DUAL RADIATOR - PARTICLE IDENTIFICATION

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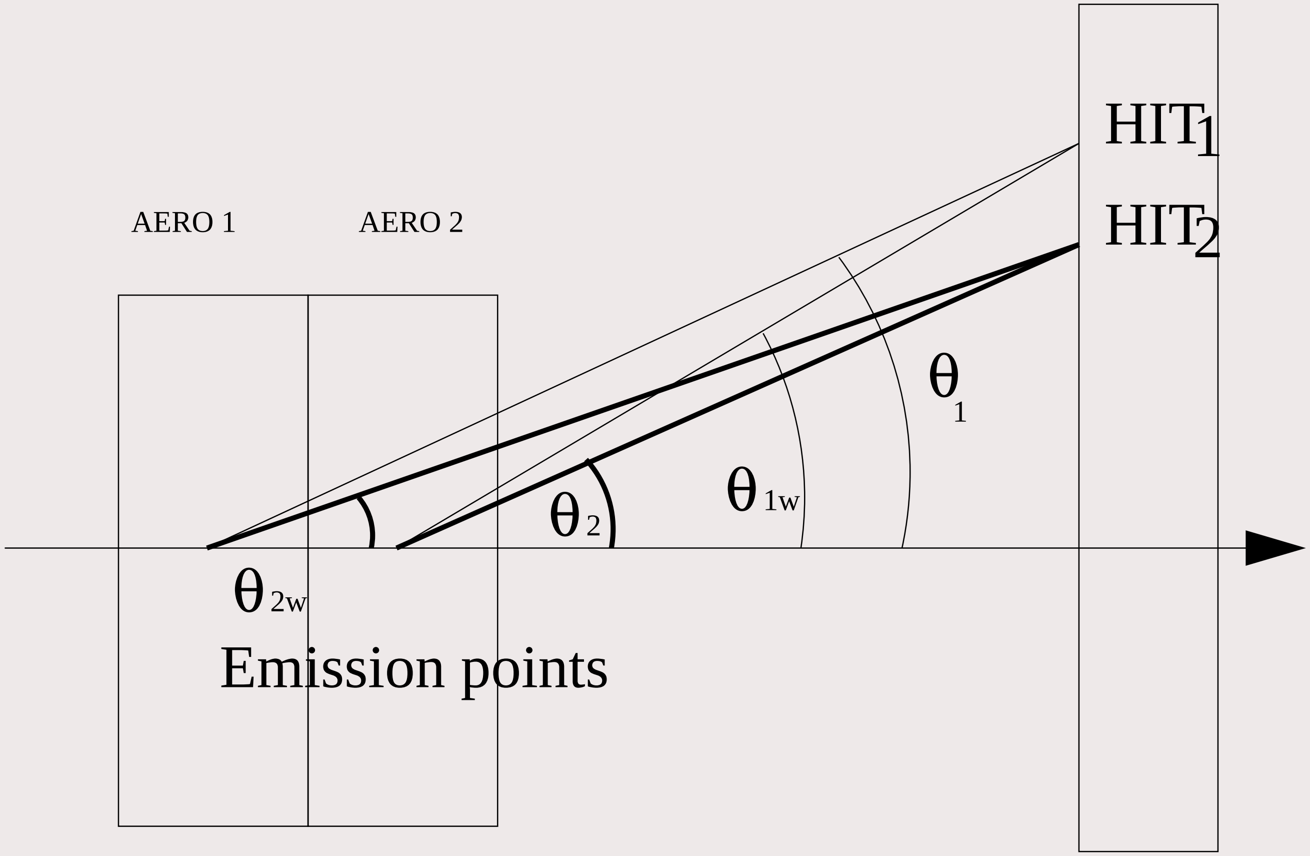
- Dual radiator → more photons compatitive widths
- The goal: choose the best particle identification method
- Dual radiator: particle identification algorithms may have problems
- standard approach: 1D formalism.
- 2D methods may have better results than 1D one
- Sketch of both formalisms →

