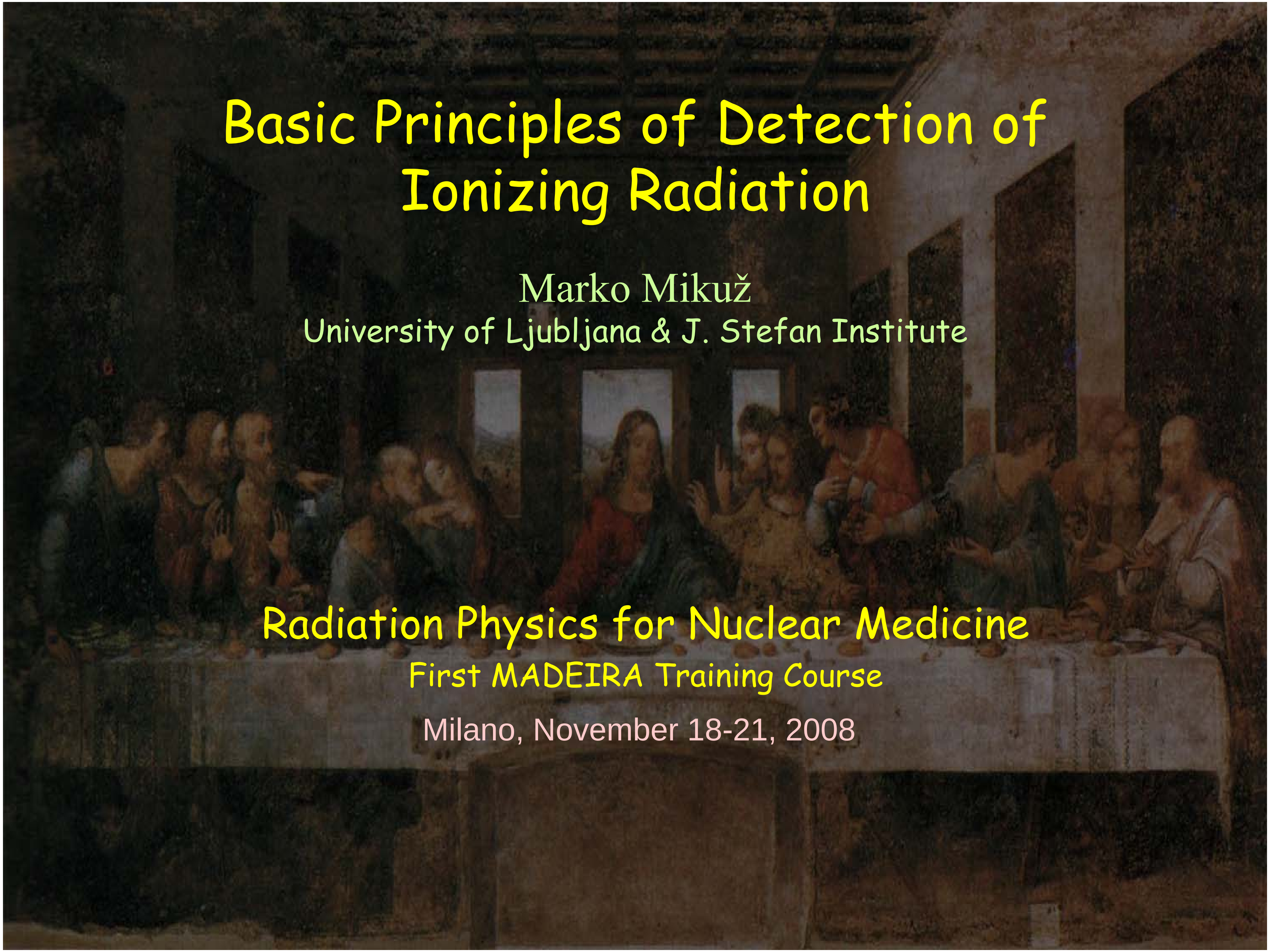

Basic Principles of Detection of Ionizing Radiation

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Radiation Physics for Nuclear Medicine

First MADEIRA Training Course

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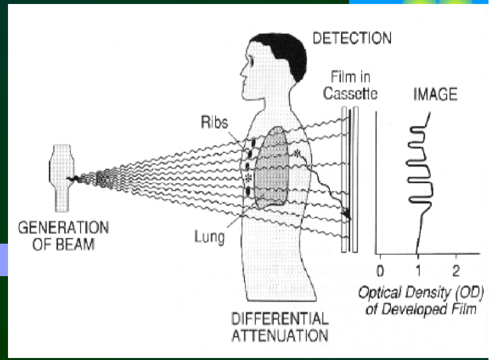
Outline

- Radiation in medical imaging
- Interaction of photons with matter
  - Photoelectric effect
  - Compton scattering
- Statistics primer
- Generic detector properties
- A (non)-typical example
  - Scatter detector of Compton camera

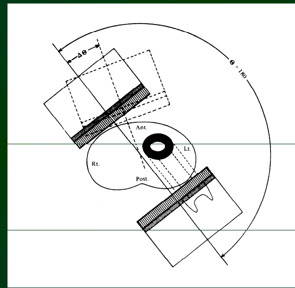
Main reference: G.F. Knoll: Radiation Detection and Measurement, J.Wiley&Sons 2000

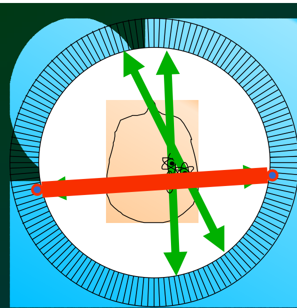
Radiation in Medical Imaging

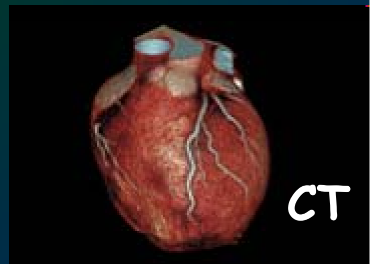


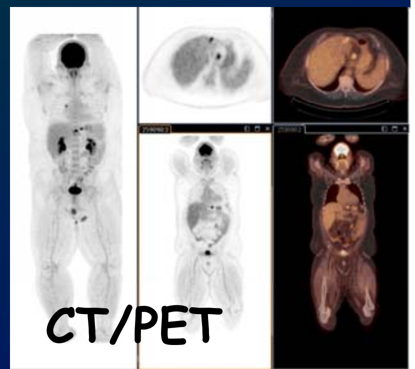
- Diagnostic imaging
  - X-rays
    - Planar X-ray
    - Transmission Computed Tomography (CT)
    - Contrast provided by absorption in body: $\mu(\underline{r})$
  - Gamma sources
    - Emission Computed Tomography
      - SPECT
      - PET
    - Contrast provided by source distribution in body: $A(\underline{r})$

✍ Both photons of $E_\gamma \sim 20 \leftrightarrow 500$ keV



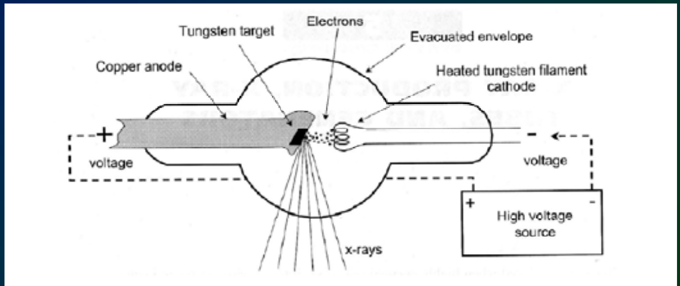




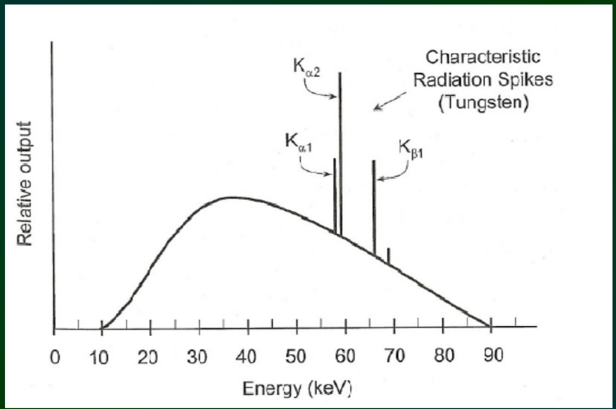


X and γ-rays

- X-ray tube



- Spectrum of W anode at 90 kV



- Typical radio-isotopes

Isotope	Energy (keV)	Half-life
^{99m}Tc	140.5	6 h
^{111}In	171 245	2 d
^{131}I	364 391	8 d
^{22}Na, ^{18}F, ^{11}C, ^{15}O	PE: 2x511	1.8 h – 3y

