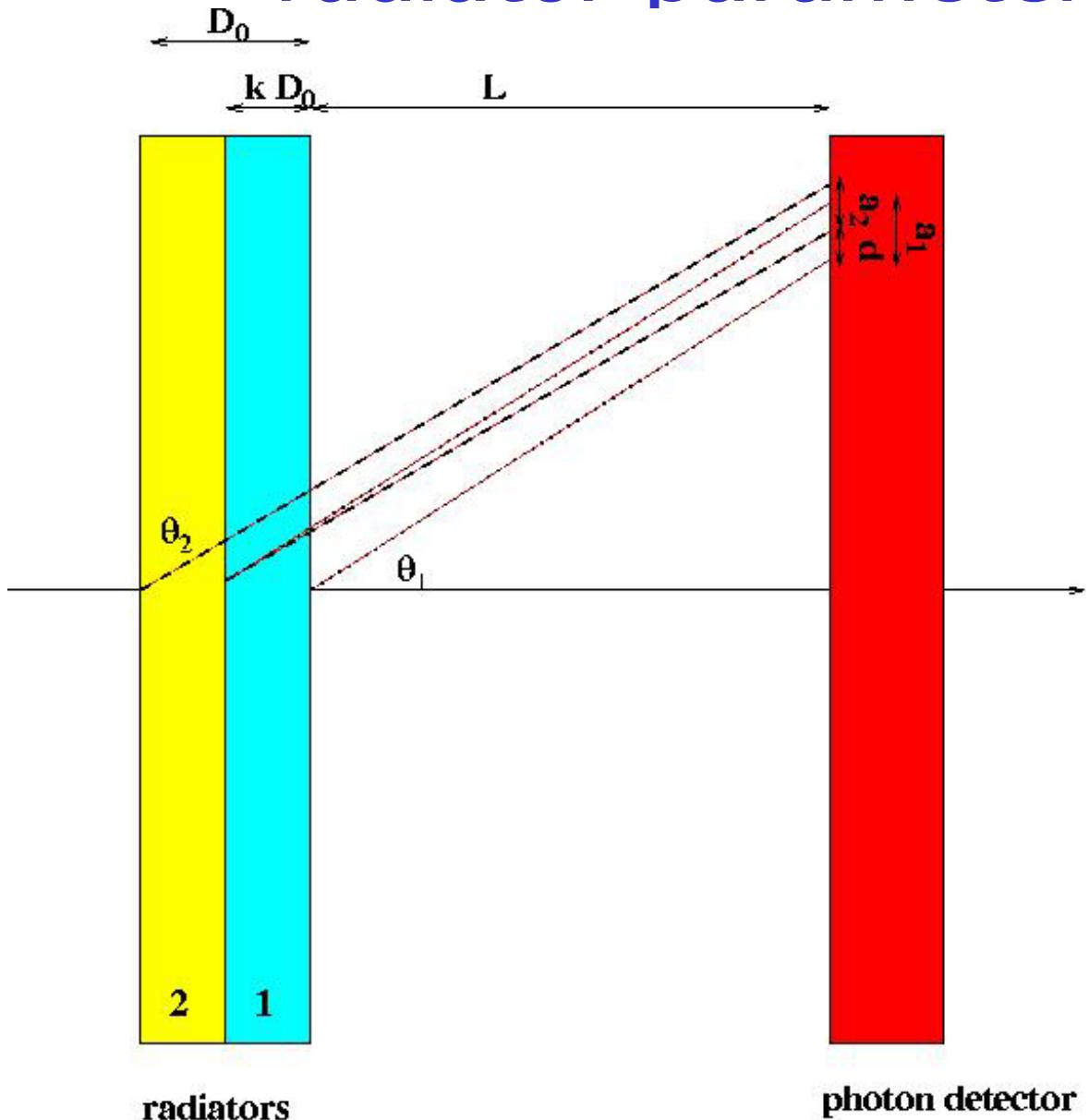


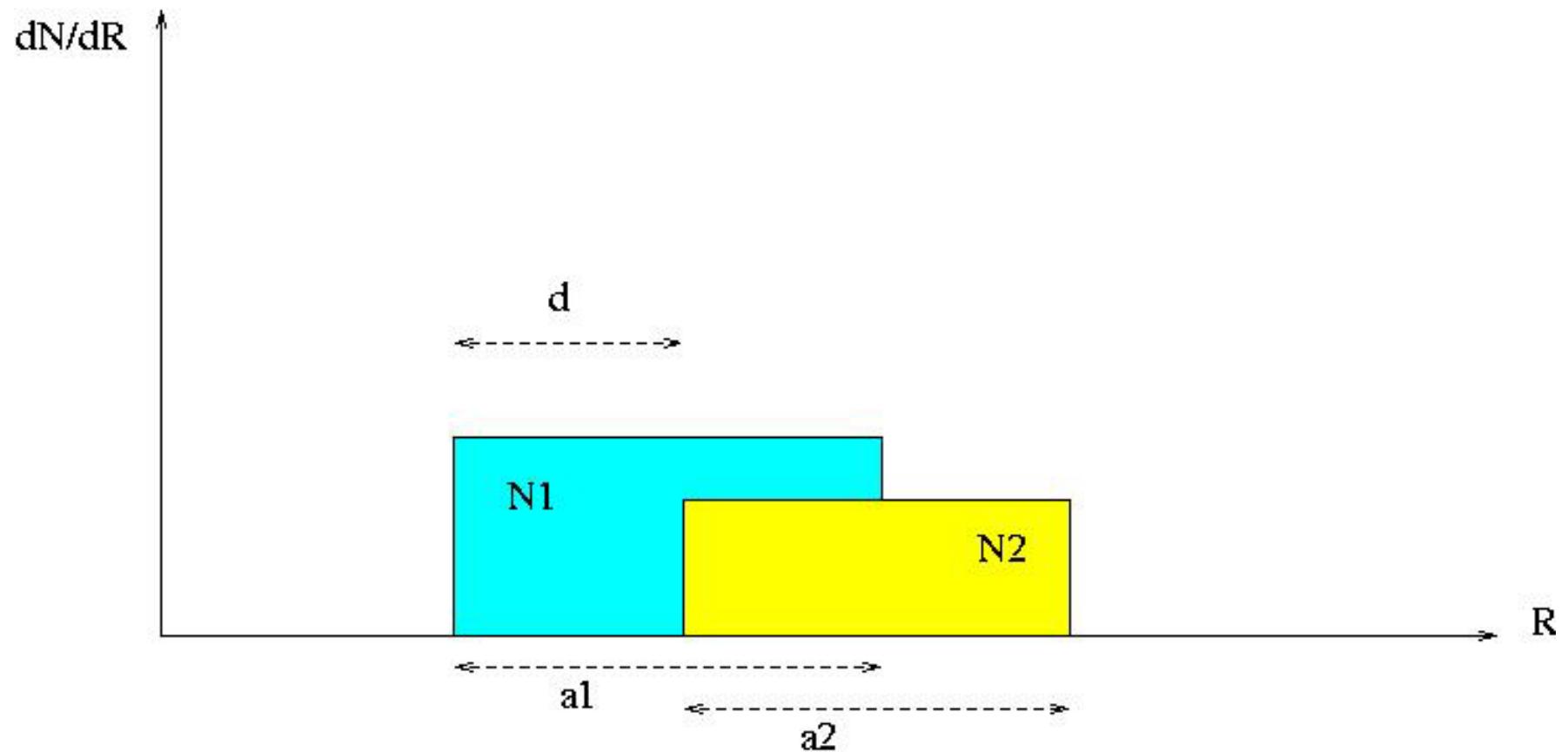
# Multiple radiator: Optimisation of radiator parameters – cont'd



Dual radiator: three parameters (if fixed space available)

- Difference in  $\theta$
- Ratio of radiator thicknesses
- Total radiator thickness

# Radial photon impact point distribution



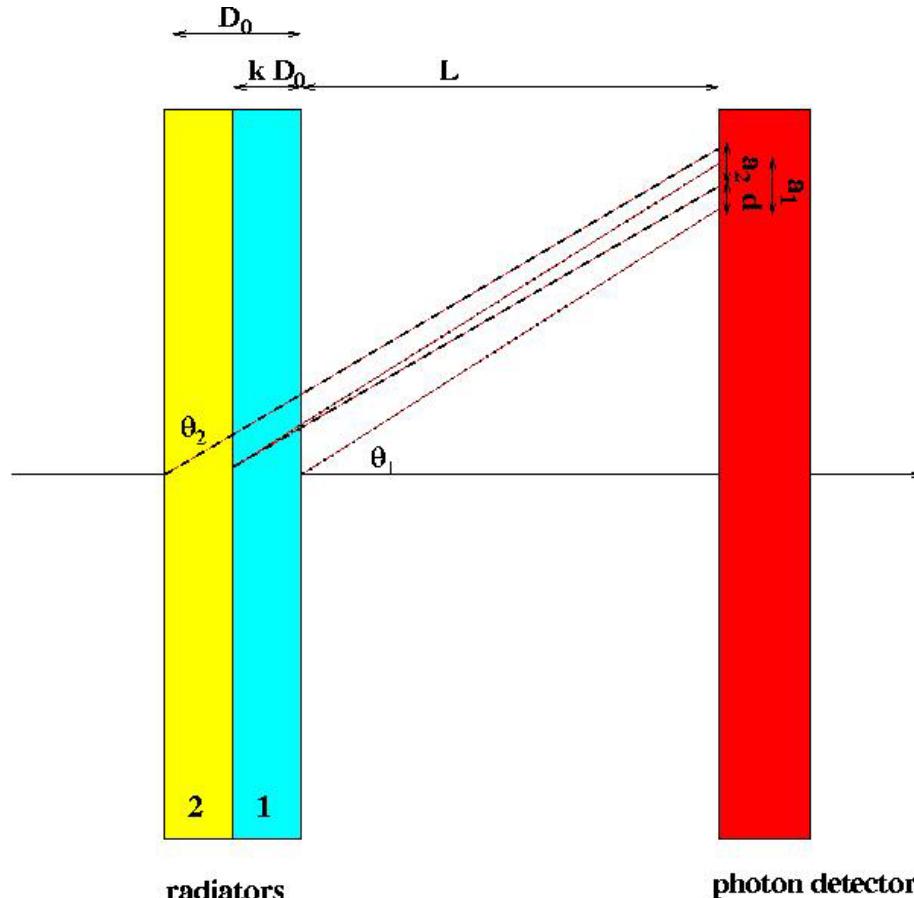
# Parameters of the counter

$$a1 = D1 * \tan[\theta_1]; a2 = D2 * \tan[\theta_1 - \delta];$$

$$d = L * \tan[\theta_1] - (L + D1) * \tan[\theta_1 - \delta]; D1 = k * D0;$$

$$D2 = D0 - D1; N1 = 50 * D1 * (\sin[\theta_1])^2 * \exp[-D1 / 2 / \lambda_{m1}];$$

$$N2 = 50 * D2 * (\sin[\theta_1 - \delta])^2 * \exp[-D2 / 2 / \lambda_{m2} - D1 / \lambda_{m1}];$$



