3.2 Transistor as an Impedance Converter

In the previous section we have realized that the amplification factor is frequency dependent, decreasing with frequency above some upper frequency limit (asymptotically to -20 dB/10f, just like a first-order low-pass system). This can help us to derive different impedance transformations from the emitter to the base circuit and back [Ref. 3.1, 3.2]. Knowing the possible transformations is extremely useful in the wideband amplifier design. We will show how the nature of the impedance changes with these transformations. A capacitive impedance may become inductive and positive impedances may occasionally become negative !

3.2.1 Common-base small-signal transistor model

As we explained in Sec. 3.1, if the voltage gain is not too high, the base-emitter capacitance C_π is the dominant cause for the frequency response roll-off at high frequencies. By considering this, we can make a simplified small-signal high-frequency transistor model as shown in Fig. 3.2.1 for the common-base configuration, where i_c ,i_e and i_b are the collector-, emitter- and base-current respectively. For this figure the DC current amplification factor is :

$$\alpha_0 = \frac{I_c}{I_e} \tag{3.2.1}$$

A more correct expression for the mutual conductance is :

$$g_m = \frac{\alpha_0}{r_e} = \frac{\beta_0}{(1+\beta_0)\, r_e} \tag{3.2.2}$$

where β_0 is the common-emitter DC-current amplification factor. If $\beta_0 \gg 1$, then $\alpha_0 \simeq 1$, so the collector current I_c is almost equal to the emitter current I_e and $g_m \simeq 1/r_e$. This simplification is often used in practice.

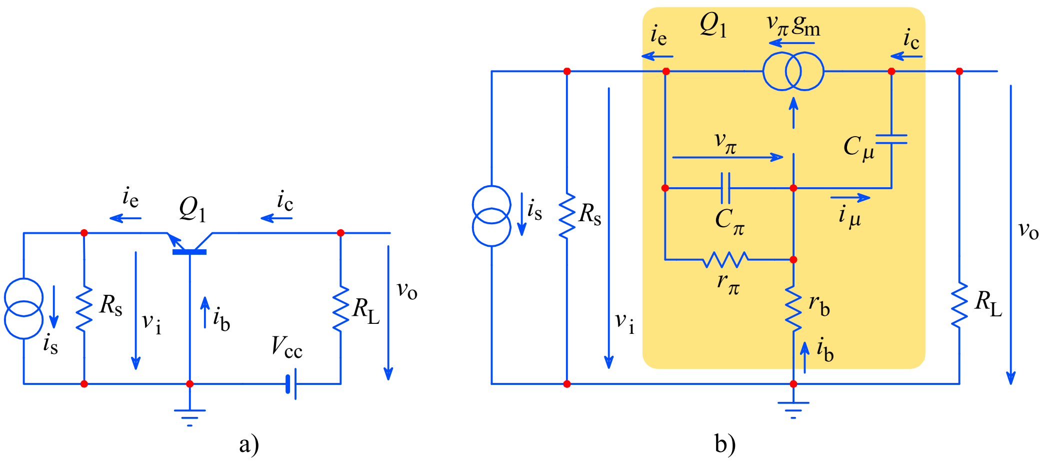


Fig. 3.2.1 : The common-base amplifier : a) circuit schematic diagram ; b) high-frequency small-signal equivalent circuit.

For the moment, let us assume that the base resistance $r_\mathrm{b} = 0$ and consider the low-frequency relations. The input resistance is :

$$r_\pi = \frac{v_\pi}{i_\mathrm{b}} \tag{3.2.3}$$

where v_π is the base-to-emitter voltage. Since the emitter current is :

$$i_\mathrm{e} = i_\mathrm{b} + i_\mathrm{c} = i_\mathrm{b} + \beta_0\, i_\mathrm{b} = i_\mathrm{b}\,(1+\beta_0) \tag{3.2.4}$$

then the base current is :

$$i_\mathrm{b} = \frac{i_\mathrm{e}}{1+\beta_0} \tag{3.2.5}$$

and :

$$r_\pi = \frac{v_\pi\,(1+\beta_0)}{i_\mathrm{e}} = r_\mathrm{e}\,(1+\beta_0) \approx \beta_0\, r_\mathrm{e} \tag{3.2.6}$$

