2.1 The Principle of Inductive Peaking

A simple common-base transistor amplifier is shown in Fig. 2.1.1. A current-step source I_s is connected to the emitter; the time scale is referenced to the current-step transition time at $t = 0$ and normalized to the system time-constant, RC. The collector is loaded by a resistor R; in addition, there is the collector-base capacitance C_{CB}, along with the unavoidable stray-capacitance C_S and the load capaciatnce C_L in parallel. Their sum is denoted as C.

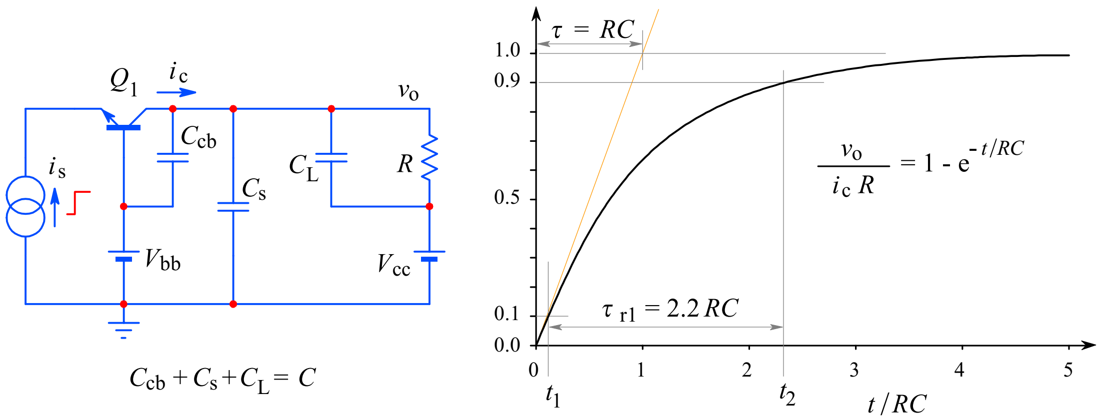

Fig. 2.1.1: A common-base amplifier with RC-load : the basic circuit and its step-response.

Because of this capacitances, the collector voltage v_o does not jump suddenly to the value $I_c R$, where I_c is the collector current. Instead, the collector voltage rises exponentially according to the formula (see Part 1, Eq. 1.6.66) :

$$v_o = I_c R \left(1 - e^{-t/RC}\right) \tag{2.1.1}$$

The time, elapsed between 10 % and 90 % of the final collector voltage value ($I_c R$), we name the *rise-time*, τ_{r1} and we calculate it by inserting these levels into the Eq. 2.1.1 :

$$0.1\, I_c R = I_c R \left(1 - e^{-t_1/RC}\right) \qquad \Rightarrow \qquad t_1 = R\,C\,ln\,0.9 \tag{2.1.2}$$

Similarly for t_2 :

$$0.9\, I_c R = I_c R \left(1 - e^{-t_2/RC}\right) \qquad \Rightarrow \qquad t_2 = R\,C\,ln\,0.1 \tag{2.1.3}$$

The rise-time is the difference between these two instants :

$$\tau_{r1} = t_2 - t_1 = R\,C\,ln\,0.9 - R\,C\,ln\,0.1 = R\,C\,ln\,\frac{0.9}{0.1} = \mathbf{2.2\,R\,C} \tag{2.1.4}$$

This value is the reference against which we will compare all other circuits in the following sections of the book.

Since in wideband amplifiers we strive to make the output voltage a replica of the input voltage (except for the amplitude !), we want to reduce the rise-time of the amplifier as much as is practically possible. As the output voltage rises, more current flows through R and less current remains to fill C. Obviously, we would achieve a shorter rise-time if we could disconnect R in some way until C is filled-up. To do so, let us introduce a switch S between the capacitor C and the load resistor R. This switch is open at time $t = 0$, when the current step starts, but it closes at time $t = RC$, as in Fig. 2.1.2. In this way we force all the available current to the capacitor, so it charges linearly to the voltage $I_c R$. But when the capacitor is fully charged, the switch S is closed, routing all the current to the loading resistor R.

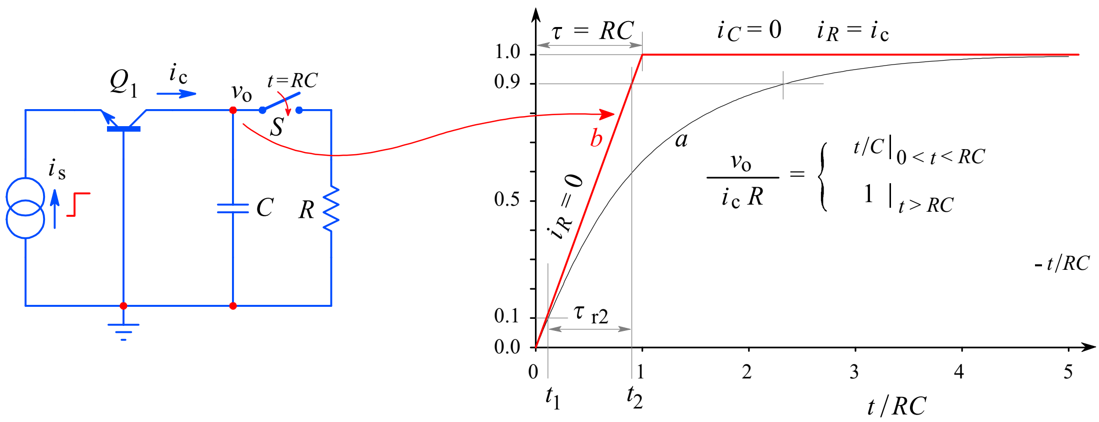

Fig. 2.1.2 : A hypothetical ideal rise-time circuit. The switch disconnects R from the circuit, so all of I_c is available to fill C, but after a time $t = RC$ the switch is closed and all I_c flows through R. The output voltage is shown in ***b*)**, in comparison to the exponential response in ***a*)**.

By comparing Fig. 2.1.1 with Fig. 2.1.2, we note a substantial decrease in rise-time τ_{r2}, which we calculate from the output voltage :

$$v_o = \frac{1}{C}\int_0^{\tau} I_c \, dt = \frac{I_c}{C}\, t\Big|_{t=0}^{t=\tau} = I_c R \quad ; \quad \tau = RC \tag{2.1.5}$$

Since the charging of the capacitor is linear, as displayed in Fig. 2.1.2, the rise-time is simply :

$$\tau_{r2} = 0.9\,RC - 0.1\,RC = 0.8\,RC \tag{2.1.6}$$

In comparison with Fig. 2.1.1, where there was no switch, the rise-time improvement factor is :

$$\eta_r = \frac{\tau_{r1}}{\tau_{r2}} = \frac{2.20\,RC}{0.8\,RC} = 2.75 \tag{2.1.7}$$

It is evident that the **rise-time** (Eq. 2.1.6) is independent of the actual value of the current I_c, but the **maximum voltage** $I_c R$ (Eq. 2.1.5) is not. On the other hand, the smaller the resistor R, the smaller is the rise-time. Clearly, the introduction of the switch S would mean a great improvement. By using a more powerful transistor and a lower value resistor R, we could (in principle, at least) decrease the rise-time at will (providing that C

remains unchanged). Unfortunately, it is impossible to make a low-on-resistance switch, functioning as in Fig. 2.1.2, which would suitably follow the signal and automatically open and close in nanoseconds or even in microseconds. So it remains only a nice idea.

But, instead of a switch, we can insert an appropriate inductance L between the capacitor C and resistor R and so **partially** achieve the effect of the switch, as shown in Fig. 2.1.3. Since the current through an inductor can not change instantaneously, more current will be charging C, at least initially. The collector network in Fig. 2.1.3 is reciprocal, so we may take the output voltage either from the resistor R or from the capacitor C. In the first case we have a *series-peaking network* while in the second case we speak of a *shunt-peaking network*. Both types of peaking networks are used in wideband amplifiers.