- Bonded to a bio-molecule

✍ Radio-tracer

Interaction of photons with matter

- Photons unlike charged particles with continuous ionization exhibit "one-off" interactions
  - Primary photon lost in this process
  - Resulting charged particles ionize and can be detected
- Photon flux is attenuated

$$\phi(x) = \phi_0 e^{-\mu x}$$

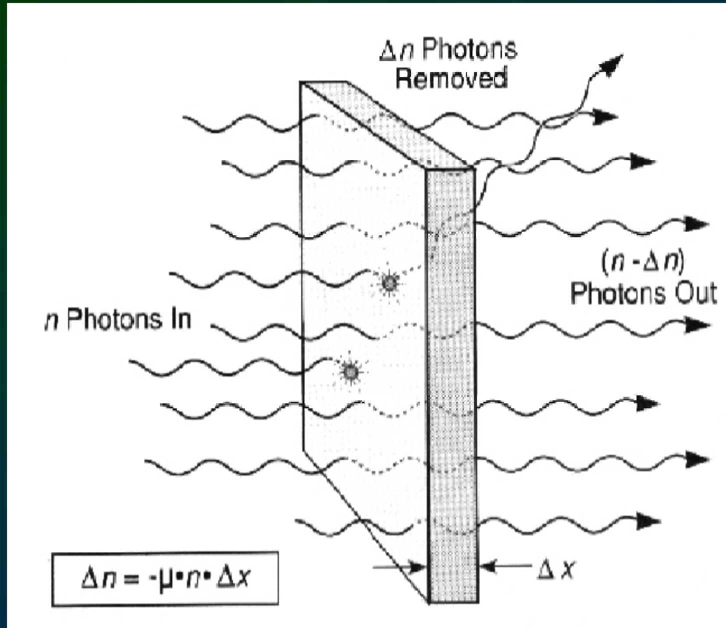
μ – linear attenuation coefficient [cm^{-1}]

$\lambda = 1/\mu$ – attenuation length, mean free path

- Attenuation scales with density

μ/ρ – mass attenuation coefficient [cm^2/g]

ρx – surface density, mass thickness [g/cm^2]



Mass attenuation coefficients

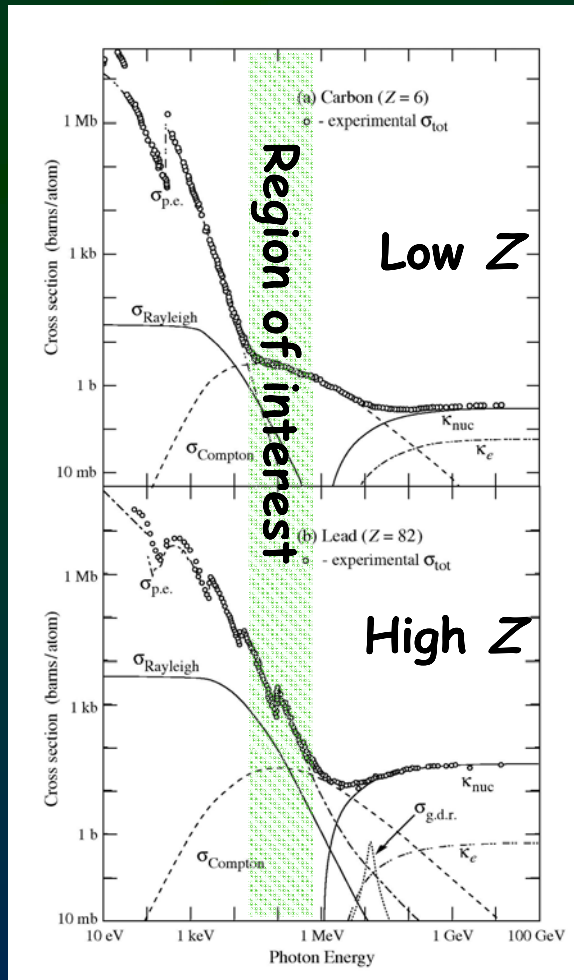
- Linked to cross section by

$$\mu = \frac{\rho N_A}{A}\sigma$$

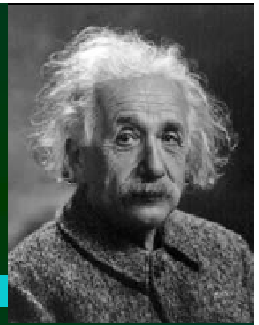
- For interesting photon energies two physical processes prevail
  - Photoelectric effect
  - Compton scattering
- High vs. low Z comparison
  - σ higher by up to 3 orders of magnitude at low $E\gamma$ for high-Z
  - Features in spectrum for high-Z

Complete set of tables for μ available at:

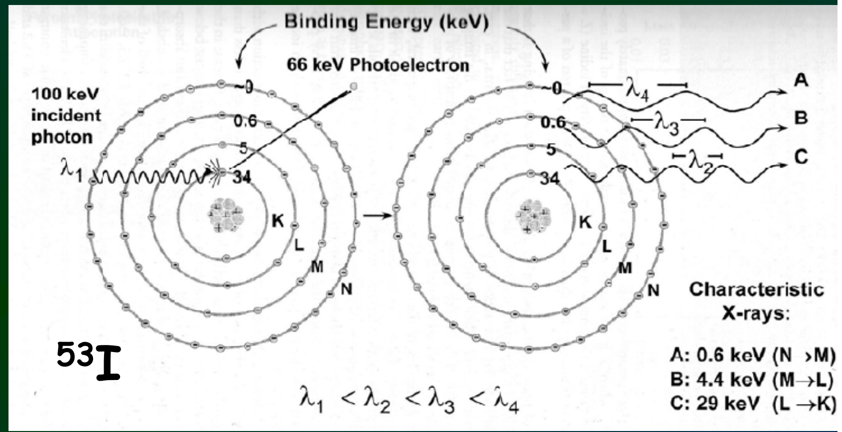
http://physics.nist.gov/PhysRefData/XrayMassCoef/cover.html



Photoelectric effect

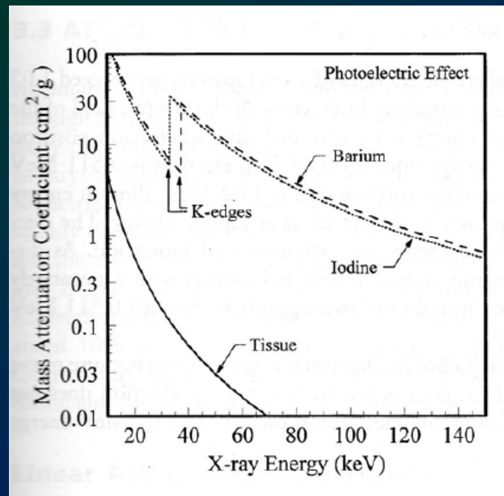


- Photon hits bound electron in atom
  - Electron takes *E*γ reduced by its binding energy
  - Momentum taken up by atom
  - Characteristic X-rays emitted
  - Tightly bound (K-shell) electrons preferred
    - Cross section rises by orders of magnitude upon crossing threshold – K-edge

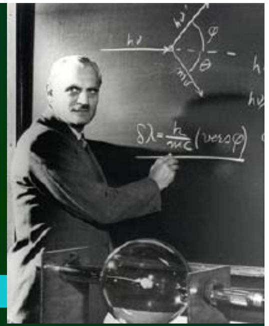


- Above K-edge

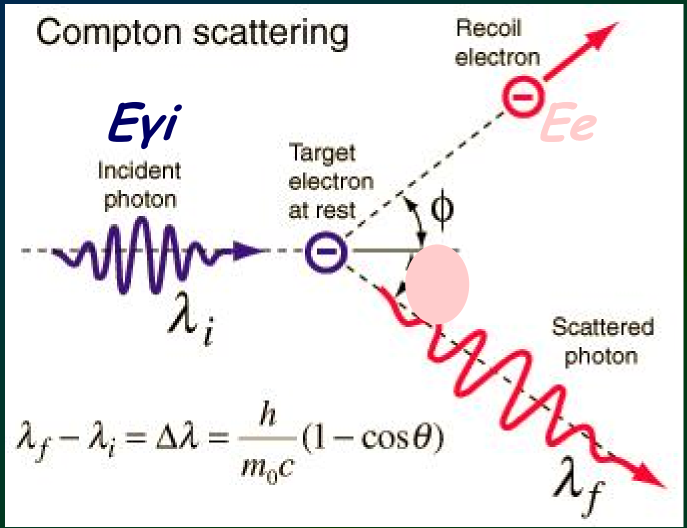
$$\sigma_{PE} \propto Z^5 / E^{3.5}$$



Compton scattering



- Photon elastic scattering on (quasi)-free electron
  - Photon scattered and reduced energy



$$\lambda_f - \lambda_i = \Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

$$\sigma_C \propto Z/E \propto \rho_{el}/E$$

$$\sin^2\left(\frac{\theta}{2}\right) = \frac{1}{2\mu}\frac{\varepsilon}{1-\varepsilon}$$