The last simplification is valid if $\beta_0 \gg 1$. To get the input impedance at high frequencies, the parallel connection of C_π must be taken into account :

$$Z_\mathrm{b} = \frac{(1+\beta_0)\, r_\mathrm{e}}{1+(1+\beta_0)\, s\, C_\pi\, r_\mathrm{e}} \tag{3.2.7}$$

The base current is :

$$i_\mathrm{b} = \frac{v_\pi}{Z_\mathrm{b}} = v_\pi\, \frac{1+(1+\beta_0)\, s\, C_\pi\, r_\mathrm{e}}{(1+\beta_0)\, r_\mathrm{e}} \tag{3.2.8}$$

Further it is :

$$v_\pi = i_\mathrm{b}\, \frac{(1+\beta_0)\, r_\mathrm{e}}{1+(1+\beta_0)\, s\, C_\pi\, r_\mathrm{e}} \tag{3.2.9}$$

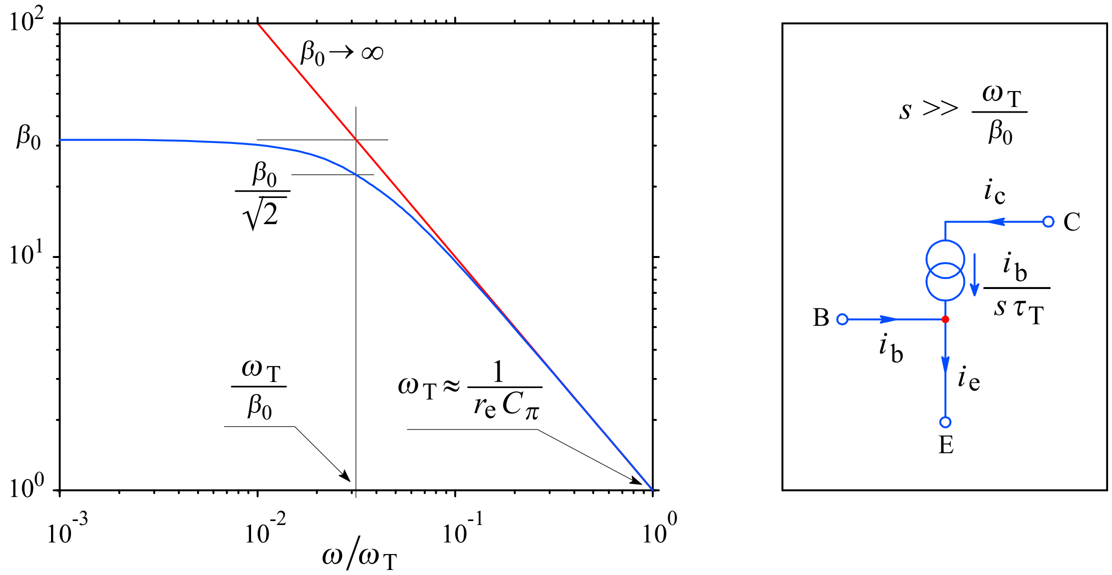
The collector current is :

$$i_\mathrm{c} = g_\mathrm{m}\, v_\pi = \frac{\beta_0}{1+\beta_0}\cdot\frac{1}{r_\mathrm{e}}\, v_\pi = \frac{\alpha_0}{r_\mathrm{e}}\, v_\pi \tag{3.2.10}$$

If we put Eq. 3.2.9 into Eq. 3.2.10, we obtain :

$$\begin{aligned} i_\mathrm{c} &= i_\mathrm{b}\, \frac{\beta_0}{1+\beta_0}\cdot\frac{1}{r_\mathrm{e}}\cdot\frac{(1+\beta_0)\, r_\mathrm{e}}{1+s\,(1+\beta_0)\, r_\mathrm{e}\, C_\pi} \\ &= i_\mathrm{b}\, \frac{1}{\dfrac{1}{\beta_0} + s\left(\dfrac{\beta_0+1}{\beta_0}\right) r_\mathrm{e}\, C_\pi} \\ &\approx i_\mathrm{b}\, \frac{1}{\dfrac{1}{\beta_0} + s\,\tau_\mathrm{T}} = i_\mathrm{b}\, \beta(s) \end{aligned} \tag{3.2.11}$$