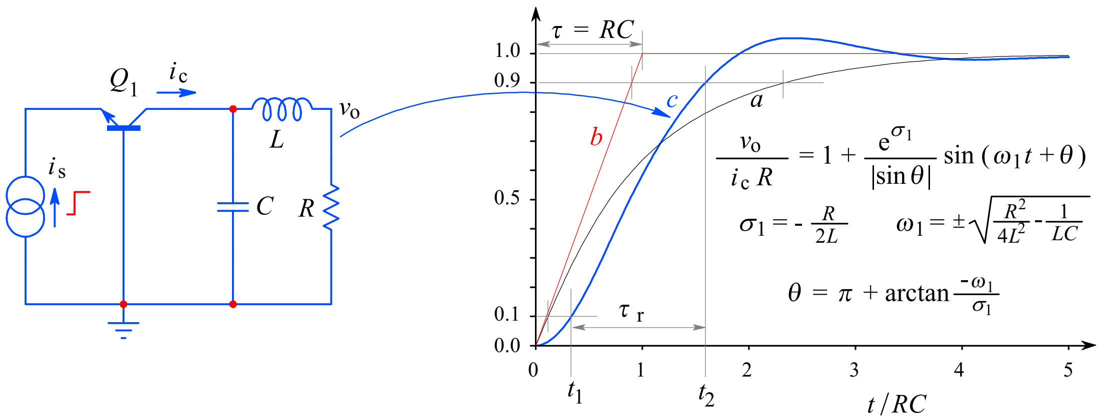

Fig. 2.1.3 : A common-base amplifier with the series-peaking circuit. The output voltage v_o (c) is compared to the exponential response (a, $L = 0$) and the ideal response (b). If we took the output voltage from the capacitor C, we would have a shunt-peaking circuit (see Sec. 2.7).

Fig. 2.1.3 is the most simple series-peaking circuit. Later, when we will discuss T-coil circuits, we will not just achieve rise-time improvements similar to the value given in Eq. 2.1.7 but, in cases where it is possible (usually it is) to split C in two parts, we will obtain a substantially greater improvement.

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2.2 Two pole series-peaking circuit

Besides the series-peaking circuit, in this section we will discuss all the significant mathematical methods, which are needed to calculate the frequency-, phase- and time-delay-response, the upper half-power frequency and the rise-time. In addition, we will derive the most important design parameters of the series-peaking circuit, which we will use also in the other sections of the book.

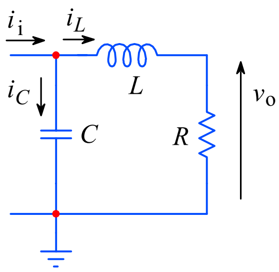

Fig. 2.2.1 : A two-pole series-peaking circuit

In Fig. 2.2.1 we have repeated the collector-loading circuit of Fig. 2.1.3. Since the inductive peaking circuits are used mostly as collector load circuits, from here on we will omit the transistor symbol ; instead, we will show the input current I_i (formerly I_c) flowing into the network, with the common ground as its drain. At first we will discuss the behavior of the network in the frequency-domain, supposing that I_i is the RMS value of the sinusoidally-changing input current. This current is split into two parts : the current through the capacitance I_C and the current through the inductance I_L . Thus we have :

$$I_i = I_C + I_L = V_i \, j\omega C + \frac{V_i}{j\omega L + R} = V_i \left(j\omega C + \frac{1}{j\omega L + R} \right) \qquad (2.2.1)$$

The output voltage is :

$$V_o = I_L \, R = V_i \, \frac{R}{j\omega L + R} \qquad (2.2.2)$$

From this we obtain the transfer function :

$$\frac{V_o}{I_i} = \frac{V_i \dfrac{R}{j\omega L + R}}{V_i \left(j\omega C + \dfrac{1}{j\omega L + R} \right)} = \frac{R}{j\omega C \, (j\omega L + R) + 1}$$

$$= \frac{R}{-\,\omega^2 L C + R \, j\omega C + 1} \qquad (2.2.3)$$

Let us set $I_i = 1 \, V/R$ and $L = m \, R^2 \, C$, where m is a dimensionless parameter ; also, let's substitute $j\omega$ with s . We obtain :

$$V_o = F(s) = \frac{1}{s^2 \, m \, R^2 C^2 + s \, R \, C + 1}$$

$$= \frac{1}{m\,R^2\,C^2} \cdot \frac{1}{s^2 + \dfrac{s}{m\,R\,C} + \dfrac{1}{m\,R^2\,C^2}} \tag{2.2.4}$$

The denominator roots, which for an efficient peaking must be complex-conjugate, like in Fig. 2.2.2 , are the poles of $F(s)$:

$$s_{1,2} = \sigma_1 \pm j\,\omega_1 = -\,\frac{1}{2\,m\,R\,C} \pm \sqrt{\frac{1}{4\,m^2 R^2\,C^2} - \frac{1}{m\,R^2\,C^2}} \tag{2.2.5}$$

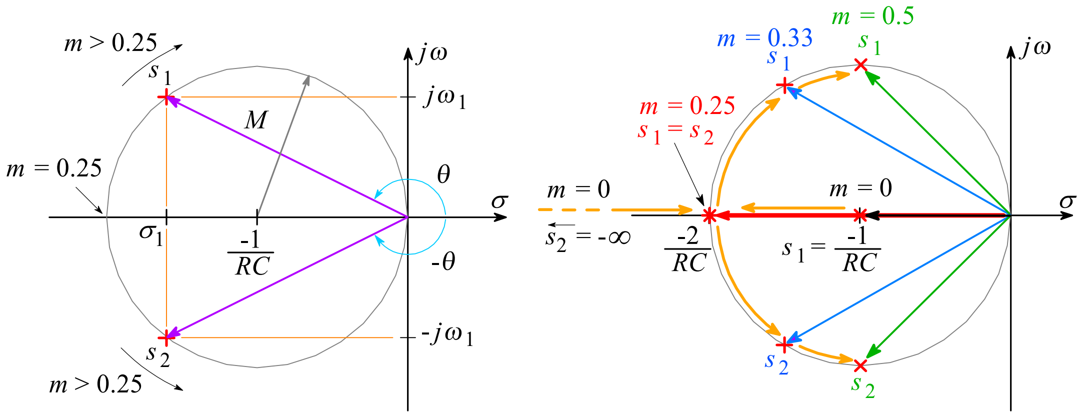

Fig. 2.2.2 : The poles s_1 and s_2 in the complex plane. By changing the parameter m , the poles travel first on the real axis, from $s_1 = -1/RC$ and $s_2 = -\infty$ $(m = 0)$, to $s_1 = s_2 = -2/RC$ $(m = 0.25)$ and then $(m > 0.25)$, as a complex-conjugate pair, on the circle, the radius of which is $r = 1/RC$ and its center at $\sigma = -r$. The figure on the right shows the four characteristic layouts, which are explained in detail in the text.

With these poles we may write Eq. 2.2.4 also in the following form :

$$F(s) = \frac{1}{m\,R\,C} \cdot \frac{1}{(s - s_1)(s - s_2)} \tag{2.2.6}$$

At DC $(s = 0)$, Eq. 2.2.6 shrinks to :

$$F(0) = \frac{1}{m\,R\,C} \cdot \frac{1}{s_1\,s_2} \tag{2.2.7}$$

By dividing Eq. 2.2.6 by Eq. 2.2.7, we get the **amplitude-normalized** transfer function :

$$F(s) = \frac{s_1\,s_2}{(s - s_1)\,(s - s_2)} \tag{2.2.8}$$

We will need this expression for the calculation of step response. By replacing both poles by their components from Eq. 2.2.5 , restricting s to the imaginary axis $j\,\omega$ and grouping the imaginary parts, we will get :

$$F(j\,\omega) = \frac{\sigma_1^2 + \omega_1^2}{[\,-\sigma_1 + j\,(\omega - \omega_1)]\,[\,-\sigma_1 + j\,(\omega + \omega_1)]} \tag{2.2.9}$$

We are often interested in the magnitude, $\left|F(\omega)\right|$, which we obtain by multiplying $F(j\,\omega)$ by its own complex-conjugate and then taking the root :

$$\left|F(\omega)\right| = \sqrt{F(j\,\omega) \cdot F^*(j\,\omega)} = \frac{\sigma_1^2 + \omega_1^2}{\sqrt{[\sigma_1^2 + (\omega - \omega_1)^2]\,[\sigma_1^2 + (\omega + \omega_1)^2]}} \tag{2.2.10}$$

The next step is the calculation of the parameter m. Its value depends on the type of poles we want to have, which in turn depend on the intended application of the amplifier. As a general rule, for sine-wave signal amplification we prefer the Butterworth poles while for pulse amplification we prefer the Bessel poles. If high bandwidth is not of primary importance, we can use a "critically-damped" system for a no-overshoot step. Other types of poles are optimized for use in filters, where our primary goal is to selectively amplify only a part of the spectrum. Poles are discussed in Part 4 (derived from some chosen optimization criteria) and Part 6 (computer algorithms).