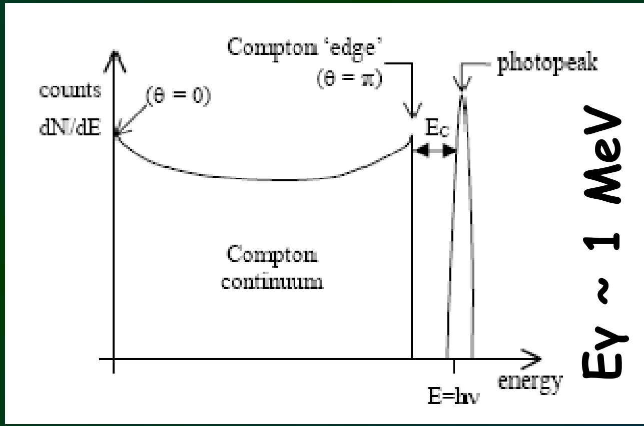
- Θ – photon scattering angle
- $\mu = E_{\gamma i} / m_e c^2$
- $\varepsilon = E_e / E_{\gamma i}$

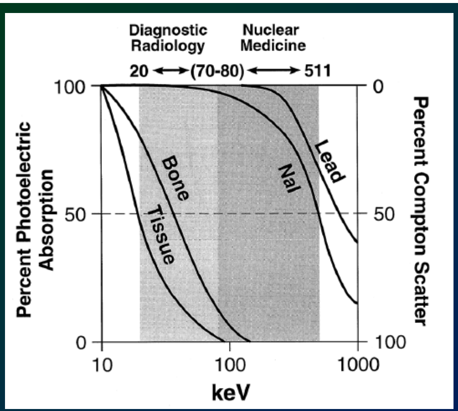
Compton scattering (cont.)

- Electron energy spectrum
  - Maximum $E\gamma$ transfer at Compton edge – backward scattering

$$T_{e,\max} = E_{\gamma} \frac{2\mu}{1+2\mu} \xrightarrow{\mu \ll 1} E_{\gamma} \frac{E_{\gamma}}{m_e c^2}$$

  - Small transfers for low $E\gamma$
  - Photons continue with ~same energy change direction
    - ☹ Bad for photon detection !
    - ☹ Even worse for imaging …





- Photoelectric vs. Compton

$$\sigma_{PE} / \sigma_C \propto Z^4 / E^{2.5}$$

- ➢ Use high Z for detectors
- ➢ Use lower $E\gamma$ for imaging

Statistics primer

- *N* independent measurements of same quantity:

$$x_1, x_2, x_3, \ldots, x_i, \ldots, x_N$$

- Frequency distribution function (discrete x)

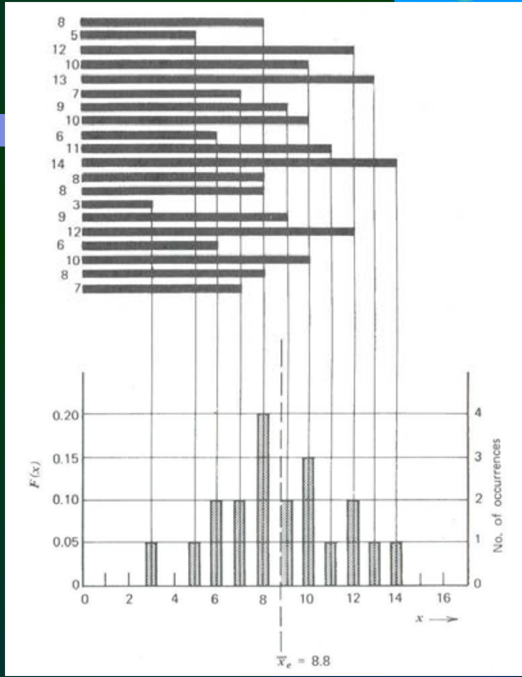
$$F(x) = \frac{N(x_i = x)}{N}; \quad \sum_{x=0}^{\infty} F(x) = 1$$

- Standard deviation from true mean

$$\sigma^2 = \overline{(x_i - \bar{x})^2} = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2 = \sum_{x=0}^{\infty}(x - \bar{x})^2 F(x) = \overline{x^2} - \bar{x}^2$$

- Experimental mean and sample variance

$$\bar{x}_e = \frac{1}{N}\sum_{i=1}^{N} x_i = \sum_{x=0}^{\infty} xF(x) \qquad \sigma_e^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x}_e)^2$$



Questions asked

- How accurate is the measurement ?
  - Best experimental estimate

$$\bar{x} = \bar{x}_e \quad \sigma^2 = \sigma_e^2 \quad \sigma_{\bar{x}}^2 = \sigma^2 / N$$

  - For u derived of non-correlated measurements of $x,y,z,\ldots$

$$u = u(x,y,z,\ldots); \quad \sigma_u^2 = \left(\frac{\partial u}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial u}{\partial y}\right)^2 \sigma_y^2 + \left(\frac{\partial u}{\partial z}\right)^2 \sigma_z^2 + \ldots$$

- Is the equipment working properly ?
  - Confront measurements to (correct) model
- Is the underlying model correct ?
  - Confront model to (proper) measurements

Statistical model - Binomial

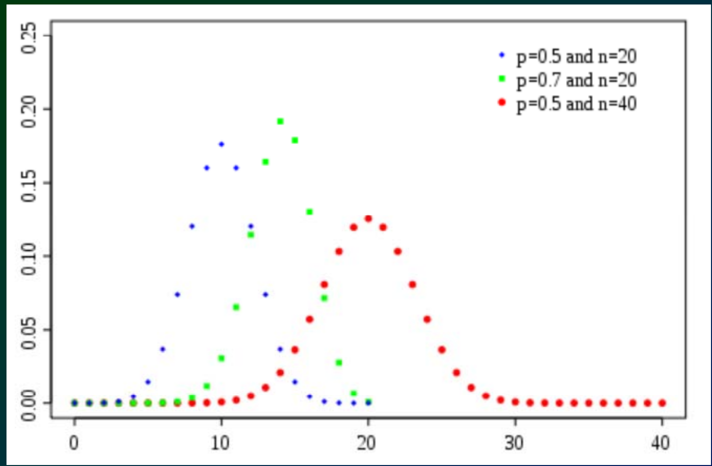
- Photon emission and detection a random (stochastic) process, like tossing a coin: *N* trials, *x* successes
- Counting experiment, integer (discrete) outcome
- *p* - success probability, e.g. *p* = 0.5 for a (fair) coin
- *x* – statistical variable, *P(x)* given by distribution:

- Binomial

$$P(x) = \frac{N!}{x!(N-x)!} p^x (1-p)^{N-x}$$

$$\bar{x} = Np$$

$$\sigma^2 = Np(1-p)$$



- Valid in general, but awkward to work with

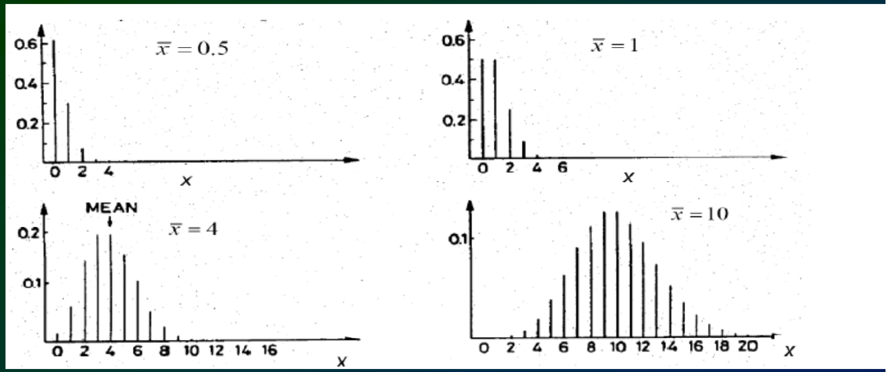
Statistical model - Poisson

- Often individual success probabilities *p* are small with a large number of trials *N*
- Binomial (*N*, *p*) → Poisson (*Np*)

$$P(x) \xrightarrow{N\to\infty,\, p\to 0,\, Np\to\bar{x}} \frac{\bar{x}^x e^{-\bar{x}}}{x!}$$

$$\sigma^2 = Np(1-p) \xrightarrow{p\to 0} Np = \bar{x}$$

$\bar{x}$ is now the only parameter !



