

The outline

- 1) Detector parameters: efficiency, geometrical acceptance, dead-time, resolution, linearity
- 2) gaseous ionization chambers
- 3) proportional counters- ionization measurement
- 4) silicon detectors for ionization measurements

Efficiency of the detector

Probability of the detector to record a particle once it goes into it. We usually try to measure efficiency by redundant detector systems

$$E_b = \frac{N(a \wedge b \wedge c)}{N(a \wedge c)}$$



The efficiency will depend on the probability of the particle to interact in the detector and probability to collect and register this type of signal. First part will depend on a particle in question.

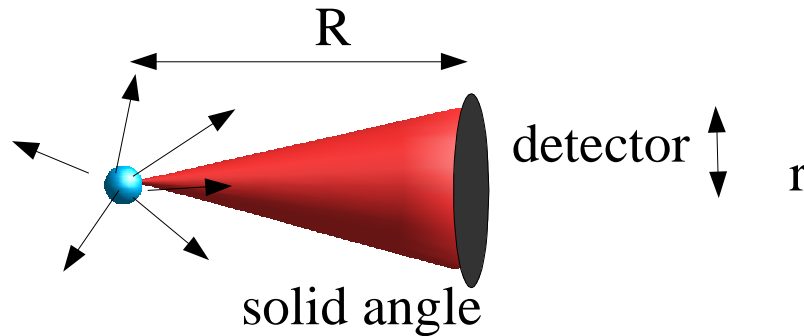
The observed efficiency of the detector for a given PROCESS giving rise to the stream of particles (point source, beam, cosmic rays) will depend on the **GEOMETRICAL ACCEPTANCE** of the detector. This will depend in general on the PROCESS in question, and it is usually calculated by Monte Carlo integration. It is defined as

$$G_A = \frac{N_{det}}{N_{tot}}$$

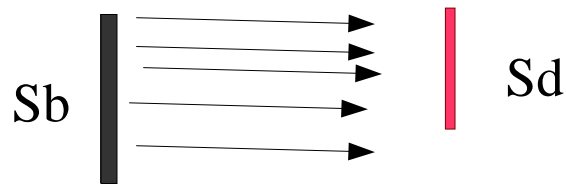
Few examples:

Geometrical acceptance

Uniform point source:



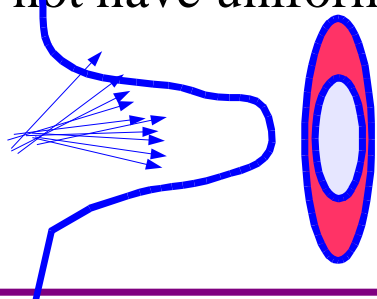
$$G_A = \frac{\Omega_D}{4\pi} \simeq \frac{\pi r^2}{4\pi R^2} = \frac{r^2}{4R^2}$$



Uniform beam perpendicular to the detector

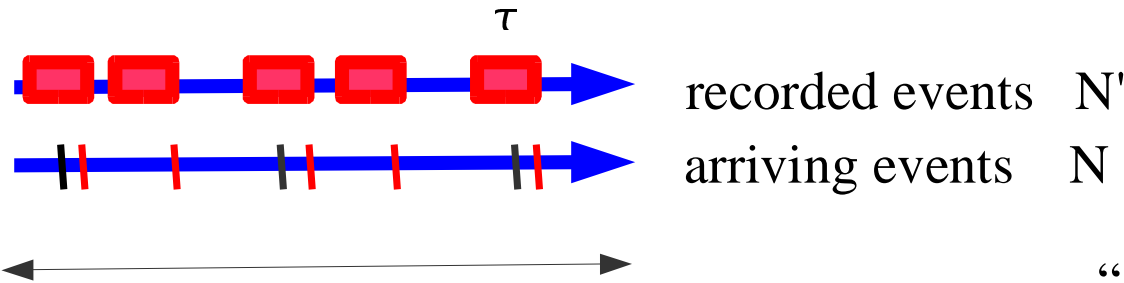
$$G_A = \frac{S_d}{S_b}$$

Usually much more complicated to calculate, the particles from the source do not have uniform distribution- and the detector does not have simple shape:



Dead-time

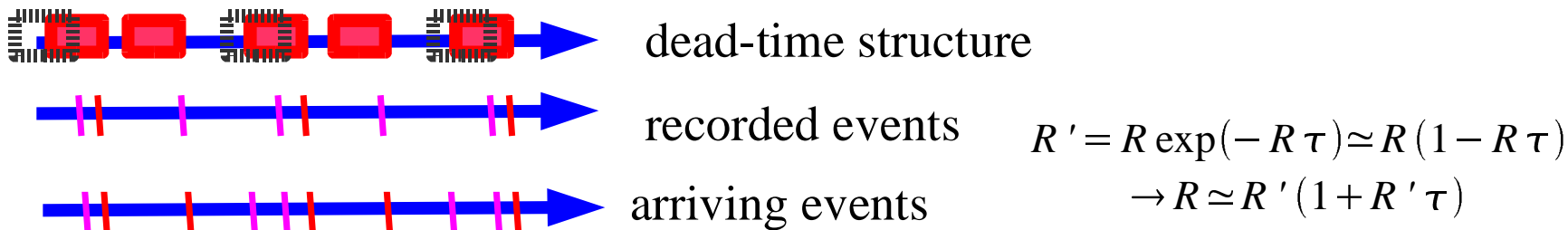
The detector (and/or) detector electronics is usually unable to register next event if it comes in time shorter than τ after the preceding one



$$\text{dead time} = N' \tau \quad \text{fraction} = N' \frac{\tau}{t} = R' \tau$$

$$\text{number} \sim \text{lost} = \frac{N}{t} * N' \tau = R * N' \tau \quad \text{'rate lost'} = R * R' \tau = R - R' \rightarrow R = \frac{R'}{1 - R' \tau} \simeq R' (1 + R' \tau)$$

Paralyzable mode- connected with noise events and trigger



“deadtime less” operation-> possible if deadtime mostly due to electronics and trigger.-> buffering

Resolution

Keywords: gaussian distribution -> FWHM versus sigma of the gaussian
relative resolution for Poissonian distribution of single measurements
Fano factor-> “improved” resolution if single measurements are correlated due to physical effects

linearity : proportionality of the response to the input in a range of inputs:

Track reconstruction pt=200 GeV

Take events with a single GEANT particle in the detector:

npar=1 (type 211): 48.4% .