2.2.1. Butterworth Poles for Maximally-Flat Amplitude Response (MFA)

We will calculate the actual values of the poles, as well as the parameter m, by using Eq. 2.2.5, where we factor out $1/2mRC$. If the square-root of Eq. 2.2.11 is imaginary, which is true for $m > 0.25$, we can also factor-out the imaginary unit :

$$s_{1,2} = \frac{1}{2mRC}\left(-1 \pm \sqrt{1-4m}\right) = \frac{1}{2mRC}\left(-1 \pm j\sqrt{4m-1}\right) \qquad (2.2.11)$$

We now compare this relation with normalized 2nd-order Butterworth pole values (the reader can find them in Part 4, Table 4.3.1, or by running the BUTTAP computer routine given in Part 6). The values obtained are $\sigma_{1t} = -0.7071$ and $\omega_{1t} = \pm 0.7071$.

Note : From now on, we will append the index "t" to the poles taken from the tables or calculated by a suitable computer program ; these values are normalized to a frequency of 1 *radian per second*.

Since both the real and imaginary axis of the Laplace plane have the dimension of frequency, the pole dimension is *radians per second* [rad/s] ; unfortunately, it has become almost a custom not to write the dimensions.

The sign is also seldom written ; instead, most authors leave to the reader to keep in mind that the poles of unconditionally stable systems always have the real part negative and the imaginary part is either zero or both positive and negative, forming a complex-conjugate pair.

To make it easier for the reader, we will always have the symbols σ and ω signed as required by the mathematical operation to be performed, while the numerical values within the symbols will always be negative for σ and positive for ω. For example, we will write a complex-conjugated pole-pair $(s_1, s_2) = (s_1, s_1^*)$ as :

$$s_1 = \sigma_1 + j\omega_1 = -0.7071 + j0.7071$$
$$s_2 = \sigma_2 + j\omega_2 = -0.7071 - j0.7071$$
$$\Rightarrow \quad s_2 = \sigma_1 - j\omega_1 = s_1^*$$

and a real pole will be given like :

$$s_3 = \sigma_3 = -1.000$$

Each σ_i and ω_i will bear the index of the pole s_i (and not their table order number). We will use the odd index for complex-conjugate pair components (with the apropriate $+/-$ sign for the imaginary part).

In order to have the same response, the poles of Eq. 2.2.11 must be proportional to those from the tables, so the imaginary-to-real part ratio must be the same :

$$\frac{\Im\{s_{1t}\}}{\Re\{s_{1t}\}} = \frac{\omega_{1t}}{\sigma_{1t}} \Rightarrow \frac{\Im\{s_1\}}{\Re\{s_1\}} = \frac{\omega_1}{\sigma_1} \Rightarrow \frac{\sqrt{4m-1}}{-1} = \frac{0.7071}{-0.7071} = -1 \tag{2.2.12}$$

and the same is true for s_2 (except the sign). From Eq. 2.2.12 follows that the value of m which satisfies our requirement for the Butterworth pole layout must be :

$$m = 0.5 \tag{2.2.13}$$

Thus, the inductance is :

$$L = m\,R^2C = 0.5\,R^2C \tag{2.2.14}$$

Finally, by inserting the value of m back into Eq. 2.2.11, the poles of our system are :

$$s_{1,2} = \sigma_1 \pm j\,\omega_1 = \frac{1}{RC}\left(-1 \pm j\right) \tag{2.2.15}$$

The value $1/RC = \omega_{\rm h}$ is equal to the upper half-power frequency of the non-peaking amplifier of Fig. 2.1.1 (at this frequency, since power is proportional to voltage squared, the voltage gain drops to $1/\sqrt{2} = 0.7071$). If we put $1/RC = 1$ (or $R = 1\,\Omega$ and $C = 1\,\mathrm{F}$, or $R = 500\,\mathrm{k}\Omega$ and $C = 2\,\mu\mathrm{F}$, or any other similar combination, provided that it can be driven by the signal source) , we get the normalized (denoted by index 'n') pole values :

$$s_{1\rm n,2n} = \sigma_{1\rm n} \pm j\,\omega_{1\rm n} = -1 \pm j \tag{2.2.16}$$

If we use normalized poles, we must also normalize the frequency : $j\,\omega/\omega_{\rm h}$ instead of $j\,\omega$.

Note : It is important not to confuse our system with normalized poles (Eq. 2.2.16) with the system having normalized Butterworth poles taken from the table ($s_{1\rm t}$, $s_{2\rm t} = -0.707 \pm j\,0.707$). Although both are Butterworth-type and both are normalized, they differ in bandwidth :

$$\sqrt{s_{1\rm t}\,s_{2\rm t}} = 1 \quad \text{while} \quad \sqrt{s_{1\rm n}\,s_{2\rm n}} = \sqrt{2} \tag{2.2.17}$$

This will become evident soon in Sec. 2.2.4, where we will calculate and plot the magnitude (absolute value) of the frequency-response.

2.2.2. Bessel Poles for Maximally-Flat Envelope-Delay (MFED) Response

In Part 4, Table 4.4.1 (or Part 6, by using the BESTAP routine), the poles for the 2nd-order system are $\sigma_{1\rm t} = -1.7544$ and $\omega_{1\rm t} = \pm 1.5000$. Then, as for the Butterworth case above, the imaginary-to-real component ratio is :

$$\frac{\Im\{s_1\}}{\Re\{s_1\}} = \frac{\sqrt{4m-1}}{-1} = \frac{\omega_{1\rm t}}{\sigma_{1\rm t}} = \frac{1.5000}{-1.7544} \tag{2.2.18}$$

Solving for m gives :

$$m = 1/3 \tag{2.2.19}$$

The inductance is :

$$L = 0.3\dot{3}\,R^2C \tag{2.2.20}$$

and the poles are :

$$s_{1,2} = \frac{1}{RC}\left(-1.5 \pm j\,0.866\right) \tag{2.2.21}$$

2.2.3. Critical Damping (CD)

In this case, both poles are real and equal, so the imaginary part in Eq. 2.2.11 (the square root) must be zero :

$$4m - 1 = 0 \qquad \Rightarrow \qquad m = 0.25 \tag{2.2.22}$$

with the inductance :

$$L = 0.25\, R^2 C \tag{2.2.23}$$

resulting in a double real pole :

$$s_{1,2} = -\frac{2}{RC} \tag{2.2.24}$$

In general, the parameter m may be calculated with the aid of Fig. 2.2.2, where both poles and the angle θ are shown. If the poles are expressed by Eq. 2.2.11 :

$$\tan\theta = \frac{\Im\{s_1\}}{\Re\{s_1\}} = \frac{\omega_1}{\sigma_1} = \frac{\sqrt{4m-1}}{-1} \tag{2.2.25}$$

and from this :

$$\boxed{m = \frac{1+\tan^2\theta}{4}} \tag{2.2.26}$$

which is also equal to $1/4\cos^2\theta$, as can be seen in some literature. We prefer Eq. 2.2.26 .

Now we have all the data needed for further calculations of the frequency-, phase-, time-delay-, and step-response.

2.2.4. Frequency-Response Magnitude

We have already written the magnitude in Eq. 2.2.10. Here we will use the normalized frequency $\omega/\omega_{\mathrm{h}}$:

$$\boxed{|F(\omega)| = \frac{\sigma_{1\mathrm{n}}^2 + \omega_{1\mathrm{n}}^2}{\sqrt{\left[\sigma_{1\mathrm{n}}^2 + (\omega/\omega_{\mathrm{h}} + \omega_{1\mathrm{n}})^2\right]\left[\sigma_{1\mathrm{n}}^2 + (\omega/\omega_{\mathrm{h}} - \omega_{1\mathrm{n}})^2\right]}}} \tag{2.2.27}$$

This is a normalized equation: in magnitude, as $|F(\omega)| = 1$ for $\omega = 0$ and in frequency to the upper half-power frequency of the non-peaking system, ω_{h} .

Inserting the pole-types of MFA, MFED and CD and the frequency in the range $0.1 < (\omega/\omega_{\mathrm{h}}) < 10$, we obtain the diagrams in Fig. 2.2.3 .

