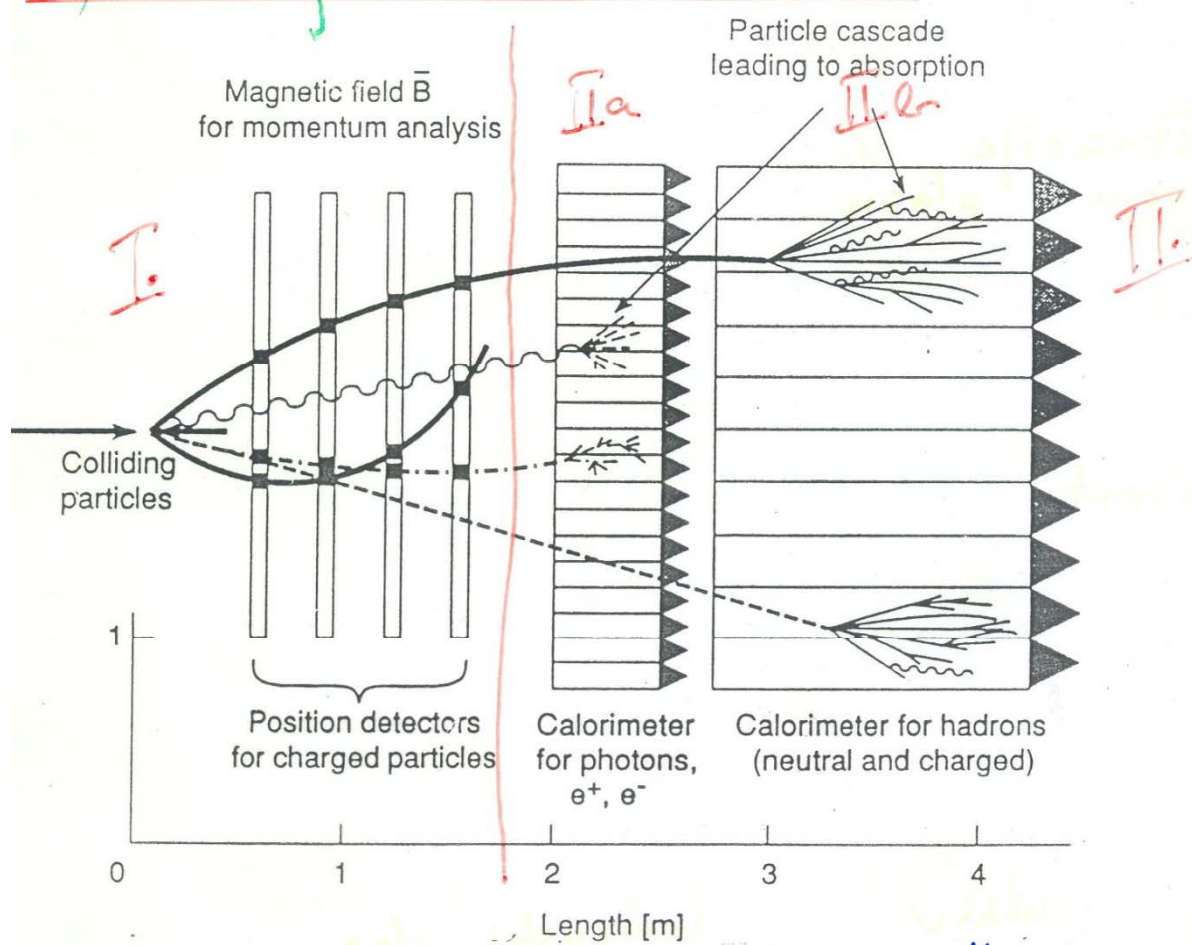


# Merjenje energije

## -osnove kalorimetrije

- a) zakaj kalorimetri?
- b) EM plaz  $\rightarrow$  EM kalorimeter
- c) ločljivost EM kalorimetrov
- d) vzorčevanje
- e) hadronski plaz  $\rightarrow$  hadronski kalorimeter
- f) ločljivost hadronskih kalorimetrov
- g) identifikacija s. kalorimetri

a) zakaj kalorimetri?



eksperiment  $\rightarrow$  meritev  $P^\mu = (E, \vec{p})$

$\vec{p}, E$  ali  $\vec{p}, m$  ali  $\vec{p}, v$

I. -  $p = e B r$  sledilni sistem

II. - meritev  $E$  s popolno absorbcijo in zažnavo odložene energije

kalorimeter IIa EM  $e, \gamma$

IIb hadronski  $\pi, K, p, n$

# odziv kalorimetra

- energija } delca
- vrsta }

absorbicija (npr. število sek. delcev -  $N$ )  
 statistični proces  $\rightarrow \frac{N}{E}$   $\frac{1}{\sqrt{E}}$   
 povprečni odziv | ponovljivost - ločljivost

## prednosti kalorimetra za visoke E

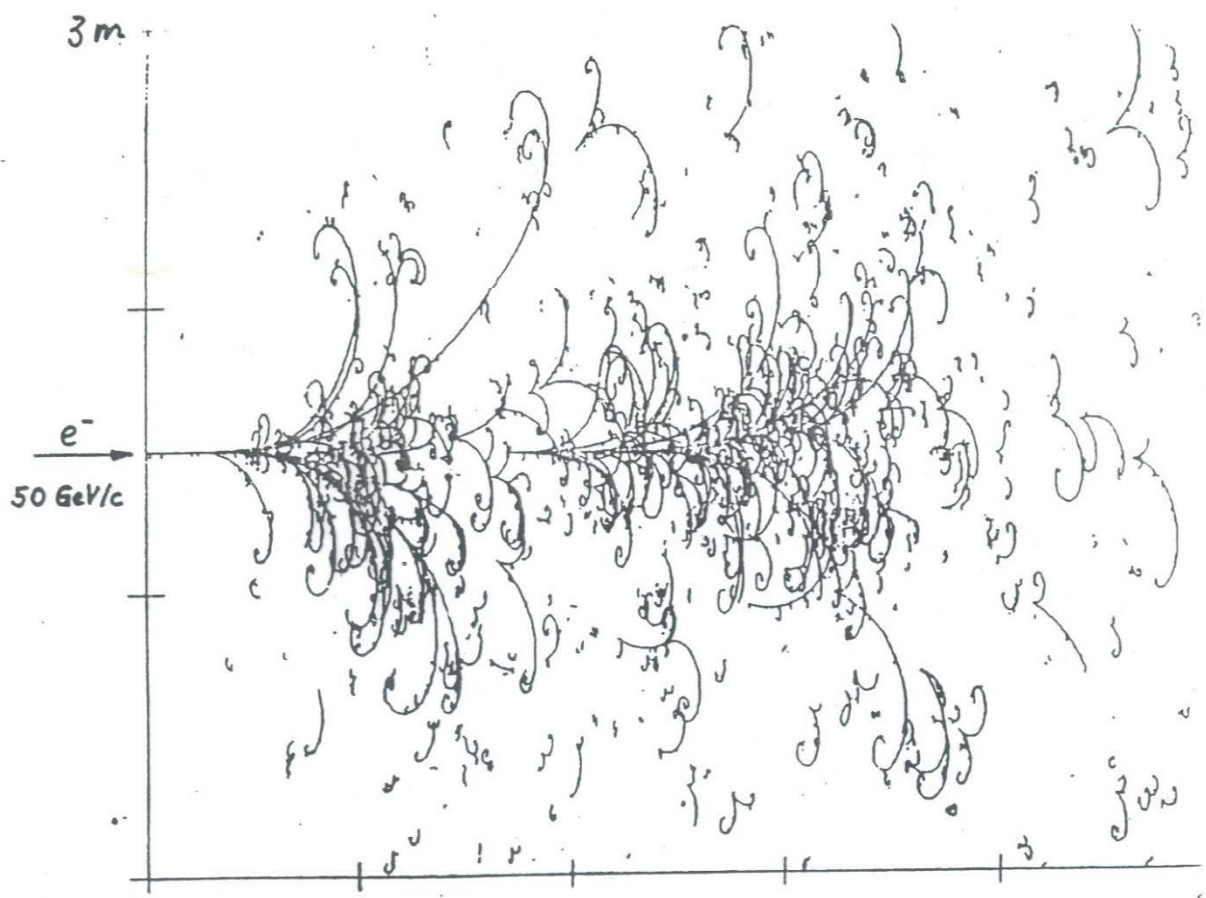
- |  |  |
|--|--|
| <ul style="list-style-type: none"> <li>- <math>\frac{\sigma_E}{E} \propto \frac{\sigma_N}{N} = \frac{1}{\sqrt{N}} \propto \frac{1}{\sqrt{E}}</math><br/>ločljivost se izboljšuje!</li> <li>- <math>L \propto \ln E</math><br/>velikost narašča</li> <li>- <math>e, \gamma, \pi^+, \pi^0, K^+, K^0, p, n</math><br/>izmeri vse delce</li> <li>- pljusk: <math>E_{jet}</math><br/>preprosta rekonstrukcija pljuskov <math>\Rightarrow</math> proženje</li> <li>- hermetičnost celoten dogodek <math>\Rightarrow</math> nevtrini, SUSY</li> </ul> | <ul style="list-style-type: none"> <li>- <math>\frac{\sigma_p}{p} \propto p/L^2</math><br/>se slabša!</li> <li>- <math>L \propto \sqrt{p}</math><br/>počasneje!</li> <li>- le nabiti</li> <li>- <math>\vec{p}_{jet} = \sum_{jet} \vec{p}_i</math></li> </ul> |
|--|--|

### III EM plaz

3

$e$  ali  $\gamma$  z visoko energijo  $E_0 \gg E_c$

pari  $\rightarrow$  zavorno sevanje  $\rightarrow$  pari  $\rightarrow$  .....