## What comes today?

- Repeat the optimisation study with  $\sigma_{\text{rest}}$
- Extend the full calculation to the multilayer case
- Study the robustness of the optimum

# Minimized: error per track

$$\sigma_{track} = \frac{1}{\sqrt{N_{det}}} \sqrt{\sigma_{emp}^2 + \sigma_{det}^2 + \sigma_{rest}^2}$$

$$\begin{aligned}
 & \frac{1}{(L + D_0 / 2)} \leftarrow \text{Distance to photon detector} \\
 & \sqrt{\left( \frac{1}{(N1 + N2)} \right)} \leftarrow \text{Number of photons} \\
 & \left( \frac{1}{12 (N1 + N2)^2} \right) \leftarrow \text{Emission point error} \\
 & (-3 (a1 N1 + (a2 + 2 d) N2)^2 + \\
 & 4 (N1 + N2) (a1^2 N1 + (a2^2 + 3 a2 d + 3 d^2) N2)) + \\
 & \text{pad}^2 / 12 \Big) \Big) \leftarrow \text{Pad size contribution}
 \end{aligned}$$

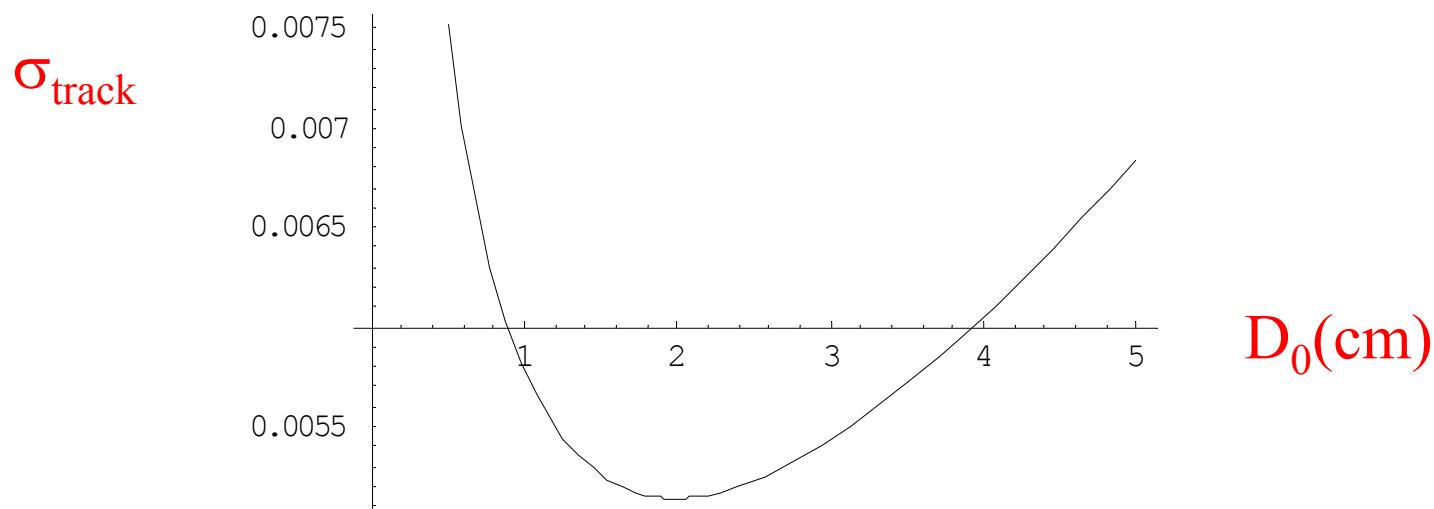
Now assume  $\sigma_{rest}=0.006$

Check the single radiator result – NIM paper

Vary total radiator thickness  $D_0$

Input data:

- total length  $L+D_0$  is fixed (20cm)
- $d_n=0.0$
- Cherenkov angle = 0.3, pad size 6mm, trans. len.= 4.5cm



Minimum at 2cm: OK

Minimize track error vs.

Relative radiator thickness  $k$  and refractive index difference  $dn$   
and total thickness  $D_0$

Transm. length 3cm

**$dn=0.0046$**

**$k=0.43$**

**$D_0 = 2.7 \text{ cm}$**

$\sigma$  at minimum 0.0047

Transm. length 4.5cm

**$dn=0.0053$**

**$k=0.44$**

**$D_0 = 3.0 \text{ cm}$**

$\sigma$  at minimum 0.0044

Available space in front of photon detector: **20cm**

$\sigma_{\text{rest}}=6\text{mrad}$

Minimize track error vs.

Relative radiator thickness  $k$  and refractive index difference  $d_n$   
and total thickness  $D_0$