2.2.5. Upper Half-Power Frequency

An important amplifier parameter is its upper half-power frequency, which we will name ω_{H} for the peaking amplifier (in contrast to the ω_{h} of the non-peaking case). This is the frequency at which the output voltage V_{o} drops to $V_{\mathrm{oDC}}/\sqrt{2}$, where V_{oDC} is the output voltage at d.c. ($\omega = 0$), or, if normalized, to $1\,\mathrm{V}/\sqrt{2}$. Since the power is proportional to the voltage-squared, the normalized output power $P_\mathrm{o} = (1\,\mathrm{V})^2/2$, which is one half of the output power at DC. We can calculate the upper half-power frequency from Eq. 2.2.27, by inserting $\omega = \omega_\mathrm{H}$; the result must be $1/\sqrt{2}$:

$$\left|F(\omega_\mathrm{H})\right| = \frac{\sigma_1^2 + \omega_1^2}{\sqrt{\left[\sigma_1^2 + (\omega_\mathrm{H} + \omega_1)^2\right]\left[\sigma_1^2 + (\omega_\mathrm{H} - \omega_1)^2\right]}} = \frac{1}{\sqrt{2}} \tag{2.2.28}$$

We will use the term *upper half-power frequency* intentionally, rather than the term *upper -3 dB frequency*, which is commonly found in literature. While it has become a custom to express the amplifier gain in dB, the dB scale (the log of the output-to-input power ratio) implies that the driving circuit, which supplies the current I_i to the input, has the same internal resistance as the loading resistor R. This is not the case in most of the circuits which we will discuss.

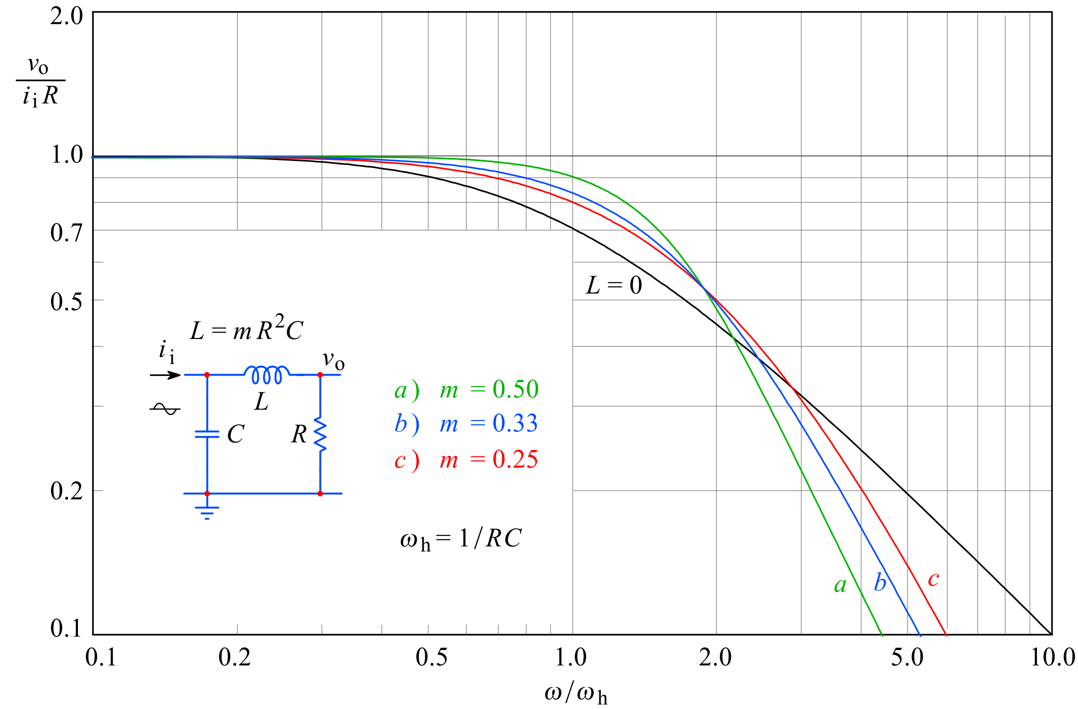

Fig. 2.2.3 : Frequency-response magnitude of the two-pole series-peaking circuit for some characteristic values of m: ***a)*** $m = 0.5$ is the maximally-flat amplitude (MFA) response; ***b)*** $m = 0.33$ is the maximally-flat envelope-delay (MFED) response; ***c)*** $m = 0.25$ is the critical damping (CD) case; the non-peaking case ($m = 0 \Rightarrow L = 0$) is the reference. Note the bandwidth improvement of all peaking responses, compared to the non-peaking bandwidth ω_h at $v_\mathrm{o} = 0.7071$.

For a series-peaking circuit the calculation of ω_H is relatively easy. The calculation becomes progressively more difficult for more sophisticated networks, where more poles and sometimes even zeros are introduced. In such cases it is better to use a computer and in Part 6 we have presented the development of routines which the reader can use to calculate the various response functions.

If we solve Eq. 2.2.28 for $\omega_\mathrm{H}/\omega_\mathrm{h}$, we get [Ref. 2.2, 2.4] :

$$\boxed{\eta_\mathrm{b} = \frac{\omega_\mathrm{H}}{\omega_\mathrm{h}}} \tag{2.2.29}$$

The value η_b is the cut-off frequency improvement factor, defined as the ratio of the system upper half-power frequency against that of the non-peaking amplifier (and, since the lower half-power frequency of a wideband amplifier is generally very low, usually down to d.c., we may call η_b also the *bandwidth improvement factor*). In Table 2.2.1 at the end of this section the bandwidth improvement factors and other data for different values of the parameter m are given.

2.2.6. Phase-Response

We can calculate the phase-angle φ of the output voltage V_o referred to the input current I_i by finding the phase-shift of each pole $s_k = \sigma_k \pm j\omega_k$ and then sum them :

$$\varphi_k(\omega) = \arctan \frac{\omega \mp \omega_k}{\sigma_k} \tag{2.2.30}$$

In Eq. 2.2.30 we have the ratio of the imaginary part to the real part of the pole, so the pole value may be either exact or normalized. For normalized values we must also normalize the frequency variable as ω/ω_h. Our frequency-response function (Eq. 2.2.8) has two coplex-conjugated poles. Therefore, the phase-response is :

$$\boxed{\varphi(\omega) = \arctan \frac{\omega/\omega_h - \omega_{1n}}{\sigma_{1n}} + \arctan \frac{\omega/\omega_h + \omega_{1n}}{\sigma_{1n}}} \tag{2.2.31}$$

In Fig. 2.2.4 the phase plots, corresponding to the same values of m as in Fig. 2.2.3 , are shown :

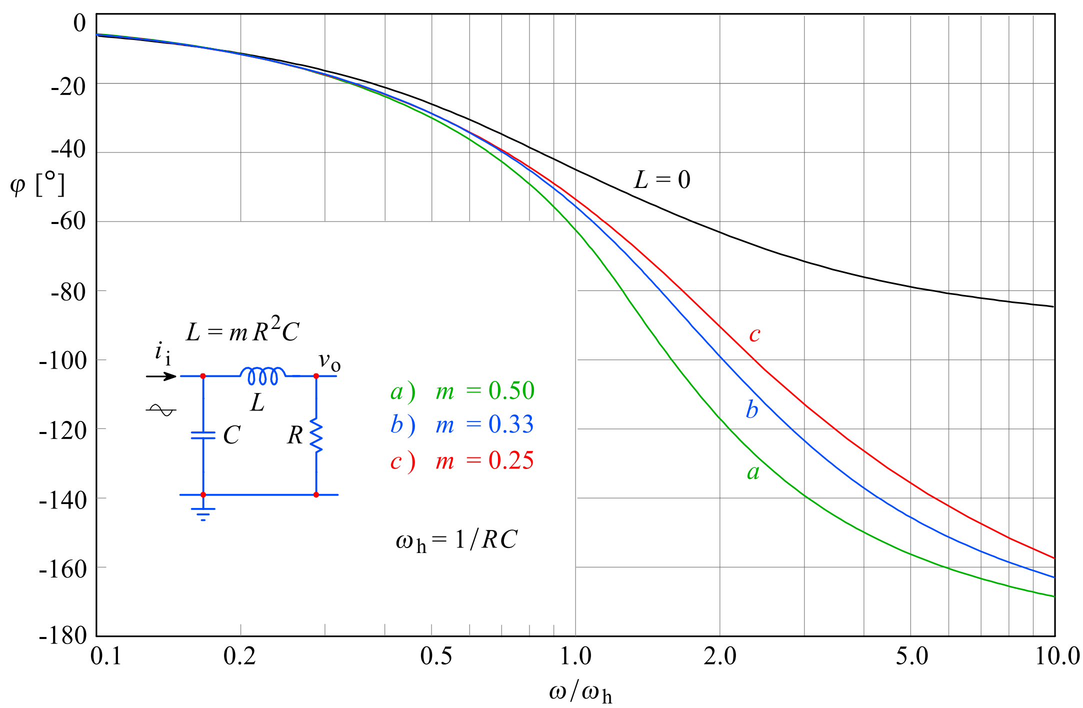

Fig. 2.2.4 : Phase response of the series peaking circuit for ***a*)** MFA, ***b*)** MFED and ***c*)** CD case, compared with the non-peaking response ($L = 0$). The phase angle scale was converted from radians to degrees by multiplying by $180/\pi$. The non-peaking (single-pole) response, has the asymptote at 90° for $\omega \to \infty$, while the second-order peaking systems have the asymptote at 180°.

