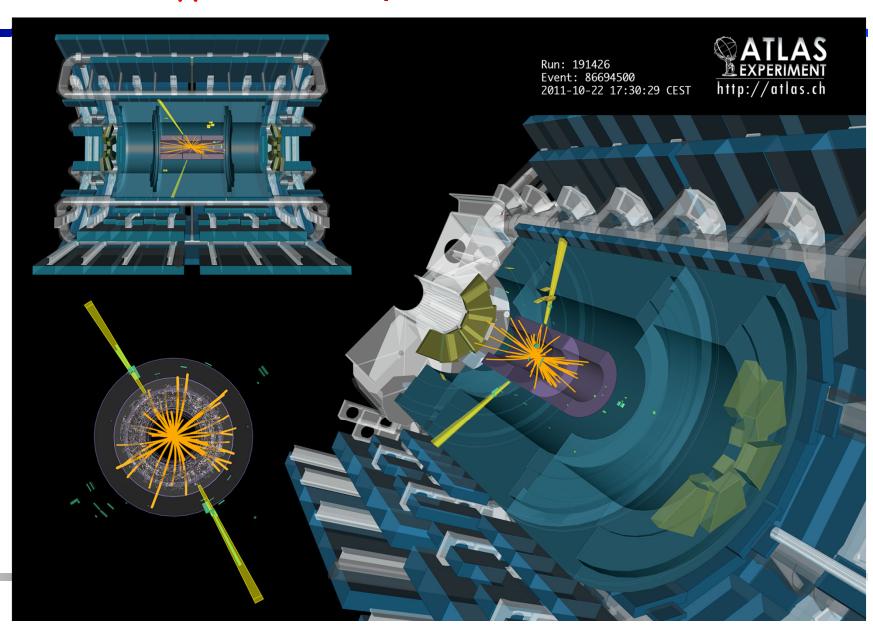
Detection of photons: scintillator crystal + photosensor

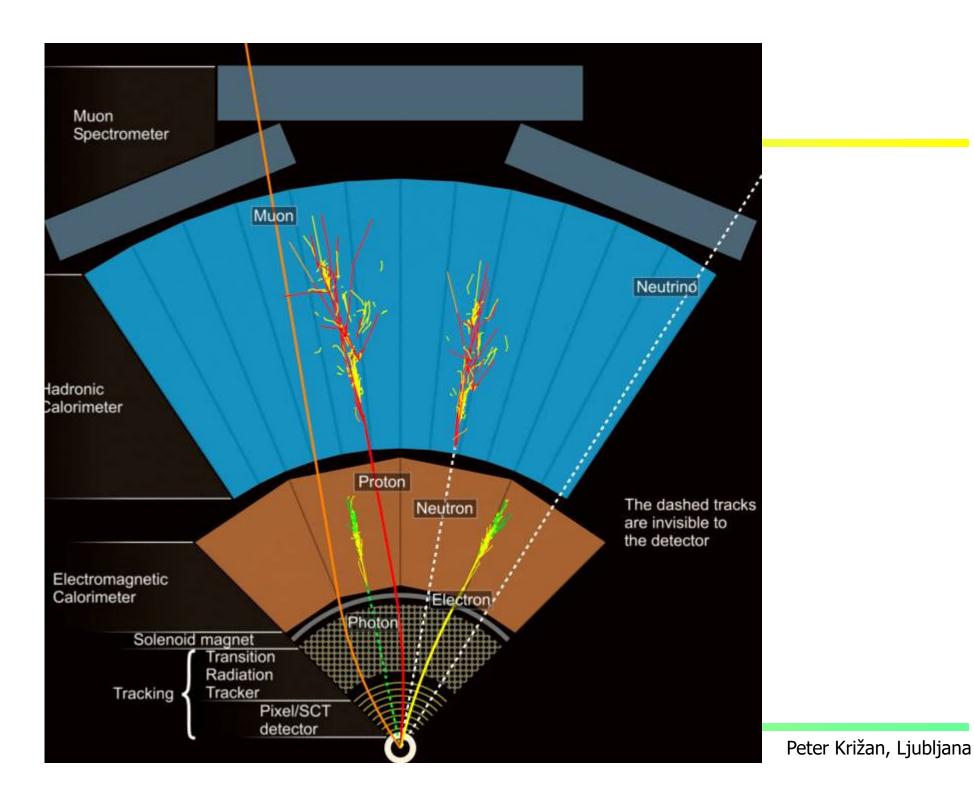


Need:

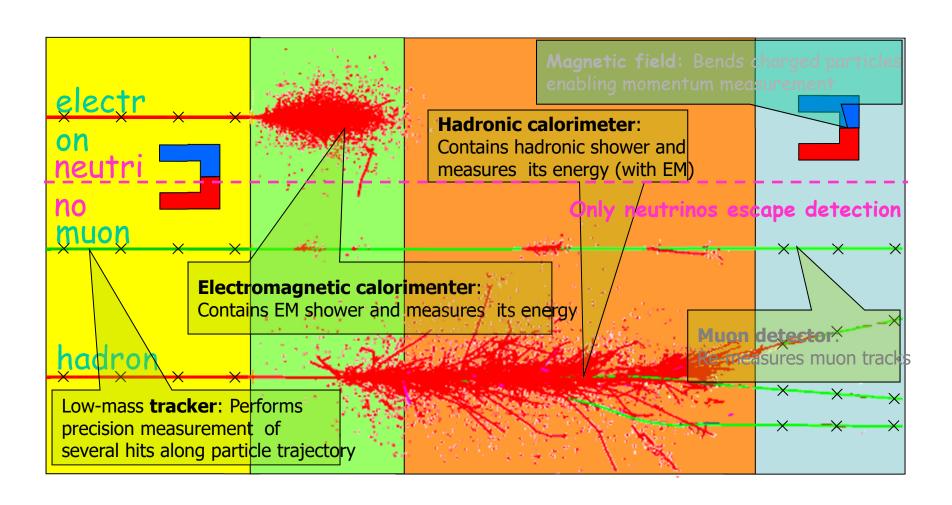
- High light yield (many scintillation photons) $\leftarrow \sigma(E)/E \propto N^{-1/2}$
- photo-sensor with low noise (noise spoils resolution)

Higgs boson decay to two high-energy gammas, $H \rightarrow \gamma \gamma$, as seen by the ATLAS detector





Generic LHC Detector for all Particles



Neutrino detection

Use inverse beta decay

$$v_e$$
+ n \rightarrow p + e⁻

$$\overline{v}_e$$
+ p \rightarrow n + e⁺

$$v_{\mu} + n \rightarrow p + \mu^{-}$$

$$\overline{\nu}_{\mu} + p \rightarrow n + \mu^{+}$$

$$v_{\tau} + n \rightarrow p + \tau$$

$$\overline{\nu}_{\tau} + p \rightarrow n + \tau^{+}$$

However: cross section is very small!

6.4 10⁻⁴⁴ cm² at 1MeV

Probability for interaction in 100m of water = $4 \cdot 10^{-16}$

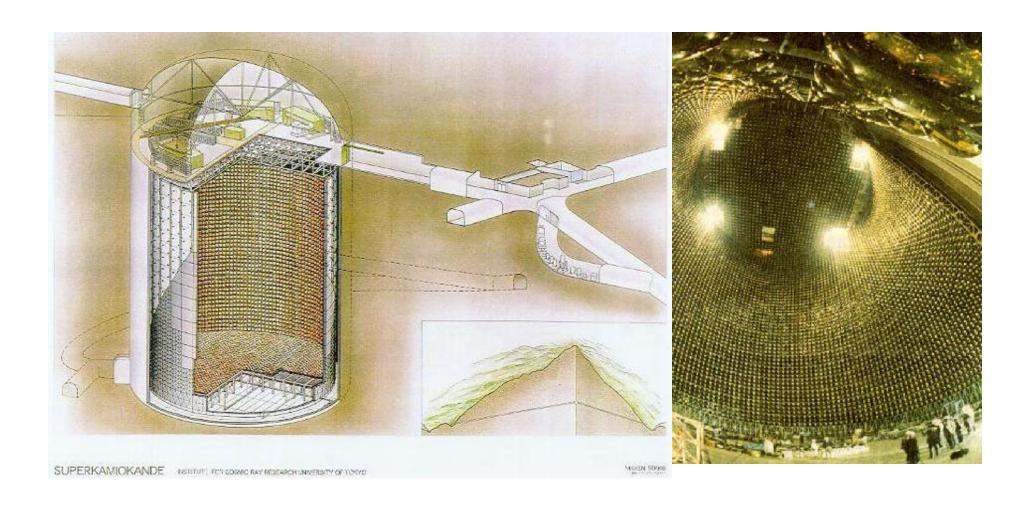
Not much better at high energies:

0.67 10⁻³⁸ E/1GeV cm² per

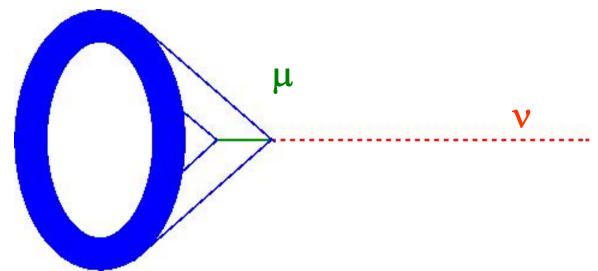
nucleon

At 100 GeV, still 11 orders below the proton-proton cross section

Superkamiokande: an example of a neutrino detector



Superkamiokande: detection of electrons and muons



The muon or electron emits Cerenkov light

- → ring at the detector walls
- Muon ring: sharp edges
- Electron ring: smeared

Superkamiokande: detection of neutrinos by measureing Cherenkov photons



Light detectors: HUGE photomultiplier tubes

M. Koshiba

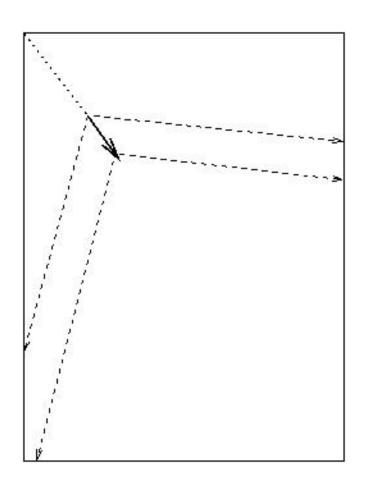
Muon vs electron

Cherenkov photons from a muon track:

Example: 1GeV muon neutrino

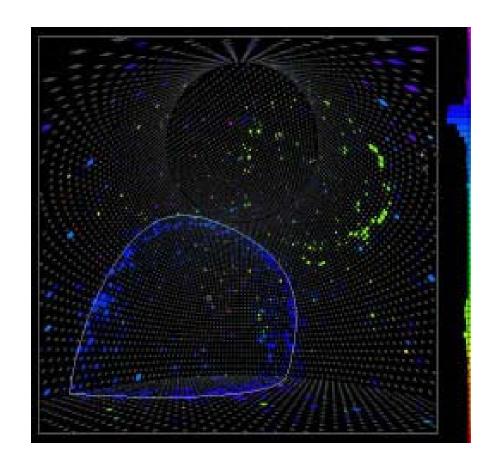
Track length of the resulting muon:
L=E/(dE/dx)=
=1GeV/(2MeV/cm)=5m

→ a well defined "ring" on the walls

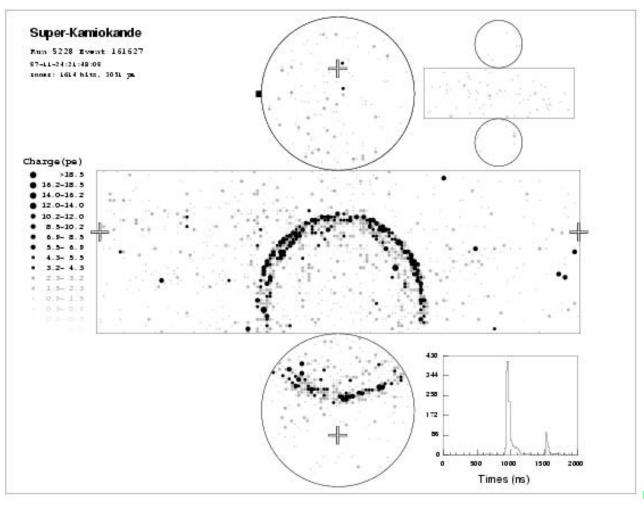


Superkamiokande: muon event

Muon 'ring' as seen by the photon detectors



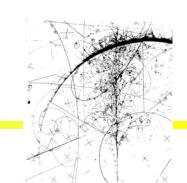
Muon event: photon detector cillinder walls



neutrino detection

Peter Križan, Ljubljana

Cherenkov photons from an electron track



Electron starts a shower!

Cherenkov photons from an electron generated shower

Example: 1GeV el. neutrino

Shower length:

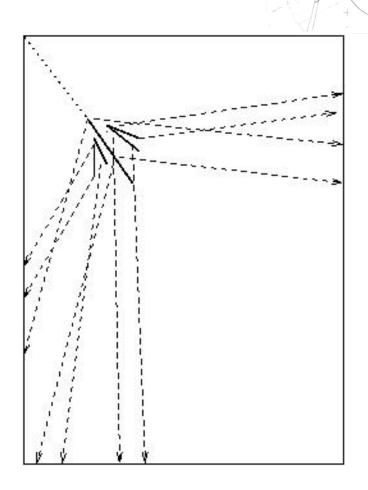
$$L=X_0*log_2(E/E_{crit})=$$

$$36cm*log_2(1GeV/10MeV)$$

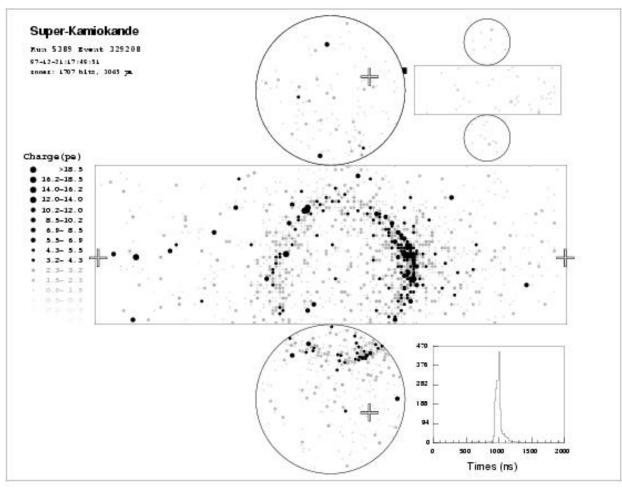
$$=2.5m$$

Shower particles are not parallel to each other

-> a blurred, less well defined "ring" on the walls



Electron event: blurred ring



neutrino detection

Peter Krizan, Ljubijana

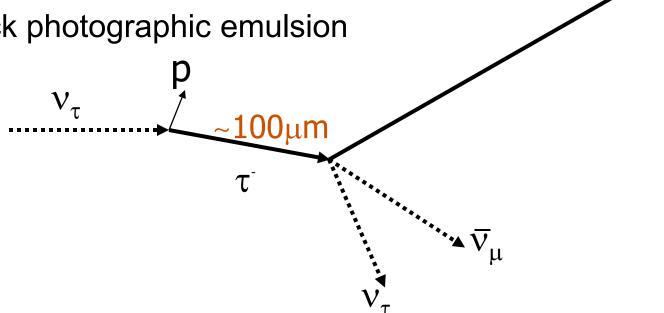
Backup slides

Detection of τ neutrinos

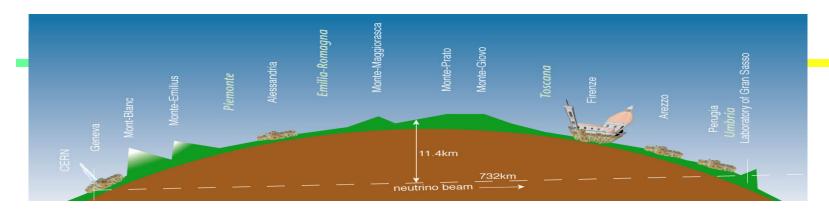
$$v_{\tau} + n \rightarrow p + \tau^{-} \qquad \tau^{-} \rightarrow \mu^{-} v_{\mu} v_{\tau}$$

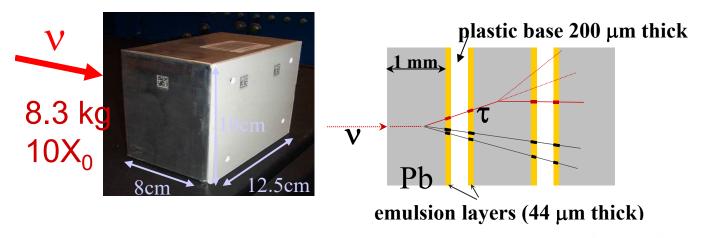
- Detect and identify mion
- Extrapolate back

Check for a 'kink' in the sensitive volume –
 e.g. a thick photographic emulsion

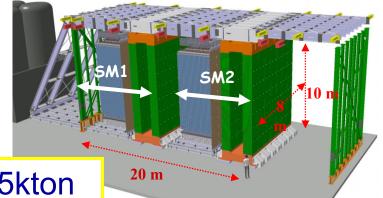


Detection of τ neutrinos: OPERA





Detection unit: a brick with 56 Pb sheets (1mm) + 57 emulsion films

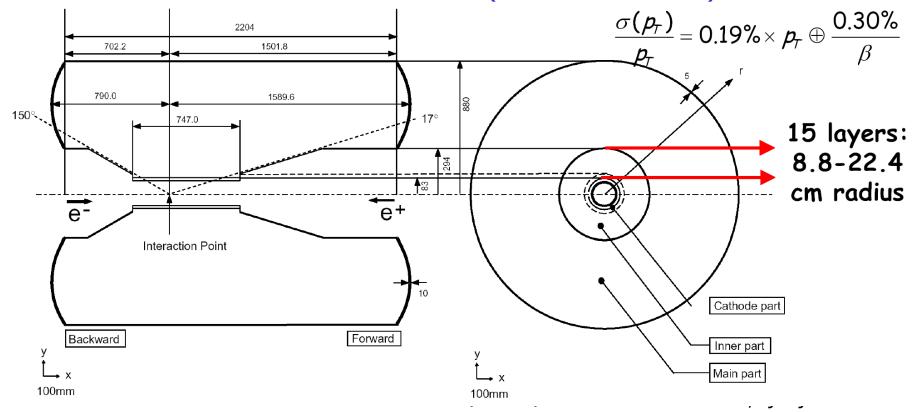


155000 bricks, detector tot. mass = 1.35kton

Belle central drift chamber



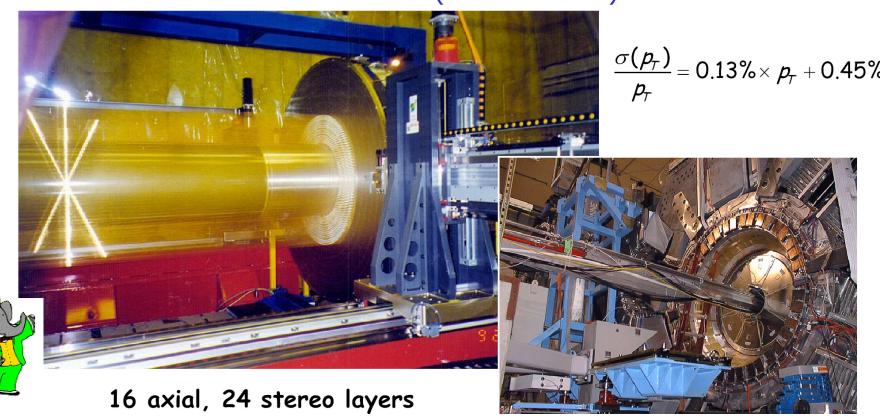
- •50 layers of wires (8400 cells) in 1.5 Tesla magnetic field
- •Helium:Ethane 50:50 gas, Al field wires, CF inner wall with cathodes, and preamp only on endplates
- •Particle identification from ionization loss (5.6-7% resolution)



Tracking: BaBar drift chamber



40 layers of wires (7104 cells) in 1.5 Tesla magnetic field Helium:Isobutane 80:20 gas, Al field wires, Beryllium inner wall, and all readout electronics mounted on rear endplate Particle identification from ionization loss (7% resolution)



6. ATOMIC AND NUCLEAR PROPERTIES OF MATERIALS

Table 6.1 Abridged from pdg.lbl.gov/AtomicNuclearProperties by D. E. Groom (2007). See web pages for more detail about entries in this table including chemical formulae, and for several hundred other entries. Quantities in parentheses are for NTP (20° C and 1 atm), and square brackets indicate quantities evaluated at STP. Boiling points are at 1 atm. Refractive indices n are evaluated at the sodium D line blend (589.2 nm); values $\gg 1$ in brackets are for $(n-1) \times 10^6$ (gases).