$$\sigma_{\text{rest}} = 6 \text{ mrad}$$

$$\sigma_{\text{rest}} = 8 \text{ mrad}$$

$$d_n = 0.0053$$

$$d_n = 0.0056$$

$$k = 0.44$$

$$k = 0.44$$

$$D_0 = 3.0 \text{ cm}$$

$$D_0 = 3.2 \text{ cm}$$

$$\sigma \text{ at minimum } 0.0043$$

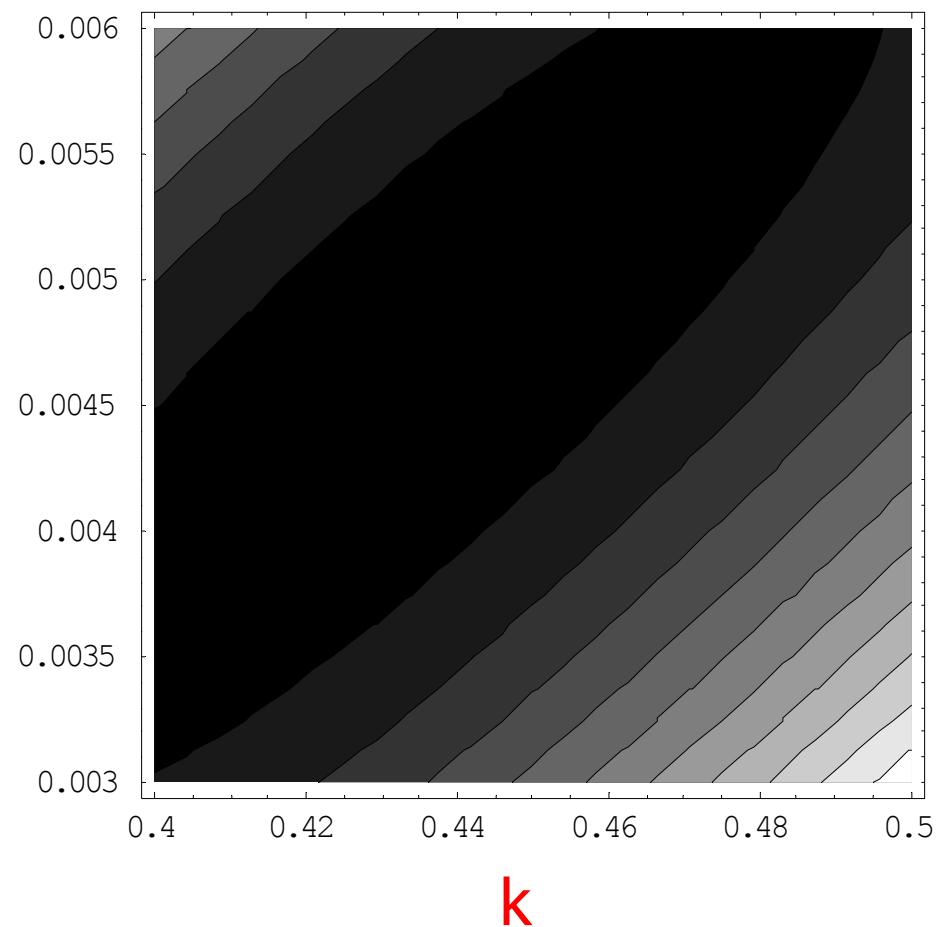
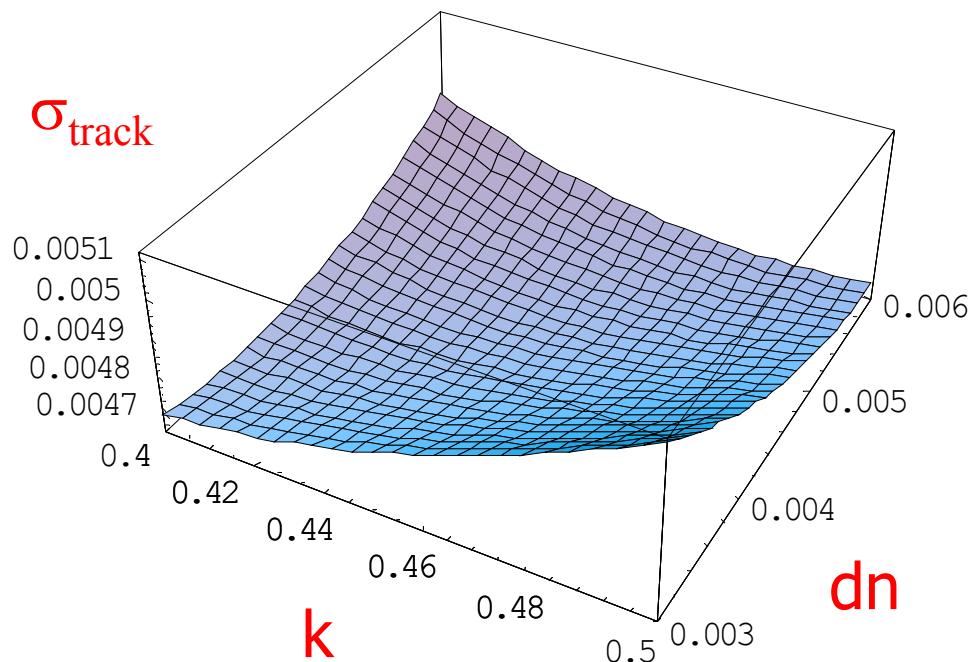
$$\sigma \text{ at minimum } 0.0047$$