In the very last expression we assumed that $\beta_0 \gg 1$ and $\tau_\mathrm{T} = r_\mathrm{e}\, C_\pi = 1/\omega_\mathrm{T}$, where $\omega_\mathrm{T} = 2\pi f_\mathrm{T}$ is the angular frequency, at which the current amplification factor β decreases to unity. The parameter τ_T (and consequently ω_T) depends on the internal configuration and structure of the transistor. Fig. 3.2.2 shows the frequency dependence of β and the equivalent current generator.



a) b)

Fig. 3.2.2 : a) The transistor gain as a function of frequency, modeled by the Eq. 3.2.11 ; b) the equivalent HF current generator.

In order to correlate Fig. 3.2.2 with Eq. 3.2.11, we rewrite it as :

$$\frac{i_c}{i_b} \approx \beta_0 \frac{-\dfrac{\omega_T}{\beta_0}}{s - \left(-\dfrac{\omega_T}{\beta_0}\right)} = \beta_0 \frac{-s_1}{s - s_1} \tag{3.2.12}$$

where s_1 is the pole at $-\omega_T/\beta_0$. This relation will become useful later, when we will apply one of the peaking circuits (from Part 2) to the amplifier. At very high frequencies, or if $\beta_0 \gg 1$, the term $s\,\tau_T$ prevails. In this case, from Eq. 3.2.11 :

$$\frac{i_c}{i_b} = \beta(s) \approx \frac{1}{s\,\tau_T} = \frac{1}{j\,\omega\, r_e\, C_\pi} \tag{3.2.13}$$

Obviously, β is decreasing with frequency. By definition, at $\omega = \omega_T$, the current ratio $i_c/i_b = 1$; then the capacitance C_π can be found as :

$$C_\pi \approx \frac{1}{\omega_T\, r_e} \tag{3.2.14}$$

This simplified relation represents the -20 dB$/(10\,f/f)$ asymptote in Fig. 3.2.2a.

3.2.2 The conversion of impedances

We can use the result of Eq. 3.2.11 to transform the transistor internal (and the added external) impedances from the emitter to the base circuitry and vice versa. Suppose, we have the impedance Z_e in the emitter circuit as displayed in Fig. 3.2.3a. We are interested what is the corresponding base impedance Z_b .

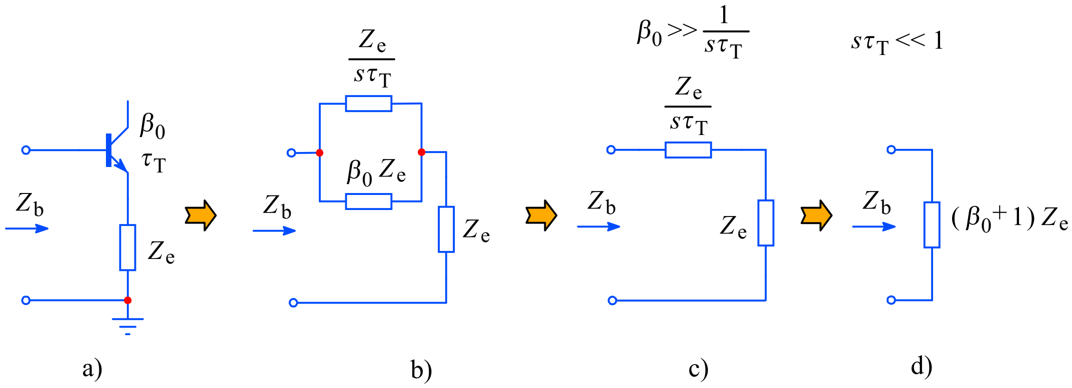


Fig. 3.2.3 : Emitter to base impedance conversion : a) schematic ; b) equivalent circuit; c) simplified for high β_0 ; d) simplified for low frequencies.