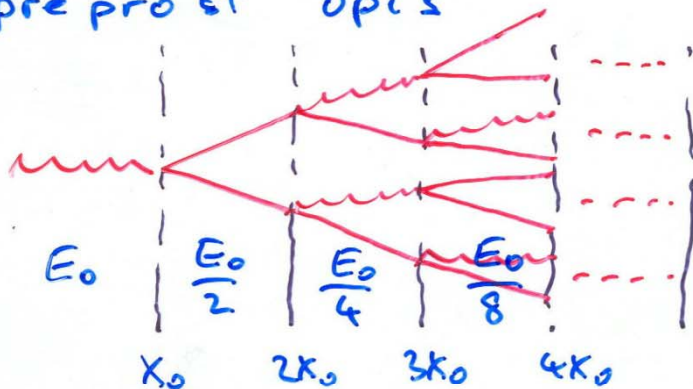
BEBC, Ne/H<sub>2</sub> (70/30%),  $B=3T$ .

ELECTROMAGNETIC SHOWER DEVEL.



plazma raste dokler  $e^{\pm}$  izgubljajo več energije z zavornim sevanjem kot z ionizacijo -  $E_c$   
 statistični proces

preprost opis



$$N = 2^{\frac{x}{x_0}}$$

$$E = \frac{E_0}{N}$$

maksimalna dolžina  $E = E_c$

$$x_{max} = \frac{\ln \frac{E_0}{E_c}}{\ln 2} \quad (44)$$

$$N_{max} = \frac{E_0}{E_c} \quad (45)$$

dolžina (velikost detektorja) narača kot  $\ln E_0 \rightarrow$  kalorimetri

poznamo posamične procese  $\Rightarrow$

MC simulacija EM plazm  
 EGS ( $\rightarrow$  V GEANT)

longitudinalna oblika

34

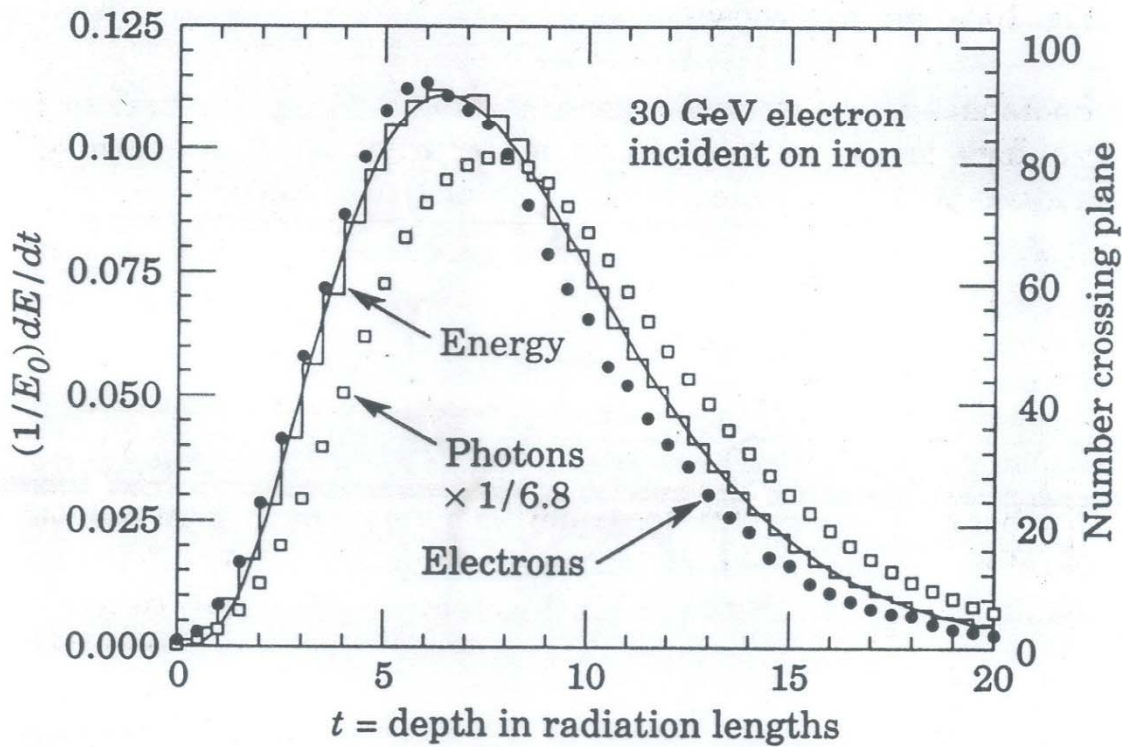
$$N = N_0 \left(\frac{x}{x_0}\right)^a e^{-b \frac{x}{x_0}} \quad (49)$$

$a, b$  : parametra, šibko odvisna od  $z$

zajetje 95%  $E_0$

$$L\left(\frac{\Delta E}{E_0} = 95\%\right) \sim \left[\ln\left(\frac{E_0}{E_c} - 1\right) + 12\right] x_0 \quad (50)$$

$\sim 21 x_0$  pri 100 GeV



analitični opis - Rossi '64  
(Rossijeva aproksimacija B)

- $\frac{dE}{dx}$  ionizacija =  $-\frac{E_c}{X_0} \neq f(E)$
- ni VCS, plaz v 1-D
- ni Comptonovega sipanja

vzdolžni profil

$$\frac{dE}{dt} = \frac{E_0 b^{z+1}}{\Gamma(z+1)} t^z e^{-bt} \quad t = \frac{x}{X_0}$$

max. pri  $t_{\max} = \frac{d}{b} = \ln \gamma e^{z+0,5}$

$$\gamma = \frac{E_0}{E_c}$$

skupna dolžina poti:  $e^+$

$$T = \gamma X_0 = \frac{E_0}{E_c} \cdot X_0$$

$\Rightarrow$  EM plaz odvisen le od

$$t = \frac{x}{X_0}$$

$$\gamma = \frac{E_0}{E_c}$$

$$\frac{1}{X_0} \sim \frac{Z(Z+1)}{A} \cdot \frac{\ln(287/\sqrt{Z})}{716,4 \text{ g cm}^{-2}} \quad E_c \sim \frac{600 \text{ MeV}}{Z+1,2}$$

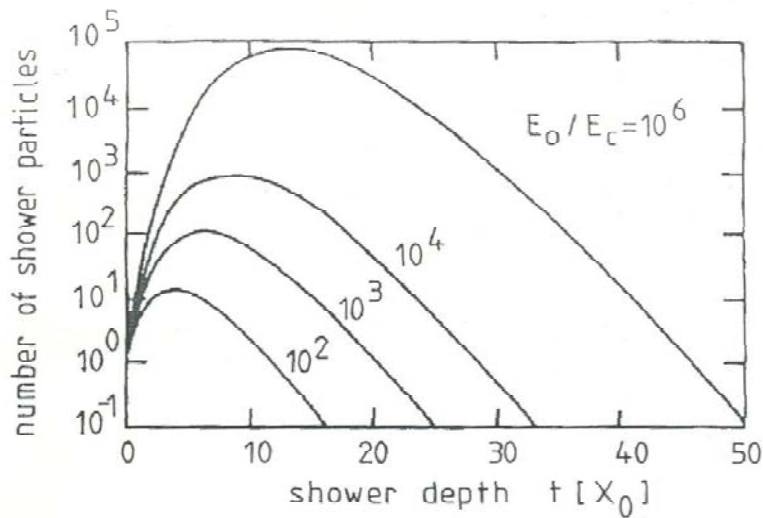


Fig. 7.21. Longitudinal shower development of electromagnetic cascades ( $E_c$  - critical energy) [509, 503].