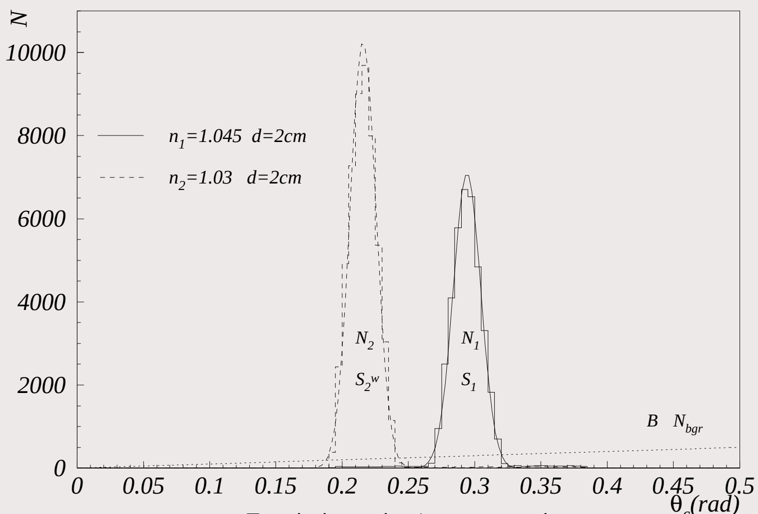
Cherenkov angle reconstruction



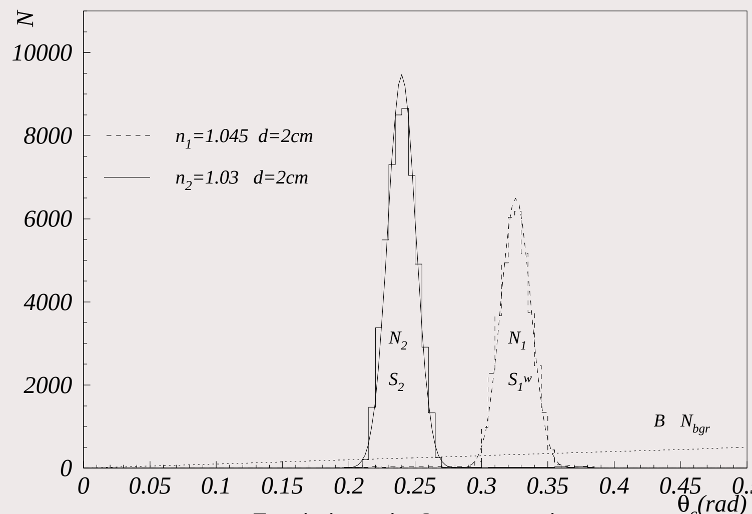
- θ_1 emitted in the aerogel 1 and reconstructed in the aerogel 1
- θ_{1w} emitted in the aerogel 1 and reconstructed in the aerogel 2
- θ_2 emitted in the aerogel 2 and reconstructed in the aerogel 2
- θ_{2w} emitted in the aerogel 2 and reconstructed in the aerogel 1

One dimensional likelihood - dual radiator type

Probability density for two different emission point assumptions:



Emission point 1 reconstruction



Emission point 2 reconstruction

The probability densities for a particular hit i assuming emission in the first (second) aerogel:

$$p_1^{hyp}(\theta_i) = N_1 S_1(\theta_i, hyp) + N_2 S_2^w(\theta_i, hyp) + N_{bgr} B(\theta_i) \quad (1)$$

$$p_2^{hyp}(\theta_i) = N_1 S_1^w(\theta_i, hyp) + N_2 S_2(\theta_i, hyp) + N_{bgr} B(\theta_i) \quad (2)$$

- Note the superscript ^{w} on the signal functions S (wrong reconstruction).
- Expected values $\theta_1^{w,hyp}$ and $\theta_2^{w,hyp}$ are different from the calculated θ_1^{hyp} and θ_1^{hyp}.

One dimensional likelihood functions, cont.

The probabilities are normalized to expected number of photons:

$$\int_{\theta_0}^{\theta_1} p_r^{hyp}(\theta) d\theta = N_1 + N_2 + N_{bgr} = n_e \tag{3}$$

The Signal functions are approximate Gasussians with different means and widths:

$$S_1(\theta_{i,1}, hyp) = G(\theta_{i,1}, \theta_1^{hyp}, \sigma_{i,1}) \tag{4}$$

$$S_1^w(\theta_{i,1}, hyp) = G(\theta_{i,1}, \theta_1^{w,hyp}, \sigma_{i,1}^w) \tag{5}$$

$$S_2(\theta_{i,2}, hyp) = G(\theta_{i,2}, \theta_2^{hyp}, \sigma_{i,2}) \tag{6}$$

$$S_2^w(\theta_{i,2}, hyp) = G(\theta_{i,2}, \theta_2^{w,hyp}, \sigma_{i,2}^w) \tag{7}$$

We can construct the extended likelihood function:

$$\mathcal{L}^{hyp} = \frac{(n_e)^{n_m}}{n_m!} e^{-n_e} \cdot \prod_i^{n_m} p_1^{hyp}(\theta_i) \cdot p_2^{hyp}(\theta_i) \tag{8}$$

Problems: $\theta_1^{w,hyp}$ depends on the incidence angle of the track, refr.index

Two-dimensional method for a dual radiator case

For diffrent mass hypotheses (hyp= e, μ, π, K, p) the probability can be constructed for each track.