We know that :

$$Z_b = \beta(s)\, Z_e + Z_e = \left[\beta(s) + 1\right] Z_e \tag{3.2.15}$$

If we insert $\beta(s)$ according to Eq. 3.2.11, we obtain :

$$Z_b = \frac{Z_e}{\dfrac{1}{\beta_0} + s\,\tau_T} + Z_e \tag{3.2.16}$$

The admittance of the first part of this equation is :

$$Y = \frac{\dfrac{1}{\beta_0} + s\,\tau_T}{Z_e} = \frac{1}{\beta_0\, Z_e} + \frac{s\,\tau_T}{Z_e} \tag{3.2.17}$$

and this represents a parallel connection of impedances $\beta_0\, Z_e$ and $Z_e / s\,\tau_T$. By adding the series impedance Z_e , as in Eq. 3.2.16, we get the equivalent circuit of Fig. 3.2.3b. At medium frequencies and with a high value of β_0 , we can assume that $1/\beta_0 \ll s\,\tau_T$, so we can delete the impedance $\beta_0\, Z_e$ and simplify the circuit, as in Fig. 3.2.3c. On the other hand, at low frequencies, where $s\,\tau_T \ll 1$, we can neglect the $Z_e / s\,\tau_T$ component and get a very basic equivalent circuit, displayed in Fig. 3.2.3d.

Eq. 3.2.11 is equally useful when we want to transform the impedance from the base into the emitter circuit as shown in Fig. 3.2.4a. In this case we have :

$$Z_e = \frac{Z_b}{\beta(s) + 1} \tag{3.2.18}$$

Again we calculate the admittance, which is :

$$Y_e = \frac{\beta(s)+1}{Z_b} = [\beta(s)+1]\,Y_b = \beta(s)\,Y_b + Y_b \tag{3.2.19}$$

The first part of this admittance is :

$$Y = \frac{\beta(s)}{Z_b} = \frac{Y_b}{\dfrac{1}{\beta_0} + s\,\tau_T} = \frac{1}{Z_b}\cdot\frac{1}{\dfrac{1}{\beta_0} + s\,\tau_T} \tag{3.2.20}$$

and the impedance :

$$Z = \frac{Z_b}{\beta_0} + s\,\tau_T\,Z_b \tag{3.2.21}$$

Thus the transformed impedance Z_e is composed of three elements: the series connected Z_b/β_0 and $s\,\tau_T\,Z_b$, with the parallel connection of the impedance Z_b .

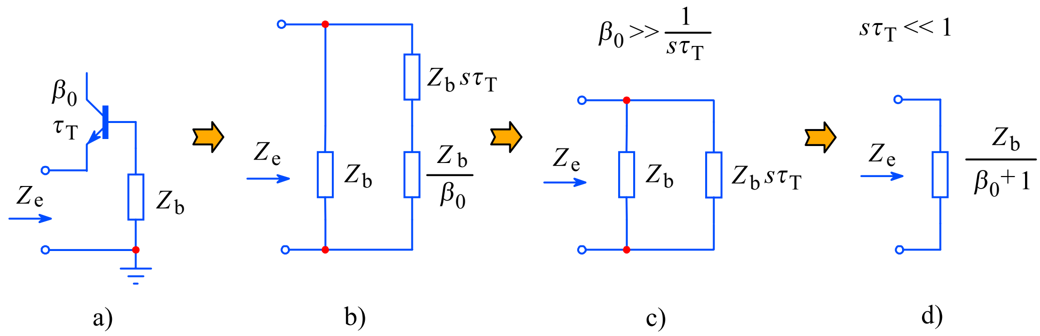


Fig. 3.2.4 : Base to emitter impedance conversion : a) schematic ; b) equivalent circuit ; c) simplified for high β_0 or for $f \simeq f_T$; d) simplified for low frequencies.

The equivalent emitter impedance is shown in Fig. 3.2.4b.