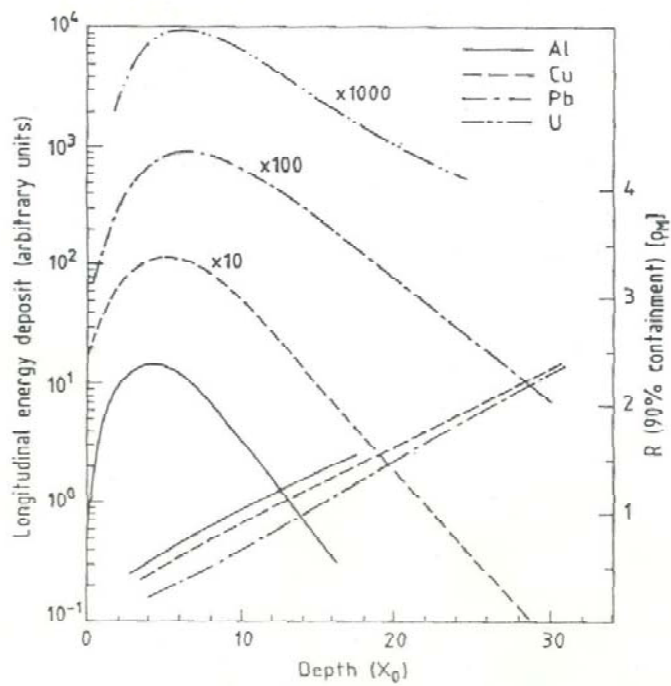


Fig. 3: Longitudinal shower development (left ordinate) of 6 GeV/c electrons in four very different materials, showing the scaling in units of radiation lengths  $X_0$ . On the right ordinate the shower radius for 90% containment of the shower is given as a function of the shower depth. In the later development of the cascade, the radial shower dimensions scale with the Molière radius  $\rho_M \sim 7A/Z$ . [Al, Cu, and Pb, adapted from G. Bathow et al., Nucl. Phys. B20:592 (1970). Uranium data from G. Barbiellini et al., Ref. [127].



prečne dimenzije

začetek - zavorno sevanje  $\propto d \left(\frac{r}{m_e}\right)^{-1}$   
- pari

kasneje - VCS

Poliere

$$S_M = X_0 \frac{21 \text{ MeV}}{E_c}$$

različni materiali - str. 7, sl. 3., leva skala

vzdolžno širjenje plaz

1 GeV  $e^-$   
na Al

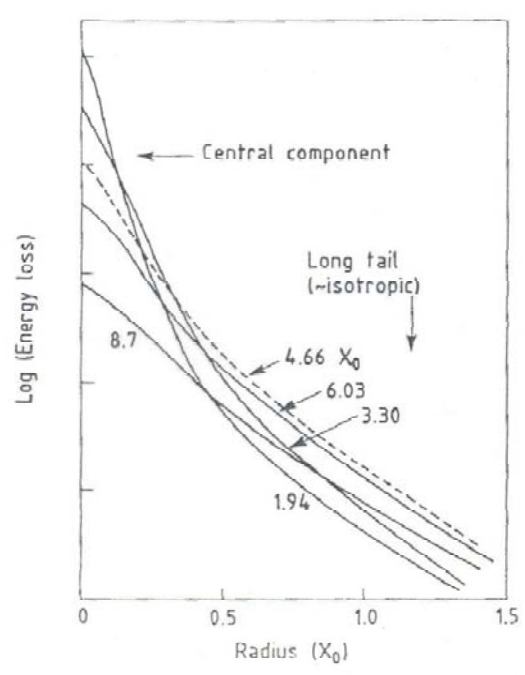


Fig. 4: Radial shower profile of 1 GeV electrons in aluminium; a pronounced central core, surrounded by 'halo', gradually widens with increasing depths of the shower [17].

## c) lozljivost EM kalorimetrov

$E \rightarrow$  plaz  $\rightarrow$  ionizacija (T)

$\rightarrow$  signal: detektiramo  $(T_v)$

- naboj (Si, LAr)
- scint. fotone (Na $\uparrow$ , Cs $\uparrow$ )
- Čerenkove - $\ll$  (Pb steklo)

prag za detekcijo  $\eta \Rightarrow T_v < T$

$$T_v = F\left(\frac{\eta}{E_c}\right) \frac{E}{E_c} X_0$$

$$F\left(\frac{\eta}{E_c}\right) = \left(1 + \xi \ln \frac{E}{1,53}\right) e^{\xi}$$

$$\xi = 2,29 \frac{\eta}{E_c}$$

ocena  $N = \frac{E}{\eta}$  ;  $\frac{\sqrt{E}}{E} = \frac{1}{\sqrt{N}}$

Pb steklo  $\eta \sim 0,7 \text{ MeV}$  ;  $\frac{\sqrt{E}}{E} = \frac{2,5\%}{\sqrt{E[\text{GeV}]}}$

še statistika fotoelektronov

$$2,5\% \rightarrow 4\%$$

Na $\uparrow$   $E_c = 11,8 \text{ MeV}$   $\frac{\eta}{E_c} = 0,04$

račun  $\left(\frac{\sqrt{E}}{E}\right)_{\text{intrinsic}} = \frac{0,4\%}{\sqrt{E[\text{GeV}]}}$

(nedoseženi) cilj graditeljev EM kalorimetrov

ločljivost je slabša od  $\left(\frac{\sigma_E}{E}\right)$  intrinsic

- statistika  $\frac{\sigma_E}{E} \propto \frac{1}{\sqrt{E}}$
- puščanje
- nehomogenosti
- kalibracija
- šum elektronike  $\sigma_E = konst$

skupaj

$$\frac{\sigma_E}{E} = \frac{a}{\sqrt{E}} \oplus b \oplus \frac{c}{E}$$

nizke E a, c

visoke E b

druge parametri za cije

$$\frac{\sigma_E}{E} \propto E^{-1/4} \quad Na\gamma, Cs\gamma$$

nehomogenosti v kristalu ( $\sim a \oplus c$ )

## d) vzorčevanje

homogen detektor - cena  
- zbiranje signala

absorber (pasivni)  $Pb, W, Fe$  & detektor  $scintilator, LAr, MWPC$

$\Rightarrow$  vzorčevalni kalorimeter

**(+)** absorber: - majhen  $X_0$  - kompakten  
- poceni  
detektor: - segmentiran - profil plazni  
- lažje zbiranje (svetloba, el. signal)

**(-)** dodatne fluktuacije - slabša  $\frac{\sigma_E}{E}$

cena: štejemo prehode skozi det. ravni

$$N = \frac{T}{d} \quad d - \text{razdalja med det. rav.}$$

$$N = \frac{E}{E_c} \frac{X_0}{d} = \frac{E}{\Delta E} \quad \Delta E - \text{izguba v eni plasti}$$

$$\frac{\sigma_E}{E} = \frac{1}{\sqrt{N}} = \frac{1}{\sqrt{1000}} \sqrt{\frac{\Delta E (\text{MeV})}{E (\text{GeV})}} = 3,2\% \sqrt{\frac{\Delta E (\text{MeV})}{E (\text{GeV})}}$$

$$\text{tipično: } d \sim X_0 \rightarrow \Delta E \sim E_c \sim 10 \text{ MeV} \Rightarrow \frac{\sigma_E}{E} \sim \frac{10\%}{\sqrt{E (\text{GeV})}}$$

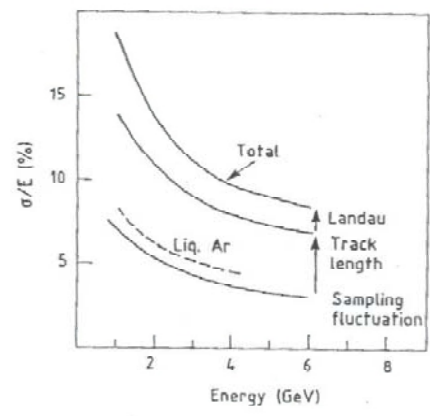


lozljivost poslabša

- VCS :  $\langle d \rangle = \frac{d}{\langle \cos \theta \rangle}$

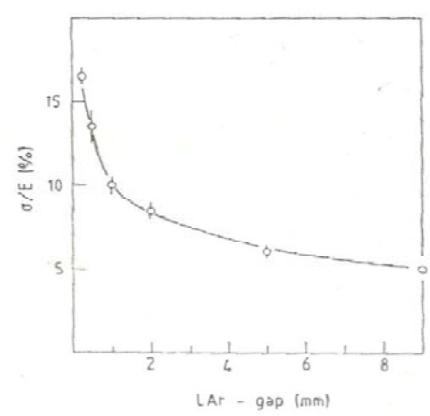
- zaradi parov  $e^+e^-$   $N_{eff} < N$

- plinski detektor : - sipanje v ravnino detektorja } 10x! slabša.  
 - fluktuacije (Landau)



plin

Fig. 7b: Contributions of sampling, path length, and Landau fluctuations to the energy resolution of a lead/MWPC sampling calorimeter. The latter contribute approximately equally (~ 12% at E = 1 GeV), and combined quadratically with the sampling fluctuations (~ 7%) they account for the overall resolution of ~ 18%/sqrt(E) [14].