- Possible to estimate both the mean and error from a single counting measurement !

$$\bar{x} = x \pm \sqrt{x}$$

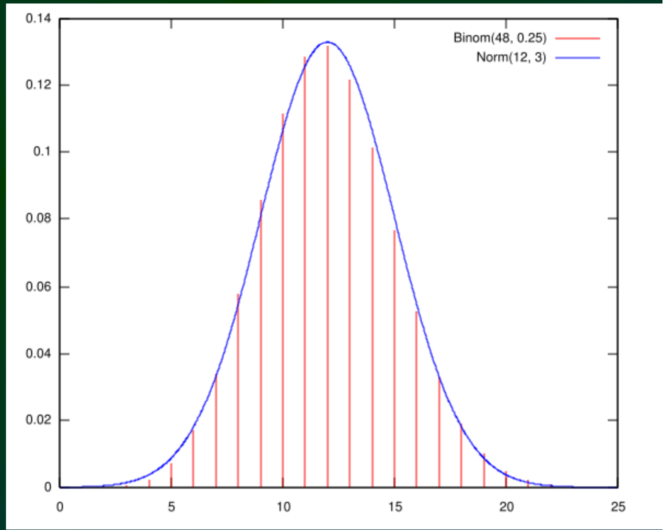
Statistical model - Gaussian

- If mean value of Poisson distribution ≥ 20
- Poisson → Gaussian

$$P(x) \xrightarrow{\bar{x} >> 1} \frac{1}{\sqrt{2\pi\bar{x}}} \exp\left(-\frac{(x-\bar{x})^2}{2\bar{x}}\right)$$

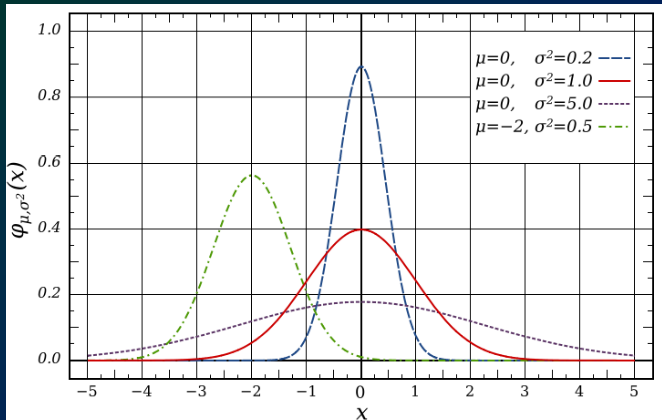
$$\sigma^2 = \bar{x}$$

$\bar{x}$ still the only parameter !



- Combination of measurements, due to Central Limit Theorem, leads to Gaussian distribution

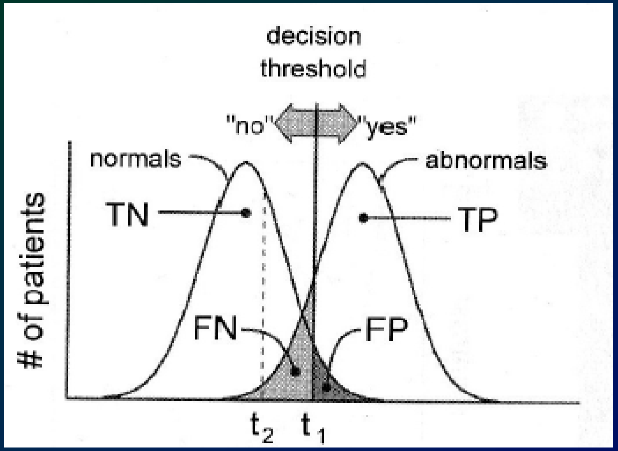
$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\bar{x})^2}{2\sigma^2}\right)$$



- Two parameters (mean, width)
- x can be a continuous variable

Statistical tests

- Confront measurement *F(x)* to model *P(x)*
  - Ignorant's attitude: Compare by eye ?
  - Scientific approach: Conduct a statistical test !
    - Most used: χ^2 test
    - Test yields probability *P* experiment matches model
    - If probability too low (e.g. $P < 0.05$)
      - *a) Question measurement if believe in model ?*
      - *b) Question model if believe in experiment ?*
      - *c) Accept lower probability ?*
      - *d) Take different model ?*
      - *e) Repeat measurement ?*
      - *f) Conduct other tests ?*
      - *…*
      - *z) Compare by eye ??*
- Eternal frustration of statistics
  - ☹ False positives vs. False negatives



Generic radiation detector

- For any γ-ray detection the following sequence applies
  - γ interacts in detector material resulting in an energetic electron (and eventual additional photons)
  - Electron ionizes detector material, creating additional electron-ion (or electron-hole) pairs – very fast process
  - Applied electric field in detector separates charges which drift towards collecting electrodes
    - Alternative: charges recombine at specific centers producing (visible) light- scintillation
  - Moving charges induce current on electrodes according to Shockley-Ramo theorem – collection time from ns to ms

$$i(t) = q\vec{v}\cdot\vec{E}_w = q\mu\vec{E}\cdot\vec{E}_w$$

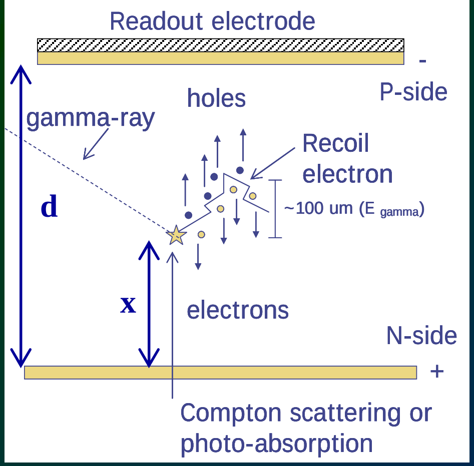
$i(t)$ - induced current

q - charge moving with velocity $\vec{v}$

$\vec{E}_w$ - weighting (Ramo) field in detector

$\vec{E}$ - electric field in detector

μ - charge mobility



Parallel electrode pair, no ρ_e

$$\vec{E}_w = 1/d \quad \vec{E} = V/d$$

$$i_+(t) = q\frac{\mu_+ V}{d^2} \quad 0 < t < \frac{(d-x)d}{\mu_+ V}$$

$$i_-(t) = q\frac{\mu_- V}{d^2} \quad 0 < t < \frac{xd}{\mu_- V}$$

$$i(t) = i_+(t) + i_-(t)$$

d - electrode spacing = detector thickness

V - applied voltage

x - γ interaction distance from + electrode

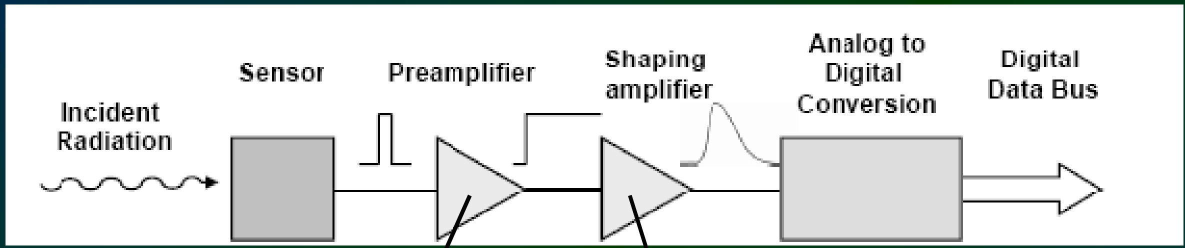
- Sometimes *E* is strong enough to provoke further ionization – charge multiplication
- Current signal gets processed and analyzed in front-end and read-out electronics

What do we want to measure ?

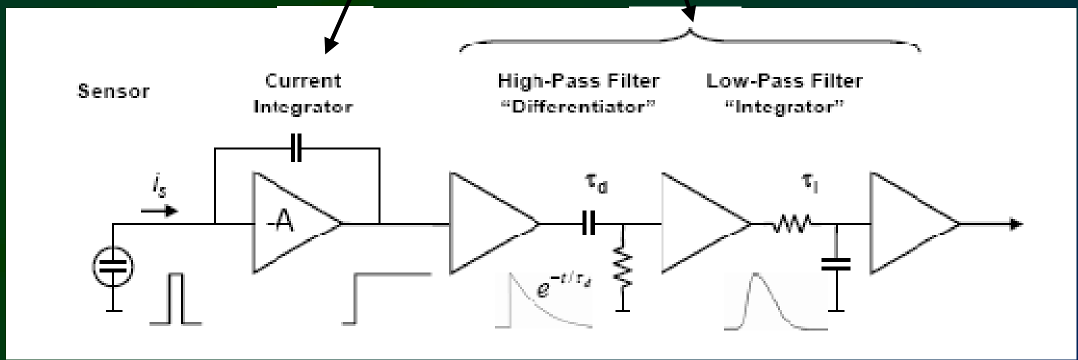
- Signal from detector - time-dependant current pulse
  - No charge trapping and no amplification ⇒ collected charge $Q = \int i(t)dt = Q_{ionization} \propto E_e$
    - $E_e \sim E_\gamma$ in photopeak
    - Handle on Compton scattering !
  - Q build-up during charge collection time
    - $t_{coll} \sim d^2/(\mu V)$ can be some ns for thin semiconductor detectors
    - Fast timing – narrow coincidences – reject random background in PET !
- Good reasons to count individual pulses, extracting Q and t
- Still for dosimetry applications average current measurement can be sufficient ($\propto$ dose-rate)