In other events the pion "multiplied"
probably due to interactions.

In 98.1% of such events a track
was reconstructed (ntkr=1)

Look at $(pt_{gen}-pt_{track})/pt_{gen}$. (cut tails)

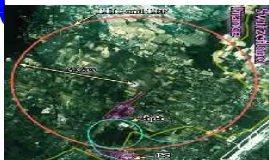
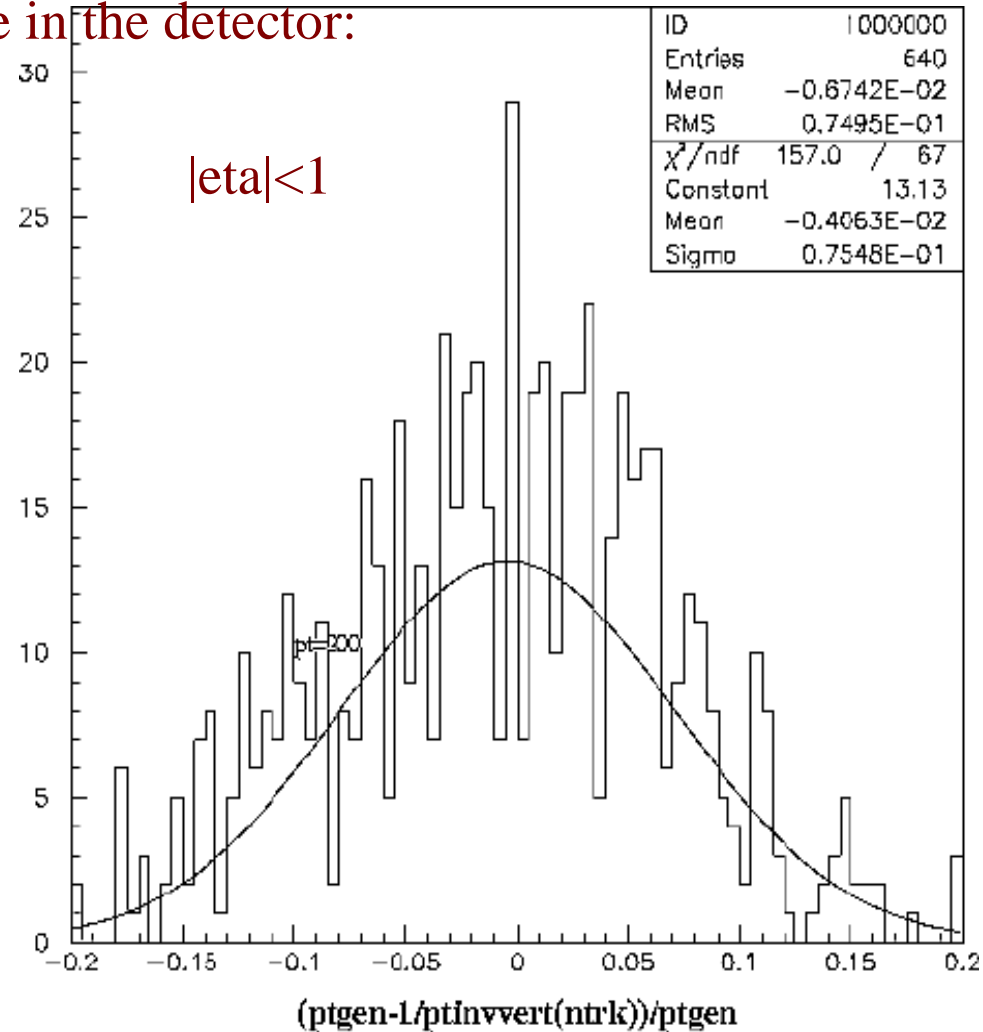
$|\eta| < 1$ $\sigma = (7.4 \pm 0.4)\%$

mean = (0.4 ± 0.4)

$2 > |\eta| > 1$ $\sigma = (11.4 \pm 1.2)\%$

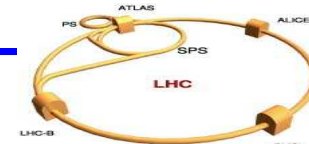
mean = (2.6%)

Conclusions: probably consistent with
Technical Proposal ($\sigma = 20\%$ for $pt = 500$)



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Single pions from DC1
reconstructed in 5.1.0



jet reconstruction pt=200 GeV

Take events with a single GEANT particle in the detector:

npar=1 and ntrk=1 and SINGLE CLUSTER nc=1

and jet_num=1. (99.9% of events with npar=1)

SIMPLEST POSSIBLE JETS.

Verify that jet_etrack=pttrack

Verify that jet_e = jet_ecal0

Look at $(pt_{gen} - jet_e / \cosh(jet_eta)) / pt_{gen}$.