Available space in front of photon detector: 20cm  
Transm. length 4.5cm

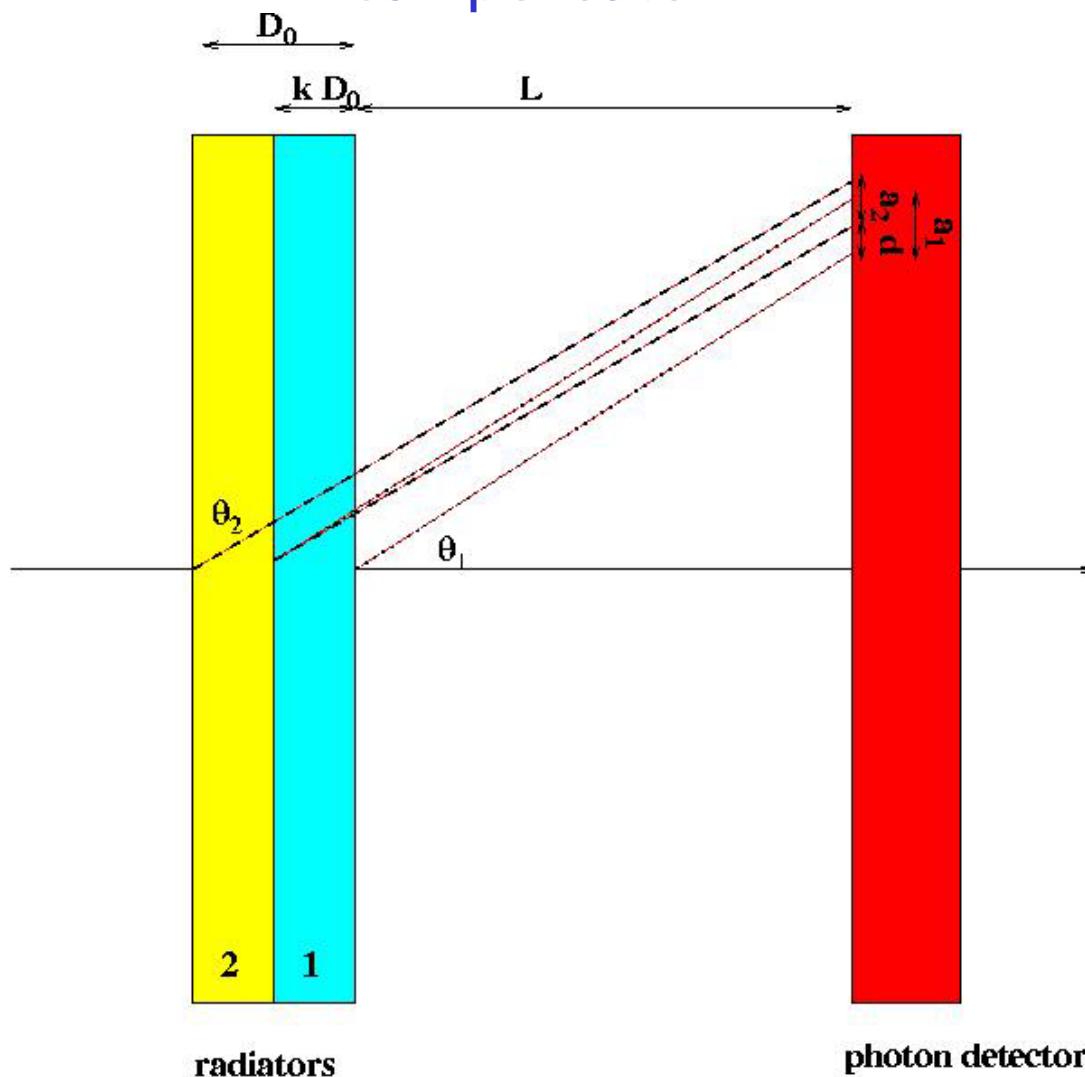
# Robustness of the optimum

Take the case with transm. length 3cm,  $\sigma_{\text{rest}}=6\text{mrad}$ , with  $dn=0.0046$ ,  $k=0.43$ ,  $D_0 = 2.7 \text{ cm}$ ,  $\sigma$  at minimum 0.0047.

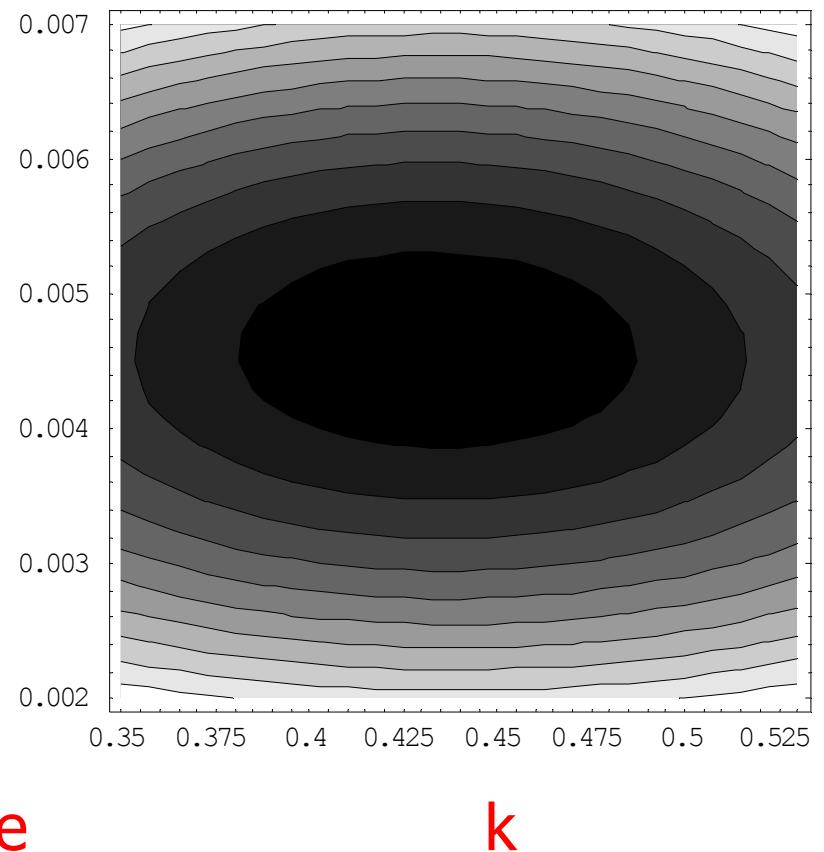
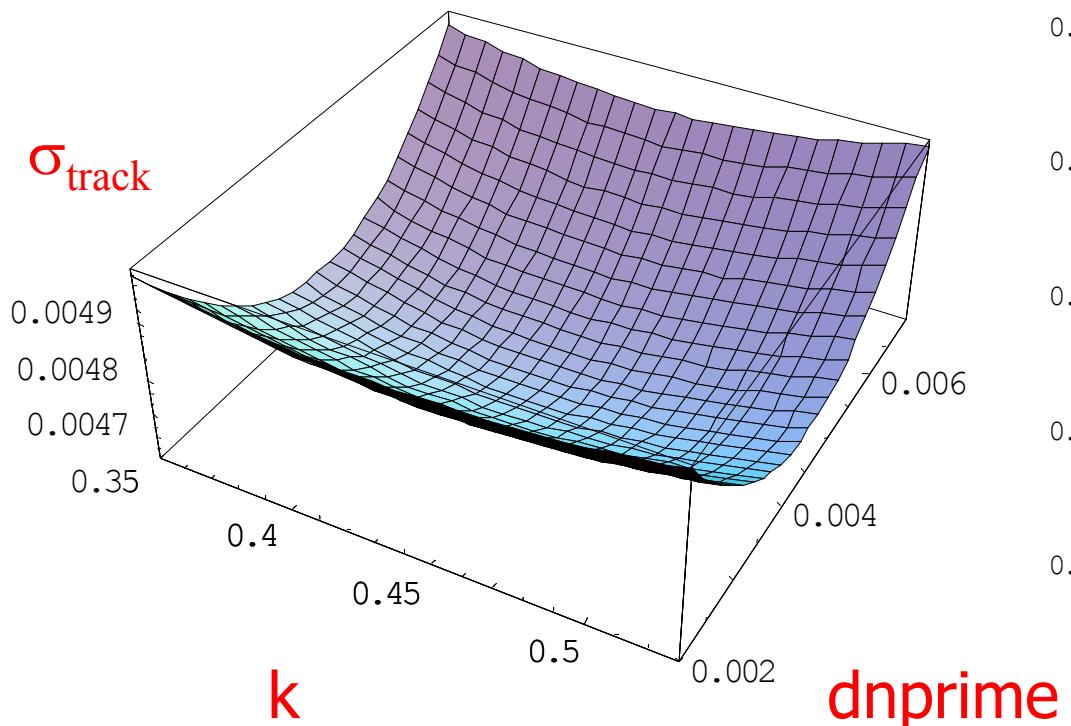
Fix  $D_0$ , vary  $k$  and  $dn$ .