2.2.7. Phase- and Envelope-Delay

For each pole, the *phase-delay* (or the *phase-advance* for any zero) is :

$$\tau_\varphi = \frac{\varphi}{\omega} \tag{2.2.32}$$

If ω is the positive angular frequency by which the input signal phasor rotates, then the angle φ, by which the output signal phasor lags the input, is defined in the direction opposite to ω, meaning that, for a phase-delay, φ will be negative, as in Fig. 2.2.4; consequently, τ_φ will also be negative. Note that τ_φ has the dimension of time.

Now, τ_φ is obviously frequency-dependent, so, in order to evaluate the time-domain performance of a wideband amplifier on a fair basis, we are much more interested in the "specific" phase-delay, known as the *envelope-delay* (also *group-delay*) and it is a frequency-derivative of the phase-angle as function of frequency :

$$\tau_e = \frac{d\varphi}{d\omega} \tag{2.2.33}$$

Here, too, a negative result means a delay and a positive result an advance against the input signal.

In Fig. 2.2.5 a tentative explanation of the difference between the phase-delay and the envelope-delay is displayed both in time-domain and as a phasor diagram.

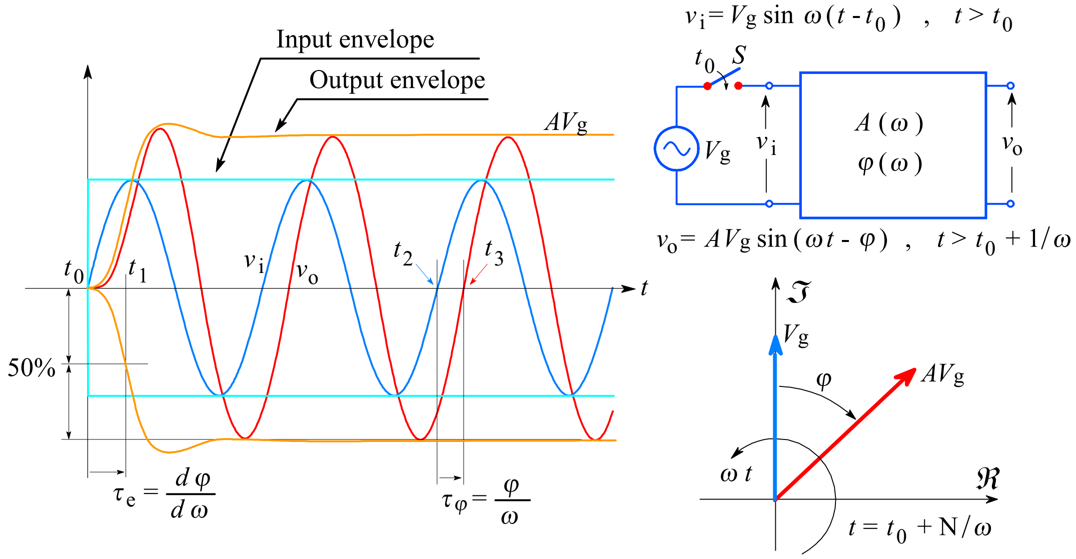

Fig. 2.2.5 : Phase- and envelope-delay definition. The switch S is closed at time t_0, to apply a sinusoidal voltage with amplitude V_g to the input of the amplifier having a frequency-dependent amplitude-response $A(\omega)$ and associated phase-response $\varphi(\omega)$. The input signal envelope is a unit step. The output envelope lags the input by $\tau_e = d\varphi/d\omega$, measured from t_0 to t_1, where t_1 is the time at which the output envelope reaches 50% of its final value. A number of periods later (N/ω), the phase-delay can be measured as the time between the input and output zero crossing, indicated by t_2 and t_3 and is expressed as $\tau_\varphi = \varphi/\omega$. The corresponding phasor diagram is shown along.

In the phase-advance case, when zeros dominate over poles, the name suggests that the output voltage will change before input, which is impossible, of course. To see what actually happens, we apply a sinewave to two simple RC networks, low-pass and high-pass (with a zero at $s = 0$), as shown in Fig. 2.2.6. Compare the phase-advance case, $V_{\rm ohp}$, with the phase-delay case, $V_{\rm olp}$. The input signal frequency equals the network cut-off, $1/RC$.

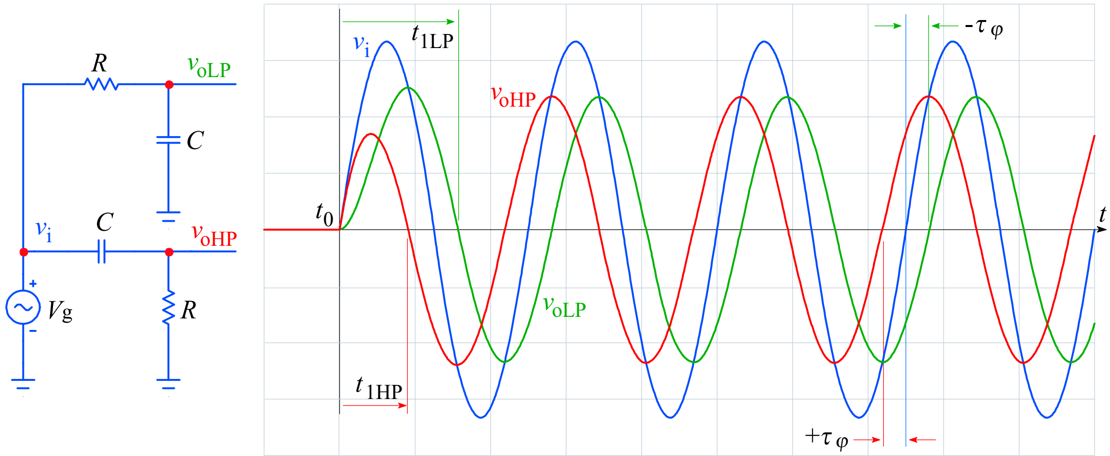

Fig. 2.2.6 : Phase delay and phase advance. It is evident that both output signals undergo a phase modulation during the first half period. The time from t_0 and the first zero-crossing of the output at t_1 is shortened for V_{ohp} and extended for V_{olp} . However, both envelopes lag the input envelope. On the other hand, the phase, measured after a number of periods, exhibits an advance of $+\tau_\varphi$ for the high-pass network and a delay of $-\tau_\varphi$ for the low-pass network.

Returning to the envelope-delay for the series peaking circuit, in accordance with Eq. 2.2.33 , we must differentiate Eq. 2.2.30 . For each pole we have :

$$\frac{d\varphi}{d\omega} = \frac{d}{d\omega}\left[\arctan\frac{\omega \mp \omega_{\rm i}}{\sigma_{\rm i}}\right] = \frac{\sigma_{\rm i}}{\sigma_{\rm i}^2 + (\omega \mp \omega_{\rm i})^2} \tag{2.2.34}$$

and, as for the phase-delay, the total envelope-delay is the sum of the contribution of each pole (and zero, if any). Again, if we use normalized poles and the normalized frequency, we get the normalized envelope-delay, $\tau_{\rm e}\,\omega_{\rm h}$, resulting in a unit-delay at d.c.

For the 2-pole case we have :

$$\boxed{\tau_{\rm e}\,\omega_{\rm h} = \frac{\sigma_{\rm 1n}}{\sigma_{\rm 1n}^2 + (\omega/\omega_{\rm h} - \omega_{\rm 1n})^2} + \frac{\sigma_{\rm 1n}}{\sigma_{\rm 1n}^2 + (\omega/\omega_{\rm h} + \omega_{\rm 1n})^2}} \tag{2.2.35}$$

The plots for the same values of m as before, after Eq. 2.2.35 , are in Fig. 2.2.7 .