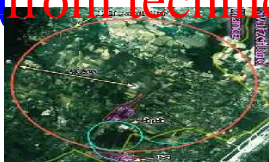
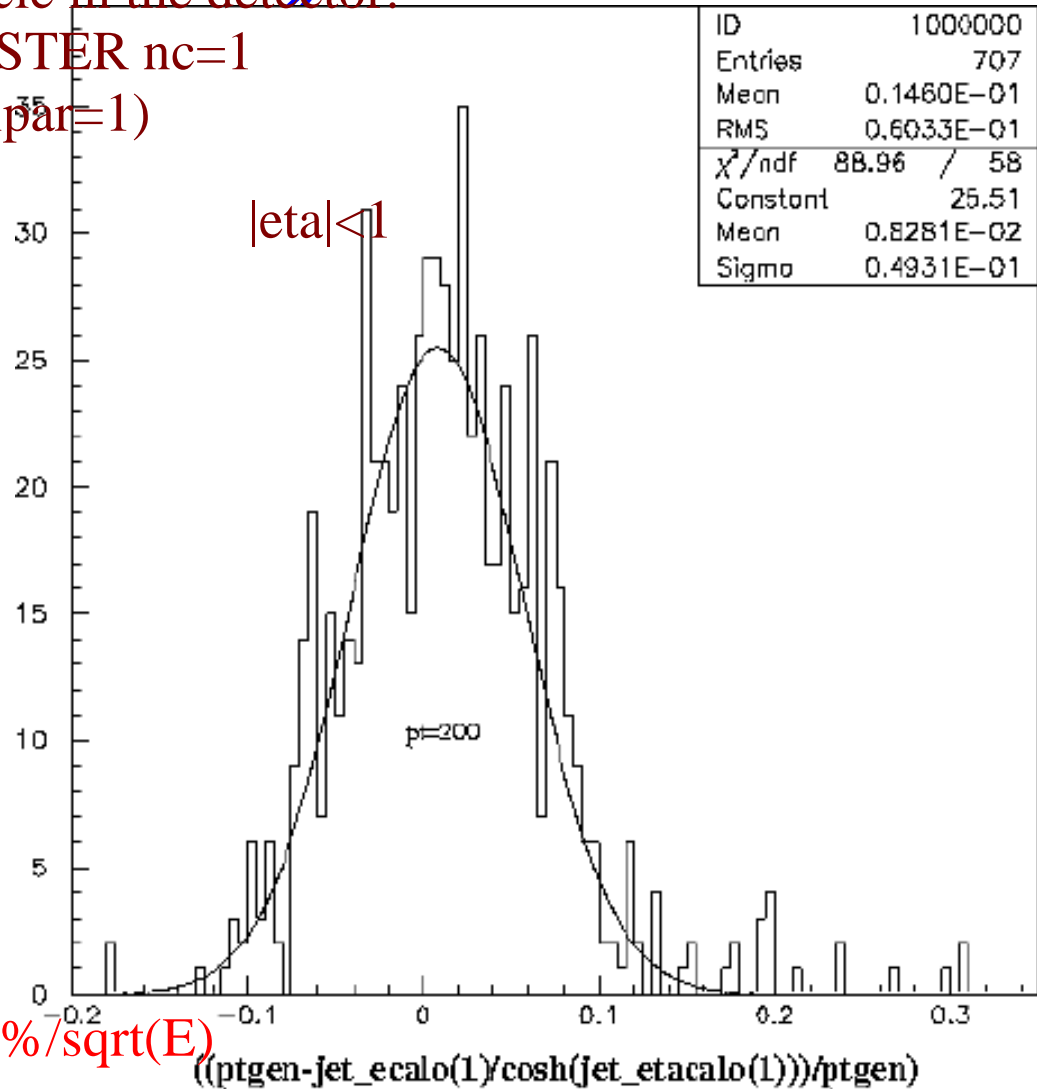
$|\eta| < 1$ $\sigma = (5 \pm 1)\%$

mean = $(1.0 \pm 0.1)\%$!!

all eta $\sigma = (5 \pm 1)\%$

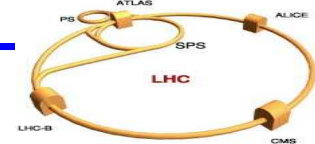
mean = $(2 \pm 1)\%$

Conclusions: the shift of cluster energy was corrected, and the resolution was slightly improved. However, the result corresponds to $85\%/\sqrt{E}$ instead of $50\%/\sqrt{E}$ from technical proposal



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Single pions from DC1
reconstructed in 5.1.0



Modes of operation of gaseous detectors

ionization chamber mode: low voltage just to induce some drift of electrons and ions, which “overcomes” recombination. Only primary ionization is collected. The signal is weak.

proportional mode, or wire chamber mode: Higher field applied, and it can be particularly strong close to the wires. The electrons are accelerated until they become ionizing themselves. At relatively low field the resultant signal still remains proportional to the primary ionization-proportional mode. In most of practical devices amplification factor is of the order of 10^{*6} - 10^{*5} .

semi-proportional mode: field is increased and amplification in number of electrons becomes nonlinear.

saturated mode: positive and negative charges produced in the secondary shower become so large that they screen much of the charge from the applied field. Large resulting signal does not depend any more on the original ionization.

Geiger-Mueller mode: recombination of ion pairs in the shower produces gamma rays which ionize further everywhere in the detector volume. If field is increased any further there will be breakdown even in the absence of primary ionization

Ionization chambers

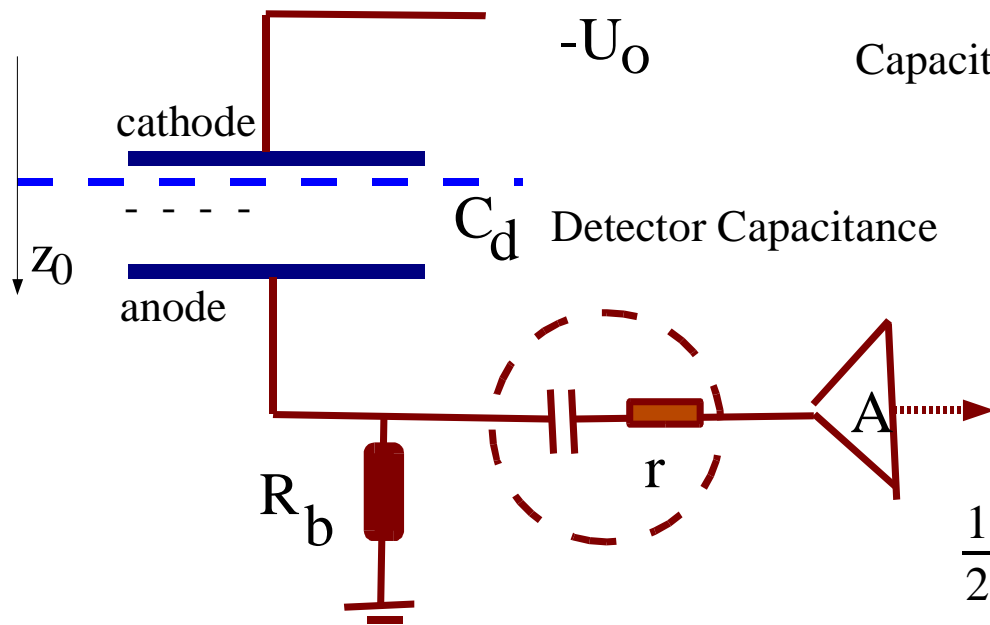
Typical number of ions left by a minimum ionizing particle is 120/cm. Thus ionization chambers are usually not sensitive to single minimum ionizing particles, more suitable to detect heavy (slow) ions, which ionize much more- remember Bethe-Bloch formula.