LAr

Fig.7a: Energy resolution versus thickness of the active liquid-argon layer for 1 GeV electrons in an iron/argon sampling calorimeter.

12a

Table 28.5: Resolution of typical electromagnetic calorimeters.  $E$  is in GeV.

Technology (Experiment)	Depth	Energy resolution	Date
NaI(Tl) (Crystal Ball)	$20X_0$	$2.7\%/E^{1/4}$	1983
$\text{Bi}_4\text{Ge}_3\text{O}_{12}$ (BGO) (L3)	$22X_0$	$2\%/\sqrt{E} \oplus 0.7\%$	1993
CsI (KTeV)	$27X_0$	$2\%/\sqrt{E} \oplus 0.45\%$	1996
CsI(Tl) (BaBar)	$16-18X_0$	$2.3\%/E^{1/4} \oplus 1.4\%$	1999
CsI(Tl) (BELLE)	$16X_0$	1.7% for $E_\gamma > 3.5$ GeV	1998
$\text{PbWO}_4$ (PWO) (CMS)	$25X_0$	$3\%/\sqrt{E} \oplus 0.5\% \oplus 0.2/E$	1997
Lead glass (OPAL)	$20.5X_0$	$5\%/\sqrt{E}$	1990
Liquid Kr (NA48)	$27X_0$	$3.2\%/\sqrt{E} \oplus 0.42\% \oplus 0.09/E$	1998
Scintillator/depleted U (ZEUS)	$20-30X_0$	$18\%/\sqrt{E}$	1988
Scintillator/Pb (CDF)	$18X_0$	$13.5\%/\sqrt{E}$	1988
Scintillator fiber/Pb spaghetti (KLOE)	$15X_0$	$5.7\%/\sqrt{E} \oplus 0.6\%$	1995
Liquid Ar/Pb (NA31)	$27X_0$	$7.5\%/\sqrt{E} \oplus 0.5\% \oplus 0.1/E$	1988
Liquid Ar/Pb (SLD)	$21X_0$	$8\%/\sqrt{E}$	1993
Liquid Ar/Pb (HI)	$20-30X_0$	$12\%/\sqrt{E} \oplus 1\%$	1988
Liquid Ar/depl. U (DØ)	$20.5X_0$	$16\%/\sqrt{E} \oplus 0.3\% \oplus 0.3/E$	1993
Liquid Ar/Pb accordion (ATLAS)	$25X_0$	$10\%/\sqrt{E} \oplus 0.4\% \oplus 0.3/E$	1996

e) hadronski plaz → HCAL

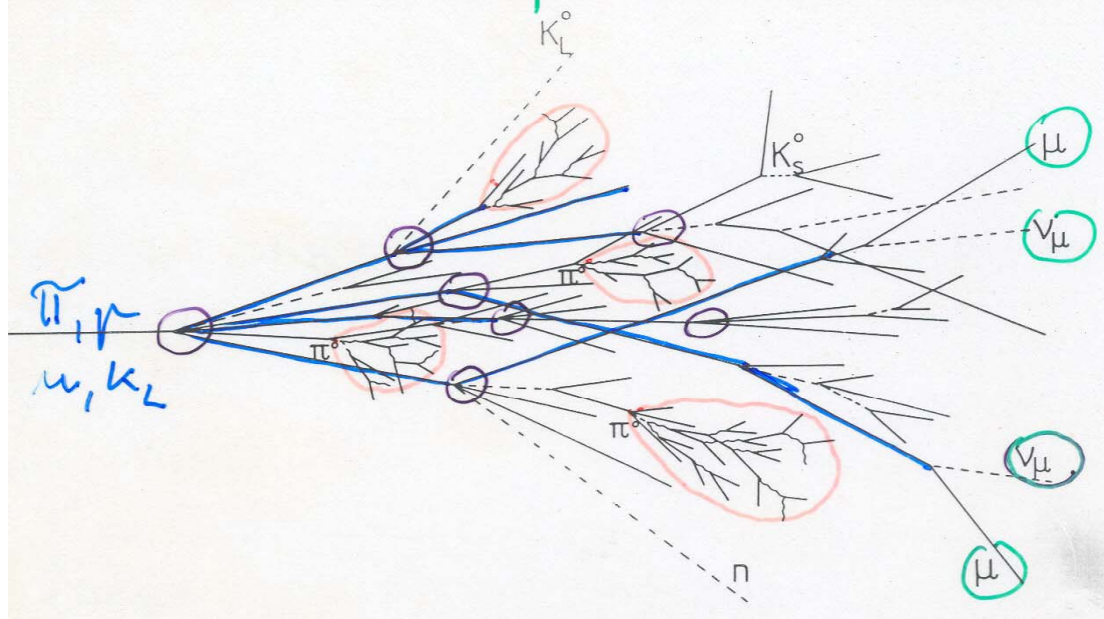


Fig. 7.32. Sketch of a hadron cascade in an absorber.

Razdelitev energije ( $f(E)$ , tipično za 10 GeV)

- ~ 50% visokoenergijski hadroni (~ pioni)

$\pi^+$

hadronski plaz ( $X_T$ )

$\pi^-$

hadronski plaz ( $X_T$ )

$\pi^0$  lokalni EM plaz ( $X_0$ )

$\gamma, \gamma$
- ~ 30% počasni nukleoni  $T_V < T$  → signal ?
- ~ 20% vzbujena jedra →  $p, n, \gamma$  1-~30 keV
- ~ 5% razpad  $\pi^\pm \rightarrow \mu^\pm \nu$  pobegnejo



raznolikost procesov

⇒ več možnosti za fluktuacije

→ slabša ločljivost

karakteristična dolžina

$\pi^+ \pi^-$  (hadroni) interakcijska dolžina

$$\lambda_I \sim 35 A^{1/3} \frac{g}{cm^2}$$

$$t = \frac{x}{\lambda_I}$$

$\pi^0 \rightarrow 2\gamma \rightarrow E\pi$  plaz  $X_0$

	$\lambda_I [cm]$	$X_0 [cm]$
Na	41	2,6
BGO	23	1,1
Fe	17	1,76
Pb	18,5	0,56
U	12	0,32

⇒ Fe tipičen absorber v hadronskih kalorimetrih  
 + cena  
 + povratni jarem magneta