Signal (pulse) processing

- Basic elements of a pulse-processing chain

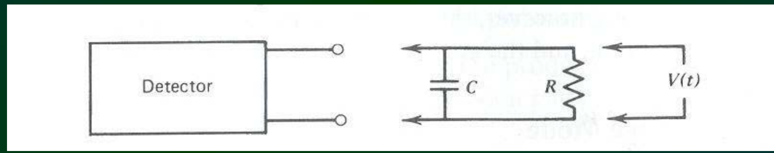


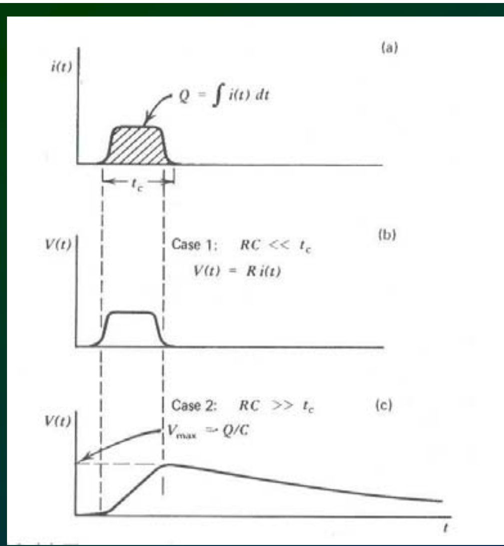
- Expanded view of preamplifier and shaper

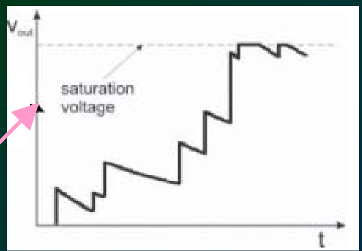


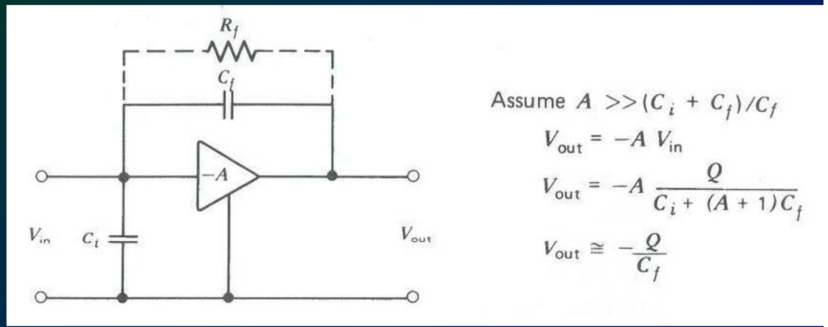
Preamplifier

- Possible simple configuration
  - R – amplifier input resistance
  - C – sum of C_{det}, C_{cable} and C_{amp}
  - $RC \ll t_{coll}$: current sensitive
  - $RC \gg t_{coll}$: charge sensitive
    - $t_{rise} \sim t_{coll}$
    - $t_{fall} \sim RC$
    - $V_{max} \sim Q/C$
  - C is dominated by C_{det}, which can exhibit variations
- Useful configuration – feedback integrator
  - $A \times C_f \gg C_{det}$: V independent of C_{det}
  - R_f needed for restoration to base-line, preventing pile-up









Energy resolution

- Intrinsic resolution
- Statistical noise in charge generation by radiation
  - Expect a stochastic process with variance

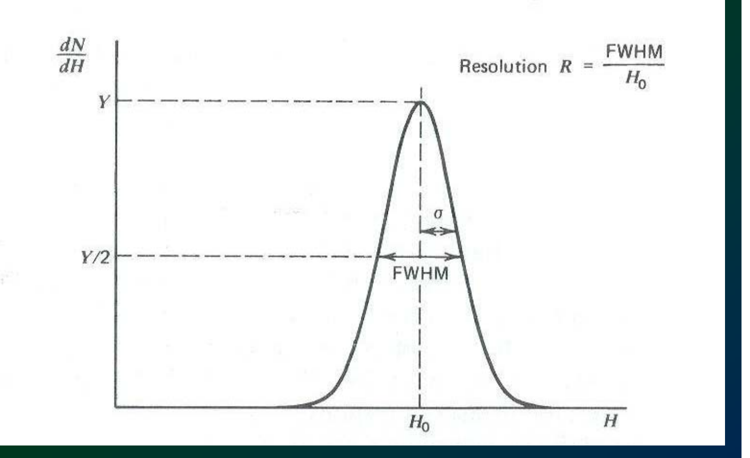
$$\sigma^2 = N_e = Q/q_0 \xrightarrow{photopeak} E_\gamma / E_{ion}$$

$$\frac{\sigma}{E_\gamma} = \frac{\sqrt{N_e}}{N_e} = \frac{1}{\sqrt{N_e}} = \sqrt{\frac{E_{ion}}{E_\gamma}}$$

  - Lower average ionization energy (e.g. Si or Ge) gives better resolution
  - Process not truly stochastic; all E lost must sum up to E_γ! Corrected by Fano factor F

$$\sigma^2 = FN_e \qquad \frac{\sigma_E}{E_\gamma} = \sqrt{\frac{FE_{ion}}{E_\gamma}}$$

  - F depends on E sharing between competing processes (ionization, phonons)
  - Measured $F \sim 0.1$ in Si & Ge; resolution improved by factor 3 !



- Full-Width at Half Maximum → universally accepted FOM for resolution
  - For Gaussian distribution

$$FWHM = 2\sigma\sqrt{2\ln 2} = 2.355\sigma$$

  - So the energy resolution R is

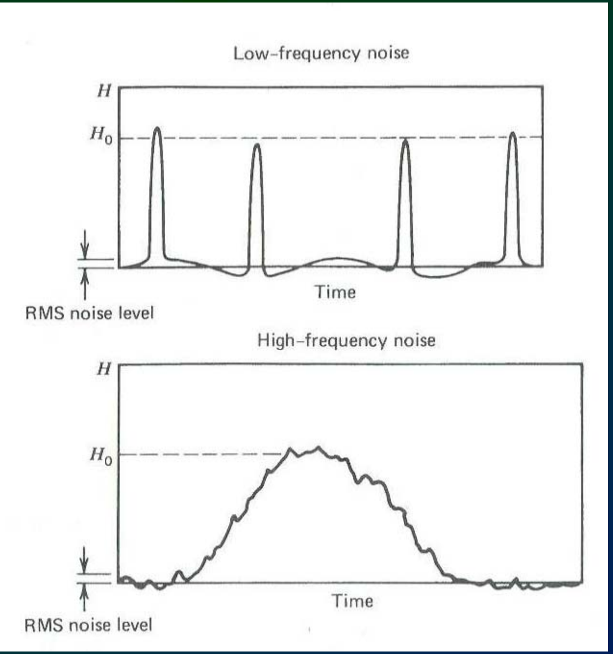
$$R = \frac{FWHM}{E_\gamma} = 2.355\sqrt{\frac{F}{N_e}} = 2.355\sqrt{\frac{FE_{ion}}{E_\gamma}}$$

Noise considerations

- Intrinsic resolution deteriorates with additional noise sources in read-out
- The signal and its noise; two sources

$$i \propto Qv \quad \Rightarrow \quad \left(\frac{\sigma_i}{i}\right)^2 = \left(\frac{\sigma_v}{v}\right)^2 + \left(\frac{\sigma_Q}{Q}\right)^2$$

  - Fluctuations in velocity – thermal noise
  - Fluctuation in charge
    - ✓ Intrinsic fluctuations
    - Fluctuations in underlying leakage current if injected (or generated) discretely – Shot noise
- Noise characterized by noise power spectrum - *dP/dv*
- Thermal and Shot noise have white spectra: *dP/dv = K*



- The signal gets conditioned by the preamplifier
- For charge sensitive pre-amp
  - Thermal noise → equivalent voltage noise source
  - Shot noise → equivalent current noise source
- Pre-amp (and other parts of the system) add their own noise sources
  - Sources (mostly) uncorrelated → noise contributions add in quadrature

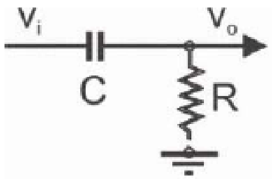
Shaper

- White spectra – noise at all frequencies
- Signal – frequencies around $1/t_{coll}$ only
- ➢ Filter out low and high frequencies to improve *S/N*
- Task of the shaper
  - Also shape signal so amplitude and time can be determined
- Basic functionality: *CR* and *RC* filters