As in the previous example, for some specific conditions the circuit can be simplified. At medium frequencies and a high β_0, we can assume $\beta_0 \gg 1/s\,\tau_T$ and therefore neglect the impedance Z_b/β_0 , as in Fig. 3.2.4c. At low frequencies, where $s\,\tau_T \ll 1$, the impedance $Z_b/(\beta_0+1)$ prevails and we can neglect the parallel impedance Z_b , as in Fig. 3.2.4d.

3.2.3 Examples of impedance transformations

The most interesting examples are the transformation of capacitive emitter impedance into base and the transformation of the inductive base impedance into emitter. In the first case we have $Z_e = 1/s\,C$, where C is the emitter-to-ground capacitance.

To get the base impedance, we apply Eq. 3.2.5 :

$$Z_\mathrm{b} = \frac{\beta(s)+1}{s\,C} = \left[\frac{1}{\dfrac{1}{\beta_0}+s\,\tau_\mathrm{T}}+1\right]\frac{1}{s\,C} = \frac{\dfrac{1}{\beta_0}+s\,\tau_\mathrm{T}+1}{\left(\dfrac{1}{\beta_0}+s\,\tau_\mathrm{T}\right)s\,C}$$

$$= \frac{s\,\tau_\mathrm{T}+\left(1+\dfrac{1}{\beta_0}\right)}{s^2\tau_\mathrm{T}\,C+\dfrac{s\,C}{\beta_0}} \tag{3.2.22}$$

The inverse of Z_b is the admittance:

$$Y_\mathrm{b} = \frac{s^2\tau_\mathrm{T}\,C+\dfrac{s\,C}{\beta_0}}{s\,\tau_\mathrm{T}+\left(1+\dfrac{1}{\beta_0}\right)} \tag{3.2.23}$$

Let us synthesize this expression by a simple continued fraction expansion [Ref. 3.27]:

$$\frac{s^2\tau_\mathrm{T}\,C+\dfrac{s\,C}{\beta_0}}{s\,\tau_\mathrm{T}+\left(1+\dfrac{1}{\beta_0}\right)} = s\,C - \frac{s\,C}{s\,\tau_\mathrm{T}+\left(1+\dfrac{1}{\beta_0}\right)} \tag{3.2.24}$$

The fraction on the right is a negative admittance with the corresponding impedance:

$$Z'_\mathrm{b} = -\,\frac{s\,\tau_\mathrm{T}+\left(1+\dfrac{1}{\beta_0}\right)}{s\,C} = -\,\frac{\tau_\mathrm{T}}{C} - \frac{1+\dfrac{1}{\beta_0}}{s\,C} \tag{3.2.25}$$

It is evident that this impedance is a series connection of a negative resistance:

$$R_\mathrm{n} = -\,\frac{\tau_\mathrm{T}}{C} = -\,r_\mathrm{e}\,\frac{C_\pi}{C} \tag{3.2.26}$$

and a negative capacitance:

$$C_\mathrm{n} = -\,\frac{C}{1+\dfrac{1}{\beta_0}} = -\,\frac{\beta_0}{1+\beta_0}\,C = -\,\alpha_0\,C \tag{3.2.27}$$

By adding the positive parallel capacitance C, as required by Eq. 3.2.24, we obtain the equivalent circuit which is shown in Fig. 3.2.5. Since we deal with an active (transistor) circuit, it is quite normal to encounter negative impedances. The complete base admittance is then:

$$Y_\mathrm{b} = s\,C - \frac{1}{\dfrac{\tau_\mathrm{T}}{C}+\dfrac{1}{s\,\alpha_0\,C}} \tag{3.2.28}$$

By rearranging this expression and substituting $s = j\omega$, we can separate the real and imaginary part, obtaining :

$$Y_b = \Re\{Y_b\} + j\Im\{Y_b\} = G_b + j\omega C_b$$
$$= -\frac{\dfrac{\tau_T}{C}}{\tau_T^2 + \dfrac{1}{\omega^2\alpha_0^2}} - j\omega C \frac{\tau_T^2 - \dfrac{\alpha_0 - 1}{\omega^2\alpha_0^2}}{\tau_T^2 + \dfrac{1}{\omega^2\alpha_0^2}} \tag{3.2.29}$$