maksimum plaz pri

$$t_{max} \sim 0,2 \ln(E [GeV]) + 0,7$$



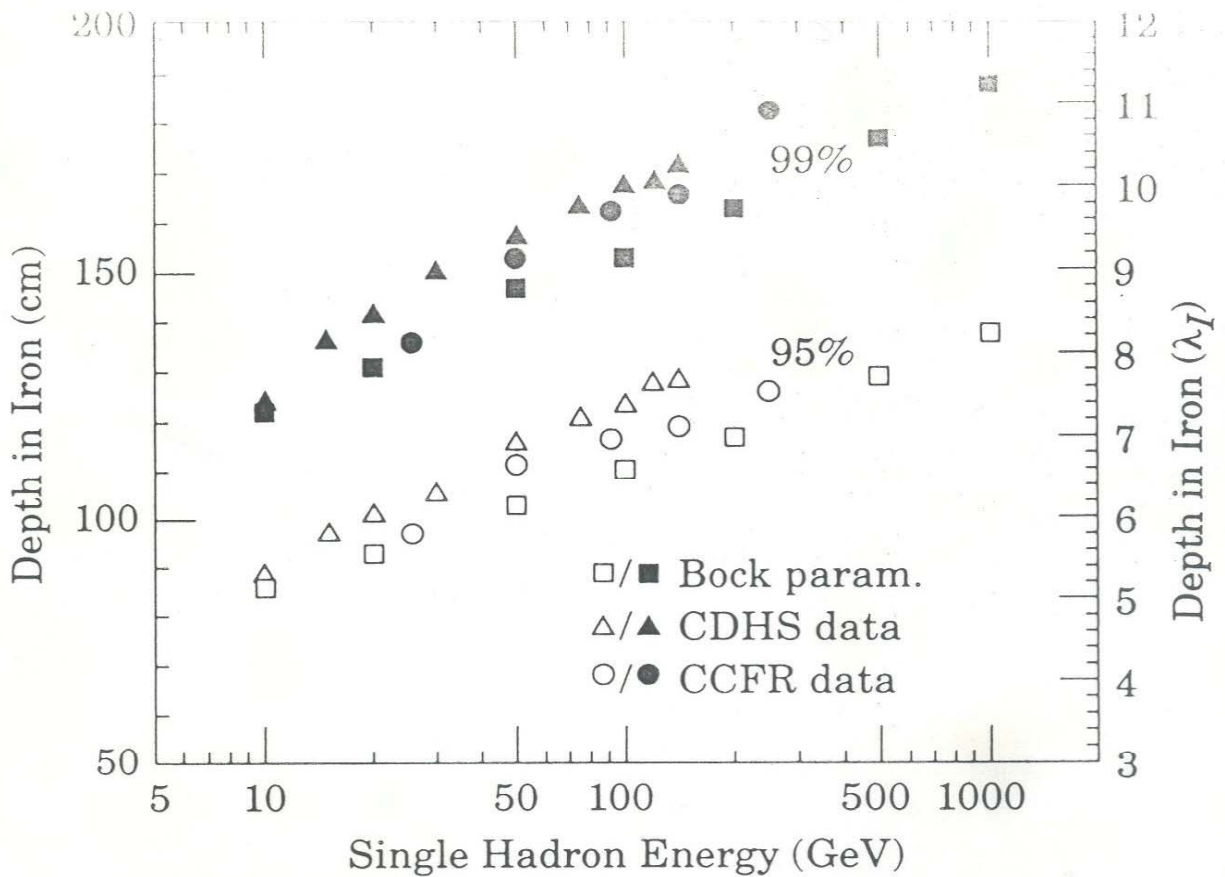


Figure 24.2: Required calorimeter thickness for 95% and 99% hadronic cascade containment in iron, on the basis of data from two large neutrino detectors and the parametrization of Bock et al. [44].

dolžina plazmu?

a) 95% energije  
 v povprečju  
 (ugašanje kot  $e^{-t/\lambda_{att}}$ )

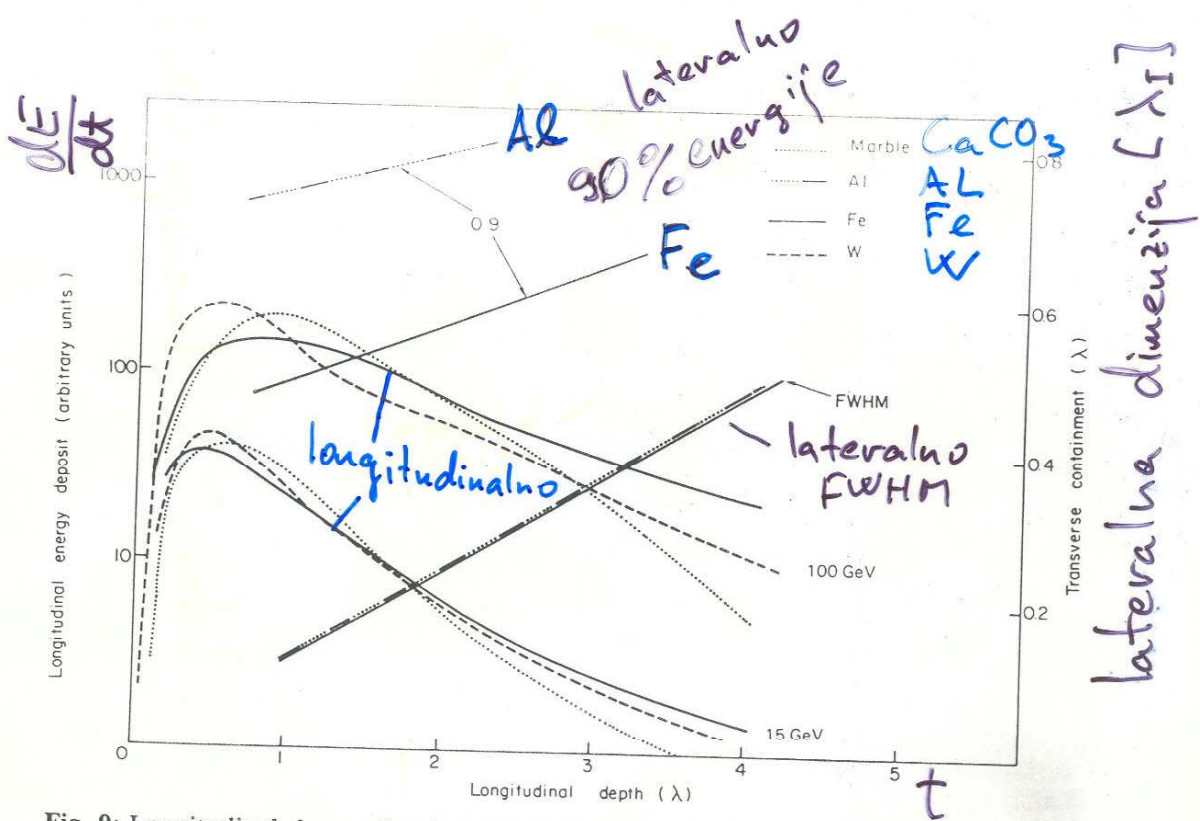
$$t_{95} \sim t_{max} + 2,5 \lambda_{att}$$

$$\lambda_{att} \sim (E [GeV])^{0,13}$$

b) 95% energije  
 v 95% primerov

$$t_{95/95} \sim t_{95} + 3$$

↑  
 fluktuacije



**Fig. 9:** Longitudinal shower development (left ordinate) induced by hadrons in different materials, showing approximate scaling in absorption length  $\lambda$ . The shower distributions are measured from the vertex of the shower and are therefore more peaked than those measured with respect to the face of the calorimeter. For the transverse distributions as a function of shower depth, scaling in  $\lambda$  is found for the narrow core (FWHM) of the showers. The radius of the cylinder for 90% lateral containment is much larger and does not scale in  $\lambda$ . [10 GeV/c  $\pi$ 's: B. Friend et al., Nucl. Instrum. Methods 136:505 (1976)]. Note that marble and aluminium have almost identical absorption and radiation lengths [Marble: M. Jonker et al., Nucl. Instrum. Methods 200:183 (1982); Fe: M. Holder et al., Nucl. Instrum. Methods 151:69 (1978); W: D.L. Cheshire et al., Nucl. Instrum. Methods 141:219 (1977)].

lateralna dimenzija

- narašča s  $t$

- FWHM (sredica) se umeri z  $\lambda_I$

- 90% E (repi) ni umerjen z  $\lambda_I$

MC simulacije - opis mnogo slabši  
(GHEISHA, FLUKA) kot za EM plaz (EGS)

19

razlike med različnimi MC

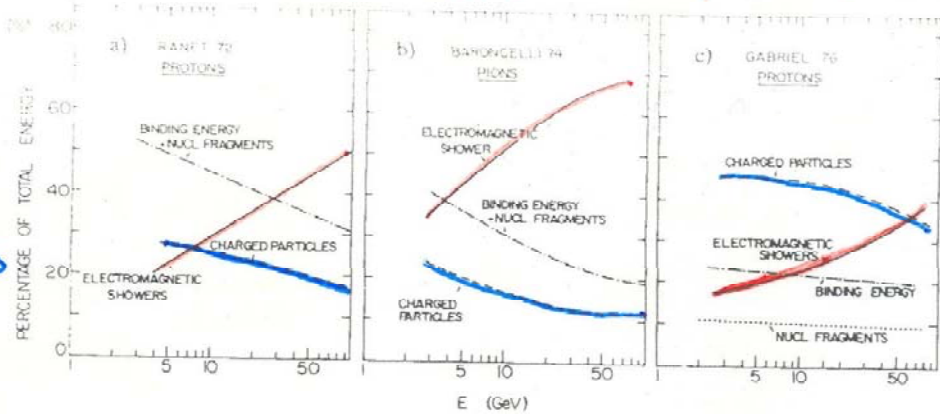


Fig. 8: Relative contributions of the most important processes to the energy dissipated by hadronic showers, as evaluated from three representative Monte Carlo calculations [25].