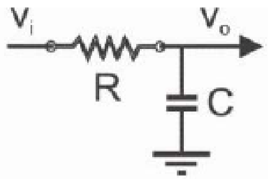
Differentiator
(CR, high-pass filter)

$$V_o = \int_0^t \frac{dV_i}{dt} e^{(t-t')/\tau} dt'$$

$$\Rightarrow V_o = V_i e^{t/\tau}$$

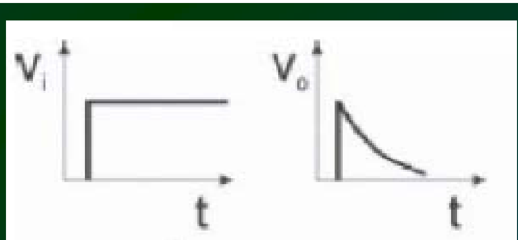


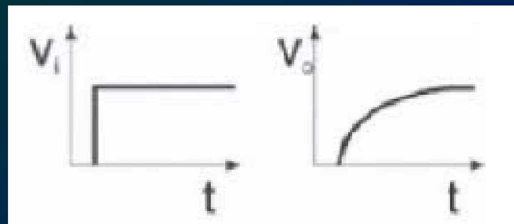
Integrator
(RC, low-pass filter)



$$V_o = -\frac{1}{\tau}\int_0^t V_i e^{(t-t')/\tau} dt'$$

$$\Rightarrow V_o = V_i (1 - e^{t/\tau})$$

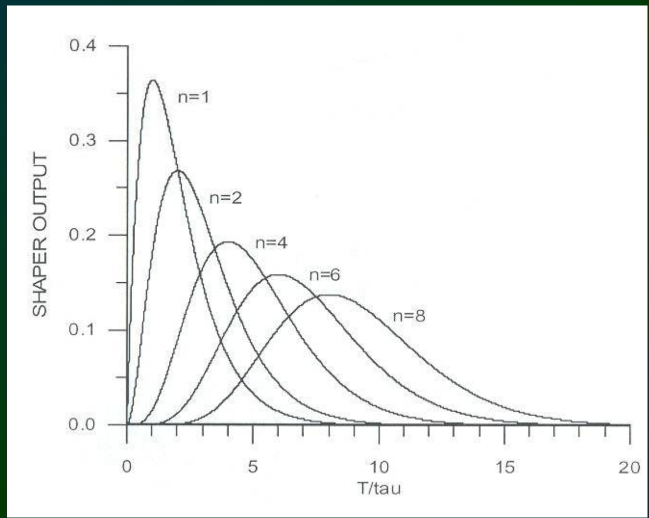


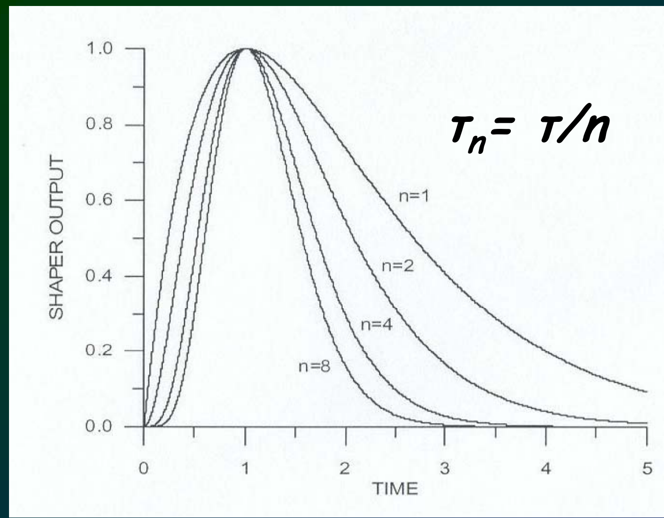


Shaper (cont.)

- Several CR and RC filters in sequence, decoupled by op-amps: *CR-RC*, *CR-RC^n^*, …
- Response of *CR-RC^n^* to step function V_0

$$V_{out} = V_0 \left(\frac{t}{\tau} \right)^n e^{-t/\tau}$$





- For equal peaking time
  - *CR-RC* fastest rise-time – best for timing
  - *CR-RC^n^* with $n > 4$ symmetric – faster return to baseline – high rates

Noise of detection system

- Shaper with peaking time τ reduces bandwidth
- Noise of detector & read-out turned into equivalent charge fluctuations at input – equivalent noise charge *ENC*
- FOM is signal to noise *S/N = Q/ENC*
- For charge sensitive pre-amp
  - Thermal (voltage) noise
  - Shot (current) noise
- No universal recipe

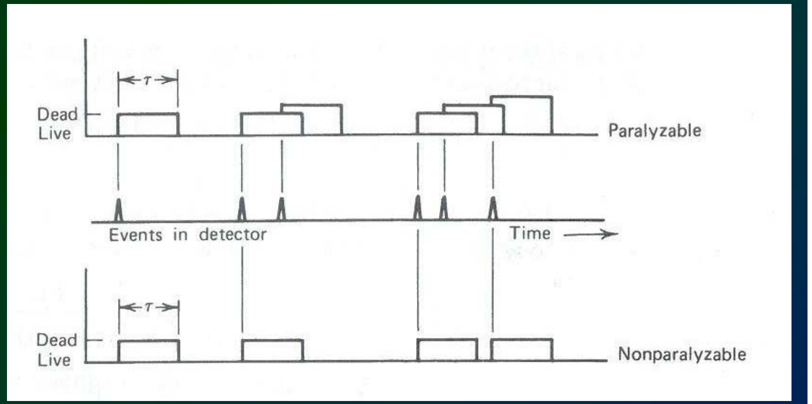
➢ Optimize τ case-by-case

$$ENC_{thermal} \propto \frac{C_{input}}{\sqrt{\tau}}$$

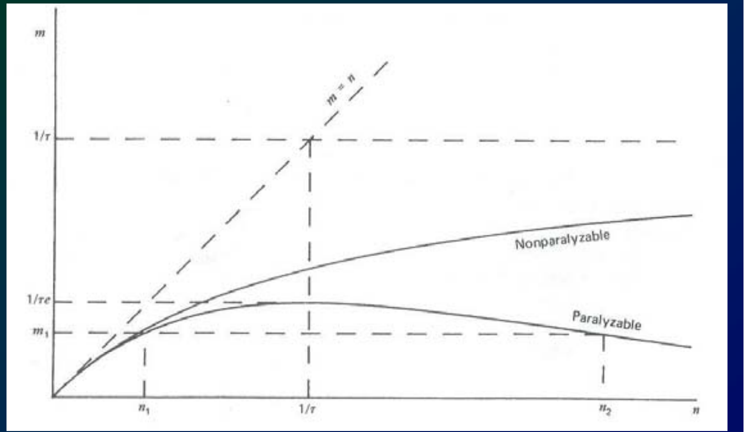
$$ENC_{Shot} \propto \sqrt{I_{leak}\tau}$$

Dead time

- Detection system can be inactive for dead-time τ for various reasons
  - Detector bias recharge (GM)
  - ADC conversion time
- Two models of interference
  - Signals during dead-time pass by unnoticed
    - Non-paralyzable model
  - Signals during dead-time lost & induce own dead-time
    - Paralyzable model

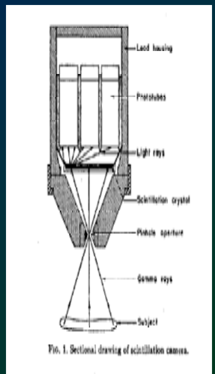


- Relation between observed pulse rate m and true rate n
  - Non-paralyzable model
    $$n = \frac{m}{1 - m\tau}$$
  - Paralyzable model
    - Solve for n iteratively
    - Two ambiguous solutions
    $$m = n \times e^{-n\tau}$$



Anger Camera – Mechanical Collimation

- SPECT imager – Anger camera
- Need collimator to reconstruct photon direction



Anger 1957 Siemens 2000

Typical collimator properties

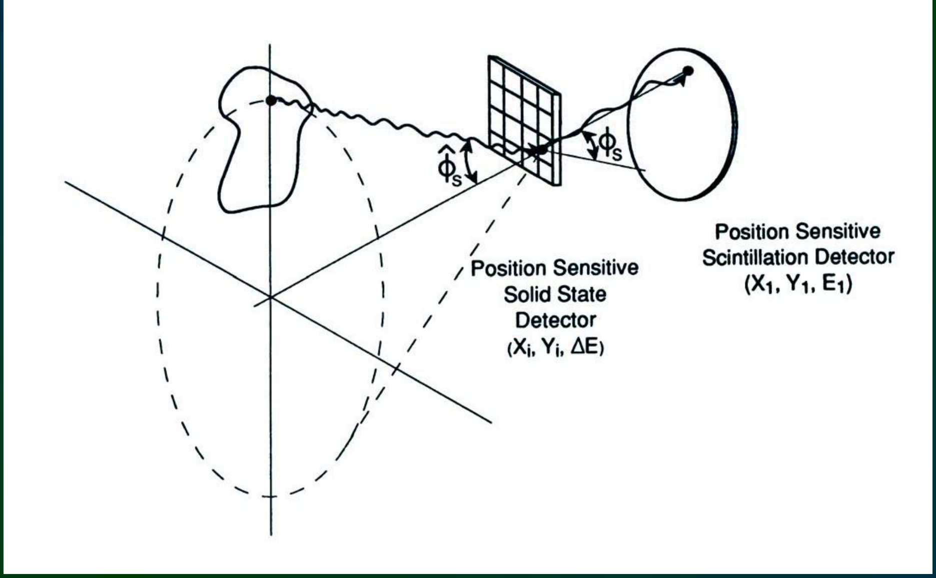
Parallel plate collimators	Efficiency	Resolution at 10 cm
High sensitivity low energy	5.7×10^{-4}	13.2 mm
High resolution low energy	1.8×10^{-4}	7.4 mm
High sensitivity medium energy	1.1×10^{-4}	15.9 mm
High resolution medium energy	4.0×10^{-5}	10.5 mm