Correlation between  $dn$  and  $k$ : larger  $k$  means thicker radiator 1 (downstream) -> need a larger  $dn$  to compensate.

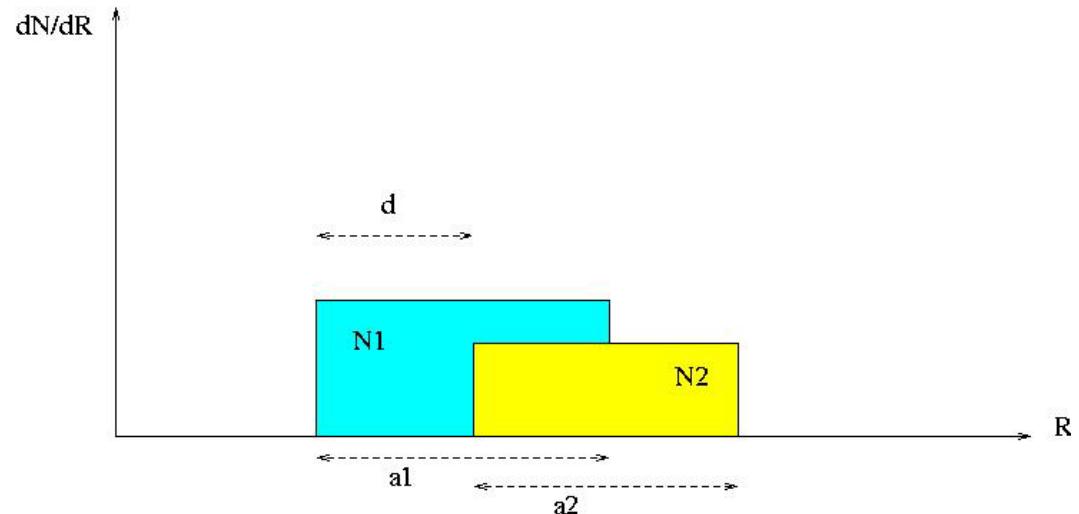
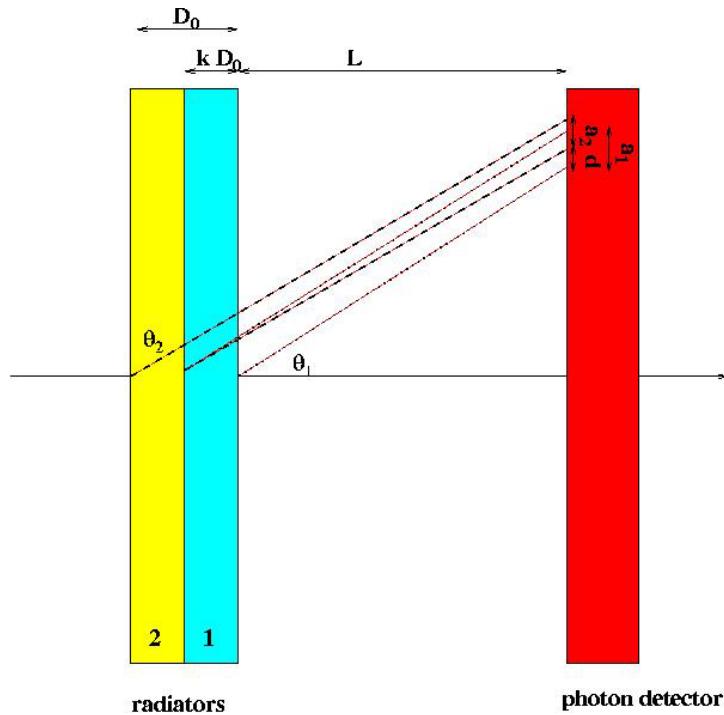