atmosferen

hadronski

EM

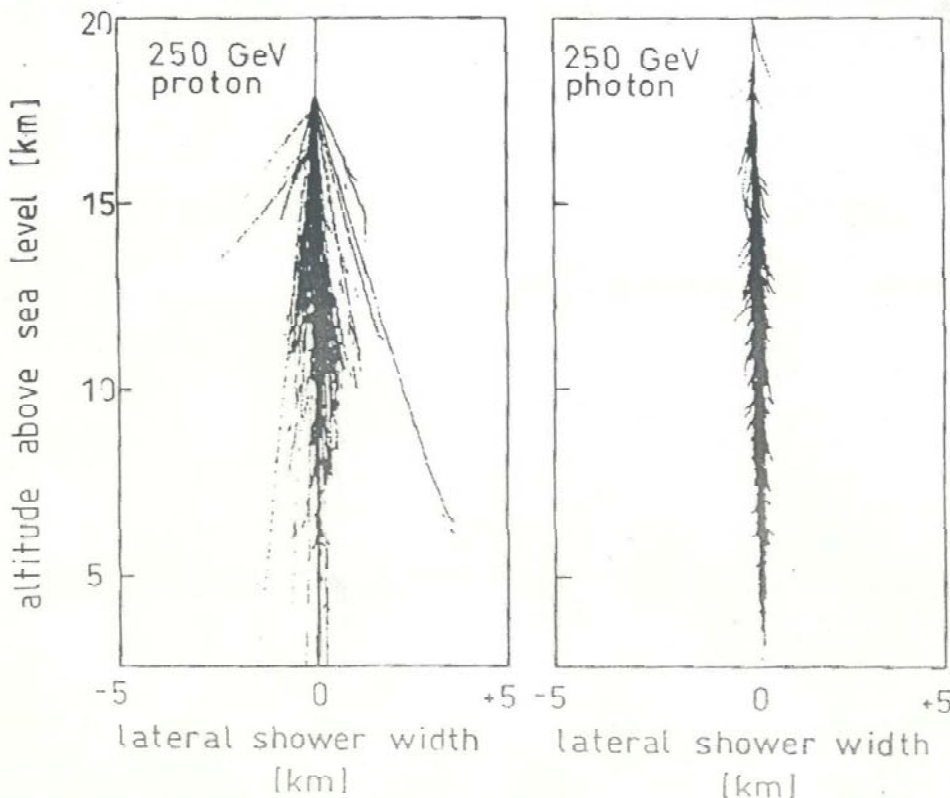


Fig. 7.34. Monte Carlo simulations of the different development of hadronic and electromagnetic cascades in the earth's atmosphere, induced by 250 GeV protons and photons [542].



# ločljivost hadronskih kalorimetrov

18

omejitev  
fluktuacije v  
razvoju plazme

- EM ( $\pi^0$ )  $\uparrow$
- neutroni  $\downarrow$
- fragmenti  $\downarrow$

ločljivost  
mnogo slabša  
kot pri EM

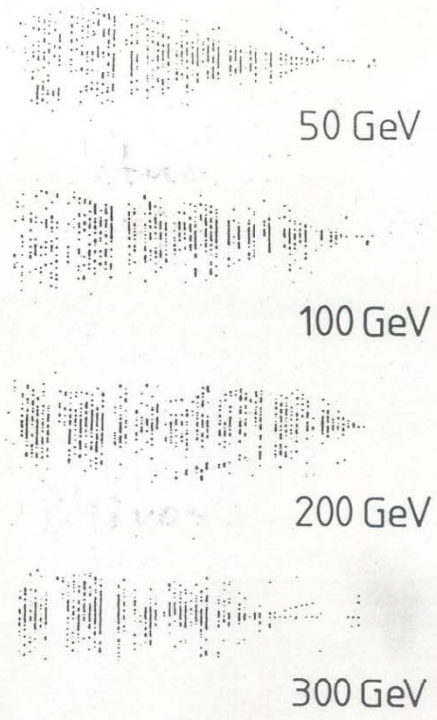
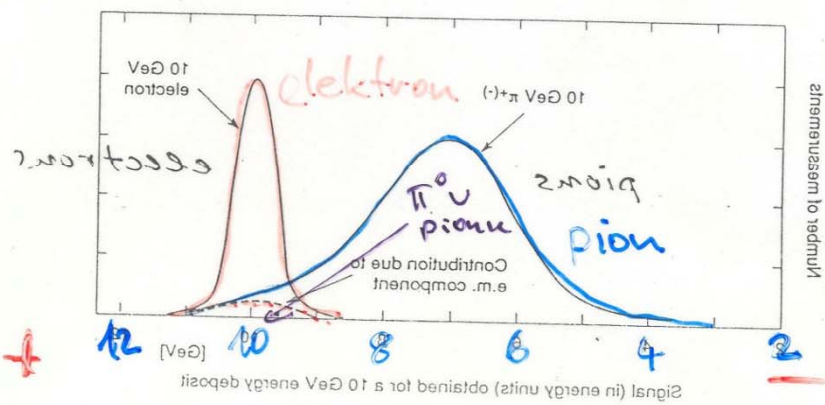


Fig. 7.42 b. Digital pattern of high energy hadron cascades (50 - 300 GeV) a flash-chamber calorimeter [560].

e/h odziv  
(meritev e/h)

hadronski plazma  
 $R_e \rightarrow \pi^0$   
 $R_h \rightarrow \text{ostalo}$



$$\frac{C}{h} > 1$$

odziv  $\uparrow$

$$\pi = e R_e + h R_h$$

$\uparrow$  delež  $\pi^0$

$$e = e \cdot 1$$



število  $\pi^0$   
( $E > 5 \text{ GeV}$ )

$$n_{\pi^0} \approx 5 \ln(E[\text{GeV}]) - 4.6$$

$n_{\pi^0}$  majhuo (7 pri 10 GeV)  $\sigma_{n_{\pi^0}} \sim \sqrt{n_{\pi^0}}$  velike  
fluktuacije  $R_e$  med dogodki

$e/h \neq 1$  -  $e/\pi = f(E)$  nelinearnost

( $\sim 1,4$ ) - odziv nesimetričen (rep)  $\Rightarrow \frac{\sigma_E}{E} = a + \frac{b}{E}$

- ločljivost slabša

$$\frac{\sigma_E}{E} \sim \frac{45\%}{\sqrt{E(\text{GeV})}}$$

*intrinsic*

rešitev: kompenzacija

$$\frac{e}{h} \rightarrow 1$$

$$\left(\frac{\sigma_E}{E}\right)_{e/h=1}^{\text{intrinsic}} \sim \frac{20\%}{\sqrt{E(\text{GeV})}} + \text{ni repov!}$$

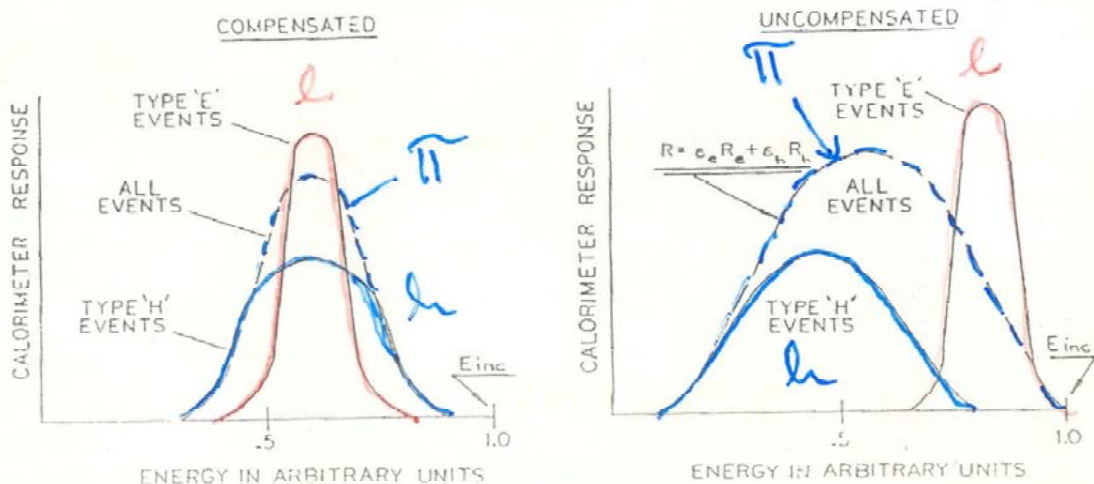


Figure 7: Schematic illustration of the effect of compensation on calorimeter response.