$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Material	Z	A	$\langle Z/A \rangle$	Nucl.coll.	Nucl.inter.	Rad.len.	$dE/dx _{\mathrm{m}}$	•••	Melting	Boiling	Refract.
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$					length λ_T	length λ_I	X_0	{ MeV	$\{ g \text{ cm}^{-3} \}$	point	point	index
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$					$\{\mathrm{g~cm}^{-2}\}$	$\{\mathrm{g~cm}^{-2}\}$	$\{\mathrm{g~cm^{-2}}\}$	$\mathrm{g}^{-1}\mathrm{cm}^2$	$\{(\{g\ell^{-1}\})\}$	(K)	(K)	(@ Na D)
He 2 $4.002602(2)$ 0.49967 51.8 71.0 94.32 (1.937) $0.125(0.166)$ 4.220 $1.02[35.6]$ Li 3 $6.941(2)$ 0.43221 52.2 71.3 82.78 1.639 0.534 453.6 $1615.$ 82.8 4 $9.012182(3)$ 0.44384 55.3 77.8 65.19 1.595 1.848 $1560.$ $2744.$ C diamond 6 $12.0107(8)$ 0.49955 59.2 85.8 42.70 1.725 3.520 2.42 C graphite 6 $12.0107(8)$ 0.49955 59.2 85.8 42.70 1.725 3.520 2.210 $8.274.$ 8.278 8.278 1.639 $9.687(1.165)$ 63.15 9.242 $1.20[298]$ 1.20	H_2	1	1.00794(7)	0.99212	42.8	52.0	63.04	(4.103)	0.071(0.084)	13.81	20.28	1.11[132.]
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	D_2	1	2.01410177803(8)	0.49650	51.3	71.8	125.97	(2.053)	0.169(0.168)	18.7	23.65	1.11[138.]
Be 4 9.012182(3) 0.44384 55.3 77.8 65.19 1.595 1.848 1560. 2744. C diamond 6 12.0107(8) 0.49955 59.2 85.8 42.70 1.725 3.520 2.42 C graphite 6 12.0107(8) 0.49976 61.1 89.7 37.99 (1.825) 0.807(1.165) 63.15 77.29 1.20[298 O2 8 15.9994(3) 0.50002 61.3 90.2 34.24 (1.801) 1.141(1.332) 54.36 90.20 1.22[271 F2 9 18.9984032(5) 0.47372 65.0 97.4 32.93 (1.676) 1.507(1.580) 53.53 85.03 [195] Ne 10 20.1797(6) 0.49555 65.7 99.0 28.93 (1.724) 1.204(0.839) 24.56 27.07 1.09[67.1 Al 13 26.9815386(8) 0.48181 69.7 107.2 24.01 1.615 2.699 933.5 2792	He	2	4.002602(2)	0.49967	51.8	71.0	94.32	(1.937)	0.125(0.166)		4.220	1.02[35.0]
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Li	3	6.941(2)	0.43221	52.2	71.3	82.78	1.639	0.534	453.6	1615.	
$ \begin{array}{c} \text{C graphite} & 6 & 12.0107(8) \\ \text{N}_2 & 7 & 14.0067(2) & 0.49976 & 61.1 & 89.7 & 37.99 & (1.825) & 0.807(1.165) & 63.15 & 77.29 & 1.20[298 \\ \text{N}_2 & 8 & 15.9994(3) & 0.50002 & 61.3 & 90.2 & 34.24 & (1.801) & 1.141(1.332) & 54.36 & 90.20 & 1.22[271.52] \\ \text{F}_2 & 9 & 18.9984032(5) & 0.47372 & 65.0 & 97.4 & 32.93 & (1.676) & 1.507(1.580) & 53.53 & 85.03 & [195.] \\ \text{Ne} & 10 & 20.1797(6) & 0.49555 & 65.7 & 99.0 & 28.93 & (1.724) & 1.204(0.839) & 24.56 & 27.07 & 1.09[67.1 & 2.017] \\ \text{Al} & 13 & 26.9815386(8) & 0.48181 & 69.7 & 107.2 & 24.01 & 1.615 & 2.699 & 933.5 & 2792. \\ \text{Si} & 14 & 28.0855(3) & 0.49848 & 70.2 & 108.4 & 21.82 & 1.664 & 2.329 & 1687. & 3538. & 3.95 \\ \text{Cl}_2 & 17 & 35.453(2) & 0.47951 & 73.8 & 115.7 & 19.28 & (1.630) & 1.574(2.980) & 171.6 & 239.1 & [773.] \\ \text{Ar} & 18 & 39.948(1) & 0.45059 & 75.7 & 119.7 & 19.55 & (1.519) & 1.396(1.662) & 83.81 & 87.26 & 1.23[281.7] \\ \text{Ti} & 22 & 47.867(1) & 0.45961 & 78.8 & 126.2 & 16.16 & 1.477 & 4.540 & 1941. & 3560. \\ \text{Cu} & 29 & 63.546(3) & 0.45636 & 84.2 & 137.3 & 12.86 & 1.403 & 8.960 & 1358. & 2835. \\ \text{Ge} & 32 & 72.64(1) & 0.44053 & 86.9 & 143.0 & 12.25 & 1.370 & 5.323 & 1211. & 3106. \\ \text{Sn} & 50 & 118.710(7) & 0.42119 & 98.2 & 166.7 & 8.82 & 1.263 & 7.310 & 