For pulse amplification, the importance of achieving a flat envelope-delay cannot be overstated. A flat delay means that all the important frequencies will reach the output with unaltered phase, preserving the shape of the input signal as much as possible for the given bandwidth, thus resulting in minimal overshoot of the step-response (see the next section). Also, a flat delay means that, since it is a phase derivative, the phase must be a linear function of frequency up to the cut-off. This is why Bessel systems are often being refered to as "linear-phase" systems. This property can not be seen in the log-scale used here, but, if plotted against a linearly scaled frequency, it would be. We leave to the curious reader to try it by himself.

In contrast, the Butterworth system shows a pronounced delay near the cut-off frequency. Conceivable, this will expose the system resonance upon the step excitation.

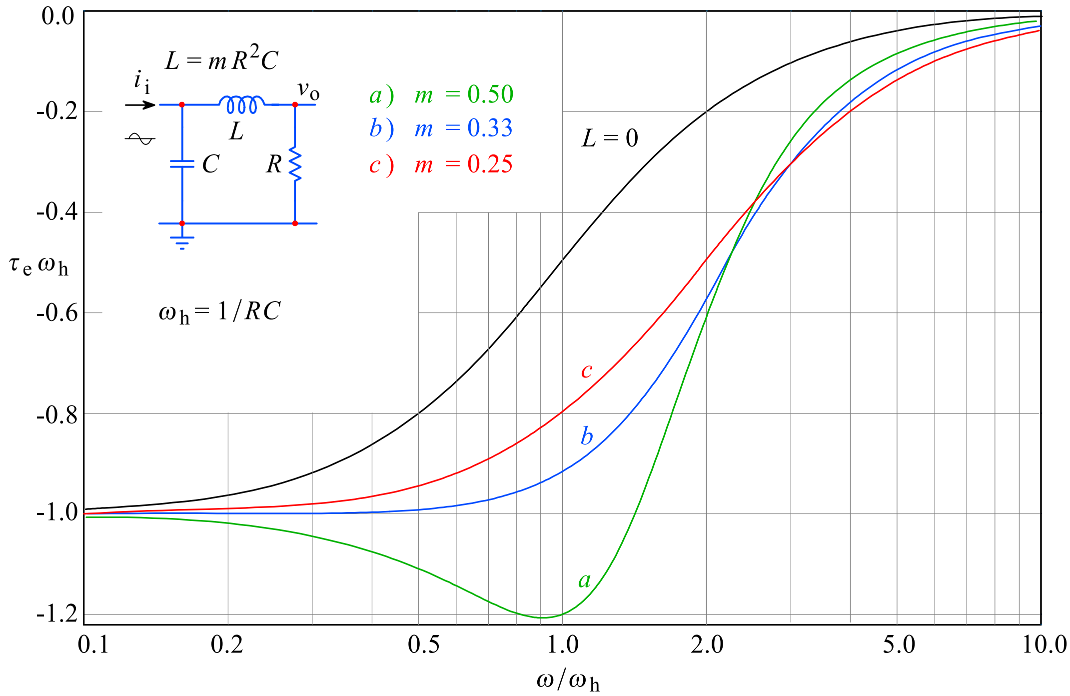

Fig. 2.2.7 : Envelope-delay of the series peaking circuit for the same characteristic values of m as before : ***a)*** MFA, ***b)*** MFED, ***c)*** CD. Note the maximally-flat envelope-delay plot b being flat up to nearly $0.5\,\omega_h$.

2.2.8. Step-Response

We have already derived the formula for the step-response in Part 1, Eq. 1.13.29 :

$$g(t) = 1 + \frac{1}{|\sin\theta|}\,\mathrm{e}^{\sigma_1 t}\sin(\omega_1 t + \theta) \tag{2.2.36}$$

where θ is the pole angle in radians : $\theta = \arctan(-\omega_1/\sigma_1) + \pi$ (read the following Note!).

Note : We are often forced to calculate some of the circuit parameters from the trigonometric relations of the real and imaginary components of the pole. The Cartesian coordinates of the pole s_1 in Laplace plain are σ_1 on the real axis and ω_1 on the imaginary axis. In polar coordinates, the pole is expressed by its modulus (the distance of the pole from the complex-plane origin) :

$$M = \sqrt{(\sigma_1 + j\omega_1)(\sigma_1 - j\omega_1)} = \sqrt{\sigma_1^2 + \omega_1^2}$$

and its argument (angle) θ , defined so that : $\tan\theta = \dfrac{\omega_1}{\sigma_1}$

Now, a mathematically correct definition of the positive-valued angle is counter-clockwise from the positive real axis ; so, if σ_1 is negative, θ will be greater than $\pi/2$. However, the *tangent* function is defined within the range of $\mp\pi/2$ and then repeats for values between $\pi \pm k\pi/2$. Therefore, by taking the *Arctangent*, $\theta = \arctan\dfrac{\omega_1}{\sigma_1}$, we loose the information in which half of the complex plane the pole actually lies and consequently a sign can be wrong. This is bad, because the left (negative) side of the real axis is associated with energy-dissipative, that is, resistive circuit action, while the right (positive) side is associated with energy-generative action. This is why unconditionally stable circuits have the poles always in the left-half of the complex plane.

To keep our analytical expressions simple, we will keep tracking the pole layout and correct the sign and value of the arctan( ) by adding π radians to the angle θ wherever necessary. But, in order to avoid any confusion, our computer algorithm should use a different form of equation (see Part 6).

Please, see Appendix 2.3 for more details.

To use the normalized values of poles in Eq. 2.2.36 we must also enter the normalized time, t/T, where T is the system time-constant, $T = RC$. Thus we obtain :

a) for Butterworth poles (MFA) :

$$g_a(t) = 1 + \sqrt{2}\,\mathrm{e}^{-t/T}\sin\left(t/T + 0.785 + \pi\right) \tag{2.2.37}$$

b) for Bessel poles (MFED) :

$$g_b(t) = 1 + 2\,\mathrm{e}^{-1.5\,t/T}\sin\left(0.866\,t/T + 0.5236 + \pi\right) \tag{2.2.38}$$

c) for Critical damping (CD) :

Eq. 2.2.36 was derived for simple poles, so it is not valid for a critical damping (CD), because here we have a double pole at s_1. To calculate the step-response for the function with a double pole, we start with Eq. 2.2.8, insert the same (real!) value $(s_1 = s_2)$ and multiply it with the unit-step operator $1/s$. The resulting equation :

$$\boxed{G(s) = \frac{s_1^2}{s\,(s - s_1)^2}} \tag{2.2.39}$$

has the time-domain function :

$$g(t) = \mathcal{L}^{-1}\{G(s)\} = \sum res\,\frac{s_1^2\,\mathrm{e}^{st}}{s\,(s - s_1)^2} \tag{2.2.40}$$

There are two residues, res_0 and res_1 :

$$res_0 = \lim_{s \to 0} s\left[\frac{s_1^2\,\mathrm{e}^{st}}{s\,(s - s_1)^2}\right] = 1$$

For res_1 we must use Eq. 1.10.12 in Part 1 :

$$res_1 = \lim_{s \to s_1} \frac{d}{ds}\left[(s - s_1)^2\,\frac{s_1^2\,\mathrm{e}^{st}}{s\,(s - s_1)^2}\right] = \mathrm{e}^{s_1 t}\,(s_1 t - 1)$$

and their sum is :

$$\boxed{g(t) = 1 + \mathrm{e}^{\sigma_1 t}\,(\sigma_1 t - 1)} \tag{2.2.41}$$

Eq. 2.2.39 has a double **real** pole $s_1 = \sigma_1 = -2/RC$ or, normalized, $\sigma_{1n} = -2$. We insert this in the Eq. 2.2.41 to get the CD step-response plot (curve c, $m = 0.25$) :

$$g_c(t) = 1 - e^{-2t/T}\,(1 + 2t/T) \tag{2.2.42}$$

The step-response plots of all three cases are shown in Fig. 2.2.8. Also shown is the non-peaking response as the reference. The MFA overshoot is $\delta = 4.3\,\%$, while for MFED it is 10 times smaller !