Kompenzacija: a) dvojica  $h$  } hardware  
 b) zmanjšanje  $e$  }  
 c) meritev  $R_e$  in  $R_h$  dogodek po dogodek in uteževanje (software)

a) in b) le v vzorčevalnih kalorimetih

dvojica  $h$  - detekcija neutronov  
 fisija ( $^{235}\text{U}$ )  $\rightarrow$   $n, \gamma$   
 nitek  $z$  ( $\text{H}$ )  $\rightarrow$   $\Delta E = E_n$

zmanjšanje  $e$  - absorber velik  $z$   
 (fotoefekt nizkoenergijskih  $\gamma$   
 $\rightarrow$  - 40% energije EM plazm)  
 - aktivna snov majhen  $z$   
 (prozorna za nizkoen.  $\gamma$ )

$e/h \sim 1$  Pb/scintilator = 4:1  
 U/scintilator = 1:1

ločljivost zaradi vzorčevanja

$$\left(\frac{\Delta E}{E}\right)_{\text{sample}} \sim 3\% \sqrt{\frac{\Delta E [\text{neV}]}{E [\text{GeV}]}}$$

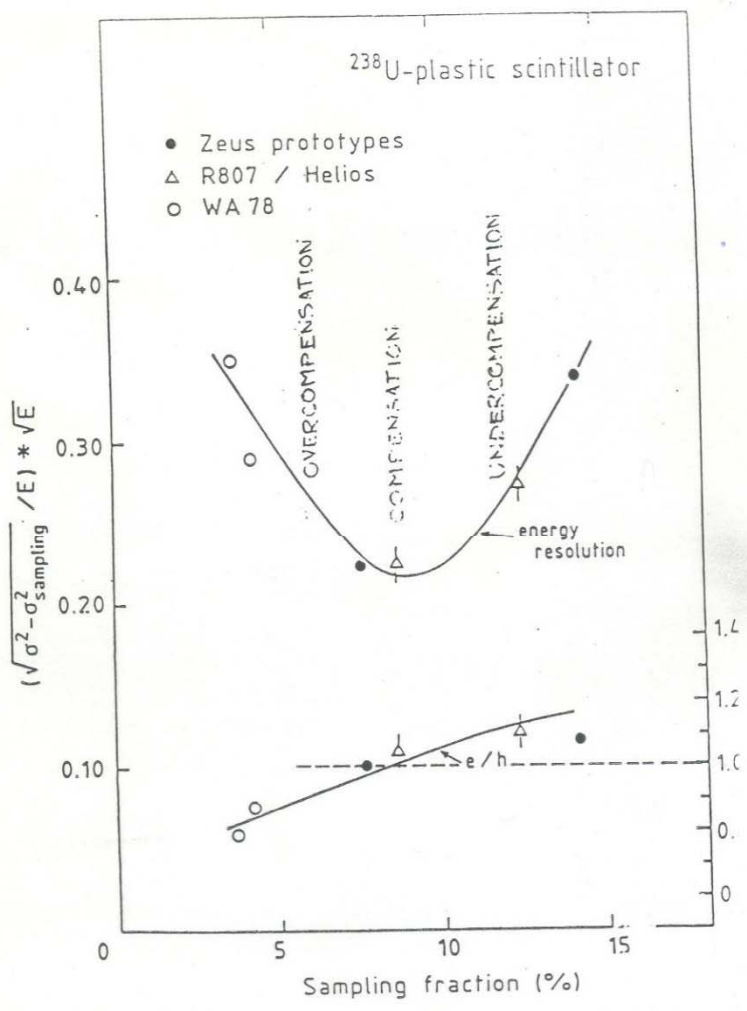
(c.f. 3,2% za EM)

$$\ll 45\% \sqrt{E}$$

↓  
 hadronski kalorimetri  
vzorčevalni

# Kompenzacija i varijaciju razmerja u scintilatoru

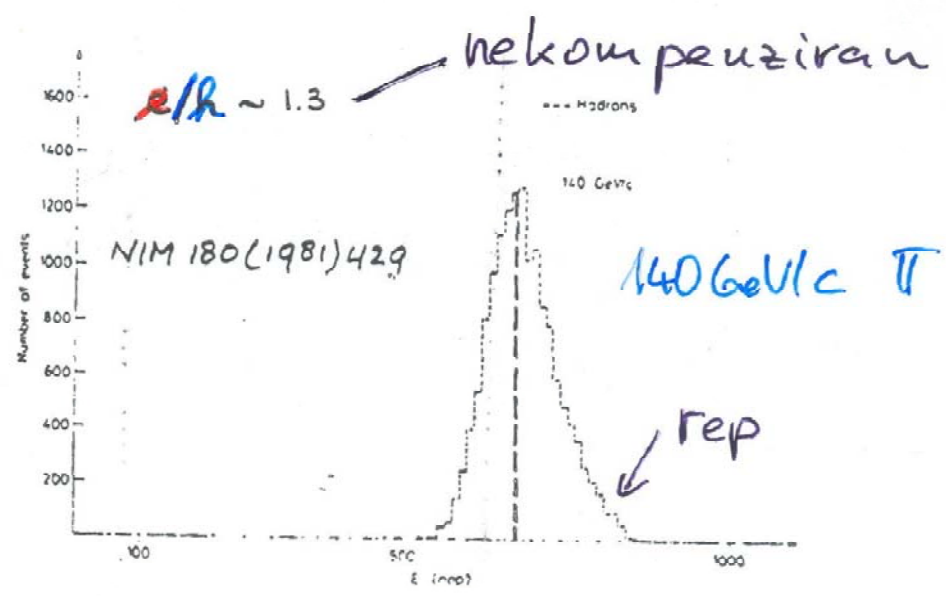
$\sigma(E)$  AND  $e/h$  AS FUNCTION OF SAMPLING FRAC  
(U-SCINTILLATOR CAL.)



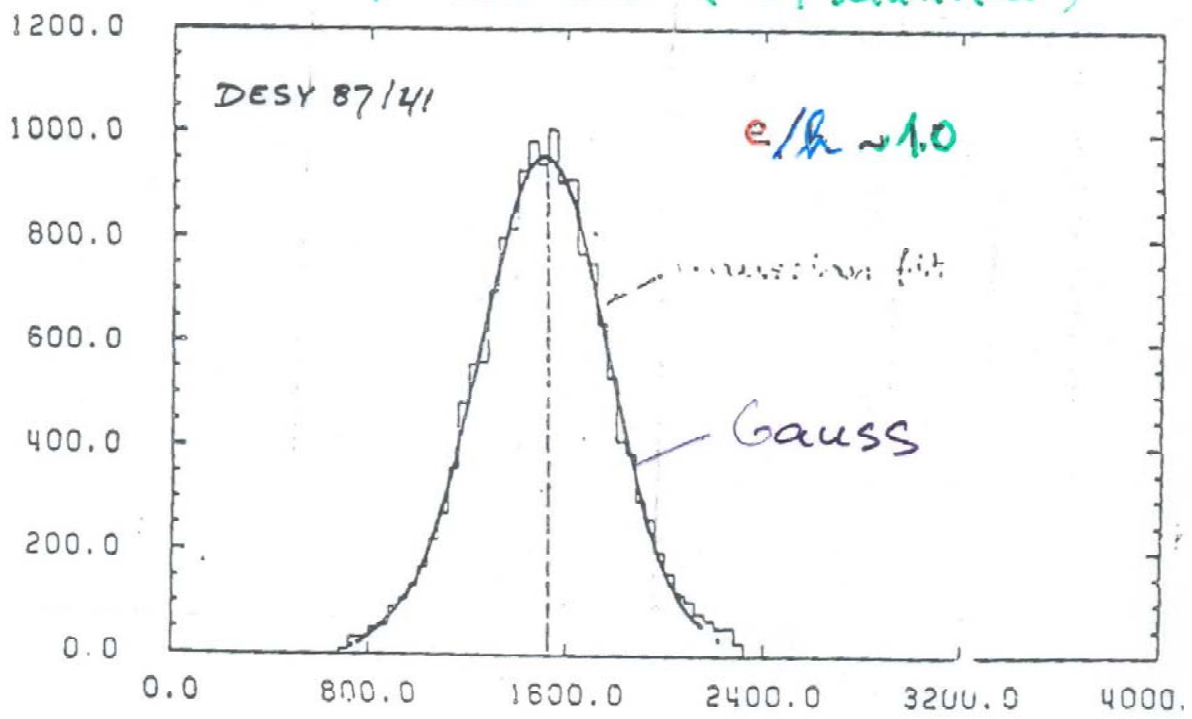
optimumu blizu  $\frac{20\%}{\sqrt{E}}$  - teoretična meja

! mogućnost pre-kompenzacije ( $e/h < 1$ )

### HADRONIC LINE SHAPE



### Kompenziran (u / scintilator)





Kompensacija z uteženjem  
 (R<sub>z</sub> in R<sub>z</sub> iz longitudinalnega profila plazme) 23