Number of observed photons obeys Poissonian statistics. Probability that a particular pixel i is hit:

$$P(n_e^i, n_{bgr}^i, n_m^i) \quad = \quad \frac{e^{-(n_e^i + n_{bgr}^i)} (n_e^i + n_{bgr}^i)^{n_m^i}}{n_m^i !} \tag{9}$$

- The total number of expected signal photoelectrons from track $N_{det} = \sum n_e^i$
- A probability that a particlar pixel is a background hit is n_{bgr}^i. $N_{bgr} = \sum_i n_{bgr}^i$
- Normalization: $\int\limits_x \int\limits_y P(n_e^i, n_{bgr}^i, n_m^i) dx dy = N_{det} + N_{bgr}$

We don't measure the number of hits, we register whether the pixel was hit one ore more times (probability for a hit $\ll 1$):

$$P(n_e^i, n_{bgr}^i, n_m^i) = \begin{cases} e^{-(n_e^i + n_{bgr}^i)} & \text{for } n_m^i = 0, \\ 1 - e^{-(n_e^i + n_{bgr}^i)} & \text{for } n_m^i > 0. \end{cases} \tag{10}$$

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Likelihood function is constructed as a product of the hit probabilities:

$$\mathcal{L} = \prod_{pixels\, i} \left(\frac{e^{-(n_e^i + n_{bgr}^i)} (n_e^i + n_{bgr}^i)^{n_m^i}}{n_m^i!} \right) \quad (11)$$

$$\mathcal{L} = \prod_{\text{not hit}\, i} e^{-(n_e^i + n_{bgr}^i)} \cdot \prod_{\text{hit}\, i} \left(1 - e^{-(n_e^i + n_{bgr}^i)}\right) \quad (12)$$

where n_m^i is the measured number of photons. n_e^i is the expected number of photons and is calculated as the sum of contriibutions from different radiators:

$$n_e^i = \sum_r \varepsilon_i n_{r,emm}^{hyp} \int_{pixel\, i} S_{r,\phi}^{hyp}(\theta_r, \phi_r) d\theta_r d\phi_r \quad (13)$$

with ε_i detection efficiency of the pixel and $n_{r,emm}^{hyp}$ total number of photons emmited in the radiator r and transmitted to the exit of the last radiator. $S_{r,\phi}^{hyp}(\theta, \phi)$ is the fraction of the number of photons of the hit i corresponding to the radiator r

$$S_{r,\phi}^{hyp}(\theta_r, \phi_r) = \frac{1}{2\pi} \frac{1}{\sqrt{2\pi}\sigma_{\theta_r}} e^{-\frac{(\theta_r - \theta_r^{hyp})^2}{2\sigma_{\theta_r}^2}} \quad (14)$$

Putting all together, the likelihood fuction can be written as

$$log\,\mathcal{L} = -\sum_{\text{not hit } i} \left(n_e^i + n_{bgr}^i\right) + \sum_{\text{hit } i} log\left(1 - e^{-(n_e^i + n_{bgr}^i)}\right) \quad (15)$$

$$log\,\mathcal{L} = -(N_{det} + N_{bgr}) + \sum_{\text{hit } i} \left(n_e^i + n_{bgr}^i\right) + \sum_{\text{hit } i} log\left(1 - e^{-(n_e^i + n_{bgr}^i)}\right) \quad (16)$$

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Coming soon

- implement both methods in the simulation program
- check the algorithms in sigle radiator case.
- single radiator: 2D method in principle the same, 1D much simpler since the formalism is not complicated by the wrong emission assumption.

Focusing vs. Defocusing type:

- Likelihood formalism is the same for the focusing and defocusing radiator type
- in the defocusing type the surface where background photons might spoil the reconstruction is larger than in the focusing one.
- in the 2D method background hits might have more dramatic effects than in 1D method, where the background is integrated.