New variable -> easier to see



$$\text{dn} = \text{dnprime} + 0.025 * (k - 0.433)$$

## Extend the full calculation to the multilayer case



Use the same method to calculate the spread on the photon detector as for the two radiator case

- Triple radiator case
- Quadruple radiator case

# Triple radiator

Transm. length 3cm,  $\sigma_{\text{rest}} = 6\text{mrad}$

**dn1=0.0033**

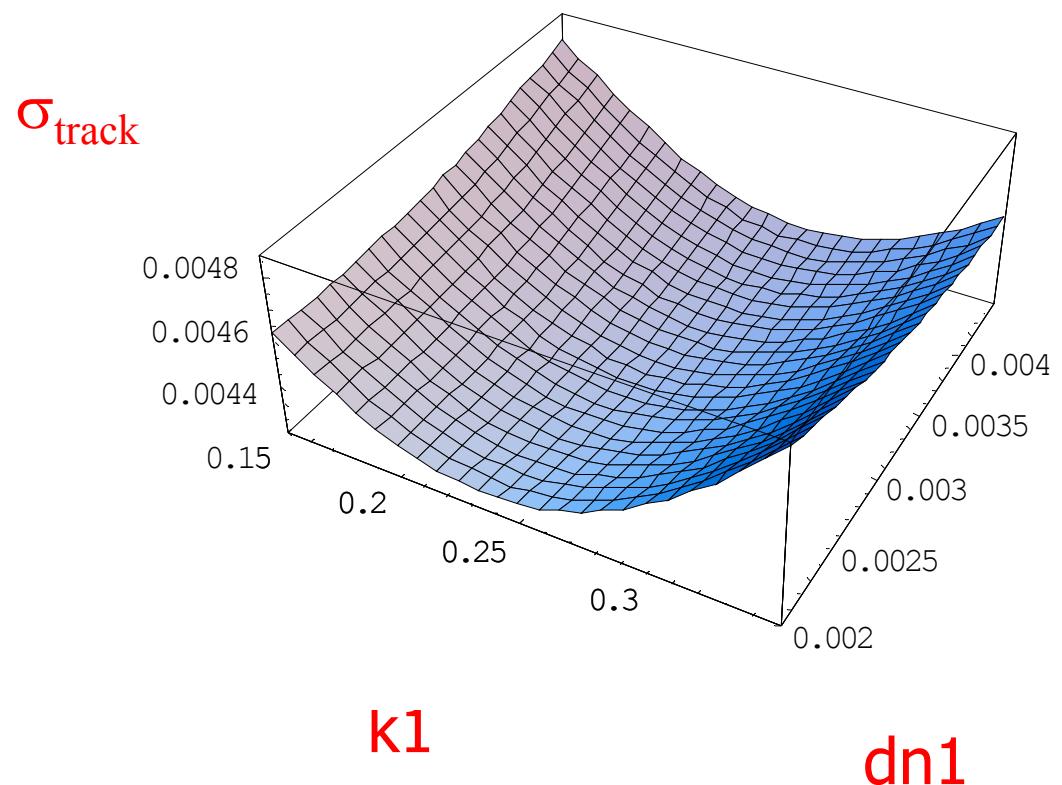
**dn2=0.0081**

**k1=0.25**

**k2=0.34**

**D<sub>0</sub> = 3.4 cm**

$\sigma$  at minimum 0.0043



# Quadruple radiator

Transm. length 3cm,  $\sigma_{\text{rest}} = 6\text{mrad}$

**dn1=0.0020**

**dn2=0.0062**

**dn3=0.0105**

**k1=0.16**

**k2=0.24**

**k3=0.28**

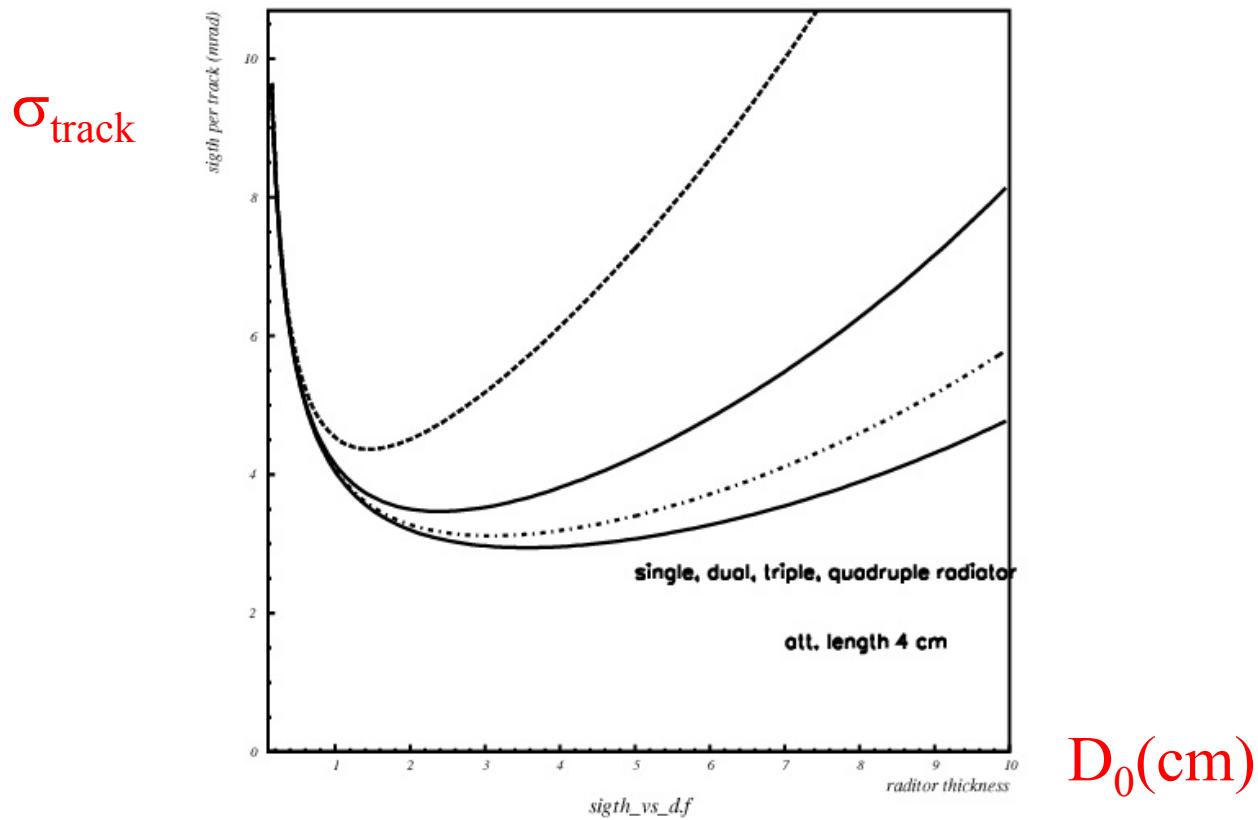