If liquid is used rather than gas-> liquid argon -> the higher density and lower ionization potential lead to increase in the signal. Used are: liquid argon (cooling) and for example room temperature liquids like TMP (tetramethyl pentane)

Ionization chambers operate usually in one of two modes

- pulse mode-> individual particles are counted- voltage pulse measured
- current mode -> only current from passing particles is measured, it is proportional to the number of passing particles

Time structure of the signal



Capacitor energy $E = \frac{1}{2} C_d U^2$

What is the change of the voltage on the capacitance of the detector due to moving N charges from position z_0 to z ?

$$U = U_0 + \delta U$$

$$\frac{1}{2} C_d U^2 = \frac{1}{2} C U_0^2 - N \int_{z_0}^z q E_z dz =$$

$$= \frac{1}{2} C_d U_0^2 - N \frac{U_0}{d} \int_{z_0}^z dz = \frac{1}{2} C_d U_0^2 - N \frac{U_0}{d} (z - z_0)$$

$$\frac{1}{2} C_d U^2 - \frac{1}{2} C_d U_0^2 = \frac{1}{2} C_d (U - U_0)(U + U_0) = -N \frac{U_0}{d} (z - z_0) \approx C_d U_0 \delta U$$

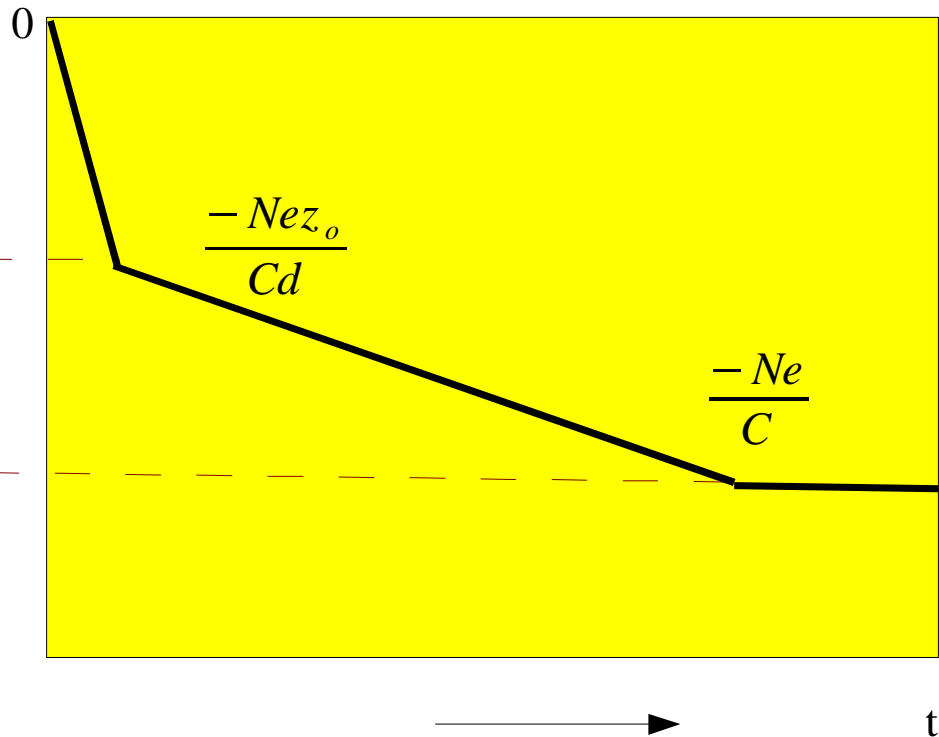
$$\delta U = -N \frac{q}{C_d d} (z - z_0)$$

$$\delta U^+ = -N \frac{e}{C_d d} v^+ \delta t^+$$

$$\delta U^+ = -N \frac{(-e)}{C_d d} (-v^-) \delta t^-$$

Argon STP 500V/cm,
d=5cm , ion drift
1ms, and microsecond
for electrons.