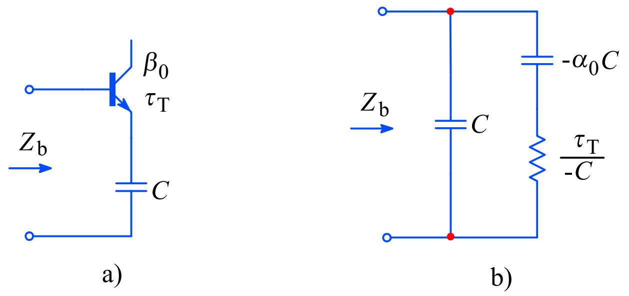


Fig. 3.2.5 : Capacitive load is reflected into the base (junction) with **negative** components.

The negative input (base) conductance G_b can cause ringing on steep signals or even continuous oscillations if the signal source impedance has an emphasized inductive component. We will thoroughly discuss this effect and its compensation later, when we will analyze the emitter-follower (= common-collector) and the JFET source-follower amplifiers.

Now let's derive the emitter impedance Z_e in case when the base impedance is inductive (sL). Here we apply Eq. 3.2.18 :

$$Z_e = \frac{sL}{\beta(s) + 1} = \frac{sL}{\dfrac{1}{\dfrac{1}{\beta_0} + s\tau_T} + 1} = \tag{3.2.30}$$

$$= \frac{sL\left(\dfrac{1}{\beta_0} + s\tau_T\right)}{1 + \dfrac{1}{\beta_0} + s\tau_T} = \frac{s^2 L\tau_T + \dfrac{sL}{\beta_0}}{s\tau_T + \left(1 + \dfrac{1}{\beta_0}\right)} \tag{3.2.31}$$

By continued fraction expansion we get :

$$\frac{s^2 L\tau_T + \dfrac{sL}{\beta_0}}{s\tau_T + \left(1 + \dfrac{1}{\beta_0}\right)} = sL - \frac{sL}{s\tau_T + \left(1 + \dfrac{1}{\beta_0}\right)} \tag{3.2.32}$$

The negative part of the result can be inverted to get the admittance :

$$Y_e' = -\frac{s\tau_T + \left(1 + \dfrac{1}{\beta_0}\right)}{sL} = -\frac{\tau_T}{L} - \frac{1 + \dfrac{1}{\beta_0}}{sL} \tag{3.2.33}$$

This means, we have two parallel impedances. The first one is a **negative** resistance :

$$R_x = -\frac{L}{\tau_T} \tag{3.2.34}$$

and the second one is a **negative** inductance :

$$L_x = -\frac{L}{1+\dfrac{1}{\beta_0}} = -\frac{\beta_0}{1+\beta_0}L = -\alpha_0 L \tag{3.2.35}$$

As required by Eq. 3.2.32, we must add in series the inductance L, thus arriving at the equivalent emitter impedance shown in the figure below :

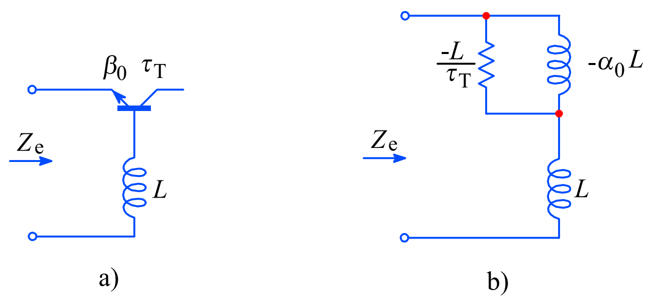


Fig. 3.2.6 : Inductive source is reflected into the emitter with **negative** components.

We have just analyzed an important aspect of a common-base amplifier, with an inductance (= long lead!) between the base and ground. The negative resistance, as given by Eq. 3.2.34 , may become the reason of ringing or oscillations if the driving circuit seen by the emitter has a capacitive character. We will discuss this problem more thoroughly when we will analyze the cascode circuit.