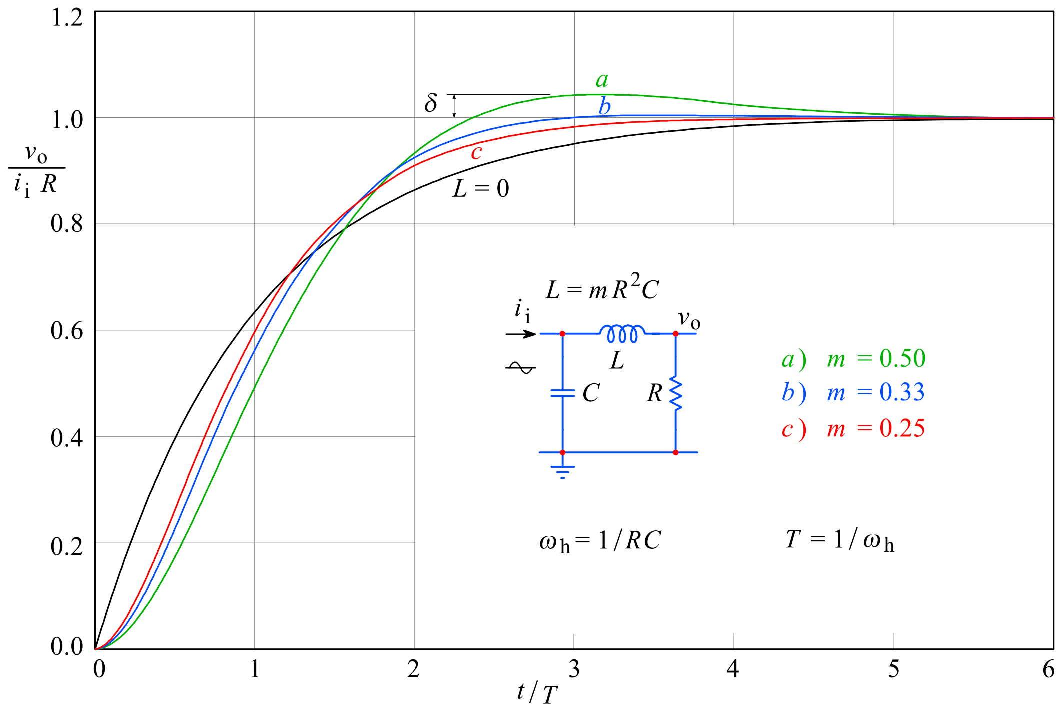

Fig. 2.2.8 : Step-response of the series peaking circuit for the four characteristic values of m : ***a)*** MFA, ***b)*** MFED, ***c)*** CD. The MFA overshoot is $\delta = 4.3$ %, while for MFED it is only $\delta = 0.43\,\%$.

2.2.9. Rise-time

The most important parameter, by which the time-domain performance of a wideband amplifier is evaluated, is the rise-time. As we have already seen in Fig. 2.1.1, this is the difference of instants at which the step-response crosses the 90 % and 10 % levels of the final value. For the non-peaking amplifier, we have labeled this time as τ_r and we have calculated it already by Eq. 2.1.4, obtaining the value $2.20\,RC$. The peaking amplifier rise-time is labeled τ_R.

The calculation for τ_R goes basically by Eq. 2.1.4. For more complex circuits, the step-response function can be rather complicated, consequently the analytical calculation becomes difficult and in such cases it is better to use a computer (see Part 6).The rise-time improvement against a non-peaking amplifier is :

$$\boxed{\eta_r = \frac{\tau_r}{\tau_R}} \tag{2.2.43}$$

The values for the bandwidth improvement η_b and for the rise-time improvement η_r are similar but in general **they are not equal**. In practice, we more often use η_b, the calculation of which is easier. If the step-response overshoot is not too large, we can **approximate** the rise-time from the formula :

$$\omega_{\mathrm{h}} = 2\pi f_{\mathrm{h}} = \frac{1}{RC} \qquad \text{and further} \qquad f_{\mathrm{h}} = \frac{1}{2\pi RC}$$

where ω_{h} is the upper half-power frequency in *rad/s*, while f_{h} is the upper half-power frequency in Hz. We have already calculated the non-peaking rise-time τ_{r} by Eq. 2.2.4 and found it to be $2.20\,RC$. From this we obtain $\tau_{\mathrm{r}} f_{\mathrm{h}} = 2.20/2\pi = 0.35$, and this relation we meet very frequently in practice :

$$\boxed{\tau_{\mathrm{r}} = \frac{0.35}{f_{\mathrm{h}}}} \tag{2.2.44}$$

By replacing f_{h} with f_{H} in this equation, we get (an estimate of) the rise-time of the **peaking** amplifier. But note that by doing so, we miss the fact that Eq. 2.2.44 is exact only for the single-pole amplifier, where the load is the parallel RC connection. For all other cases, **it can be used as an approximation if the overshoot does not exceed some 2 %**. The overshoot of a Butterworth two-pole network amounts to 4.3 % and is getting larger with increasing the number of poles, thus calculating the rise-time by Eq. 2.2.43 will be in error. Even greater error will result for networks with Chebyshev and Cauer (elliptic) system poles. In such cases we must compute the actual system step-response and find the rise-time from it. For Bessel poles, the error is tolerable; never-the-less, using a computer to calculate the rise-time from the step-response will give greater precision.

2.2.10. Input Impedance

We will use the series-peaking network also as an addition to T-coil peaking. This is possible, since the T-coil network has a constant input impedance (the T-coil is discussed in Sec. 2.4, 2.5 and 2.6). Therefore, it is useful to know the input impedance of the series-peaking network. From Fig. 2.2.1 it is evident that the input impedance is a capacitor C in parallel with the serially connected L and R :

$$Z_{\mathrm{i}} = \frac{1}{j\omega C + \dfrac{1}{j\omega L + R}} = \frac{j\omega L + R}{1 - \omega^2 LC + j\omega RC} \tag{2.2.45}$$

It would be inconvenient to continue with this expression. To simplify, we substitute: $L = mR^2C$ and $\omega_{\mathrm{h}} = 1/RC$, obtaining :

$$Z_{\mathrm{i}} = R\,\frac{1 + m\,j\omega/\omega_{\mathrm{h}}}{1 - m\,(\omega/\omega_{\mathrm{h}})^2 + j\omega/\omega_{\mathrm{h}}} \tag{2.2.46}$$

By making the denominator real and with some further rearrangement we get :

$$Z_{\mathrm{i}} = R\,\frac{1 + j\,\omega/\omega_{\mathrm{h}}[(m-1) - m^2(\omega/\omega_{\mathrm{h}})^2]}{1 + (1-2m)(\omega/\omega_{\mathrm{h}})^2 + m^2(\omega/\omega_{\mathrm{h}})^4} \tag{2.2.47}$$

and the phase-angle is :

$$\varphi = \arctan\frac{\Im\{Z_{\mathrm{i}}\}}{\Re\{Z_{\mathrm{i}}\}} = \arctan\left\{\omega/\omega_{\mathrm{h}}\left[(m-1) - m^2(\omega/\omega_{\mathrm{h}})^2\right]\right\} \tag{2.2.48}$$

The normalized impedance modulus is :

$$\frac{|Z_\mathrm{i}|}{R} = \sqrt{\Re\left\{\frac{Z_\mathrm{i}}{R}\right\}^2 + \Im\left\{\frac{Z_\mathrm{i}}{R}\right\}^2} = \frac{\sqrt{1+(\omega/\omega_\mathrm{h})^2[(m-1)-m^2(\omega/\omega_\mathrm{h})^2]^2}}{1+(1-2m)(\omega/\omega_\mathrm{h})^2+m^2(\omega/\omega_\mathrm{h})^4} \tag{2.2.49}$$

In Fig. 2.2.9 the plots of Eq. 2.2.49 and Eq. 2.2.48 for the same values of m as before are shown :