505.1 & 2875. \\ \text{W} & 74 & 138.384(1) & 0.40525 & 110.4 & 191.9 & 6.76 & 1.145 & 19.300 & 3695. & 5828. \\ \text{Pt} & 78 & 195.084(9) & 0.39983 & 112.2 & 195.7 & 6.54 & 1.128 & 21.450 & 2042. & 4098. \\ \text{Au} & 79 & 196.966569(4) & 0.4018 & 112.5 & 196.3 & 6.46 & 1.134 & 19.320 & 1337. & 3129. \\ \text{Pb} & 82 & 207.2(1) & 0.39575 & 114.1 & 199.6 & 6.37 & 1.122 & 11.350 & 600.6 & 2022. \\ \text{U} & 92 & [238.02891(3)] & 0.38651 & 118.6 & 209.0 & 6.00 & 1.081 & 18.950 & 1408. & 4404. \\ \text{Air} (dry, 1 atm) & 0.49919 & 61.3 & 90.1 & 36.62 & (1.815) & (1.205) & 78.80 \\ \text{Shielding concrete} & 0.50274 & 65.1 & 97.5 & 26.57 & 1.711 & 2.300 \\ \end{array}$	$_{ m Be}$	4	9.012182(3)	0.44384	55.3	77.8	65.19	1.595	1.848	1560.	2744.	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	C diamond	6	12.0107(8)	0.49955	59.2	85.8	42.70	1.725	3.520			2.42
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	C graphite	6	12.0107(8)	0.49955	59.2	85.8	42.70	1.742	2.210			
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	N_2	7	14.0067(2)	0.49976	61.1	89.7	37.99	(1.825)	0.807(1.165)	63.15	77.29	1.20[298.]
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	O_2	8	15.9994(3)	0.50002	61.3	90.2	34.24	(1.801)	1.141(1.332)	54.36	90.20	1.22[271.]
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	F_2	9	18.9984032(5)	0.47372	65.0	97.4	32.93	(1.676)	1.507(1.580)	53.53	85.03	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Ne	10	20.1797(6)	0.49555	65.7	99.0	28.93	(1.724)	1.204(0.839)	24.56	27.07	1.09[67.1]
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Al	13	26.9815386(8)	0.48181	69.7	107.2	24.01	1.615	2.699	933.5	2792.	
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Si	14	28.0855(3)	0.49848	70.2	108.4	21.82	1.664	2.329	1687.	3538.	3.95
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Cl_2	17	35.453(2)	0.47951	73.8	115.7	19.28	(1.630)	1.574(2.980)	171.6	239.1	[773.]
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Ar	18	39.948(1)	0.45059	75.7	119.7	19.55	(1.519)	1.396(1.662)	83.81	87.26	1.23[281.]
Cu 29 63.546(3) 0.45636 84.2 137.3 12.86 1.403 8.960 1358. 2835. Ge 32 72.64(1) 0.44053 86.9 143.0 12.25 1.370 5.323 1211. 3106. Sn 50 118.710(7) 0.42119 98.2 166.7 8.82 1.263 7.310 505.1 2875. Xe 54 131.293(6) 0.41129 100.8 172.1 8.48 (1.255) 2.953(5.483) 161.4 165.1 1.39[701. W 74 183.84(1) 0.40252 110.4 191.9 6.76 1.145 19.300 3695. 5828. Pt 78 195.084(9) 0.39983 112.2 195.7 6.54 1.128 21.450 2042. 4098. Au 79 196.966569(4) 0.40108 112.5 196.3 6.46 1.134 19.320 1337. 3129. Pb 82 207.2(1) 0.39575	Ti	22		0.45961		126.2					3560.	
Ge 32 72.64(1) 0.44053 86.9 143.0 12.25 1.370 5.323 1211. 3106. Sn 50 118.710(7) 0.42119 98.2 166.7 8.82 1.263 7.310 505.1 2875. Xe 54 131.293(6) 0.41129 100.8 172.1 8.48 (1.255) 2.953(5.483) 161.4 165.1 1.39[701.0] W 74 183.84(1) 0.40252 110.4 191.9 6.76 1.145 19.300 3695. 5828. Pt 78 195.084(9) 0.39983 112.2 195.7 6.54 1.128 21.450 2042. 4098. Au 79 196.966569(4) 0.40108 112.5 196.3 6.46 1.134 19.320 1337. 3129. Pb 82 207.2(1) 0.39575 114.1 199.6 6.37 1.122 11.350 600.6 2022. U 92 [238.02891(3)] 0.38651 118.6 209.0 6.00 1.081 18.950 1408. 4404. Air (dry, 1 atm)	Fe	26	55.845(2)	0.46557	81.7	132.1	13.84	1.451	7.874	1811.	3134.	