Time structure of the signal for large bias R

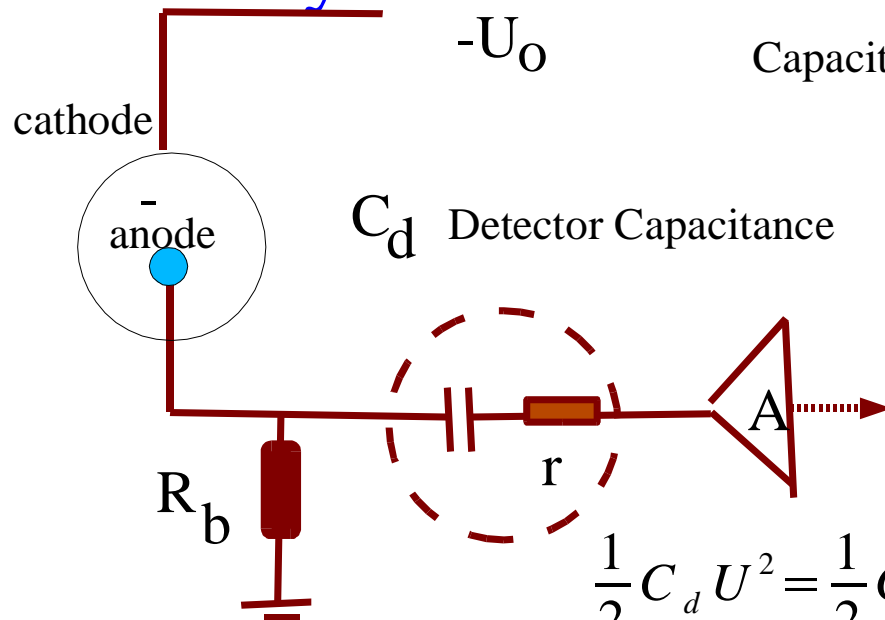


$$-\delta U$$

For very large bias resistance the voltage development will be as marked. However if the $R_b C_d$ is too small the detector will “discharge” thru the bias resistance before all the ions will arrive. Keeping very large bias resistance is not practical for counting particles- too long time for the detector to discharge. In practice a capacitor is introduced before amplifier which gives RC of amplifier-capacitor branch long enough for electrons to arrive, but not enough to collect the pulse from ions. In this case the pulse amplitude depends on the distance from anode where the electrons were produced.

Current mode :
average detector current $I = -Ne/(RC)$

Signal for cylindrical chamber (straw tubes)



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$$\frac{1}{2} C_d U^2 = \frac{1}{2} C U_0^2 - N \int_{r_a}^{r_0} q E_r dr =$$

$$= \frac{1}{2} C_d U_0^2 - N q \frac{U_0}{\ln(r_c/r_a)} \int_{r_a}^{r_0} \frac{dr}{r} = \frac{1}{2} C_d U_0^2 - N q \frac{U_0}{\ln(r_c/r_a)} \ln(r_0/r_a)$$

$$\frac{1}{2} C_d U^2 - \frac{1}{2} C_d U_0^2 \simeq C_d U_0 \delta U$$

$$\delta U^+ = -N \frac{q}{C_d \ln(r_a/r_c)} \ln(r_c/r_0)$$

$$\delta U^- = -N \frac{q}{C_d \ln(r_a/r_c)} \ln(r_a/r_0)$$

for $r_c/r_a \simeq 10^3 \rightarrow \frac{\delta U^+}{\delta U^-} \simeq 0.1$

Self-Study for Thursday

A swarm of electrons and positive ions (mainly argon) was produced by a passing particle 2cm from another wire and 1 cm from cathode of a chamber operating at 1kV/cm in argon (93%) isobutane (7%) mixture under normal conditions.

Using the data from your course book, notes, web, whatever you like find out :

- 1) find out- calculate- what should be the mean free path of electrons and mobility of ions in argon-isobutane mixture above
- 2) the time of arrival of positive ions to cathode and electrons to anode.
- 3) assuming that the electric field has no influence on diffusion of electrons and ions calculate the size of ion cloud at the arrival of the cathode and the size of the electron cloud at the arrival of the anode. You can neglect the effect of isobutane on the diffusion and assume that for the electrons we have approximately:

$$D = \frac{1}{3} \frac{\lambda_e^2}{\tau} = \frac{1}{3} \frac{\lambda_e^2}{(\lambda_e / u)} = \frac{1}{3} \lambda_e u$$

where D is diffusion coefficient (we assumed it is the same in all directions) and it is expressed here as a function of electron mean free path and thermal velocity. Check how it compares with the one for argon ions in argon. Continued...

Self-Study 2

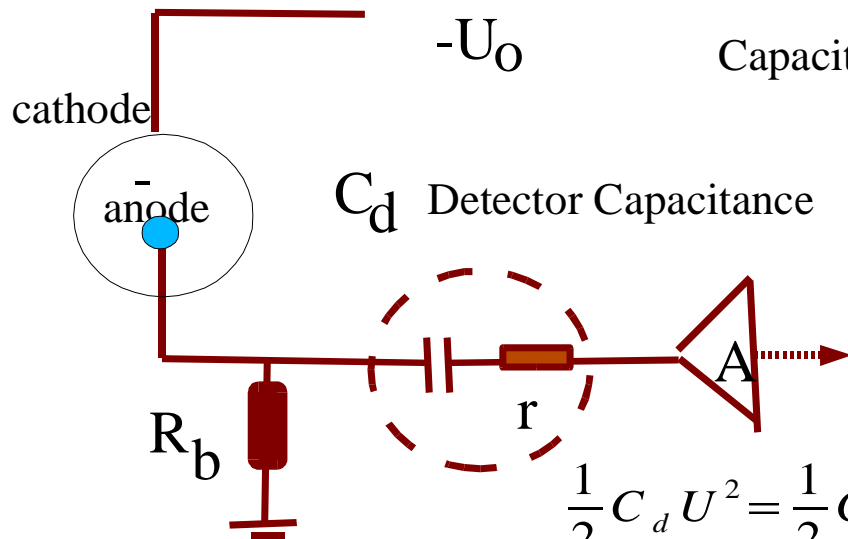
- 4) calculate how the size of the electron cloud will change if magnetic field 2 Tesla is applied parallel to the electric field. Give size along and perpendicular to the electric field.
- 5) The probability of electron capture in argon is very small. However the chamber is contaminated with 1% of oxygen. Assuming a reasonable value of electron capture probability p_a for oxygen (minimal for example, as it depends on the electron energy), and remembering that with 1% volume of oxygen the frequency of electron-oxygen collision will be reduced compared to 100% oxygen calculate the capture time t_a of electrons in the chamber. Assuming that the number of electrons surviving time t without being captured is :

$$N(t) = N(t=0) \exp\left(-\frac{t}{t_a}\right)$$

calculate what percentage of electrons is captured before arriving to anode.

Proportional gaseous counters

Typical proportional counters differ from cylindrical ionization chamber only by the electric field- potential difference applied. However r_0 where the most of charges is produced will be also different as a result.