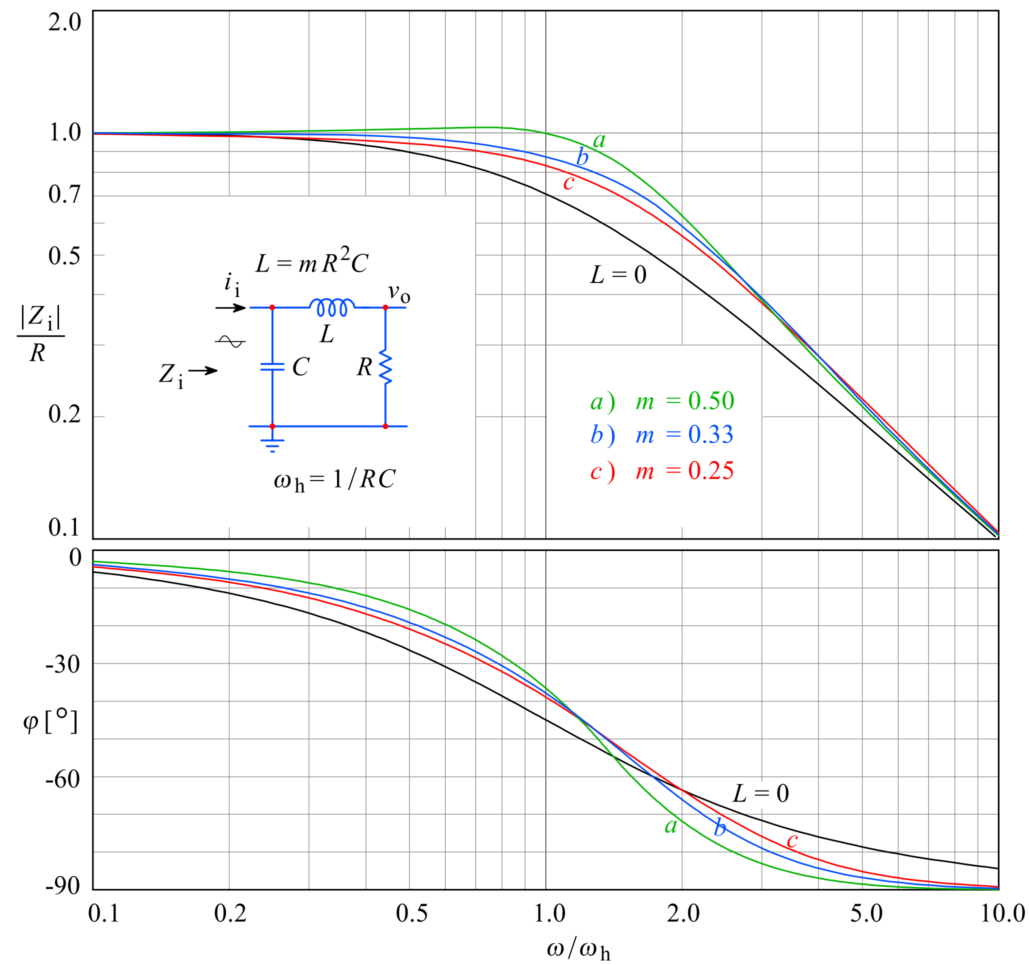

Fig. 2.2.9 : Input impedance modulus (normalized) and the associated phase angle of the series-peaking circuit, for the characteristic values of m . Note that for high frequencies the input impedance approaches that of the capacitance. ***a)*** MFA, ***b)*** MFED, ***c)*** CD.

We will use the equations derived in this section also in the following section, where we will omit the derivations. Table 2.2.1 shows all important design parameters of the two-pole series-peaking circuit :

Table 2.2.1

response	m	η_b	η_r	δ [%]
MFA	0.50	1.41	1.49	4.30
MFED	0.33	1.36	1.39	0.43
CD	0.25	1.29	1.33	0.00

Table 2.2.1 : 2^nd-order series-peaking circuit parameters summarized : m is the inductance-proportionality factor, η_b is the bandwidth improvement, η_r is the rise-time improvement and δ is the step-response overshoot.

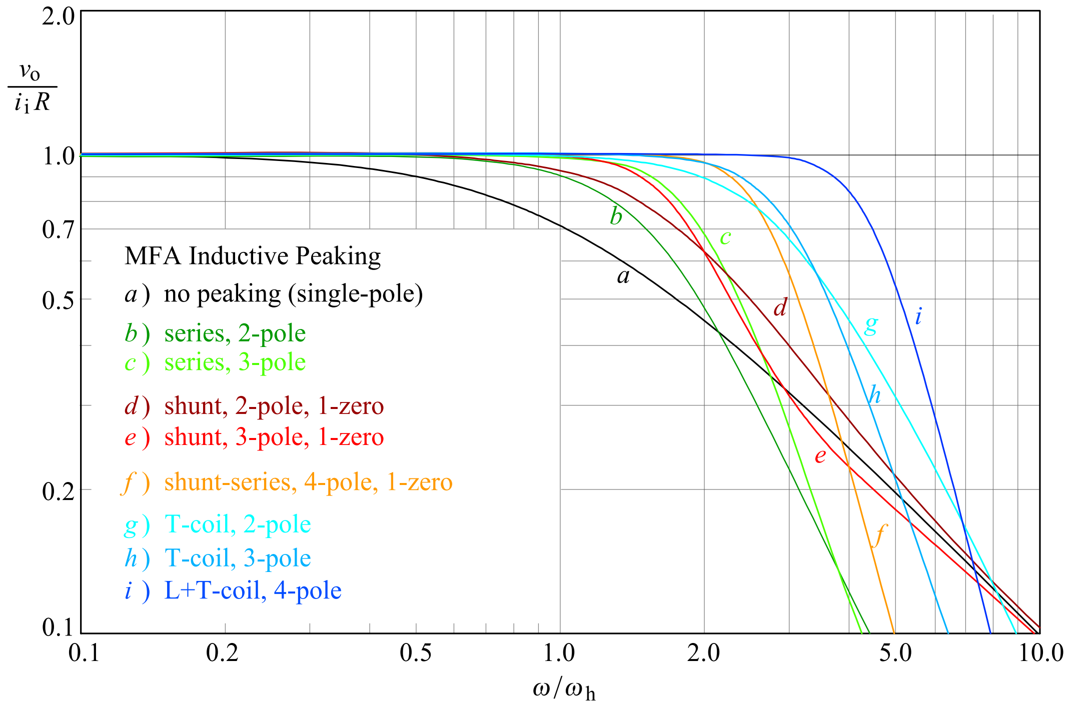

Fig. 2.10.1 : MFA frequency-responses of all the circuit configurations discussed. By far, the 4-pole T-coil response ***i)*** has the largest bandwidth.

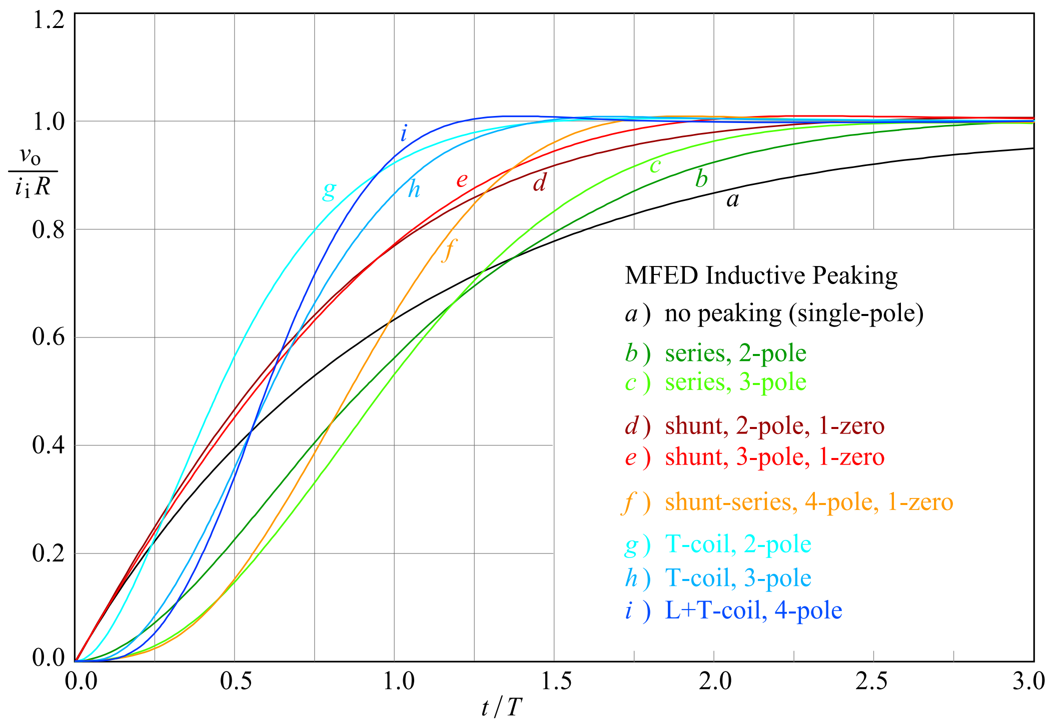

Fig. 2.10.2 : MFED step-responses of all the circuit configurations discussed. Again, the 4-pole T-coil step response ***i)*** has the steepest slope, but the 3-pole T-coil response ***h)*** is close.