**D<sub>0</sub> = 3.9 cm**

$\sigma$  at minimum 0.0041

Lower ref. index -> smaller emission point error at same thickness-> optimal if upstream radiators thicker

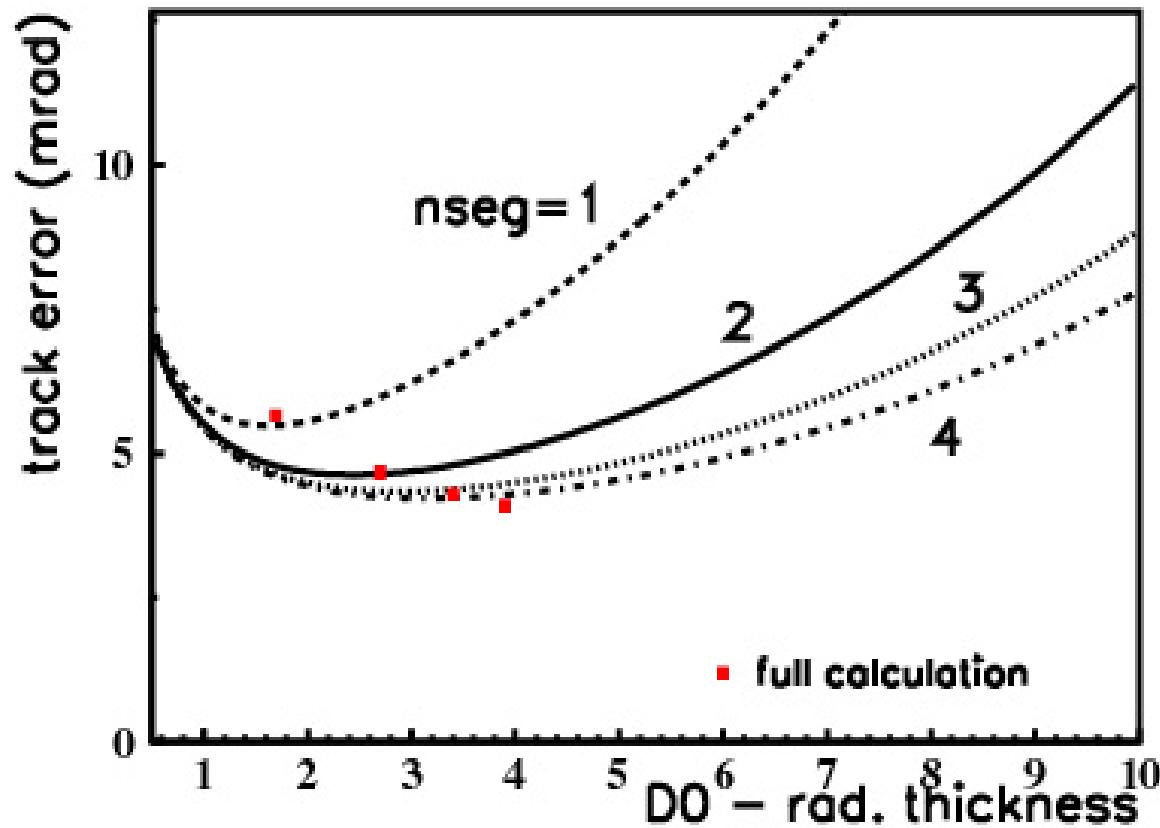
Last meeting: a simple model of what happens when we go from the single layer to focusing multilayer arrangement:

If the indices are well adjusted, the error in emission point goes down by a factor of two in the dual radiator case etc



Simple model

How good is this simple model? Contrast it with  
the full calculation



Curves: simple model  
Points: full calculation

## What comes next?

- Evaluate the multilayer results
- Continue the study of robustness of the optimum  
(need input for what is realistic error in  $n$ ,  
thickness)