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$$U = U_0 + \delta U$$

$$\frac{1}{2} C_d U^2 = \frac{1}{2} C U_0^2 - N \int_{r_a}^{r_0} q E_r dr =$$

$$= \frac{1}{2} C_d U_0^2 - N q \frac{U_0}{\ln(r_c/r_a)} \int_{r_a}^{r_0} \frac{dr}{r} = \frac{1}{2} C_d U_0^2 - N q \frac{U_0}{\ln(r_c/r_a)} \ln(r_0/r_a)$$

$$\frac{1}{2} C_d U^2 - \frac{1}{2} C_d U_0^2 \approx C_d U_0 \delta U$$

$$\delta U^+ = -N \frac{q}{C_d \ln(r_a/r_c)} \ln(r_c/r_0)$$

$$\delta U^- = -N \frac{q}{C_d \ln(r_a/r_c)} \ln(r_a/r_0)$$

for $r_c/r_a \approx 10^3 \rightarrow \frac{\delta U^+}{\delta U^-} \approx 0.1$

Gas amplification factor

In electric field close to anode wire electrons will be accelerated. In principle they can gain kinetic energy enough to ionize further. Townsend avalanche will started.

Kinetic energy between collisions at points r_1 and r_2

$$\delta T_{kin} = e \int_{r_1}^{r_2} E(r) dr = e \frac{U_0}{\ln(r_c/r_a)} \int_{r_1}^{r_2} \frac{dr}{r} = e U_0 \frac{\ln(r_2/r_1)}{\ln(r_c/r_a)}$$

If ionization is amplified by factor A , than the voltage pulse collected by electrodes will be amplified by the same factor A .

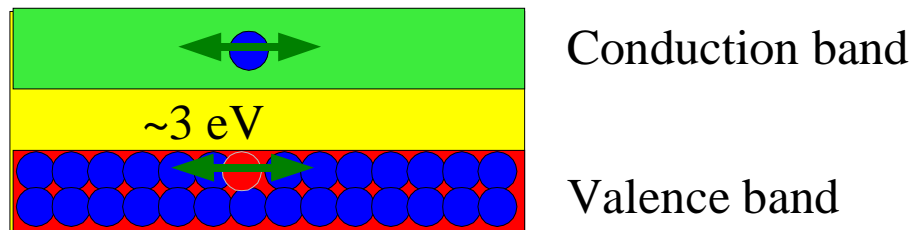
There are several approximate ways to calculate the amplification factor. It is related to the electrons mean free path on collision in such a way that the collision produces ionisation:

$$\lambda = \frac{1}{(N \sigma_i)} \quad \text{or} \quad \frac{P}{x} = \frac{1}{\lambda} \stackrel{\text{def}}{=} \alpha = N \sigma_i$$

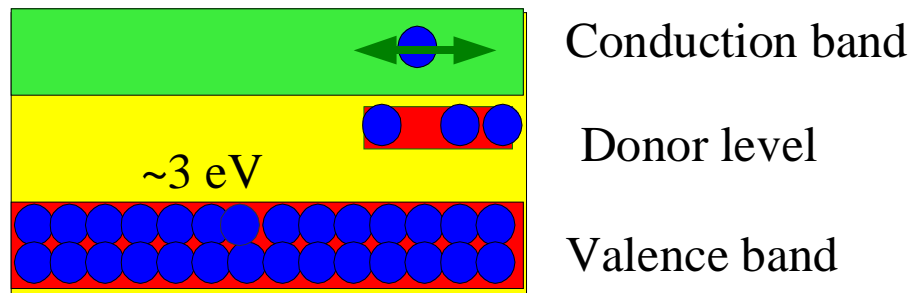
$\alpha \cdot dx$ is a probability that an electron will interact and ionize in the distance dx . If assume (wrongly !!) that this probability does not depend on the electron energy we can write :

$$dN(x) = N(x) \alpha dx \quad \rightarrow \quad N(x) = n_0 \exp(\alpha x)$$

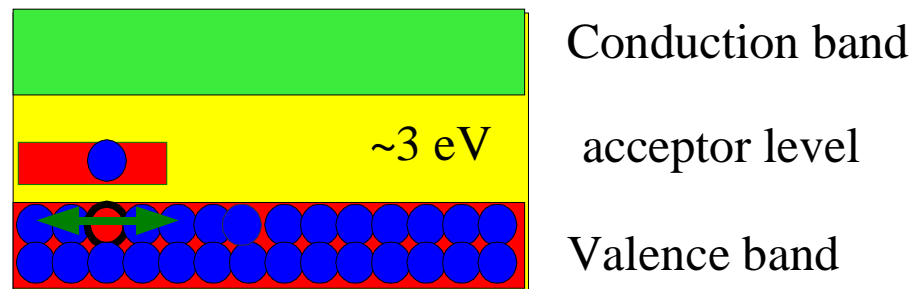
Semiconductors



n- type, with local donor levels



p- type, with local acceptor levels



The gap is 3.6 eV in silicon and 2.9 eV in germanium.

This is the ionization energy needed to create electron-hole pair.