Ge 32 72.64(1) 0.44053 86.9 143.0 12.25 1.370 5.323 1211. 3106. Sn 50 118.710(7) 0.42119 98.2 166.7 8.82 1.263 7.310 505.1 2875. Xe 54 131.293(6) 0.41129 100.8 172.1 8.48 (1.255) 2.953(5.483) 161.4 165.1 1.39[701.0] W 74 183.84(1) 0.40252 110.4 191.9 6.76 1.145 19.300 3695. 5828. Pt 78 195.084(9) 0.39983 112.2 195.7 6.54 1.128 21.450 2042. 4098. Au 79 196.966569(4) 0.40108 112.5 196.3 6.46 1.134 19.320 1337. 3129. Pb 82 207.2(1) 0.39575 114.1 199.6 6.37 1.122 11.350 600.6 2022. U 92 [238.02891(3)] 0.38651 118.6 209.0 6.00 1.081 18.950 1408. 4404. Air (dry, 1 atm)	Cu	29	63.546(3)	0.45636	84.2	137.3	12.86	1.403	8.960	1358.	2835.	
Sn 50 118.710(7) 0.42119 98.2 166.7 8.82 1.263 7.310 505.1 2875. Xe 54 131.293(6) 0.41129 100.8 172.1 8.48 (1.255) 2.953(5.483) 161.4 165.1 1.39[701.] W 74 183.84(1) 0.40252 110.4 191.9 6.76 1.145 19.300 3695. 5828. Pt 78 195.084(9) 0.39983 112.2 195.7 6.54 1.128 21.450 2042. 4098. Au 79 196.966569(4) 0.40108 112.5 196.3 6.46 1.134 19.320 1337. 3129. Pb 82 207.2(1) 0.39575 114.1 199.6 6.37 1.122 11.350 600.6 2022. U 92 [238.02891(3)] 0.38651 118.6 209.0 6.00 1.081 18.950 1408. 4404. Air (dry, 1 atm) Shielding concrete 0.50274 65.1 97.5 26.57 1.711 2.300	Ge	32				143.0	12.25					
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Sn			0.42119	98.2	166.7	8.82	1.263		505.1	2875.	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Xe	54	131.293(6)	0.41129	100.8	172.1	8.48	(1.255)	2.953(5.483)	161.4	165.1	1.39[701.]
Pt 78 195.084(9) 0.39983 112.2 195.7 6.54 1.128 21.450 2042. 4098. Au 79 196.966569(4) 0.40108 112.5 196.3 6.46 1.134 19.320 1337. 3129. Pb 82 207.2(1) 0.39575 114.1 199.6 6.37 1.122 11.350 600.6 2022. U 92 [238.02891(3)] 0.38651 118.6 209.0 6.00 1.081 18.950 1408. 4404. Air (dry, 1 atm) 0.49919 61.3 90.1 36.62 (1.815) (1.205) 78.80 Shielding concrete 0.50274 65.1 97.5 26.57 1.711 2.300	W	74	183.84(1)	0.40252	110.4	191.9	6.76	1.145	19.300	3695.	5828.	
Au 79 196.966569(4) 0.40108 112.5 196.3 6.46 1.134 19.320 1337. 3129. Pb 82 207.2(1) 0.39575 114.1 199.6 6.37 1.122 11.350 600.6 2022. U 92 [238.02891(3)] 0.38651 118.6 209.0 6.00 1.081 18.950 1408. 4404. Air (dry, 1 atm) 0.49919 61.3 90.1 36.62 (1.815) (1.205) 78.80 Shielding concrete 0.50274 65.1 97.5 26.57 1.711 2.300	Pt	78	195.084(9)	0.39983	112.2	195.7	6.54	1.128	21.450	2042.	4098.	
Pb 82 207.2(1) 0.39575 114.1 199.6 6.37 1.122 11.350 600.6 2022. U 92 [238.02891(3)] 0.38651 118.6 209.0 6.00 1.081 18.950 1408. 4404. Air (dry, 1 atm) 0.49919 61.3 90.1 36.62 (1.815) (1.205) 78.80 Shielding concrete 0.50274 65.1 97.5 26.57 1.711 2.300	Au	79		0.40108	112.5	196.3	6.46	1.134	19.320	1337.	3129.	
U 92 [238.02891(3)] 0.38651 118.6 209.0 6.00 1.081 18.950 1408. 4404. Air (dry, 1 atm) 0.49919 61.3 90.1 36.62 (1.815) (1.205) 78.80 Shielding concrete 0.50274 65.1 97.5 26.57 1.711 2.300	Pb	82		0.39575	114.1	199.6	6.37	1.122	11.350	600.6	2022.	
Shielding concrete 0.50274 65.1 97.5 26.57 1.711 2.300	U	92	[238.02891(3)]	0.38651	118.6	209.0	6.00	1.081	18.950	1408.	4404.	
	Air (dry, 1 atm) 0.49919			61.3	90.1	36.62	(1.815)	(1.205)		78.80		
Borosilicate glass (Pyrex) 0.49707 64.6 96.5 28.17 1.696 2.230	Shielding cor	icrete		0.50274								
	Borosilicate glass (Pyrex)			0.49707	64.6	96.5	28.17	1.696	2.230			