Low efficiency, coupled to resolution ($\varepsilon.\sigma^2$ ~ const.), worse @ higher E_γ, bulky

☞ standard medical imaging technique

Compton Camera – Electronic Collimation

- Replace mechanical collimator by active target (scatter detector) to Compton scatter the photon
- Detect scattered photon in position sensitive scintillator (Anger camera head w/o collimator)
- Reconstruct emitted photon from Compton kinematics



•Old idea

Todd, Nightingale, Evrett:

A Proposed γ-Camera, Nature 1974

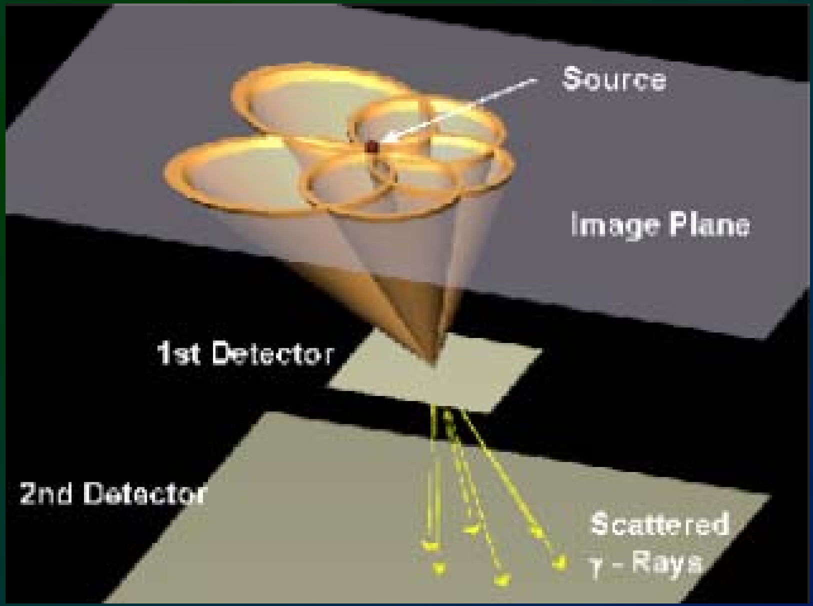
•Compton telescopes standard instrument in γ-ray astronomy

Compton Camera – The Principle

- Measure position of scattering and absorption
- Measure electron (and photon) energy

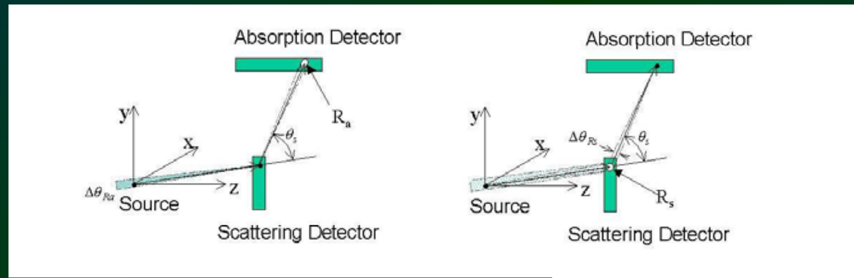
➢ Each measurement defines a cone with angle *Θ* in space

➢ Many cones provide a 3-D image of the source distribution

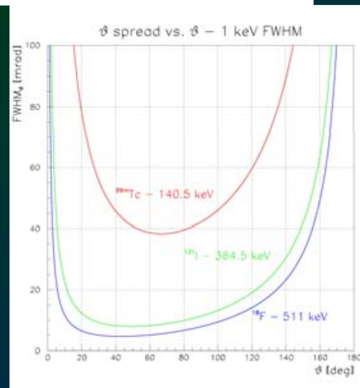


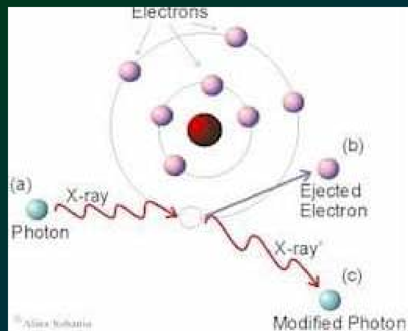
Compton Camera – The Small Print

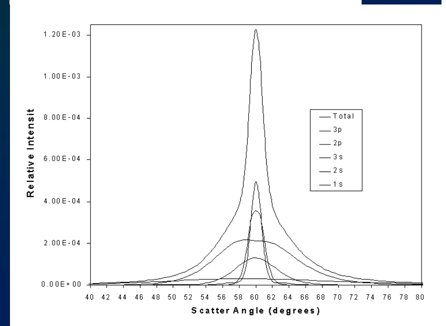
- Error on the source position results from
  - Position resolution
    - Error on cone axis
    - Place absorber far from scatter (solid angle, cost)
    - Place scatter close to source – near field imaging
  - Electron energy resolution
    - Error on cone angle
  - Doppler broadening
    - Electron bound in atoms
    - $\vec{p}_e \neq 0$, broadening in θ



$$\Delta\theta = \frac{(1+\mu(1-\cos\theta))^2}{E_{\gamma i}\,\mu \sin\theta}\,\Delta E_e$$

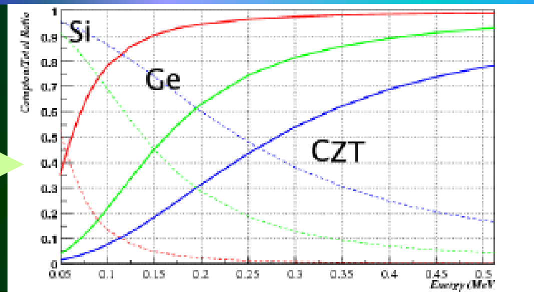


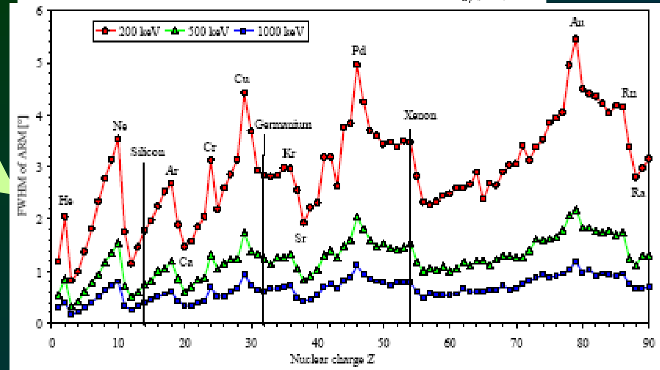


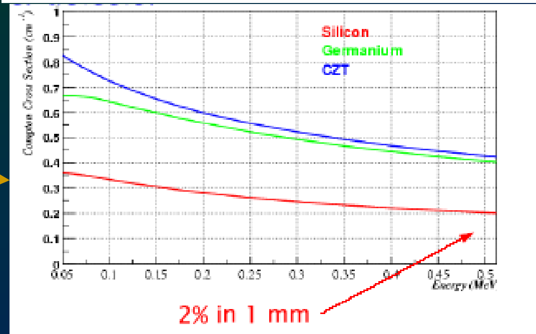


Rationale of Si as Scatter Detector

- Silicon exhibits
  - ☺ Highest Compton/total x-section ratio
  - ☺ Smallest Doppler broadening
  - ☺ Excellent energy and position resolution
  - ☺ Mature technology
  - ☺ Simple operation (hospital !)
  - ☺ Reasonable cost
  - ☹ Low efficiency ~ 0.2/cm
    - ➢ Thick detectors 0.3 → ~1 mm
    - ➢ Stack for higher efficiency