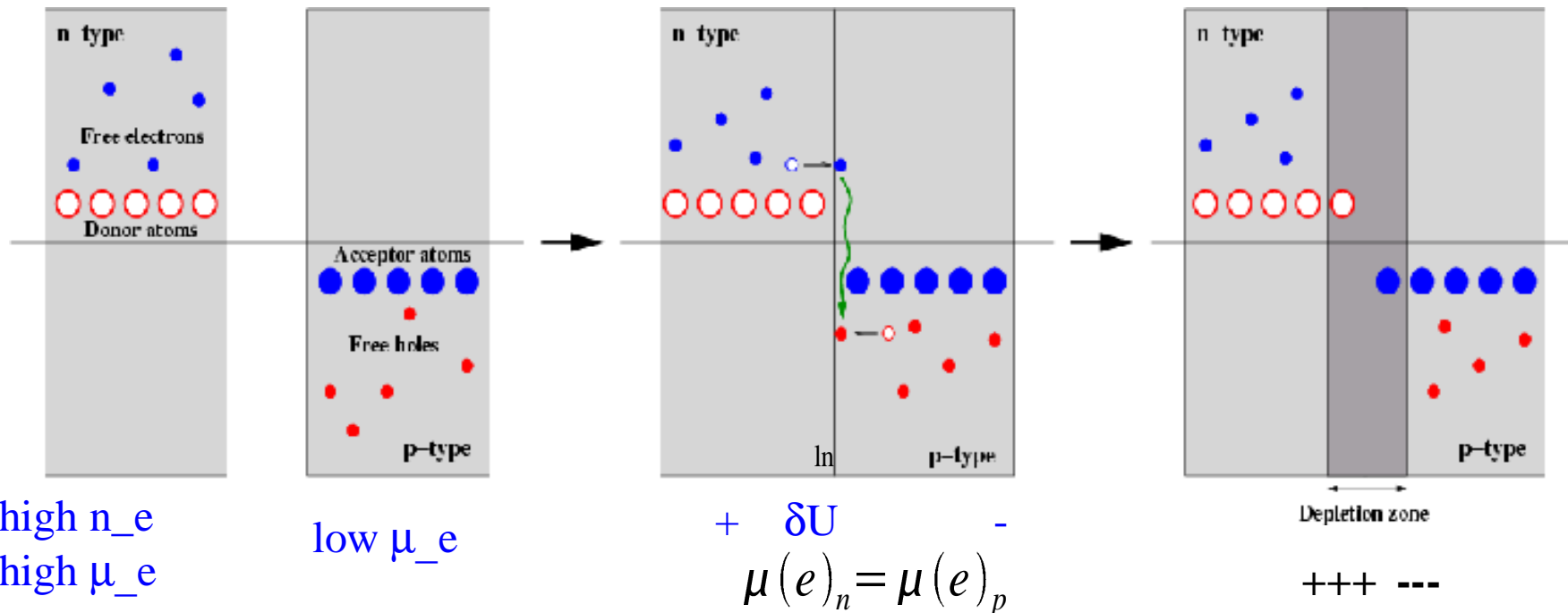
After passage of charged particle there remains a tube of plasma with a high concentration of electrons and holes (10^{15} up to 10^{17} /cm³)

Exercise: find out what is the ionization energy loss in silicon and estimate the radius of the plasma tube.

The problem is now how to separate electrons and holes before they recombine.

To build up high electric field needed to separate the produced charges p-n junctions are used.

Space charge and depletion zone in p-n junction

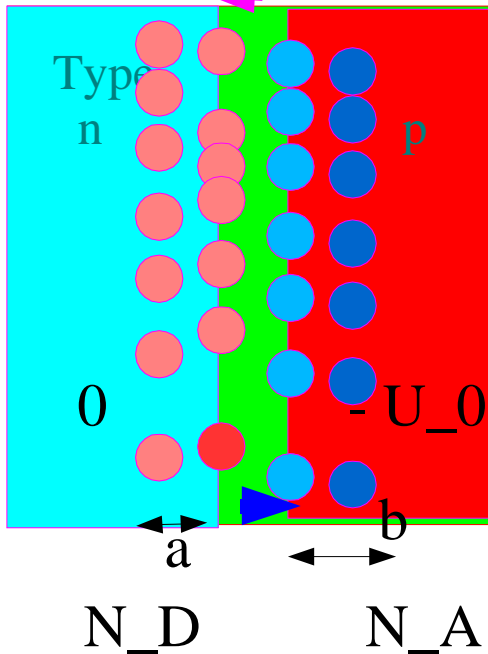


Free electrons and holes drift over p-n junction to equalize the chemical potential (Fermi level in this case). However, as they are charged, chemical potential depends not only on the concentration, but also on the potential energy of electrons and holes in the potential difference which builds up due to the drift.

Space charge builds up on the junction, positive on n-side and negative on p-side. If reverse bias is applied (+ to n side and - to the p side) the flow of majority carriers is stopped. Moreover the electric field builds up on the depletion zone, which is needed to separate electron-hole pairs produced by a passing particle.

Electric field in depletion zone

Depletion zone (where electrons fall into holes). There is double layer of space charges near to and in it. In depletion zone the concentration of charge



carriers is very small $10^{12}/\text{cm}^3$ compared to $10^{10}/\text{cm}^3$ for purified silicon. Typical junction consist of weakly doped p-silicon into which high concentration of n- impurities is introduced on one side

We will calculate now the electric field, and the thickness of depletion zone.

$$\nabla^2 U = \frac{-\rho}{\epsilon_0 \epsilon} \rightarrow \frac{d^2 U(x)}{dx^2} = \frac{-\rho(x)}{\epsilon_0 \epsilon}$$

$$\begin{aligned} \rho(x) &= eN_D & -a \leq x \leq 0 & \text{density of donors} \\ \rho(x) &= -eN_A & 0 \leq x \leq b & \text{density of acceptors} \end{aligned}$$

Boundary conditions on the U and electric field $E = -dU/dx$

$$\frac{dU(x)}{dx}(x=b) = 0 \wedge \frac{dU(x)}{dx}(x=-a) = 0 \qquad U(-a) = 0 \wedge U(b) = -U_0$$

In regions of small resistance charges drift to compensate electric field and all potential difference is deposited on the resistive area. We neglect potential difference due to the initial flow of charges

Electric field and depletion zone size

Solution:

$$U(x) = \frac{-eN_D}{2\epsilon_0\epsilon}(x+a)^2 \quad \text{for } -a < x \leq 0$$

$$U(x) = \frac{eN_A}{2\epsilon_0\epsilon}(x-b)^2 - U_0 \quad \text{for } 0 < x \leq b$$

We impose condition of continuity of U at $x=0$ and the fact that all charged which escaped from zone $(-a,0)$ have to be absorbed in zone $(0,b)$. This fixes the relative length of this zones:

$$aN_D = bN_A$$

If concentration of donors is much larger than the concentration of acceptors then the size of b zone will be much larger than the size of a zone. We have:

$$b(a+b) = \frac{2\epsilon_0\epsilon U_0}{eN_A}$$

$$d = b + a \simeq b = \sqrt{\frac{2\epsilon_0\epsilon U_0}{eN_A}}$$

we can also calculate the electric field $E_x = -dU/dx$. The electric field is maximal in $x=0$, in the transition region.

$$E_x(x=0) = \sqrt{\frac{2eN_A U_0}{\epsilon_0\epsilon}} = \frac{2U_0}{d}$$

Remarks and exercises.