W	74	183.84(1)	0.40252	110.4	191.9	6.76	1.145	19.300	3695.	5828.	
Pt	78	195.084(9)	0.39983	112.2	195.7	6.54	1.128	21.450	2042.	4098.	
Au	79	196.966569(4)	0.40108	112.5	196.3	6.46	1.134	19.320	1337.	3129.	
Pb	82	207.2(1)	0.39575	114.1	199.6	6.37	1.122	11.350	600.6	2022.	
U	92	[238.02891(3)]	0.38651	118.6	209.0	6.00	1.081	18.950	1408.	4404.	
Air (dry,	1 atm)		0.49919	61.3	90.1	36.62	(1.815)	(1.205)		78.80	
Shielding	concrete		0.50274	65.1	97.5	26.57	1.711	2.300			
Borosilica	ate glass (Py	yrex)	0.49707	64.6	96.5	28.17	1.696	2.230			
Lead glas			0.42101	95.9	158.0	7.87	1.255	6.220			
Standard	rock		0.50000	66.8	101.3	26.54	1.688	2.650			
Methane	(CH_4)		0.62334	54.0	73.8	46.47	(2.417)	(0.667)	90.68	111.7	[444.]
Ethane (C	C_2H_6		0.59861	55.0	75.9	45.66	(2.304)	(1.263)	90.36	184.5	
Propane ((C_3H_8)		0.58962	55.3	76.7	45.37	(2.262)	0.493(1.868)	85.52	231.0	
Butane (C	C_4H_{10}		0.59497	55.5	77.1	45.23	(2.278)	(2.489)	134.9	272.6	
Octane (C	C_8H_{18})		0.57778	55.8	77.8	45.00	2.123	0.703	214.4	398.8	
Paraffin (Paraffin (CH ₃ (CH ₂) _{n≈23} CH ₃)			56.0	78.3	44.85	2.088	0.930			
	Nylon (type 6, 6/6)			57.5	81.6	41.92	1.973	1.18			
Polycarbo	Polycarbonate (Lexan)			58.3	83.6	41.50	1.886	1.20			
_	Polyethylene ([CH ₂ CH ₂] _n)			56.1	78.5	44.77	2.079	0.89			
	Polyethylene terephthalate (Mylar)			58.9	84.9	39.95	1.848	1.40			
Polyimide	Polyimide film (Kapton)			59.2	85.5	40.58	1.820	1.42			
_	Polymethylmethacrylate (acrylic)			58.1	82.8	40.55	1.929	1.19			1.49
	Polypropylene			56.1	78.5	44.77	2.041	0.90			
Polystyre	ne ($[C_6H_5C]$	$[HCH_2]_n$	0.53768	57.5	81.7	43.79	1.936	1.06			1.59
Polytetra	Polytetrafluoroethylene (Teflon)			63.5	94.4	34.84	1.671	2.20			
Polyvinyl	toluene	,	0.54141	57.3	81.3	43.90	1.956	1.03			1.58
Aluminun	n oxide (sap	ophire)	0.49038	65.5	98.4	27.94	1.647	3.970	2327.	3273.	1.77
Barium fl	ouride (Bal	F_2)	0.42207	90.8	149.0	9.91	1.303	4.893	1641.	2533.	1.47
Bismuth g	germanate ((BGO)	0.42065	96.2	159.1	7.97	1.251	7.130	1317.		2.15
Carbon d	ioxide gas ((CO_2)	0.49989	60.7	88.9	36.20	1.819	(1.842)			[449.]
	lid carbon dioxide (dry ice)		0.49989	60.7	88.9	36.20	1.787	$1.563^{'}$	Sublim	es at 194.7	
	odide (CsI)	, - ,	0.41569	100.6	171.5	8.39	1.243	4.510	894.2	1553.	1.79
	luoride (LiF	?)	0.46262	61.0	88.7	39.26	1.614	2.635	1121.	1946.	1.39
	nydride (LiF	/	0.50321	50.8	68.1	79.62	1.897	0.820	965.		
	gstate (PbV	/	0.41315	100.6	168.3	7.39	1.229	8.300	1403.		2.20
		, fused quartz)	0.49930	65.2	97.8	27.05	1.699	2.200	1986.	3223.	1.46
	hloride (Na	,	0.55509	71.2	110.1	21.91	1.847	2.170	1075.	1738.	1.54
	odide (NaI)	/	0.42697	93.1	154.6	9.49	1.305	3.667	933.2	1577.	1.77
Water (H	, ,		0.55509	58.5	83.3	36.08	1.992	1.000(0.756)	273.1	373.1	1.33
Silica aero	- /		0.50093	65.0	97.3	27.25	1.740	0.200		I ₂ O, 0.97 Si	
									\	- /	-,