Energy Resolution

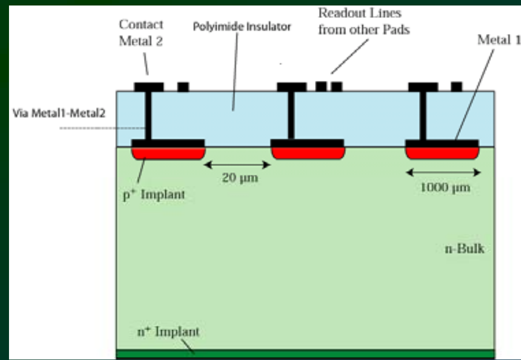
- Statistical
  - $\Delta E_{FWHM} = 2.35 \sqrt{F N}$
  - 140/511 keV: $\Delta E_{FWHM} \sim$ 55/200 e ~ 200/720 eV
- Electronics
  - Voltage noise $\propto (C_{int}+C_{det}) / \sqrt{\tau_p}$
  - Current noise $\propto \sqrt{(I_{det} \tau_p)}$

Even in optimized systems electronics noise dominates

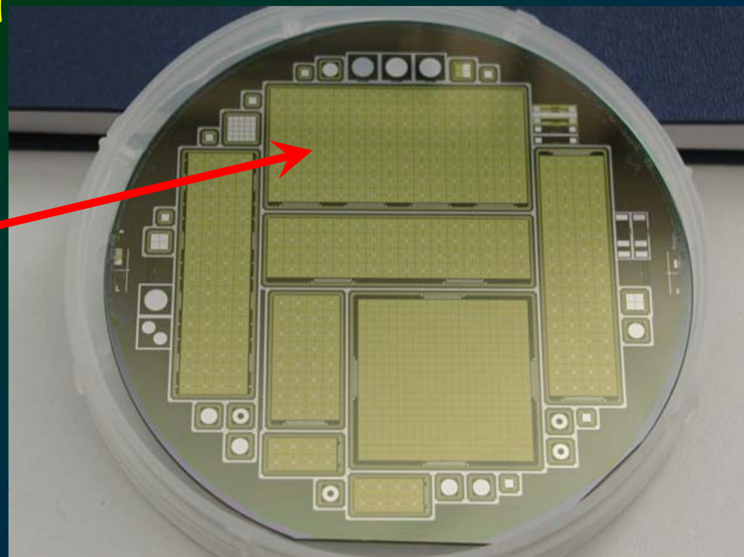
☞ 1 keV FWHM (σ_{noise} = 120 e) a challenge

Silicon Sensors

- 1 mm thick p^+-n pad sensors
- Pad dimensions 1.4 mm x 1.4 mm
- Routed to bond pads at detector edge through double metal
- Full depletion ~ 150 V for 1 mm
- Very low leakage current ~ 50 pA/pad

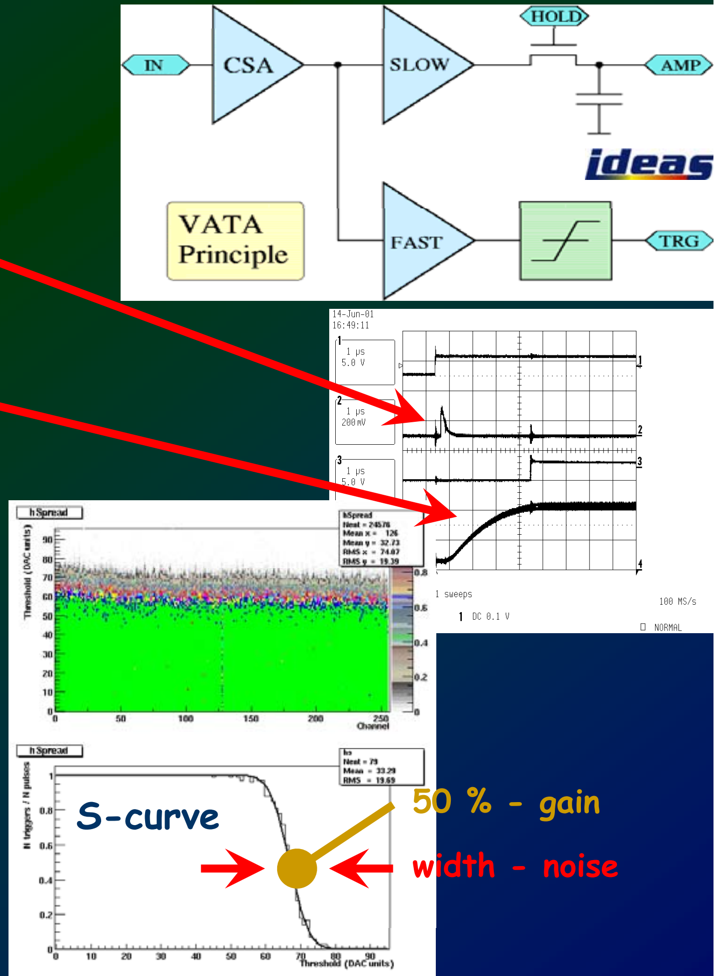


- ➢ Produced by SINTEF, Norway
- ➢ 512-pad (16x32) detectors used for this prototype
- ➢ Active area 22.4 mm x 44.8 mm

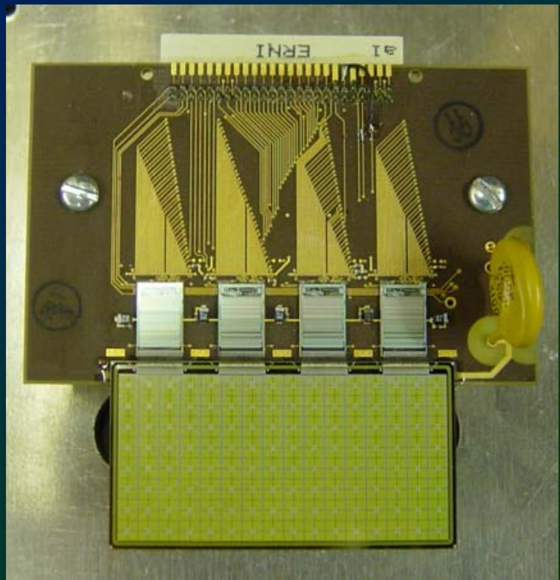


VATAGP3 Read-Out Chip

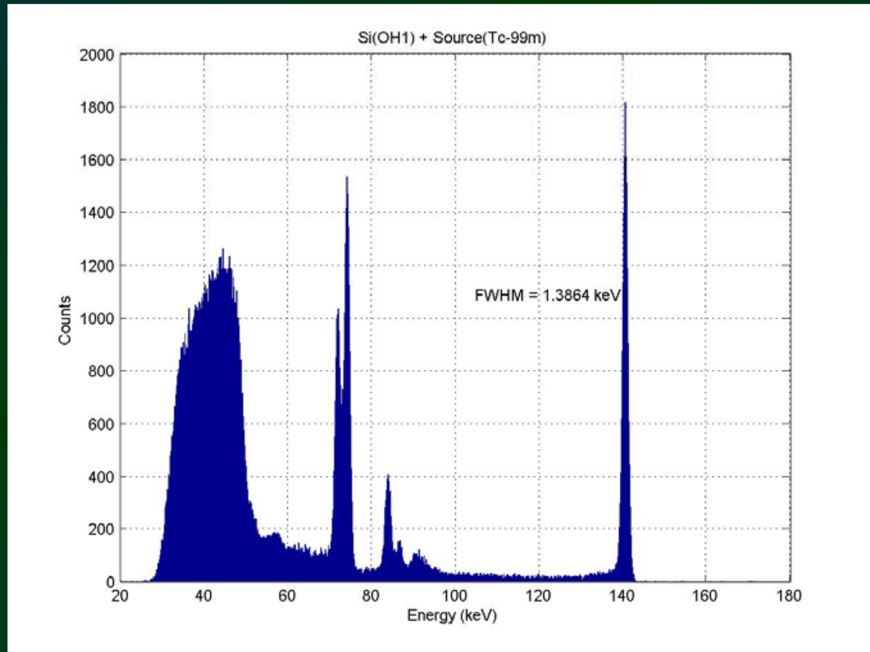
- 128-channel self-triggering ASIC produced by IDE AS, Norway
  - Charge-sensitive pre-amplifier
  - TA channel: fast-shaper (150 ns) & discriminator for self-triggering
  - Trim-DAC’s for threshold alignment
  - VA channel: low-noise slow shaper (0.5-5 μs) for energy measurement
  - Read-out of up to 16 daisy-chained chips
    - Serial: all channels
    - Sparse: channel triggering with address
    - Sparse ± specified number of neighbouring channels
  - 2 multiplexed analogue outputs (up, down)
  - Calibration circuitry for diagnostics



Silicon Pad Module



Tc-99m (140.5 keV)



- Si detector with four VATAGP3 mounted on 4-layer PCB hybrid
- Measured noise figure 170 e_0, corresponding to ΔE of 1.4 keV
- VA shaping time of 3 μs used, but noise still dominated by voltage noise
- Noise correlated to capacitance of double-layer routing lines on silicon