In a similar way as we derived the previous two results, we can transform other impedance types from emitter to base and vice versa. The Table 3.2.1 displays the six possible variations and the reader is encouraged to derive the remaining four, which we did not discuss.

Note that all the three transformations for the comon-base circuit in the table apply to the base-emitter-**junction**-to-ground only. In order to get the correct base-**terminal**-to-ground impedance the transistor base-spread resistance r_b must be added in series to the circuits shown in the table.

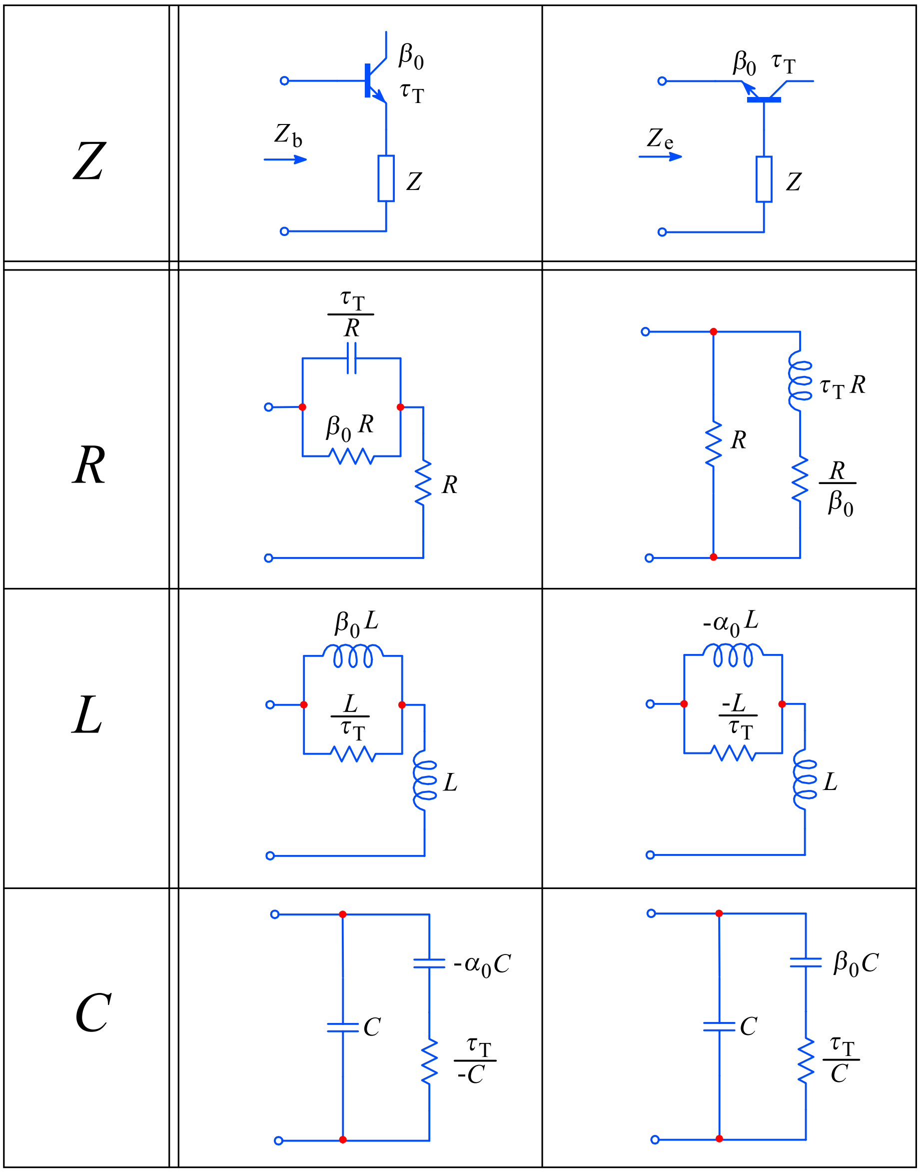


Table 3.2.1 : The Table of impedance conversions