100 GeV π

$$\frac{\sigma_E}{E} = \frac{80\%}{\sqrt{E}}$$

uteži

$$\frac{\sigma_E}{E} = \frac{50\%}{\sqrt{E}}$$

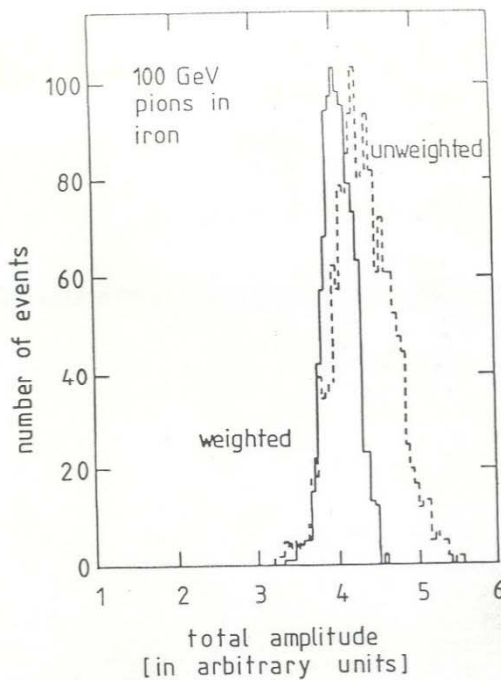


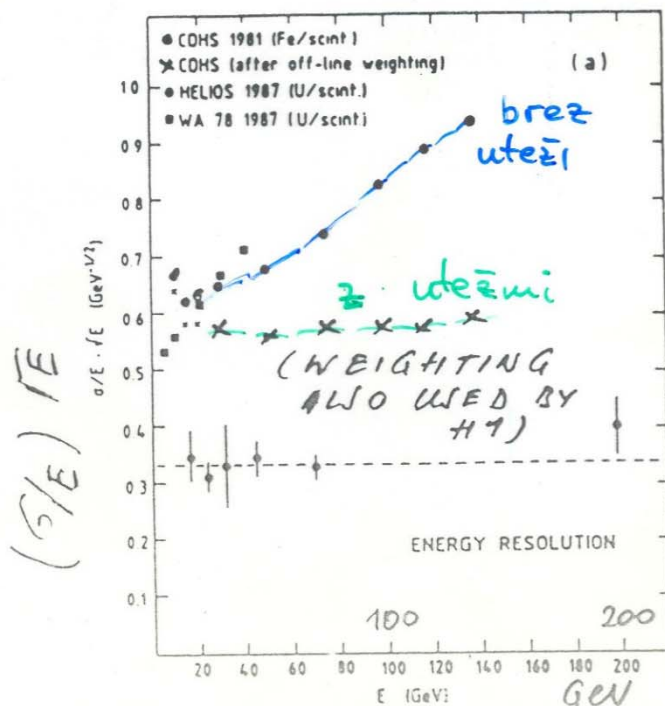
Fig. 7.45. Amplitude distribution for 100 GeV pions in an iron streamer-tube calorimeter with and without weighting factors [518]. The weighting factor was optimized for an incident energy of 100 GeV.

brez uteži

$$\frac{\sigma_E}{E} \propto E^{-\alpha} \quad \alpha < 1/2$$

z uteži

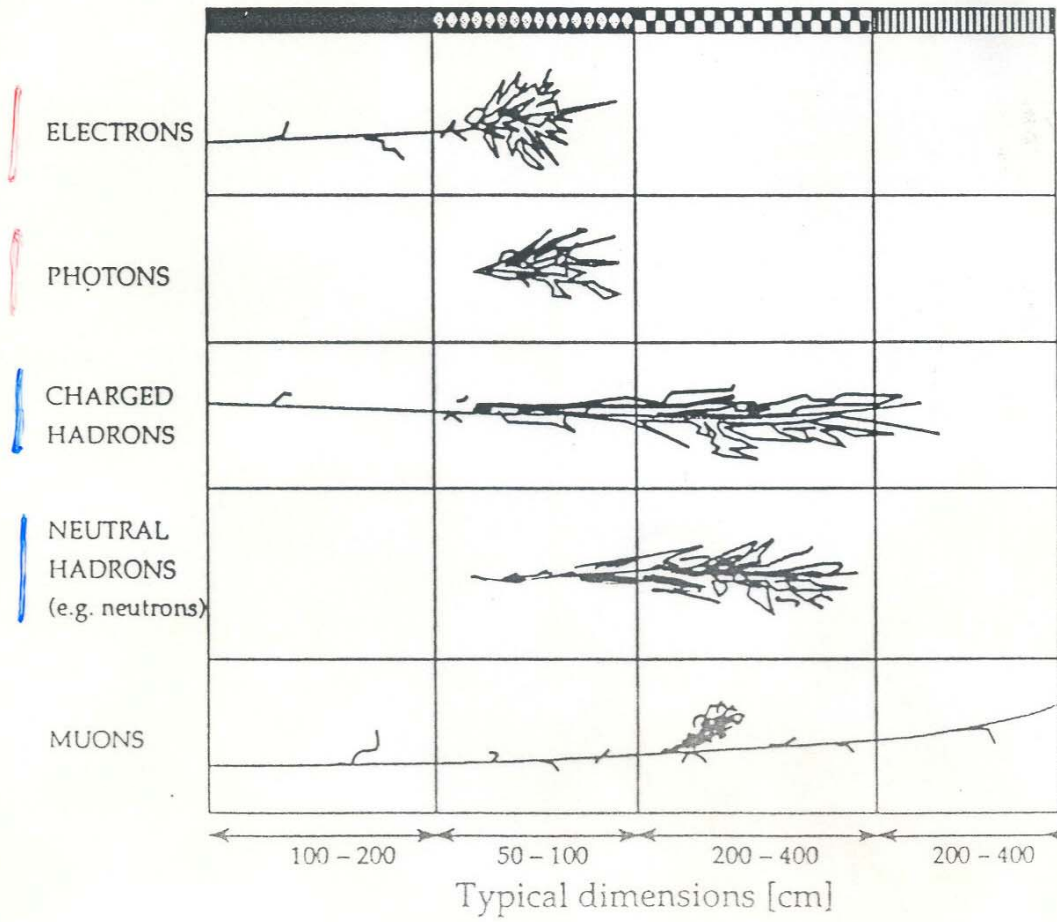
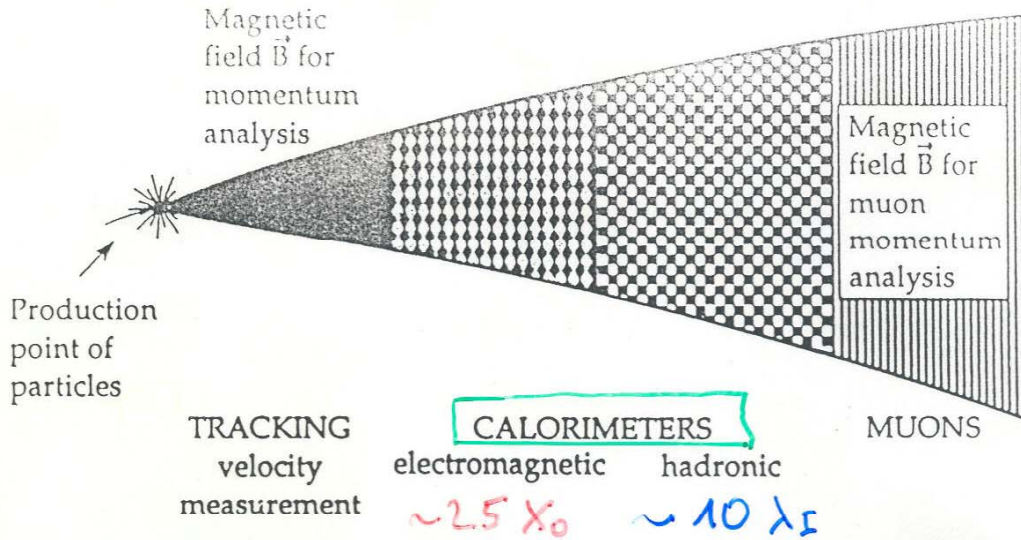
$$\frac{\sigma_E}{E} \propto \frac{1}{\sqrt{E}}$$



# g) identifikacija skalorimetri

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Osnova: različne interakcije  $e, \gamma, \pi, p, n, k, \mu$



merjenje energije - osnove kalorimetrije

$e \gamma \leftrightarrow$  hadroni

$X_0 \leftrightarrow \lambda_I$

$$\frac{X_0}{\lambda_I} = \frac{180 \text{ A}}{35 Z^2 \text{ A}^{1/3}}$$

$$\sim 8 Z^{-4/3}$$

0,6 nah

0,1 Fe

0,02 Pb

velik  $Z$  za EM kalorimeter

$\Rightarrow$  dobro ločevanje EM in hadronskega dela plazm

$\rightarrow e^\pm/\pi^\pm$  ločevanje (upr. za  $H \rightarrow Z^0 Z^0 \rightarrow e^+ e^-$ )

omejitev 1. interakcija

$\pi^\pm N \rightarrow \pi^0 N$

izmenjava naboja  
verjetnost  $10^{-2} - 10^{-3}$

$\gamma \leftrightarrow \pi^0$  ločevanje (upr. za  $H \rightarrow \gamma\gamma$ )

$\pi^0 \rightarrow 2\gamma$  pod majhnim kotom

$\Rightarrow$  zelo fino deljen prvi del ( $\sim 2X_0$ )  
EM kalorimetra  $\rightarrow$  pre shower

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