We would like to have the size of the depletion zone small enough to have high field, on the other hand it is only the depletion zone where our diode is sensitive to passing particles. Lets see what are the numbers involved

$$d = \sqrt{\frac{2 \epsilon_0 \epsilon U_0}{e N_A}} \quad N_A = 10^{10} / \text{cm}^3 \text{ (purified silicon), } U_0 = 100 \text{ V}$$

$$U = \frac{Q}{C} \quad C = \frac{Q}{U}, \quad F = \frac{C}{V} \quad \epsilon_0 = 8.9 \cdot 10^{-12} \frac{\text{F}}{\text{m}} \quad U_0 = 100 \text{ V}, \quad e = 1.6 \cdot 10^{-19} \text{ C}$$

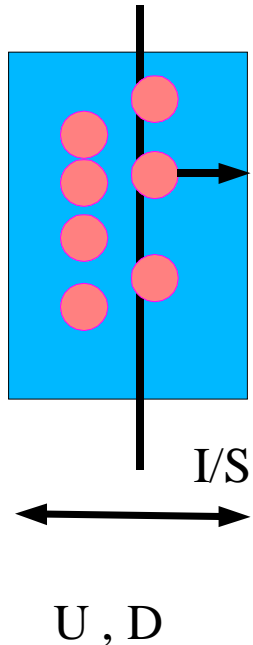
$$2 \epsilon_0 \epsilon \frac{U_0}{e N_A} = \frac{4 \cdot 9 \cdot 10^{-12} \frac{\text{C}}{\text{Vm}} \cdot 100 \text{ V}}{10^{10} / \text{cm}^3 \cdot 1.6 \cdot 10^{-19} \text{ C}} = 3.6 \cdot 10^{-9} \frac{\text{cm}^3}{\text{m} \cdot 1.6 \cdot 10^{-9}} \simeq 2 \cdot 10^{-2} \text{ cm}^2 \rightarrow d \simeq 1.4 \text{ mm}$$

For less purified silicon N_A will be much bigger due to boron contamination which is an acceptor addition. Thus the depletion zone will be much narrower. However it is hard to obtain wider depletion zone. p-i-n junction has to be used. "i" represents the part where p-impurities (boron) were neutralized with donor impurities (n)-lithium

Remarks, resistivity

$$d = \sqrt{\frac{2 \epsilon_0 \epsilon U_0}{e N_A}}$$

is often expressed in terms of resistivity (r) of the p-part



$$I/S = e N_A v_D \quad dS D \left(\frac{R}{V} \right) \left(\frac{I}{S} \right) dS = U$$

$$r \stackrel{\text{def}}{=} \left(\frac{R}{V} \right) (dS)^2 = \frac{U}{D} / \frac{I}{S} = \frac{E}{I/S} = \frac{E}{e N_A v_d} = \frac{1}{\mu e N_A}$$

$$\text{where } v_d = \mu E$$

remember that for electrons or holes parametrisation of drift velocity as a function of mobility is not very good- mobility will depend as well on the electric field. Anyway, the depletion zone can be expressed as a function of resistivity ($\text{Ohm} \cdot \text{cm}$) and mobility ($\text{cm}^2 / (\text{Vs})$)

$$d = \sqrt{\frac{2 \epsilon_0 \epsilon U_0}{e N_A}} = \sqrt{2 \epsilon_0 \epsilon U_0 r_p \mu}$$

If boron impurities are compensated with lithium impurities the density of uncompensated impurities can be decreased to $10^{19} / \text{cm}^3$ resistivity $3 \cdot 10^5 \text{ Ohm} \cdot \text{cm}$ for Si and $50 \text{ Ohm} \cdot \text{cm}$ for germanium

Electric field and collection time of produced carriers

We would like to have high enough electric field in the depletion zone to collect the carriers produced by passing particle before they recombine.

$$E_x(x=0) = \sqrt{\frac{2eN_A U_0}{\epsilon_0 \epsilon}} = \frac{2U_0}{d}$$

for $d = 100 \mu m = 0.1 mm$, $U_0 = 200V$, $E = 4 \times 10^6 \frac{V}{m}$

This field is sufficiently high to collect carriers. Collection time for counter thickness $s=1mm$ and

average electric field $E = 2 \times 10^5 \frac{V}{m}$, $\mu = 2 \times 10^4 \frac{cm^2}{Vs}$

$$\rightarrow v_d = 4 * 10^7 cm/s = 0.4 mm/ns \quad \rightarrow t_c = \frac{s}{v_d} = 2.5 ns$$

Energy resolution of semiconductor detector

Semiconductor detectors can be used to measure ionization energy loss in the depletion zone and also to measure the total energy of low energy particles- in case their energy is low enough and ionization energy losses big enough to stop in depletion zone. For example 10 MeV alpha particle will stop in 0.1mm of silicon.

In general : not only for silicon !! If “n” particles (ionization electrons, ions, electron -hole pairs) is produced by a passing particle ON AVERAGE then, if the production of each of this ionization particles is independent (Poissonian statistics) the fluctuation of the number of produced particles will be: $\sigma_n = \sqrt{n}$

$$\frac{\sigma_n}{n} = \frac{\sqrt{n}}{n} = \frac{1}{\sqrt{n}}$$

However, for silicon usually production of carriers is not independent and the fluctuations are diminished by so called Fano factor < 1

$$\sigma_n = \sqrt{Fn} \quad F = 0.06 - 0.14$$

For particles stopping in depletion zone we have approximately :

$$n = \frac{E_0}{W_i} \rightarrow \sigma_n = \sqrt{F \frac{E_0}{W_i}} \rightarrow \frac{\sigma_n}{n} = \frac{\sigma_E}{E_0} = \sqrt{\frac{FW_i}{E_0}} \quad W=3.6(2.8) \text{ eV Si(Ge)}$$

$E_0 \simeq 10 \text{ MeV} \quad \frac{\sigma_E}{E_0} \geq 2 \times 10^{-4}$ This is the minimal resolution, in practice resolutions order of magnitude bigger are observed